Q No 1

⇒ Newton's law of heating and Cooling :-

(a)

$$\frac{dT}{T_0 - T_s} = -a\,dt$$

$$\Rightarrow \int \frac{dT}{T_0 - T_s} = -\int a\,dt$$

$$\Rightarrow T_0 - T_s = e^{-at+C}$$

$$\Rightarrow T_0 - T_s = Ce^{-at}$$

$$\Rightarrow T_0 = Ce^{-at} + T_s$$

$$\Rightarrow T(t) = Ce^{-at} + T_s$$

$t = 0 \qquad T_s = 70^\circ C \qquad T_0 = 90^\circ C$

$$T(0) = Ce^{-a(0)} + 70$$

$$T(0) = C + 70$$

$$90 = C + 70$$

$$C = 20$$

At $t = 1$.

$$T(1) = 80$$

$T(1) = 20e^{-a(1)} + 170$

$T(1) = 20e^{-a} + 70$

$80 = 20e^{-a} + 70$

$80 - 70 = 20e^{-a}$

$e^{-a} = \frac{10}{20}$

$a = 0.69$

(b) Temperature at 100°C

$T(t) = 100^{\circ}C$

$100 = 20e^{0.69t}$

$\frac{30}{20} = e^{0.69t}$

$\ln(e^{0.69t}) = \ln(1.5)$

$0.69t = 0.405$

$t = 0.58 min$

$\frac{dT}{T - T_0} = -k dt$

$T = Ce^{-kt} + T_s$

At $t = 0$ $T = 28^{\circ}C$

(b) At $t = 0.004$

$T = 3e^{-0.66(0.004)} + 25$.

$T = 3(0.99) + 25$.

$T = 27.94$

$t = 0.008$

$T = 3e^{-0.66(0.008)} + 25$

$T = 27.99$

$t = 0.012$.

$T = 3.e^{-0.66(0.12)} + 25)$

$T = 27.97$

QNO#2

(c) $dN/dt = kN \ln(M/N)$

$\int dN/N \ln(M/N) = \int k \, dt$

Let $u = \ln(M/N)$, $du = 1/(M/N) - M/(N \times N)\, dN$

$du = N/M \times M/N \times N\, dN = \int k\, dt$

$-\int 1/u\, du = \int k\, dt$

$\ln|u| = -|kt + C|$

$u = e^{-kt+C}$.

$|u| = Ce^{-kt}$

$|u| = Ce^{-kt}$. $\longrightarrow A$

Putting values of u

$\ln(M/N) = Ce^{-kt}$.

As $t = 0$.

$\ln(M/N) = Ce^{-k(0)}$.

$\ln(M/N) = C$.

Putting C in equation 1.

$u = \ln(M/N)\, e^{-kt}$.

$N = Me - \ln(M/N)\, e^{-kt}$.

Q NO 3.

Given

Half life of Carbon-14 = 5370 year

Solution

As Analytical solution of $\frac{dQ}{dt} = rQ$.

$$Q = Q_0 e^{-rt} \rightarrow (A)$$

$$Q = \frac{Q_0}{2} \quad \text{at} \quad 5730 \text{ years.}$$

$$\frac{Q_0}{2} = e^{-rt}$$

$$\frac{Q_0}{2} = e^{-(0.000298)(5730)}$$

$$Q_0 = 11.0248 \rightarrow (B)$$

Now

$$Q = Q_0 e^{-rt}$$

we will find Q_0 when amount is 20%

$$Q = (11.0248)(e^{-0.000298 \times 2168})$$

$$Q = (11.0248)(e^{-21680.6})$$

$$Q = 64.18 \rightarrow (C)$$

The equation (A)

$$Q = Q_0 e^{-rt}$$

$$64.18 = 11.0248\, e^{-0.000298t}$$

$$e^{-0.000298t} = 5.8203$$

$$\ln\left[e^{-0.000298t}\right] = \ln 5.8203$$

$$0.000298t = 1.761$$

$$\boxed{t = 591.06 \text{ years}}$$

Q No 4