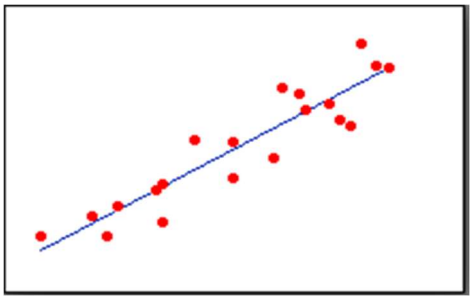
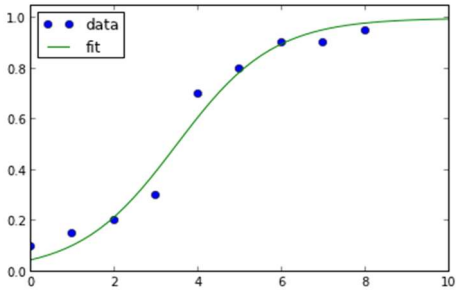
Curve Fitting

(Curve fitting by of method of least squares: fitting of straight lines and second-degree polynomials)

1. Fitting of linear curves- Straight line
2. Fitting of quadratic curves

Curve fitting is the process of finding the 'best-fit' curve for a given set of data. It is the representation of the relationship between the two variables by means of an algebraic equation. On the basis of this mathematical equation, predictions can be made in many statistical problems.





Suppose a set of n points of values (x_1, y_1), (x_2, y_2) ….,(x_n, y_n) of the two variables x and y are given. These values are plotted on a rectangular coordinate system (XY-plane). The resulting set of points is known as a scatter diagram. The scatter diagram exhibits the trend and it is possible to visualize a smooth curve approximating the data. Such a curve is known as approximating curve.

A more accurate way of finding the curve of best fit is the **least square method**.

Least square approximation – Straight line

Ex: 1 Fit a straight line **y=ax+b** to the following line:

x	1	2	3	4	6	8
Y	2.4	3	3.6	4	5	6

Solution:

Let the equation of the straight line: **y=a+bx**

The normal equations are $\sum y = na + b\sum x$ …(1)

$\sum xy = a\sum x + b\sum x^2$ …(2)

Here, n=6

x	**y**	**x^2**	**xy**
1	2.4	1	2.4
2	3	4	6
3	3.6	9	10.8
4	4	16	16
6	5	36	30
8	6	64	48
$\sum x=24$	**$\sum y=24$**	**$\sum x^2=130$**	**$\sum xy=113.2$**

Substituting these values in Eqs(1) and (2),

$24 = 6a + 24b$ …(3)

$113.2 = 24a + 130b$ ….(4)

Solving Eqs (3) and (4),

a=1.9764 b=0.5059

Hence, the required equation of the straight line is

y=a+bx

then **y=1.9764 + 0.5059x**

Ex: 2 Fit a straight line to the following data. Also, estimate the value of y at x=2.5.

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

Solution:

Let the straight line be fitted to the data be y=a+bx

The normal equations are $\sum y=na + b\sum x$ ...(1)

$\sum xy=a\sum x +b\sum x^2$ ....(2)

Here, n=5

x	y	x^2	xy
0	1	0	0
1	1.8	1	1.8
2	3.3	4	6.6
3	4.5	9	13.5
4	6.3	16	25.2
$\sum x=10$	$\sum y=16.9$	$\sum x^2=30$	$\sum xy=47.1$

Substituting these values in Eqs (1) and (2),

$16.9=5a + 10b$ ...(3)

$47.1=10a + 30b$ ...(4)

Solving eqs (3) and (4),

a=0.72 & b=1.33

Hence the required equation of the straight line is

Y=0.72 + 1.33x

At x=2.5,

$Y(2.5)=0.72+1.33(2.5)=$**4.4025**

Ex: 3 If P is the pull required to lift a load W by means of a pulley block. Find a linear approximation of the form $P = mW + c$ connecting P and W, using the following data.

P	13	18	23	27
W	51	75	102	119

Ans:

$P = 0.2309W + 0.2186$

Least Square Approximation – Quadratic Equation

Ex: 4 Fit a second degree polynomial using least square method to the following data:

X	0	1	2	3	4
Y	1	1.8	1.3	2.5	6.3

Solution:

Let the equation of the least squares quadratic curve:

$y = a + bx + cx^2$.

The normal equations are

$\sum y = na + b\sum x + c\sum x^2$

$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$

$\sum x^2 y = a\sum x^2 + b\sum x^3 + c\sum x^4$

Here, n=5

x	y	x^2	x^3	x^4	xy	x^2y
0	1	0	0	0	0	0
1	1.8	1	1	1	1.8	1.8
2	1.3	4	8	16	2.6	5.2
3	2.5	9	27	81	7.5	22.5
4	6.3	16	64	256	25.2	100.8
$\sum x$=10	$\sum y$=12.9	$\sum x^2$=30	$\sum x^3$=100	$\sum x^4$=354	$\sum xy$=37.1	$\sum x^2y$=130.3

Substituting these values in Eqs

12.9 = 5a + 10b + 30c

37.1 = 10a + 30b +100c

130.3 = 30a + 100b +354c

Solving above Eqs by Gauss-Jordan method, we can get,

a = 1.42

b = -1.07

c = 0.55

Hence, the required equation of the least squares quadratic curve is

$y = 1.42 - 1.07x + 0.55x^2$

Ex: 5 Fit a second-degree polynomial using least square method to the following data. Estimate y(2.4).

X	1	2	3	4
Y	1.7	1.8	2.3	3.2

Solution:

Let the equation of the least squares quadratic curve be

$y = a + bx + cx^2$. The normal equations are

$\sum y = na + b\sum x + c\sum x^2$

$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$

$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4$

Here, n=4

x	y	x^2	x^3	x^4	xy	x^2y
1	1.7	1	1	1	1.7	1.7
2	1.8	4	8	16	3.6	7.2
3	2.3	9	27	81	6.9	20.7
4	3.2	16	64	256	12.8	51.2
$\sum x$=10	$\sum y$=9	$\sum x^2$=30	$\sum x^3$=100	$\sum x^4$=354	$\sum xy$=25	$\sum x^2y$=80.8

Substituting these values in Eqs,

9 = 4a + 10b + 30c

25 = 10a + 30b +100c

80.8 = 30a + 100b +354c

Solving Eqs,

a =2

b = -0.5

c = 0.2

Hence, the required equation of the least squares quadratic curve is

$y = 2 - 0.5x + 0.2x^2$

$y(2.4) = 2 - 0.5(2.4) + 0.2(2.4)^2 =$ **1.952**

Ex: 6 Using the method of least squares, find the best fitting second degree curve to the following data.

x	1	2	3	4
y	6	11	18	27

Ans:

$$y = 3 + 2x + x^2$$