Linear Algebra

Name: M. Nauman Afzal
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Properties of Determinants

1. A scaler value associated with a square matrix.
2. Let $A \neq 0$ then matrix A is an inversible matrix.
3. Interchanging the determinants changes the sign of determinants.
4. If a row or column are multipeied with a scaler then determinant will also be multipeied.

Matrix Determinant

Determinant is a scaler value. It is a funtion of elements of square. It defines nature of matrix.

1. **Co-factor of matrix A**

$$C_{ij} = (-1)^{i+j} \det A_{ij}$$

$$A = \begin{bmatrix} a_{11} & \text{———} & a_{1m} \\ a_{21} & \text{———} & a_{2m} \\ a_{31} & \text{———} & a_{3m} \\ \vdots & \text{———} & \\ a_{n} & \text{———} & a_{nm} \end{bmatrix}$$

determinant of 2×2 C non Zero

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei - fh) - b(di - fg) - c(dh - eg)$$

Reflection Property

It will be as

Example

$$M = \begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$$

$$|M| = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} = \begin{vmatrix} 2 & 6 & 1 \\ -3 & 0 & 5 \\ 5 & 4 & -7 \end{vmatrix}$$

This will be defind in two cases

case 1	case 2
$\lvert M\rvert = 2(0-20) - (-3)(-4-24) + 5(30-0)$	$\lvert M\rvert = 2(-20) - (2)(-25) + 1(-12)$
$\lvert M\rvert = -40 - 138 + 150$	$= -40 + 24 - 12$
$= -28$	$= -28$

Switching property

Example

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 3 & 7 \\ 4 & 1 & 6 \end{bmatrix}$$

$|A| = 1(18-7) - 4(12-28) + 5(2-12)$
$= 1(11) - 4(-18) + 5(-10)$
$= 11 + 72 - 50$
$= 33$

After switching

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 4 & 1 & 6 \\ 2 & 3 & 7 \end{bmatrix}$$

$|A| = 1(7-18) - 4(28-12) + 5(12-2)$
$= 1(-11) - 4(18) + 5(10)$
$= -11 - 72 + 50$
$= -33$

Zero property

If any row of column of matrix is zero then determinant of matrix will also be zero.

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 4 & 0 & 5 \\ 7 & 0 & 1 \end{bmatrix}$$

$|A| = 0$

~~Reflection~~ Sum property

If elements of of any matrix A are expressed as sum of matrix then the determinant can be expressed as sum of two or more determinants.

Let

$$\begin{vmatrix} x & y & z \\ w & k & p \\ q+ai & p+bj & m+o \end{vmatrix} = \begin{vmatrix} x & y & z \\ w & k & p \\ v & n & m \end{vmatrix} + \begin{vmatrix} x & y & z \\ w & k & p \\ a & b & o \end{vmatrix}$$

Triangular property

In this case if matrix is triangular matrix than its determinant will be the Product of its diagonal matrix.

Let

$$A = \begin{bmatrix} 4 & 6 & 0 \\ 0 & -1 & 4 \\ 0 & 0 & -2 \end{bmatrix}$$

$$|A| = (4)(-1)(-2)$$
$$= (-4)(-2)$$
$$= 8$$

Invariance Property

If each element of matrix of its row and column of a determinant is added with the elements of other row and column of a matrix than the value of determinant remains unchanged.

$$|A| = \begin{vmatrix} 2 & 4 \\ 6 & 8 \end{vmatrix}, \quad |B| = \begin{vmatrix} 2 & 4 \\ 6 & 8 \end{vmatrix}$$

$$|A| = -8, \quad |B| = \begin{vmatrix} 2+2(6) & 4+2(8) \\ 6 & 8 \end{vmatrix}$$

$$|A| = -8 = |B|$$

$$|B| = \begin{vmatrix} 14 & 20 \\ 6 & 8 \end{vmatrix}$$

Multiplication property

If elements multiplied by non zero constant then determinant will also be multipeid by same constant.

$$A = \begin{bmatrix} 4 & 6 & 4 \\ 3 & 2 & 1 \\ 1 & 4 & 6 \end{bmatrix}$$

$$\text{if } 4A = \begin{bmatrix} 4(4) & 6(4) & 4(4) \\ 3(4) & 2(4) & 1(4) \\ 1(4) & 4(4) & 6(4) \end{bmatrix}$$

$$\text{so } |4A| = 4 \begin{vmatrix} 4 & 6 & 4 \\ 3 & 2 & 1 \\ 1 & 4 & 6 \end{vmatrix}$$

$= 4(8) - 6(17) + 4(10)$

$= 32 - 102 + 40$

$= -30$

$= -30 \times 4$

$= -120$