Homework 0

HW0 is a self-assessment covering topics and techniques which are assumed knowledge for this course. If you are unfamiliar with these topics we encourage you to refresh it during week 1. It does not need to be submitted and will not be graded.

PART I: Big-O notation and functions.
Suggested reading: [DPV] Chapter 0.

Problem 1

Part (a). DPV 0.1(c)

$f(n) = 100n + \log n, g(n) = n + (\log n)^2$.

Which holds: (Pick one)

- ○ $f(n) = O(g(n))$
- ○ $g(n) = O(f(n))$
- ○ $f(n) = O(g(n))$ and $g(n) = O(f(n))$

Part (b). DPV 0.1(d)

$f(n) = n \log n, g(n) = 10n \log 10n$.

Which holds: (Pick one)

- ○ $f(n) = O(g(n))$
- ○ $g(n) = O(f(n))$
- ○ $f(n) = O(g(n))$ and $g(n) = O(f(n))$

Part (c). DPV 0.1(k)

$f(n) = \sqrt{n}, g(n) = (\log n)^3$.

Which holds: (Pick one)

- ○ $f(n) = O(g(n))$
- ○ $g(n) = O(f(n))$
- ○ $f(n) = O(g(n))$ and $g(n) = O(f(n))$

Part (d). DPV 0.1(ℓ)

$f(n) = \sqrt{n}, g(n) = 5^{\log_2 n}$.

Which holds: (Pick one)

- ○ $f(n) = O(g(n))$
- ○ $g(n) = O(f(n))$
- ○ $f(n) = O(g(n))$ and $g(n) = O(f(n))$

Problem 2

(**[DPV 0.2]**) Show that $g(n) = 1 + a + a^2 + \ldots + a^n$ is $O(a^n)$ when $a > 1$ and $O(1)$ when $a < 1$. (*Hint*: You may try to prove $g(n) = \frac{a^{n+1}-1}{a-1}$ at first.)

PART II: Graphs fundamentals.

Problem 3

For all parts, $G = (V, E)$ represents an undirected, simple graph (i.e.: no multiple edges and no loops).

(a) Denote by $deg(v)$, the degree of vertex v, the number of edges incident to v. Check that

$$\sum_{v \in V} deg(v) = 2|E|.$$

(b) Review the concepts of path, cycle, connectivity.

(c) G is said to be a *tree* if it is connected and have no cycles. Think why the following three conditions are equivalent:

(i) G is a tree.

(ii) G is connected and $|E| = |V| - 1$.

(iii) G has no cycles and $|E| = |V| - 1$.

(d) A vertex is called a *leaf* if it has degree one. Show that every tree has at least two leaves. Think of an example of a tree with exactly two leaves.

PART III: Boolean functions.

Problem 4

A boolean function takes n variables $\{x_i\}_{1 \leq i \leq n}$ with values `true, false` (sometimes we use the set $\{0, 1\}$ instead) and outputs a boolean value. We will study boolean functions in *conjunctive normal form* (CNF), i.e.: our function is the conjunction of m clauses, each of which is made of the disjunction of distinct literals.

For each example below, decide which functions are in CNF and find an assignment of the variables such that the corresponding function evaluates to `true`, if such assignment exists.

- $(x \vee y \vee z) \wedge (x \vee w) \wedge (y \vee \bar{w})$
- $(\bar{x} \vee \bar{y}) \wedge (x) \wedge (z \vee \bar{z})$
- $x \wedge (y \wedge (z \vee \bar{w}))$