Study Guideline

You need to follow Four steps to study

Step 1: Watch the video link given in the start of the topic.

Step 2: Read the lecture notes attached.

Step 3: Read the topic from course book and do practice questions mention below.

Step4: Attend the online sessions on zoom.

Topic: Definition of derivative

Step 1

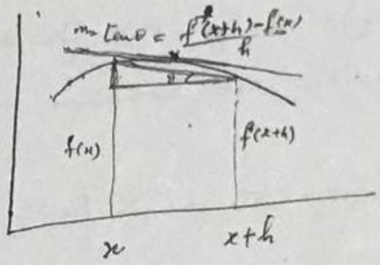
https://www.youtube.com/watch?v=8FXaiwthlW8

You can also watch other videos related to topic.

Step 2

3.2 Definition of the Derivative :-

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ derivative of f w.r.t. x.



Example 1 Find the derivative w.r.t. x of

$f(x) = x^2 + 1$, & use it to find the eq. of

the tangent line to $y = x^2 + 1$ at $x = 2$

sol:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[(x+h)^2 + 1] - [x^2 + \ldots}{h}$$

$$= \lim_{h \to 0} (2x + h) = 2x.$$

$y = mx + c$

$(x_1, y_1)\ m$

$y - y_1 = m(x - x_1)$

$\therefore\ f'(2) = 4 = m$

also at $x = 2$, $y = 5$ $\therefore$ $(2, 5)$

$\therefore$ eq. of the tangent line is

$\cancel{y = m}$ $y - y_1 = m(x - x_1)$

or $y - 5 = 4(x - 2)$

Example 5. The position ftn. for an object dropped from the Empire state building from 1250 ft. above sea level is given by. $f(t) = 1250 - 16t^2$,

a) Find the velocity ftn. of the object.

b) Find the time interval over which the velocity ftn. is valid.

c) what is the velocity of the object when it hits the ground?

soln. a) $v(t) = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h} = \lim_{h \to 0} \frac{-16(2th + h^2)}{h}$

$$= -16 \lim_{h \to 0} (2t + h) = -32t.$$

b). The velocity ftn is valid from $t = 0$ until $t_1 \equiv$ when it hits the ground, i.e

$$1250 - 16t_1^2 = 0 \Rightarrow t_1 = 8.8 \text{ sec} \quad \therefore t \in [0, 8.8]$$

c). $v(t_1) = -32t_1 = -282.8$ ft/sec.

Exercise set 3.2.

Q. 7. given $f(3)=-1$ & $f'(3)=5$, find an eq. for the tangent line to the graph of $y=f(x)$ at $x=3$.

Sol^n at $x=3$, $y=-1$ $\therefore$ $(3,-1)$

& $m=5$ given

$\therefore$ $y-y_1=m(x-x_1)$ $\Rightarrow$ $\boxed{y+1=5(x-3)}$

use definition to find $f'(x)$ & then find eq. of tang line at $x=a$

Q. 13. $f(x)=\sqrt{x+1}$; $a=8$

as $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\dfrac{\sqrt{x+h+1}-\sqrt{x+1}}{h}$

$=\lim_{h\to 0}\dfrac{x+h+1-x-1}{h}\times\dfrac{1}{\sqrt{x+h+1}+\sqrt{x+1}}$ Rationalize

$=\lim_{h\to 0}\dfrac{1}{\sqrt{x+h+1}+\sqrt{x+1}}=\dfrac{1}{2\sqrt{x+1}}=m$

$\therefore$ eq of line is . . .

Q 20. $y = \frac{1}{\sqrt{x-1}}$ use formula $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

sol.

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\frac{1}{\sqrt{x+\Delta x-1}} - \frac{1}{\sqrt{x-1}}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x-1} - \sqrt{x+\Delta x-1}}{\Delta x\sqrt{x-1}\sqrt{x+\Delta x-1}} \times \frac{\sqrt{x-1} + \sqrt{x+\Delta x-1}}{\sqrt{x-1} + \sqrt{x+\Delta x-1}}$$

$$= \lim_{\Delta x \to 0} \frac{x-1-(x+\Delta x-1)}{\Delta x\sqrt{x-1}\sqrt{x+\Delta x-1}\left(\sqrt{x-1}+\sqrt{x+\Delta x-1}\right)}$$

$$= -\lim_{\Delta x \to 0} \frac{1}{\sqrt{x-1}\sqrt{x+\Delta x-1}\left(\sqrt{x-1}+\sqrt{x+\Delta x-1}\right)}$$

$$= -\frac{1}{(x-1)\,2\sqrt{x-1}} = -\frac{1}{2(x-1)^{3/2}}$$

Q.22 Find dV/dr if $V = \frac{4}{3}\pi r^3$

sol.

$$\frac{dV}{dr} = \lim_{h \to 0} \frac{f(r+h) - f(r)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{4}{3}\pi(r+h)^3 - \frac{4}{3}\pi r^3}{h} = \lim_{h \to 0} \frac{\frac{4}{3}\pi[h^3+3r}{h}$$

$$= \lim_{h \to 0} \frac{\frac{4}{3}\pi[h^3+3r^2h+3rh^2]}{h} = \frac{4}{3}\pi(3r^2)$$

$$= 4\pi r^2$$

Q. 27. given limit represents $f'(a)$, find $f(x)$ & a.

Sol. (a) $\lim_{\Delta x \to 0} \frac{\sqrt{1+\Delta x}-1}{\Delta x}$ — (i)

Sol. as $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$.

hence $f'(a) = \lim_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{\Delta x}$. — (ii)

Comparing (i) & (ii)

$\Rightarrow f(a+\Delta x) = \sqrt{1+\Delta x}$

& $f(a) = 1$

$\therefore$ ~~$f(x) = x+1$~~ $f(x+\Delta x) = \sqrt{x+\Delta x}$

& $f(x) = \sqrt{x}$, $a = 1$

Q. 28 a) $\lim_{h \to 0} \frac{\cos(\pi+h)+1}{h}$.

Sol. here $a = \pi$, $f(x) = \cos x$.

Q 30. Find $dy/dx\big|_{x=-2}$, given that $y = (x+2)/x$.

Sol:

$$y = 1 + 2/x.$$

$$\therefore \frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\left(1 + \frac{2}{x+h}\right) - \left(1 + \frac{2}{x}\right)}{h}$$

$$= \lim_{h\to 0} \frac{2x - 2(x+h)}{h\,x\,(x+h)}$$

$$= \lim_{h\to 0} \frac{-2}{x(x+h)} = -2/x^2$$

Q31 Find an equation for the line that is tangent to the curve $y = x^3 - 2x + 1$ at the point $(0, 1)$.

Sol.

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^3 - 2(x+h) + 1 - x^3 + 2x - }{h}$$

$$= \lim_{h\to 0} \frac{h^3 + 3xh^2 + 3x^2h - \cancel{2x} - 2h \cancel{\;} + \cancel{2x}}{h}$$

$$= \lim_{h\to 0} \left(h^2 + 3xh + 3x^2 - 2\right) = 3x^2 - 2.$$

$\therefore$ slope at $(0,1)$ is $m = dy/dx\big|_{(0,1)} = -2$

$\therefore$ $y - y_1 = m(x - x_1) \Rightarrow y - 1 = -2(x - 0)$

Step 3: Read topic 3.2 from text book (Calculus by Howard Anton 8th edition)

Practice exercise 3.2 (Q.7 to Q.22, Q.27 to Q.32)