Lecture Study Guideline

You need to follow three steps to study

Step 1: Watch the topic related video uploaded on LMS.

Step 2: Read the lecture notes attached.

Step 3: Read the topic from course book and do practice of questions mention below.

Topic: Integration by Substitution

Step 1

Watch the topic related video uploaded on LMS.

Step 2

Integration by subsitution.

Two cases.

Case 1: $\int f(x)^n \cdot \frac{df}{dx} \cdot dx$

$u = f(x). \quad \frac{du}{dx} = f'(x). \quad du = f'(x)dx.$

$\int u^n du = \frac{u^{n+1}}{n+1} + c.$

Case 2:- $\int \frac{f'(x)}{f(x)} dx.$

Example :- $I = \int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}} dx.$

$u = \sin^{-1} x.$

$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}.$

$du = \frac{1}{\sqrt{1-x^2}} dx.$

$I = \int e^u du.$

$I = e^u + c$

$$\boxed{I = e^{\sin^{-1}x} + c}$$

Cases :- 1. $\int \text{Quadratic} \sqrt{\text{Linear equation}}\, dx$.

2. $\int \frac{\text{Quadratic}}{\sqrt{\text{Linear equation}}}\, dx$.

3. $\int \frac{\text{Linear equation}}{\sqrt{\text{Linear equation}}}\, dx$.

Example :- $I = \int \frac{x^2}{\sqrt{a+bx}}\, dx$.

$\sqrt{a+bx} = u$.

$a+bx = u^2$. $\quad x^2 = \left(\frac{u^2-a}{b}\right)^2$.

$b\,dx = 2u\,du$.

$dx = \frac{2u}{b}\,du$.

$$I = \int \frac{u^2-a}{u \cdot b^2} \times \left(\frac{2u}{b}\right) du$$

$$= \frac{1}{b^3}\int (u^2-a)\, du$$

$$= \frac{1}{b^3}\frac{u^3}{3} - \frac{a}{b^3}u.$$

$$\boxed{I = \frac{(a+bx)^{3/2}}{3b^3} - \frac{a}{b^3}\sqrt{a+bx} + C}.$$

Example 2 :- $\int \sin(x+9)\,dx$.

$u = x+9$.

$du = dx$.

$$\int \sin u \, du = -\cos u + c = -\cos(x+9) + C.$$

Example 4 :-

$$\int \frac{1}{(\frac{1}{3}x - 8)^5}\,dx.$$

$u = \frac{1}{3}x - 8$.

$\frac{du}{dx} = \frac{1}{3} \Rightarrow dx = 3\,du$.

$$\int \frac{1}{u^5}\,3\,du.$$

$$= 3\int u^{-5}\,du = 3 \cdot \frac{u^{-4}}{-4} + C.$$

$$= -\frac{3}{4}\left[\tfrac{1}{3}x - 8\right]^{-4} + C.$$

$$\boxed{I = -\frac{3}{4(\frac{1}{3}x-8)^4} + C}.$$

Example :- $I = \int \frac{1}{1+3x^2}\,dx$.

$u = \sqrt{3}\,x$.

$du = \sqrt{3}\,dx$.

$$\int \frac{1}{1+3x^2}\,dx = \frac{1}{\sqrt{3}}\int \frac{1}{1+u^2}\,du.$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}u + C.$$

$$\boxed{I = \frac{1}{\sqrt{3}}\tan^{-1}(\sqrt{3}x) + C}$$

Example :- $I = \int \sin^2 x \cos x\,dx$.

$u = \sin x \cdot du = \cos x\,dx$.

$$I = \int u^2\,du.$$

$$I = \frac{u^3}{3} + C$$

$$\boxed{I = \frac{\sin^3 x}{3} + C.}$$

Example :- $I = \int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$.

$u = \sqrt{x}$.

$$\frac{du}{dx} = \frac{1}{2\sqrt{x}} \qquad du = \frac{1}{2\sqrt{x}}\,dx \Rightarrow 2du = \frac{1}{\sqrt{x}}dx.$$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = \int 2e^{u}du = 2\int e^{u}du = 2e^{u}+C = 2e^{\sqrt{x}}+C$$

Example: $I = \int t^4 \sqrt[3]{3-5t^5}\,dt$

$$u = 3-5t^5$$

$$du = -25t^4\,dt$$

$$-\frac{1}{25}du = t^4 dt$$

$$I = \frac{-1}{25}\int \sqrt[3]{u}\,du$$

$$= \frac{-1}{25}\frac{u^{4/3}}{(4/3)} + C.$$

$$\boxed{I = -\frac{3}{100}(3-5t^5)^{4/3} + C}$$

Example:

$$I = \int \frac{dx}{a^2+x^2}$$

$$= \frac{1}{a}\int \frac{dx}{1+(x/a)^2}$$

$$u = x/a \qquad du = dx\, 1/a.$$

$$= \frac{1}{a} \int \frac{du}{1+u^2}$$

$$= \frac{1}{a} \tan^{-1} u + C$$

$$= \frac{1}{a} \tan^{-1}(x/a) + C$$

Generalize form :-

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} \sec^{-1} \left|\frac{u}{a}\right| + C$$

Ex 6.3.

Q No 13. $\int \sec 4x \tan 4x \, dx = I$

$u = 4x$

$du = 4\,dx \Rightarrow dx = \frac{1}{4} du$

$$I = \int \sec u \tan u \frac{du}{4}$$

$$= \frac{1}{4} \sec u + C$$

$$\boxed{I = \frac{1}{4} \sec 4x + C}$$

Q No 29. $\int \frac{e^x}{1+e^{2x}} \, dx$.

$u = e^x$.

$du = e^x \, dx$.

$I = \int \frac{du}{1+u^2} = \tan^{-1} u + C = \tan^{-1}(e^x) + C$.

Q No 38. $I = \int \tan^3(5x) \cdot \sec^2(5x) \, dx$.

$u = \tan 5x$.

$du = \sec^2 5x \cdot 5 \, dx$.

$\frac{du}{5} = \sec^2 5x \, dx$.

$I = \int u^3 \frac{du}{5} = \frac{1}{5}\left[\frac{u^4}{4}\right] + C = \frac{1}{20} \tan 5x + C$.

Q No 52. $\int e^{2\ln x} \, dx$.

$= \int e^{\ln x^2} \, dx$.

$= \int x^2 \, dx$.

$\boxed{I = \frac{x^3}{3} + C}$.

Step 3: Read topic 6.3 from text book (Calculus by Howard Anton 8th edition)

Practice exercise 6.3 (Q.1 to Q.6 , Q.9 to Q.49)