Lecture Study Guideline

You need to follow three steps to study

Step 1: Watch the topic related video uploaded on LMS.

Step 2: Read the lecture notes attached.

Step 3: Read the topic from course book and do practice of questions mention below.

Topic: Definite integrals

Step 1

Watch the topic related video uploaded on LMS.

Step 2

Definite Integral.

$$\int_a^b f(x)\,dx \longrightarrow \text{constant}.$$

(b → upper limit, a → lower limit)

First fundamental theorem of Calculus.

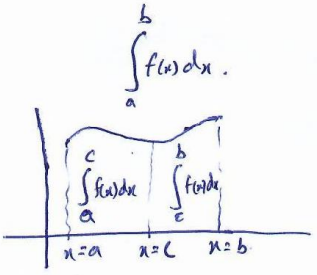
$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

where $F(x)$ is indefinite integral of $f(x)$.

Properties of definite integral :-

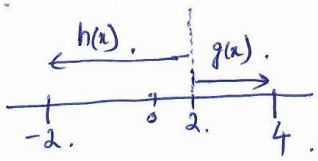
1. $$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx.$$

2. $$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$



Definit Integral of piece wise function.

$$f(x) = \begin{cases} g(x) & x \geq 2 \\ h(x) & x < 2 \end{cases}$$



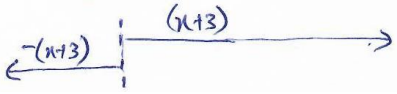
Let suppose $\int_{-2}^{4} f(x)\,dx = \int_{-2}^{2} h(x)\,dx + \int_{2}^{4} g(x)\,dx$.

Example :-

$$f(x) = |x+3| = \begin{cases} x+3 & x \geq -3 \\ -(x+3) & x < -3 \end{cases}$$

$$\int_{-6}^{6} f(x)\,dx = \int_{-6}^{-3} f(x)\,dx + \int_{-3}^{6} f(x)\,dx$$

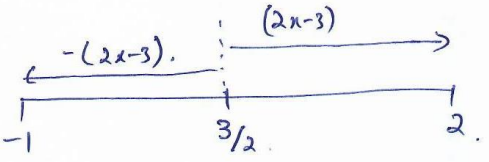
$$= \int_{-6}^{-3} -(x+3)\,dx + \int_{-3}^{6} (x+3)\,dx$$



Exercise 6.5.

Q No 13 (c). $\int_{-1}^{2} |2x-3|\,dx$.

$$|2x-3| = \begin{cases} 2x-3 & \text{if } x \geq 3/2 \\ -(2x-3) & \text{if } x < 3/2 \end{cases}$$



$$\int_{-1}^{2} |2x-3|\,dx = \int_{-1}^{3/2} -(2x-3)\,dx + \int_{3/2}^{2} (2x-3)\,dx.$$

$$= -\left[\int_{-1}^{3/2} 2x\,dx - \int_{-1}^{3/2} (-3)\,dx\right] + \int_{3/2}^{2} 2x\,dx - \int_{3/2}^{2} 3\,dx.$$

$$= -\left[\frac{2x^2}{2}\Big|_{-1}^{3/2} + 3x\Big|_{-1}^{3/2}\right] + \frac{2x^2}{2}\Big|_{3/2}^{2} - 3x\Big|_{3/2}^{2}$$

$$= -\left[(3/2)^2 - (-1)^2\right] + \left[3(3/2) - 3(-1)\right] + \left[(2)^2 - (3/2)^2\right] - \left[3(2) - 3(3/2)\right]$$

$$= -5/4 + \frac{15}{2} + \frac{7}{4} - 3/2 = 26/4 = 13/2.$$

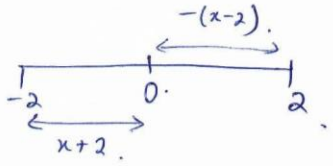
Q NO 15. Evaluate the integral given that

$$f(x) = \begin{cases} |x-2| & x \geq 0 \\ x+2 & x < 0 \end{cases}$$

$$|x-2| = \begin{cases} x-2 & \text{if } x \geq 2 \\ -(x-2) & \text{if } x < 2 \end{cases}$$

$$f(x) = \begin{cases} x-2 & \text{if } x \geq 2 \\ -(x-2) & \text{if } x < 2 \\ x+2 & \text{if } x < 0 \end{cases}$$

(b) $\int_{-2}^{2} f(x)\,dx$.



$$= \int_{-2}^{0} (x+2)\,dx + \int_{0}^{2} -(x-2)\,dx$$

$$= \frac{x^2}{2}\Big|_{-2}^{0} + 2x\Big|_{-2}^{0} - \frac{x^2}{2}\Big|_{0}^{2} + 2x\Big|_{0}^{2}$$

$$= \left[0 - \frac{(-2)^2}{2}\right] + \left[0 - 2(-2)\right] - \left[\frac{4}{2} - 0\right] + \left[4 - 0\right].$$

Step 3: Read topic 6.5 from text book (Calculus by Howard Anton 8th edition)

Practice exercise 6.5 (Q.9 to Q.22)