Lecture Study Guideline

You need to follow three steps to study

Step 1: Watch the topic related video uploaded on LMS.

Step 2: Read the lecture notes attached.

Step 3: Read the topic from course book and do practice of questions mention below.

Topic: Introduction to Integration

Step 1

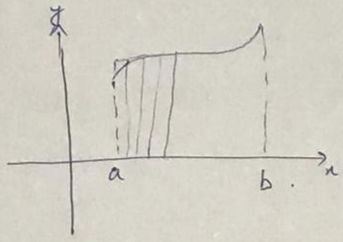
Watch the topic related video uploaded on LMS.

Step 2

Integration.

Indefinite $\int f(x)\,dx$ = function.

Definite $\int_a^b f(x)\,dx$ = Constant number.



$$A = \int_a^b f(x)\,dx.$$

$$\frac{d}{dx}(A) = f(x) \Rightarrow \int f(x)\,dx = (A).$$

Integration Rules :-

1. $\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx.$

2. $\int c\,f(x)\,dx = c\int f(x)\,dx.$

3. $\int 1 \cdot dx = x + C$ arbitrary. $\frac{d}{dx}(x+1) = 1$. $\frac{d}{dx}(x+2) = 1$.

4. Generalized power rule.

$$\int [f(x)]^n \frac{df}{dx} \cdot dx = \frac{[f(x)]^{n+1}}{n+1} + C, \quad n \neq -1.$$

$$\int [f(x)]^{-1} \frac{df}{dx} \cdot dx = \ln|f(x)| + C.$$

e.g. $\Rightarrow \int (2x+1)^{3/2} \cdot 2\, dx = \frac{(2x+1)^{3/2+1}}{3/2+1} + C.$

$$\Rightarrow \int x^{-1}\, dx = \int \frac{1}{x}\, dx$$

$$= \ln|x| + C.$$

5. Exponential function.

$$\int e^{f(x)} \cdot \frac{df}{dx}\, dx = e^{f(x)} + c.$$

e.g $\int e^{x^2} \cdot 2x\, dx = e^{x^2} + c$

$\int a^x\, dx = \frac{a^x}{\ln a} + c.$ 'a' is any constant number

e.g $\int 2^x\, dx = \frac{2^x}{\ln 2} + c.$

$\int e^{x^2} dx \longrightarrow$ Integral does not exist.

6. Integration of trignometric functions.

1.	$\int \cos x \, dx = \sin x$.	$\frac{d}{dx} \sin x = \cos x$.
2.	$\int \sin x \, dx = -\cos x$.	$\frac{d}{dx} \cos x = -\sin x$
3.	$\int \sec^2 x \, dx = \tan x$.	$\frac{d}{dx} \tan x = \sec^2 x$,
4.	$\int \operatorname{cosec}^2 x \, dx = -\cot x$.	$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$.
5.	$\int \sec x \tan x \, dx = \sec x$.	$\frac{d}{dx} \sec x = \sec x \tan x$.
6.	$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x$.	$\frac{d}{dx} \operatorname{cosec} x = \sec x \tan x$.

7.	$\int \tan x \, dx = \ln|\sec x|$	$\Rightarrow \int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$.
8	$\int \cot x \, dx = \ln|\sin x|$.	$= \int (\cos x)^{-1} \sin x \, dx$.
9.	$\int \sec x \, dx = \ln|\sec x + \tan x|$.	$= (-) \int (\cos x)^{-1} (-\sin x) \, dx$.
10	$\int \operatorname{cosec} x \, dx = \ln|\operatorname{cosec} x - \cot x|$.	$= -\ln|\cos x| = \ln|(\cos x)^{-1}|$
		$= \ln|\sec x|$.

Inverse trigonometric functions

1. $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x$.

$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$.

2. $\int \frac{-1}{\sqrt{1-x^2}}\,dx = \cos^{-1}x$.

$\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$.

3. $\int \frac{1}{1+x^2}\,dx = \tan^{-1}x$.

$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$

4. $\int \frac{-1}{1+x^2}\,dx = \cot^{-1}x$.

$\frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$.

5. $\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}|x|$

$\frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$

6. $\int \frac{-1}{x\sqrt{x^2-1}}\,dx = \operatorname{cosec}^{-1}|x|$.

$\frac{d}{dx}\operatorname{cosec}^{-1}x = \frac{-1}{x\sqrt{x^2-1}}$

Exercise 6.2.

Q No 2. Confirm that the stated formula is correct by differentiating.

$$\int x \sin x \, dx = \sin x - x \cos x + C.$$

By differentiating the right side.

$$\frac{d}{dx}\left[\sin x - x\cos x + c\right]$$

$$= \frac{d}{dx}(\sin x) - \frac{d}{dx}(x\cos x) + \frac{d}{dx}(c).$$

$$= \cos x - \left[x(-\sin x) + \cos x\right] + 0.$$

$$= \cancel{\cos x} + x\sin x - \cancel{\cos x}.$$

$$= x \sin x.$$ proved.

Q No5. Find the derivative and state corresponding integral formula.

$$\frac{d}{dx}\left[\sqrt{x^3+5}\right].$$

$$= \frac{1}{2}\left(x^3+5\right)^{\frac{1}{2}-1} \cdot \frac{d}{dx}\left(x^3+5\right).$$

$$= \frac{1}{2}\left(x^3+5\right)^{-\frac{1}{2}} \cdot \left(3x^2\right).$$

$$= \frac{3x^2}{2\sqrt{x^3+5}}.$$

As we know. $\left\{ \frac{d}{dx}(A) = f(x) \Rightarrow \int f(x)\,dx = (A) \right.$.

Integral formula $\int \frac{3x^2}{2\sqrt{x^3+5}}\,dx$.

Q No 15 Evaluate the integral.

$$\int x(1+x^3)\,dx.$$

$$= \int (x + x^4)\,dx.$$

$$= \int x\,dx + \int x^4\,dx.$$

$$= \frac{x^2}{2} + \frac{x^5}{5} + C.$$

Q No 21. $$\int \left(\frac{2}{x} + 3e^x\right)dx.$$

$$= 2\int x^{-1}\,dx + 3\int e^x\,dx.$$

$$= 2\ln|x| + 3e^x + C.$$

Step 3: Read topic 6.2 from text book (Calculus by Howard Anton 8th edition)

Practice exercise 6.2 (Q.2, Q.5 to Q.34, Q.41 to Q.44)