Lecture Study Guideline

You need to follow three steps to study

Step 1: Watch the topic related video uploaded on LMS.

Step 2: Read the lecture notes attached.

Step 3: Read the topic from course book and do practice of questions mention below.

Topic: Vectors

Step 1

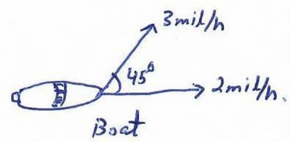
Watch the topic related video uploaded on LMS.

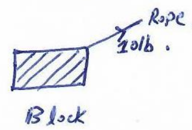
Step 2

CH # 12. (6)

Vectors :-

Vectors are used to represent all physical entities that involve both a magnitude and a direction. work, force and measurement of wind required both magnitude and direction so they are vector quantities. Whereas area, length and mass are scalar quantities because they required only magnitude.

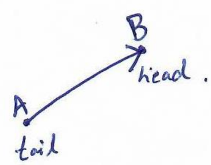


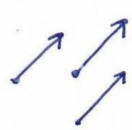


Vectors Geometrically :-

$V = \overrightarrow{AB}$

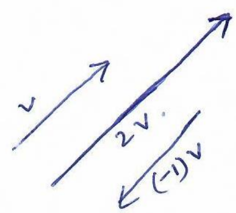
A = initial point B = final point.





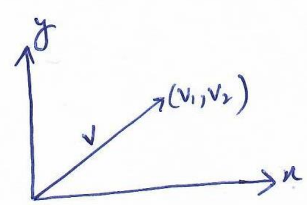
All these three vectors are equal because they have same magnitude and direction.

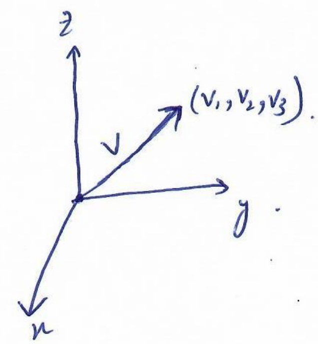
Vector and its scalar multiples :-



Vectors in coordinate System :-

Vector v is positioned with its initial point at origin of a rectangular coordinate system then its terminal point will have coordinates of the form (v_1, v_2) . or (v_1, v_2, v_3) depending on the vector in 2-space or 3-space.





Zero vector :-

2-space

$0 = \{0, 0\}$

3- space

$0 = \{0, 0, 0\}$.

Example 1. If $v = (-2, 0, 1)$ $w = (3, 5, -4)$.

(i) $v + w = \{-2, 0, 1\} + \{3, 5, -4\}$.

$v + w = \{1, 5, -3\}$.

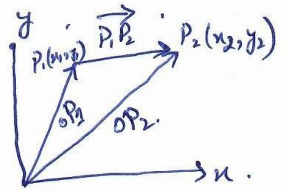
(ii) $3v = \{-6, 0, 3\}$.

Vectors with initial point not at the origin and finding the components of a vector.

$\overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1} = (x_2, y_2) - (x_1, y_1)$ P_2 is the final point.

where P_1 is the initial point

$\overrightarrow{P_1P_2} = (x_2 - x_1, y_2 - y_1)$.



Example :- In 2 space the vector from $P_1(1,3)$ to $P_2(4,-2)$ is

$\overrightarrow{P_1P_2} = \{4-1, -2-3\}$.

$$\boxed{\overrightarrow{P_1P_2} = \{3, -5\}}$$

Rules of vector Arithmetic :- u, v & w are vectors, k & l are scalars.

a. $u + v = v + u$

b. $(u + v) + w = u + (v + w)$.

e. $k(lu) = (kl)u$.

f. $k(u + v) = ku + kv$.

Magnitude of a vector :-

$$V = \{v_1, v_2\}.$$

$$|V| = \sqrt{v_1^2 + v_2^2}.$$

Example :- (i) $V = \{-2, 3\}$.

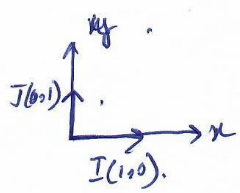
$$|V| = \sqrt{4+9} = \sqrt{13}.$$

(ii) $10V = \{-20, 30\}$.

$$|10V| = \sqrt{(-20)^2 + (30)^2} = 10\sqrt{13}.$$

<u>Unit vector</u> :-

A vector of length 1 is called unit vector,



Finding unit vector of same direction as the direction of given vector,

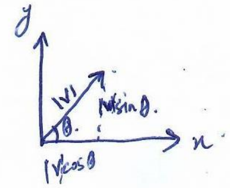
$U = \frac{1}{|V|} V.$ u is the unit in the same direction of v vector.

Example: Find the unit vector that has the same direction as $v = 2i + 2j - k$.

$$|v| = \sqrt{2^2 + 2^2 + (-1)^2} = 3.$$

$$\text{Unit vector} = u = \frac{v}{|v|} = \frac{2}{3}i + \frac{2}{3}j - \frac{1}{3}k.$$

vectors determined by length and angle :-



v can also represent vector v as.

$$v = |v|\cos\theta\, i + |v|\sin\theta\, j.$$

Example :- a. Find the vector of length 2 that makes an angle of $\pi/4$ with the positive x-axis.

$$v = 2\cos\pi/4\, i + 2\sin\pi/4\, j$$

b. Find the angle that the vector $v = -\sqrt{3}i + j$ makes with positive x-axis.

$$V = |V|\cos\theta\, i + |V|\sin\theta\, j.$$

$$V = |V|\left(\cos\theta\, i + \sin\theta\, j\right).$$

$$\frac{V}{|V|} = \cos\theta\, i + \sin\theta\, j.$$

$$\frac{V}{|V|} = \frac{-\sqrt{3}\, i + j}{\sqrt{(-\sqrt{3})^2 + 1}} = -\frac{\sqrt{3}}{2} i + \frac{1}{2} j = \cos\theta i + \sin\theta j.$$

Angle with x-axis $\cos\theta = -\frac{\sqrt{3}}{2}. \Rightarrow \theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = 5\pi/6.$

$$\boxed{\theta = 5\pi/6.}$$

Step 3: Read topic 12.2 from text book (Calculus by Howard Anton 8th edition)

Practice exercise 12.2 (Q.1 to Q.4, Q.7 to Q.15, Q.27 to Q.30, Q.51-Q52)