

Study Guideline

You need to follow three steps to study

Step 1: Watch the video link given in the start of the topic.

Step 2: Read the lecture notes attached.

Step 3: Read the topic from course book and do practice questions mention below.

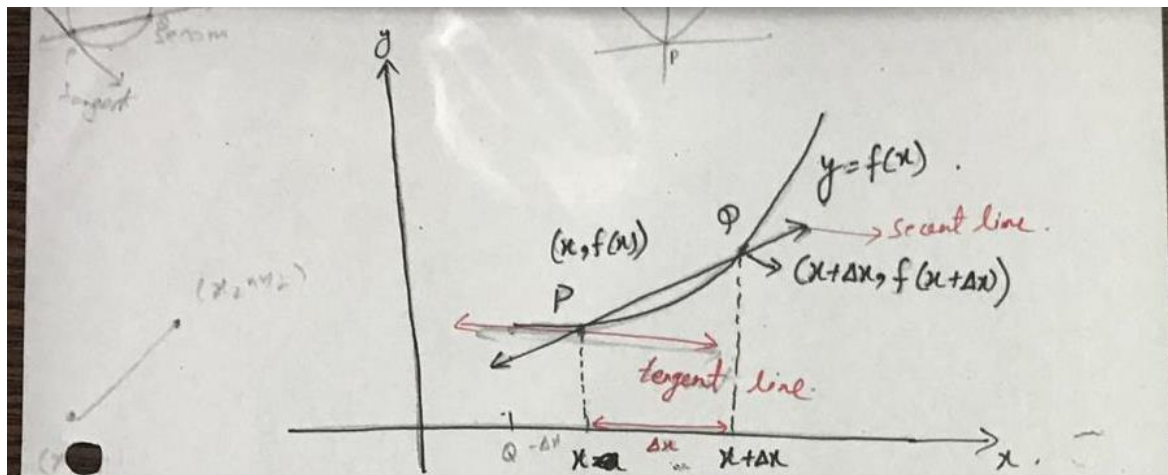
Topic: Definition of derivative

Step 1

<https://www.youtube.com/watch?v=8FXaiwthIW8>

You can also watch other videos related to topic.

Step 2



$$\frac{\Delta y}{\Delta x} = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x + \Delta x) - f(x)}{x + \Delta x - x}$$

of secant line through P and Q

$$= \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

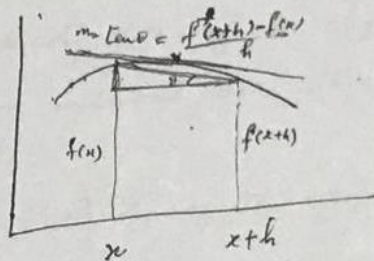
When $Q \rightarrow P \Rightarrow \Delta x \rightarrow 0$, secant \rightarrow tangent
slope of secant \rightarrow slope

$$\text{slope of tangent line at P} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \frac{dy}{dx}$$

3.2 Definition of the Derivative:-

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \text{derivative of } f \text{ w.r.t. } x.$$



Example 1 Find the derivative w.r.t. x of $f(x) = x^2 + 1$, & use it to find the eq. of the tangent line to $y = x^2 + 1$ at $x = 2$

solve

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{[(x+h)^2 + 1] - [x^2 + 1]}{h}$$

$$= \lim_{h \rightarrow 0} (2x + h) = 2x.$$

$$y = mx + c$$

$$(x_1, y_1) \rightarrow m$$

$$y - y_1 = m(x - x_1)$$

$$\therefore f'(2) = 4 = m$$

also at $x = 2, y = 5 \therefore (2, 5)$

\therefore eq. of the tangent line is

$$y - y_1 = m(x - x_1)$$

$$\text{or } y - 5 = 4(x - 2)$$

Example 5. The position ftn. for an object dropped from the Empire state building from 1250 ft. above sea level is given by, $f(t) = 1250 - 16t^2$,

- Find The velocity ftn. of the object.
- Find the time interval over which the velocity ftn. is valid.
- what is the velocity of the object when it hits the ground?

solt. a)
$$v(t) = \lim_{h \rightarrow 0} \frac{f(t+h) - f(t)}{h} = \lim_{h \rightarrow 0} \frac{-16(2t+h)}{h}$$
$$= -16 \lim_{h \rightarrow 0} (2t+h) = -32t.$$

b). The velocity ftn is valid from $\underline{t=0}$ until $t_1 \equiv$ when it hits the ground, i.e

$$1250 - 16t_1^2 = 0 \Rightarrow \underline{t_1 = 8.8 \text{ sec}} \quad \therefore t \in [0, 8.8]$$

c). $v(t_1) = -32t_1 = -282.8 \text{ ft/sec.}$

Exercise set 3.2.

Q. 7. given $f(3) = -1$ & $f'(3) = 5$, find an eq. for the tangent line to the graph of $y = f(x)$ at $x = 3$.

Soln: at $x = 3$, $y = -1 \therefore (3, -1)$

& $m = 5$ given

$$\therefore y - y_1 = m(x - x_1) \Rightarrow \boxed{y + 1 = 5(x - 3)}$$

use definition to find $f'(x)$ & then find eq. of tangent at $x = a$

Q. 13.
= $f(x) = \sqrt{x+1}$; $a = 8$

$$\text{or } f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x+h+1 - x-1}{h} \times \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}} \quad \text{Rationalize}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}} = \frac{1}{2\sqrt{x+1}} = m$$

\therefore eq. of line is ~ ~ ~

Q 20.

$$y = \frac{1}{\sqrt{x-1}}$$

use formula

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

sol.

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{\sqrt{x+\Delta x-1}} - \frac{1}{\sqrt{x-1}}}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x-1} - \sqrt{x+\Delta x-1}}{\Delta x \sqrt{x-1} \sqrt{x+\Delta x-1}} \times \frac{\sqrt{x-1} + \sqrt{x+\Delta x-1}}{\sqrt{x-1} + \sqrt{x+\Delta x-1}}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{x-1 - (x+\Delta x-1)}{\Delta x \sqrt{x-1} \sqrt{x+\Delta x-1} (\sqrt{x-1} + \sqrt{x+\Delta x-1})}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-1}{\sqrt{x-1} \sqrt{x+\Delta x-1} (\sqrt{x-1} + \sqrt{x+\Delta x-1})}$$

$$= - \frac{1}{(x-1) 2\sqrt{x-1}} = - \frac{1}{2(x-1)^{3/2}}$$

Q.22 Find dV/dr if $V = \frac{4}{3}\pi r^3$

sol. $\frac{dV}{dr} = \lim_{h \rightarrow 0} \frac{f(r+h) - f(r)}{h}$

$$= \lim_{h \rightarrow 0} \frac{\frac{4}{3}\pi(r+h)^3 - \frac{4}{3}\pi r^3}{h} = \lim_{h \rightarrow 0} \frac{\frac{4}{3}\pi[h^3 + 3r^2h + 3rh^2]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{4}{3}\pi[h^3 + 3r^2h + 3rh^2]}{h} = \frac{4}{3}\pi(3r^2) = 4\pi r^2$$

Q. 27. given limit represents $f'(a)$, find $f(x)$ & a .

~~Q. 27~~ (a) ~~$f'(x)$~~ $\lim_{\Delta x \rightarrow 0} \frac{\sqrt{1+\Delta x} - 1}{\Delta x}$ — (i)

Sol. $\therefore f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

hence $f'(a) = \lim_{\Delta x \rightarrow 0} \frac{f(a+\Delta x) - f(a)}{\Delta x}$ — (ii)

Comparing (i) & (ii)

$$\Rightarrow f(a+\Delta x) = \sqrt{1+\Delta x}$$

$$\& f(a) = 1$$

$$\therefore f(x) = \sqrt{x+1} \quad f(x+\Delta x) = \sqrt{x+\Delta x+1}$$

$$\& f(x) = \sqrt{x}, \quad a = 1$$

Q. 28 a)

$$\lim_{h \rightarrow 0} \frac{\cos(\pi+h) + 1}{h}$$

Sol. here $a = \pi$, $f(x) = \cos x$.

Q 30. Find $\frac{dy}{dx} \Big|_{x=-2}$, given that $y = (x+2)/x$.

Soln:
 $y = 1 + \frac{2}{x}$

$$\begin{aligned} \therefore \frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(1 + \frac{2}{x+h}) - (1 + \frac{2}{x})}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{2x - 2(x+h)}{x(x+h)}}{h} \\ &= \lim_{h \rightarrow 0} \frac{-2}{x(x+h)} = -\frac{2}{x^2} \end{aligned}$$

Q 31 Find an equation for the line that is tangent to the curve $y = x^3 - 2x + 1$ at the point $(0, 1)$.

Soln:

$$\begin{aligned} \frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^3 - 2(x+h) + 1 - x^3 + 2x - 1}{h} \\ &= \lim_{h \rightarrow 0} \frac{h^3 + 3xh^2 + 3x^2h - 2x - 2h + 2x}{h} \\ &= \lim_{h \rightarrow 0} (h^2 + 3xh + 3x^2 - 2) = 3x^2 - 2 \end{aligned}$$

\therefore slope at $(0, 1)$ is $m = \frac{dy}{dx} \Big|_{(0,1)} = -2$

$\therefore y - y_1 = m(x - x_1) \Rightarrow y - 1 = -2(x - 0)$

Step 3: Read topic 3.2 from text book (Calculus by Howard Anton 8th edition)

Practice exercise 3.2 (Q.7 to Q.22, Q.27 to Q.32)