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Foundations of Finance: Excel Project
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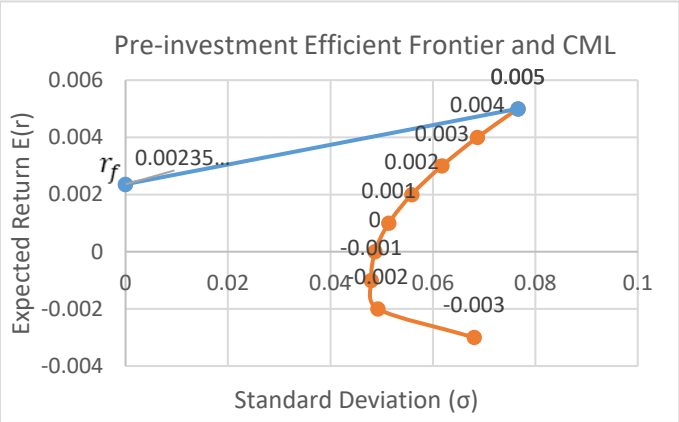
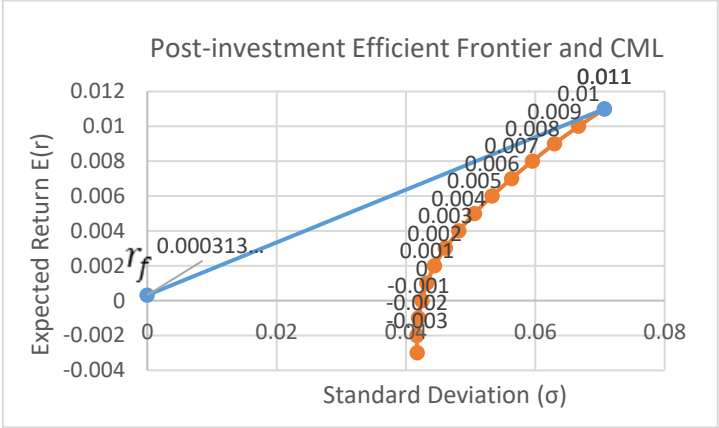
Part A Word

To begin exploring the concept of portfolio optimization, I selected a portfolio of four stocks, BP., Tesco, Barclays, and Marks and Spencer, due to the diversity of their goods and services, which lowers correlation. This was proven by the low covariance, which is closely related to correlation, between my stocks found on the Variance Covariance Matrix. Therefore, I am able to decrease the possibility of all of my stocks being negatively affected by events that take place in only one industry. Furthermore, I picked two high risk and two low risk stocks, BP. and Tesco have a standard deviation of around 0.6 versus Barclays and Marks and Spencer whose hovers around 0.1. Both of these considerations hopefully decrease the volatility of the overall portfolio while increasing returns. Below I have placed the Efficient Frontier graphs and the Capital Market Line of my two investment periods.





The terminal wealth found by using the optimum portfolio weights in the post-investment time period, in which I have perfect hind-sight, was much higher than that of the pre-investment time period. This is because the first period optimum portfolio was composed of only Tesco and MKS, which had the lowest risks, while the second period optimum portfolio consisted of nearly all BP shares and a small fraction of Barclays, which had the highest risks. As higher risk equates to greater returns, the optimum portfolio weights in the second period led to a higher terminal wealth.

Furthermore, the pre-investment minimum variance portfolio yielded a terminal wealth that was higher than that of the pre-investment optimum portfolio. This is because the minimum variance portfolio has the lowest risk, and therefore returns, but also uses the covariance variance matrix to find stocks with the least correlation, which decreases the volatility of the portfolio. As a result of the decreased volatility and low correlation, the terminal wealth for pre-investment minimum variance portfolio was higher than the initial investment, unlike that of the optimum portfolio which was significantly below. However, the post-investment minimum variance portfolio terminal wealth was lower than that of the post-investment optimum portfolio, as the post-investment optimum portfolio consisted nearly only of BP., who had the highest risk but also the highest return.

Additionally, the equal weighted portfolio, with no rebalancing, outperformed the pre-investment optimum portfolio but not the pre-investment minimum variance portfolio. Usually when equally weighted portfolios beat the market it is due to the "small stock effect", which shows that smaller firms outperform large companies, like BP., due to their high potential for growth. However, in my case the stocks that went down in value were MKS and Tesco, both of which are smaller than BP. and Barclays, however they are also not small enough to take on this effect. Therefore the equal weighted portfolio's terminal wealth was higher than the optimum portfolio most likely due to chance, as this way of allocation does not consider historical data or any other factors.

Finally, the terminal wealth of the stock market index (FTSE 100) and the risk free investment was higher than that of my pre-investment portfolio but lower than that of my post-investment portfolio. FTSE 100 is highly diversified, allowing for moderate to high returns without a lot of risk, and follows the movement of its 100 indexes which have grown over the past few years, as the UK is not in a recession, allowing it to outperform my pre-investment portfolio. However, indexes are passively managed and does not allow an investor to change the weights of the companies, like I did to find the pre-investment optimum portfolio which outperformed the FTSE

100. Moving onto the risk free investment, risk free assets have low volatility and returns; but as my pre-investment optimum portfolio resulted in negative returns it was outperformed by the risk free investment which increased by the interest rate of about 0.04%, at the time of my investment, over two years. However, the low returns, due to the low risk, were unable to outperform my post-investment optimum portfolio, which was largely composed of the stock with the highest risk and return.

Part B Word

By using mean variance optimization, I am able to find the portfolio allocation among my four stocks that has the best risk-return tradeoff for our firm. After calculating the returns, based off of the prices, I found the expected return and volatility, denoted by the variance and standard deviation, of each stock. To find how the two stocks vary with one another and their unstandardized correlation, I used the "covar" function in excel and created a Variance Covariance Matrix that found the covariance of each stock against the other three and then itself.

To find the measurements of the portfolio, I created a table of the four stocks, their expected returns, and weights, which are arbitrarily filled in but sum to one. Then I used the Variance Covariance Matrix weights to calculate the portfolio expected return, variance, and standard deviation through matrix multiplication. I then manipulated the expected return and created a list of possible expected returns, incremented or decremented by 0.001 in my case. Then using the Solver Tool in Excel, I found the minimum risk that must be taken to achieve those possible returns, as represented by standard deviation, and how to allocate my funds across the stocks to achieve those returns, as represented by the weights. However, before doing so I created a list of constraints that the solver had to follow in finding those three values. (1) All of the weights have to sum to 1, or 100% of the portfolio. (2) It takes the expected return, a constant that I change myself. (3)The weights themselves have to be greater than or equal to zero, because I hope that the stock prices increase rather than decrease.

Using the expected returns and their associated minimum risks, I created a coordinate system from which to draw the Efficient Frontier. The Efficient Frontier is a curve that shows me the highest expected returns that can be achieved by a portfolio with a certain amount of risk, sigma, or the lowest risk necessary to receive those returns. The Capital Market Line on the other hand shows me the expected returns for each level of risk after a portfolio has been infused with a risk free asset, which is Libor in my case. The CML line starts at the risk free rate, the minimum return I expect, and extends to the point on the efficient frontier which represents the optimum market portfolio, which yields the highest risk-adjusted return.

To find that optimal market portfolio and where the CML touches the Efficient Frontier, I have to find the slope of the CML. To do this I first need to find the risk free rate of the Libor risk free asset by taking the given interests rates and dividing by 1200, as it is in annual rates and in percentages. To find the slope, I find the Sharp Ratio of each risk and expected return. The Sharp Ratio tells me how much more return I can receive for taking on a certain amount of extra volatility besides simply investing in the risk free asset, as any amount of risk I take should be justified by the possibility of an increased return. The Sharp Ratio is therefore found by deducting the risk free rate, the rate of return you receive from investing in Libor the risk free asset, from the expected return and dividing that by the riskiness of that return to find the risk-adjusted return. The portfolio with the highest Sharp Ratio is the market portfolio; and its Sharp Ratio is the slope of the CML. Then I draw the CML line on the Efficient Frontier Curve with that slope starting from the risk free rate to the market portfolio, whose coordinates are its standard deviation and the expected return. Where the CML line is tangent to the Efficient Frontier is the market portfolio. Finally, to find the allocation of my investment among my four stocks that represent the market portfolio, I run the Solver Tool again but this time using the expected return of the newly found market portfolio. Those given weights are the optimum weights.

After being enlightened by the post-investment period historical data and various terminal values, I would not invest my money based on the optimum weights as given by the mean variance optimization technique. Based on the optimum weights, I found the terminal wealth, the final value after the investment period, of my portfolio after two years of investing 1,000 pounds. Afterwards, I looked at the returns of the stocks during the two years after my investing period was over and found the optimum market portfolio and weights for that time period. Based on these new weights, I calculated the new terminal wealth of my portfolio and found that it was higher than that of my pre-investment period. This shows that historical data alone may not be able to accurately predict future patterns, as the terminal wealth based on the pre-investment period, which is based on past data, was lower than the initial investment and nearly half that of the market portfolio I found when I had perfect hindsight.