

$$\begin{array}{c}
\frac{1:\epsilon p, 0:\mathit{here}\mathcal{K}_a(p \rightarrow q), 0:\mathit{here}\mathcal{K}_{ap}, (0, \Box)\mathsf{R}_a^\epsilon(1, \Box) \Rightarrow 1:\epsilon p, 1:\epsilon q,}{0:\mathit{here}\mathcal{K}_a(p \rightarrow q), 0:\epsilon\mathcal{K}_{ap}, (0, \Box)\mathsf{R}_a^\epsilon(1, \Box) \Rightarrow 1:\epsilon q, 1:\epsilon p,} \quad (init) \quad \frac{0:\mathit{here}\mathcal{K}_a(p \rightarrow q), 0:\mathit{here}\mathcal{K}_{ap}, (0, \Box)\mathsf{R}_a^\epsilon(1, \Box) \Rightarrow 1:\epsilon p, 1:\epsilon q,}{1:\epsilon p \rightarrow q, 0:} \\
\hline
\end{array}$$

Table 1: G3PAL'

(Initial Sequent)

$$x:A, \Gamma ==> \Delta, x:A$$

(Rules for propositional connectives)

$$\frac{\Gamma ==> \Delta, x(\alpha):A}{x(\alpha):\sim A, \Gamma ==> \Delta} (L\sim) \quad \frac{x(\alpha):A, \Gamma ==> \Delta}{\Gamma ==> \Delta, x(\alpha):A} (R\sim)$$

$$\frac{\Gamma ==> \Delta, x(\alpha):A \quad x(\alpha):B, \Gamma ==> \Delta}{x(\alpha):A \multimap B, \Gamma ==> \Delta} (L\multimap) \quad \frac{x(\alpha):A, \Gamma ==> \Delta}{\Gamma ==> \Delta, x(\alpha):A} (R\multimap)$$

(Rules for knowledge operators)

$$\frac{x(\alpha):\#a(\beta, y)A, \Gamma ==> \Delta, xR_a(\alpha)y, \quad y(\alpha):A, x(\alpha):\#a(\beta, y)A}{x(\alpha):\#a(\beta)A, \Gamma ==> \Delta} (L\#)$$

$$\frac{xR_a(\alpha)y, \Gamma ==> \Delta, y(\alpha):A}{\Gamma ==> \Delta, x(\alpha):\#a(\beta)A} (R\#)^\ddagger$$

 $^\dagger y$ does not appear in β . $^\ddagger y$ does not appear in the lower sequent

(Rules for PAL)

$$\frac{x(\alpha):A, x(\alpha):p, \Gamma ==> \Delta}{x(\alpha, A):p, \Gamma ==> \Delta} (Lat) \quad \frac{\Gamma ==> \Delta, x(\alpha):A}{\Gamma ==> \Delta, x(\alpha, A):A} (R\Box)$$

$$\frac{\Gamma ==> \Delta, x(\alpha):A \quad x(\alpha, A):B, \Gamma ==> \Delta}{x(\alpha):[A]B, \Gamma ==> \Delta} (L[.]) \quad \frac{x(\alpha):A, \Gamma ==> \Delta}{\Gamma ==> \Delta, x(\alpha):[A]A} (R[.])$$

$$\frac{x:A, y:A, xR_a(\alpha)y, \Gamma ==> \Delta}{xR_a(\alpha, A)y, \Gamma ==> \Delta} (Lrel)$$

$$\frac{\Gamma ==> \Delta, x:A \quad \Gamma ==> \Delta, y:A \quad \Gamma ==> \Delta, xR_a(\alpha, A)y}{\Gamma ==> \Delta, xR_a(\alpha, A)y} (Rrel)$$

(Rules for propositional connectives)

$$\frac{}{x(\alpha):bot, \Gamma ==> \Delta} (Lbot) \quad \frac{}{\Gamma ==> \Delta, x(\alpha):top} (Rtop)$$

$$\frac{x(\alpha):A, x(\alpha):B, \Gamma ==> \Delta}{x(\alpha):A \& B, \Gamma ==> \Delta} (L\&) \quad \frac{\Gamma ==> \Delta, x(\alpha):A \quad \Gamma ==> \Delta, x(\alpha):B}{\Gamma ==> \Delta, x(\alpha):A \& B} (R\&)$$

$$\frac{x(\alpha):A, \Gamma ==> \Delta \quad x(\alpha):B, \Gamma ==> \Delta}{x(\alpha):A \vee B, \Gamma ==> \Delta} (Lv) \quad \frac{\Gamma ==> \Delta, x(\alpha):A \quad \Gamma ==> \Delta, x(\alpha):B}{\Gamma ==> \Delta, x(\alpha):A \vee B} (Rv)$$

(Rules for knowledge operators)

$$\frac{x(\alpha, A, B):C, \Gamma ==> \Delta}{x(\alpha, A \& [A]B):C, \Gamma ==> \Delta} (Lcmp) \quad \frac{\Gamma ==> \Delta, x(\alpha, A):C}{\Gamma ==> \Delta, x(\alpha, A \& [A]B):C} (Rcmp)$$

other

$$\frac{x(\alpha):A \multimap B, x(\alpha):B \multimap A, \Gamma ==> \Delta}{x(\alpha):A \multimap \multimap B, \Gamma ==> \Delta} (L\multimap\multimap) \quad \frac{\Gamma ==> \Delta, x(\alpha):A \multimap B \quad \Gamma ==> \Delta, x(\alpha):B \multimap A}{\Gamma ==> \Delta, x(\alpha):A \multimap \multimap B} (R\multimap\multimap)$$