



Tab. 1: G3PAL'

(Initial Sequent)

$$x:A, \Gamma \Rightarrow \Delta, x:A$$

(Rules for propositional connectives)

$$\frac{\Gamma \Rightarrow \Delta, x(\alpha):A}{x(\alpha):\sim A, \Gamma \Rightarrow \Delta} (L\sim) \quad \frac{x(\alpha):A, \Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, x(\alpha):\sim A} (R\sim)$$

$$\frac{\Gamma \Rightarrow \Delta, x(\alpha):A \quad x(\alpha):B, \Gamma \Rightarrow \Delta}{x(\alpha):A -> B, \Gamma \Rightarrow \Delta} (L->) \quad \frac{x(\alpha):A, \Gamma \Rightarrow \Delta, x(\alpha):B}{\Gamma \Rightarrow \Delta, x(\alpha):A -> B} (R->)$$

(Rules for knowledge operators)

$$\frac{x(\alpha):\#a(\beta, y)A, \Gamma \Rightarrow \Delta, xR_a(\alpha)y, \quad y(\alpha):A, x(\alpha):\#a(\beta, y)A, \Gamma \Rightarrow \Delta}{x(\alpha):\#a(\beta)A, \Gamma \Rightarrow \Delta} (L\#a)^\dagger$$

$$\frac{xR_a(\alpha)y, \Gamma \Rightarrow \Delta, y(\alpha):A}{\Gamma \Rightarrow \Delta, x(\alpha):\#a(\beta)A} (R\#a)^\ddagger$$

 $^\dagger$   $y$  does not appear in  $\beta$ . $^\ddagger$   $y$  does not appear in the lower sequent.

(Rules for PAL)

$$\frac{x(\alpha):A, x(\alpha):p, \Gamma \Rightarrow \Delta}{x(\alpha, A):p, \Gamma \Rightarrow \Delta} (Lat) \quad \frac{\Gamma \Rightarrow \Delta, x(\alpha):A \quad \Gamma \Rightarrow \Delta, x(\alpha):p}{\Gamma \Rightarrow \Delta, x(\alpha, A):p} (Rat)$$

$$\frac{\Gamma \Rightarrow \Delta, x(\alpha):A \quad x(\alpha, A):B, \Gamma \Rightarrow \Delta}{x(\alpha):[A]B, \Gamma \Rightarrow \Delta} (L[.]) \quad \frac{x(\alpha):A, \Gamma \Rightarrow \Delta, x(\alpha, A):B}{\Gamma \Rightarrow \Delta, x(\alpha):[A]B} (R[.])$$

$$\frac{x:A, y:A, xR_a(\alpha)y, \Gamma \Rightarrow \Delta}{xR_a(\alpha, A)y, \Gamma \Rightarrow \Delta} (Lrel)$$

$$\frac{\Gamma \Rightarrow \Delta, x:A \quad \Gamma \Rightarrow \Delta, y:A \quad \Gamma \Rightarrow \Delta, xR_a(\alpha)y}{\Gamma \Rightarrow \Delta, xR_a(\alpha, A)y} (Rrel)$$

(Rules for propositional connectives)

$$\frac{}{x(\alpha):bot, \Gamma \Rightarrow \Delta} (Lbot) \quad \frac{}{\Gamma \Rightarrow \Delta, x(\alpha):top} (Rtop)$$

$$\frac{x(\alpha):A, x(\alpha):B, \Gamma \Rightarrow \Delta}{x(\alpha):A \& B, \Gamma \Rightarrow \Delta} (L\&) \quad \frac{\Gamma \Rightarrow \Delta, x(\alpha):A \quad \Gamma \Rightarrow \Delta, x(\alpha):B}{\Gamma \Rightarrow \Delta, x(\alpha):A \& B} (R\&)$$

$$\frac{x(\alpha):A, \Gamma \Rightarrow \Delta \quad x(\alpha):B, \Gamma \Rightarrow \Delta}{x(\alpha):A \vee B, \Gamma \Rightarrow \Delta} (Lv) \quad \frac{\Gamma \Rightarrow \Delta, x(\alpha):A, x(\alpha):B}{\Gamma \Rightarrow \Delta, x(\alpha):A \vee B} (Rv)$$

(Rules for knowledge operators)

$$\frac{x(\alpha, A, B):C, \Gamma \Rightarrow \Delta}{x(\alpha, A \& [A]B):C, \Gamma \Rightarrow \Delta} (Lcmp) \quad \frac{\Gamma \Rightarrow \Delta, x(\alpha, A, B):C}{\Gamma \Rightarrow \Delta, x(\alpha, A \& [A]B):C} (Rcmp)$$

other

$$\frac{x(\alpha):A -> B, x(\alpha):B -> A, \Gamma \Rightarrow \Delta}{x(\alpha):A <-> B, \Gamma \Rightarrow \Delta} (L<->) \quad \frac{\Gamma \Rightarrow \Delta, x(\alpha):A -> B \quad \Gamma \Rightarrow \Delta, x(\alpha):B -> A}{\Gamma \Rightarrow \Delta, x(\alpha):A <-> B} (R<->)$$