

Calculus: Integration References

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Trigonometric Function References

Pythagorean Identity:

$$\sin(\theta)^2 + \cos(\theta)^2 = 1$$

Divide by $\sin(\theta)^2$ or $\cos(\theta)^2$:

$$\tan(\theta)^2 + 1 = \sec(\theta)^2, \quad 1 + \cot(\theta)^2 = \csc(\theta)^2$$

Sum of Angle Formulas:

$$\sin(\alpha + \beta) = \sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha) \cos(\beta) - \cos(\alpha) \sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta)$$

$$\cos(\alpha - \beta) = \cos(\alpha) \cos(\beta) + \sin(\alpha) \sin(\beta)$$

Power Reduction Derivation:

$$\cos(2\theta) = \cos(\theta)^2 - \sin(\theta)^2$$

$$= \cos(\theta)^2 - (1 - \cos(\theta))^2 = (1 - \sin(\theta))^2 - \sin(\theta)^2$$

$$\cos(\theta)^2 = \frac{1 + \cos(2\theta)}{2}, \quad \sin(\theta)^2 = \frac{1 - \cos(2\theta)}{2}$$

Product to Sum Derivation:

$$\sin(\alpha + \beta) - \sin(\alpha - \beta) = 2 \cos(\alpha) \sin(\beta)$$

Cofunction Identities:

$$\text{Identity: } f\left(\frac{\pi}{2} - \theta\right) = g(\theta) \leftrightarrow g\left(\frac{\pi}{2} - \theta\right) = f(\theta)$$

$$\sin \leftrightarrow \cos, \quad \tan \leftrightarrow \cot, \quad \sec \leftrightarrow \csc$$

$$\frac{d}{dx} \sin(x) = \cos(x), \quad \frac{d}{dx} \csc(x) = -\csc(x) \cot(x)$$

$$\frac{d}{dx} \cos(x) = -\sin(x), \quad \frac{d}{dx} \sec(x) = \sec(x) \tan(x)$$

$$\frac{d}{dx} \tan(x) = \sec(x)^2, \quad \frac{d}{dx} \cot(x) = -\csc(x)^2$$

$$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx} \cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\int \sec(x) dx = \ln|\sec(x) + \tan(x)|$$

$$\int \csc(x) dx = \ln|\csc(x) - \cot(x)|$$

$$\int \tan(x) dx = \ln|\sec(x)|$$

$$\int \cot(x) dx = \ln|\sin(x)|$$

Substitution

$$\text{Let } u = g(x), du = g'(x)dx$$

$$\int f(g(x))g'(x) dx = \int f(u) du$$

$$\int_a^b f(g(x))g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$$

Integration by Parts

$$\int u dv = uv - \int v du$$

Trigonometric Functions

$$\int \sin(x)^m \cos(x)^n dx$$

If m is odd:

$$= \int \sin(x)^{m-1} \cos(x)^n d(-\cos(x))$$

$$\sin(x)^{m-1} = (1 - \cos(x)^2)^{\frac{m-1}{2}}$$

If n is odd:

$$= \int \sin(x)^m \cos(x)^{n-1} d(\sin(x))$$

$$\cos(x)^{n-1} = (1 - \sin(x)^2)^{\frac{n-1}{2}}$$

If both are even:

Use Power Reduction

$$\int \tan(x)^m \sec(x)^n dx$$

If m is odd:

$$= \int \tan(x)^{m-1} \sec(x)^{n-1} d(\sec(x))$$

$$\tan(x)^{m-1} = (\sec(x)^2 - 1)^{\frac{m-1}{2}}$$

If n is even:

$$= \int \tan(x)^m \sec(x)^{n-2} d(\tan(x))$$

$$\sec(x)^{n-2} = (\tan(x)^2 + 1)^{\frac{n-2}{2}}$$

If m is even and n is odd:Use $\tan(x)^{2k} = (\sec(x)^2 - 1)^k$

Trigonometric Substitutions

Let $x = c \tan(\theta)$

$$\sqrt{x^2 + c^2} = c \sec(\theta)$$

Let $x = c \sec(\theta)$

$$\sqrt{x^2 - c^2} = c \tan(\theta)$$

Let $x = c \sin(\theta)$

$$\sqrt{c^2 - x^2} = c \cos(\theta)$$

Partial Fraction Decomposition

Let $R(x) = \frac{P(x)}{Q(x)}$ be proper rational function.Factorize $Q(x)$ into linear factors or quadratic factors:

$$Q(x) = [\dots (a_i x + b_i)^{r_i}] [\dots (a_i x^2 + b_i x + c_i)^{r_i}]$$

For each factor, decompose into:

$$(ax + b)^r \rightarrow \frac{C_1}{(ax + b)^1} + \frac{C_2}{(ax + b)^2} + \frac{C_3}{(ax + b)^3} + \dots + \frac{C_r}{(ax + b)^r}$$

$$(ax^2 + bx + c)^r \rightarrow \frac{C_1 x + D_1}{(ax^2 + bx + c)^1} + \frac{C_2 x + D_2}{(ax^2 + bx + c)^2} + \frac{C_3 x + D_3}{(ax^2 + bx + c)^3} + \dots + \frac{C_r x + D_r}{(ax^2 + bx + c)^r}$$

Then, $Q(x)$ is equal to the sum of all decomposed factors.