Solutions to *Mathematica* For The Constantly Busy

Nathan Chapman
10/7/2016

Part 2 Syntax

- **1) Use a function in the Mathematica catalog.**

  For this question, you can choose any function that *Mathematica* knows.

In[92]:=
```
Factorial[0]
```

Out[92]= 1

- **2) Make a function that multiplies the variable by a special constant.**

In[93]:=
```
f[x_] := π x
```

In[94]:=
$$f\left[\frac{\pi}{2}\right]$$

Out[94]=
$$\frac{\pi^2}{2}$$

- **3) Clear the function from the previous problem and make a function of two variables using the name of the previous function. Then make function that take a set as an argument.**

In[95]:=
```
Clear[f]
f[x_, y_] := x y
f[ⅇ, π]
g[{a_, b_}] := λ {a, b}
g[{1, 2}]
```

Out[97]= $e\,\pi$

Out[99]= $\{\lambda, 2\,\lambda\}$

Part 3 Math! (Again, not factorial)

- **1) Analytically solve an algebraic equation.**

In[100]:=
$$\text{Solve}\left[\pi\, x^2 + e\, x + \phi == x,\ x\right]$$

Out[100]=
$$\left\{\left\{x \to \frac{1 - e - \sqrt{1 - 2\,e + e^2 - 4\,\pi\,\phi}}{2\,\pi}\right\}, \left\{x \to \frac{1 - e + \sqrt{1 - 2\,e + e^2 - 4\,\pi\,\phi}}{2\,\pi}\right\}\right\}$$

2) Numerically solve a triginometric equation.

In[101]:=
```
NSolve[Cos[x] == 3, x]
```

NSolve::ifun : Inverse functions are being used by NSolve,
so some solutions may not be found; use Reduce for complete solution information. ≫

Out[101]=
```
{{x → 0. - 1.76275 i}, {x → 0. + 1.76275 i}}
```

To see the difference between the analytic and the numerical solutions we see

In[102]:=
```
Solve[Cos[x] == 3, x]
```

Solve::ifun : Inverse functions are being used by Solve,
so some solutions may not be found; use Reduce for complete solution information. ≫

Out[102]=
```
{{x → -ArcCos[3]}, {x → ArcCos[3]}}
```

3) Add the first 100 natural numbers.

In[103]:=
```
Sum[k, {k, 100}]
```

Out[103]=
```
5050
```

4) Make a function that adds the previous two values of the function up to a certain amount of times.

In[104]:=
```
fib[0] = 0;
fib[1] = 1;
fib[n_] := fib[n - 1] + fib[n - 2]
```

Fun fact: This function produces the Fibonacci sequence

In[107]:=
```
Table[{Fibonacci[n], fib[n]}, {n, 0, 10}]
```

Out[107]=
```
{{0, 0}, {1, 1}, {1, 1}, {2, 2}, {3, 3}, {5, 5}, {8, 8}, {13, 13}, {21, 21}, {34, 34}, {55, 55}}
```

5) Find the closed form solution to the series of an infinite degree polynomial.

In[108]:= Sum[x^p, {p, 1, ∞}]

Out[108]= $-\frac{x}{-1+x}$

6) Find the derivative of the solution to the previous problem.

In[109]:= $D\left[-\frac{x}{-1+x}, x\right]$

Out[109]= $-\frac{1}{-1+x}+\frac{x}{(-1+x)^2}$

7) Find the derivative of the factorial function.

In[110]:=
```
D[Factorial[n], n]
```

Out[110]=
```
Gamma[1 + n] PolyGamma[0, 1 + n]
```

8) Find the integral of the solution to #6.

In[111]:= $\text{Integrate}\left[-\frac{1}{-1+x}+\frac{x}{(-1+x)^2}, x\right]$

Out[111]= $-\frac{1}{-1+x}$

■ 9) Create a 2x2 matrix, a vector in R^2. Then multiply them.

In[112]:=
```
Clear[A, x, X]
A = {{a1, a2}, {a3, a4}}
X = {x1, x2}
A.X // MatrixForm
```

Out[113]= $\{\{a_1, a_2\}, \{a_3, a_4\}\}$

Out[114]= $\{x_1, x_2\}$

Out[115]//MatrixForm=

$$\begin{pmatrix} a_1 x_1 + a_2 x_2 \\ a_3 x_1 + a_4 x_2 \end{pmatrix}$$

■ 10) Find the determinant of the inverse of the transpose of a matrix of your choice.

In[116]:=
```
Det[Inverse[Transpose[{{a1, a2}, {a3, a4}}]]]
```

Out[116]= $-\frac{a_2 a_3}{(-a_2 a_3 + a_1 a_4)^2} + \frac{a_1 a_4}{(-a_2 a_3 + a_1 a_4)^2}$

■ 11) Row reduce a 10x10 matrix of odd numbers.

In[117]:=
```
Table[2 k + t, {k, 1, 10}, {t, 1, 20, 2}] // MatrixForm
RowReduce[%] // MatrixForm
```

Out[117]//MatrixForm=

$$\begin{pmatrix} 3 & 5 & 7 & 9 & 11 & 13 & 15 & 17 & 19 & 21 \\ 5 & 7 & 9 & 11 & 13 & 15 & 17 & 19 & 21 & 23 \\ 7 & 9 & 11 & 13 & 15 & 17 & 19 & 21 & 23 & 25 \\ 9 & 11 & 13 & 15 & 17 & 19 & 21 & 23 & 25 & 27 \\ 11 & 13 & 15 & 17 & 19 & 21 & 23 & 25 & 27 & 29 \\ 13 & 15 & 17 & 19 & 21 & 23 & 25 & 27 & 29 & 31 \\ 15 & 17 & 19 & 21 & 23 & 25 & 27 & 29 & 31 & 33 \\ 17 & 19 & 21 & 23 & 25 & 27 & 29 & 31 & 33 & 35 \\ 19 & 21 & 23 & 25 & 27 & 29 & 31 & 33 & 35 & 37 \\ 21 & 23 & 25 & 27 & 29 & 31 & 33 & 35 & 37 & 39 \end{pmatrix}$$

Out[118]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & -1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

■ 12) Find the dimension of the nullspace of the space spanned by the eigenvectors of a matrix.

In[119]:=
```
Clear[A, B]
Dimensions[NullSpace[
  A Eigenvectors[{{a, b}, {c, d}}][[1]][[1]] + B Eigenvectors[{{a, b}, {c, d}}][[1]][[2]]]]
```

Out[120]= {1}

- **13) Solve a matrix equation whose matrix is composed of complex numbers and the solution is composed of real numbers.**

In[121]:=
```
LinearSolve[Table[k^p ⅈ, {k, 1, 3}, {p, 1, 3}], Table[k, {k, 3}]]
```

Out[121]= $\{-i, 0, 0\}$

- **14) Solve the ordinary differential equation $A\,u''(x) + B\,u'(x) + C\,u(x) = u(x)$**

In[122]:=
```
DSolve[{A u''[x] + B u'[x] + C u[x] == u[x], u[0] == 0, u'[0] == Γ}, u[x], x]
```

Out[122]= $$\left\{\left\{u[x] \to -\frac{A\left(e^{\frac{\left(-B-\sqrt{4A+B^2-4AC}\right)x}{2A}} - e^{\frac{\left(-B+\sqrt{4A+B^2-4AC}\right)x}{2A}}\right)\Gamma}{\sqrt{4A+B^2-4AC}}\right\}\right\}$$

- **15) Solve the partial differential equation $A\,u^{(1,0)}(x,t) + B\,u^{(0,1)}(x,t) + C\,u(x,t) = u(x,t)$**

In[123]:=
```
DSolve[
 {A D[u[x, t], x] + B D[u[x, t], t] + C u[x, t] == u[x, t], u[x, 0] == f[x]}, u[x, t], {x, t}]
```

Out[123]= $$\left\{\left\{u[x,t] \to e^{-\frac{(-1+C)x}{A}-\frac{(-1+C)(At-Bx)}{AB}}\, f\left[-\frac{At-Bx}{B}\right]\right\}\right\}$$

- **16) Using the solutions to problems 14 and 15, evaluate new expressions.**

- **14**

In[124]:=
```
Solve[
 μ u[x] + b == μ /. DSolve[{A u''[x] + B u'[x] + C u[x] == u[x], u[0] == 0, u'[0] == Γ}, u[x], x], μ]
```

Out[124]= $$\left\{\left\{\mu \to \frac{b\sqrt{4A+B^2-4AC}}{\sqrt{4A+B^2-4AC} + A\,e^{\frac{\left(-B-\sqrt{4A+B^2-4AC}\right)x}{2A}}\,\Gamma - A\,e^{\frac{\left(-B+\sqrt{4A+B^2-4AC}\right)x}{2A}}\,\Gamma}\right\}\right\}$$

- **15**

In[125]:=
```
Solve[μ u[x, t] + 7 == μ /. DSolve[
   {A D[u[x, t], x] + B D[u[x, t], t] + C u[x, t] == u[x, t], u[x, 0] == f[x]}, u[x, t], {x, t}],
  μ
 ]
```

Out[125]= $$\left\{\left\{\mu \to \frac{7\,e^{\frac{(-1+C)x}{A}+\frac{(-1+C)(At-Bx)}{AB}}}{e^{\frac{(-1+C)x}{A}+\frac{(-1+C)(At-Bx)}{AB}} - f\left[-\frac{At-Bx}{B}\right]}\right\}\right\}$$

Part 4 Data (Again, note the Android)

In[126]:=
```
Data = Table[k, {k, 100}];
```

- **1) Find the length of Data**

In[127]:=
```
Length[Data]
```

Out[127]= 100

- **2) Find the depth of Data**

In[128]:=
```
Depth[Data]
```

Out[128]= 2

- **3) Select the elements of Data that are greater than 1.**

In[129]:=
```
Select[Data, # > 1 &]
```

Out[129]= {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100}

- **4) Find the length and depth of the solution to exercise 3.**

In[130]:=
```
Length[Select[Data, # > 1 &]]
Depth[Select[Data, # > 1 &]]
```

Out[130]= 99

Out[131]= 2

- **5) Create a set of data which only has one level. Then repeat exercises 1-3 for this new data set.**

In[132]:= data = Table$\left[\frac{1}{k}, \{k, 100\}\right]$;

- **a) Length**

In[133]:=
```
Length[data]
```

Out[133]= 100

- **b) Depth**

In[134]:=
```
Depth[data]
```

Out[134]= 2

- **c) Select**

In[135]:=
```
Select[data, # > 1 &]
```

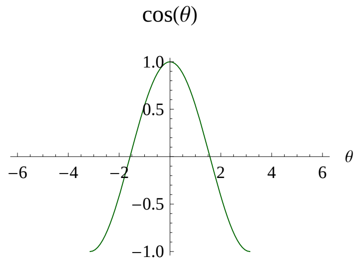
Out[135]= { }

Part 5 Important Functions

- **1) Create a plot of your favorite function with the axes labeled appropriately, the line your favorite color, the graph very jagged, and blank space on either side of the graph.**

In[166]:=
```
Plot[Cos[θ], {θ, -π, π},
 AxesLabel → Automatic,
 PlotLabel → Cos[θ],
 PlotStyle → Darker[Green, 3 / 5],
 PlotPoints → 2,
 PlotRange → {{-2 π, 2 π}, Automatic},
 ImageSize → Small
]
```

Out[166]=



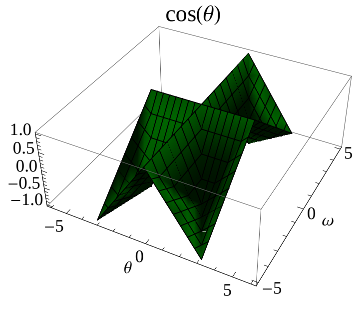
Note: I also added the "PlotLabel" option. This is a very important option when presenting your work.

Note: Intending to make the plot jagged, I used the minimum number of plot points. In this case, it doesn't alter the plot.

- **2) Create a 3D plot of a sphere with similar options as above.**

In[165]:=
```
Plot3D[Cos[ω θ], {θ, -π, π}, {ω, -5, 5},
 AxesLabel → Automatic,
 PlotLabel → Cos[θ],
 PlotStyle → Darker[Green, 3 / 5],
 PlotPoints → 2,
 PlotRange → {{-2 π, 2 π}, Automatic},
 ImageSize → Small
]
```

Out[165]=



- **3) Create a table of odd number between 3 and 71.**

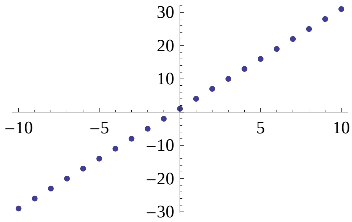
In[138]:= `Table[2 k + 1, {k, 1, 70/2}]`

Out[138]= {3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71}

- **4) Create a set of pairs of real numbers, such that when plotted they form a line.**

In[169]:=
```
Table[{n, 3 n + 1}, {n, -10, 10}]
ListPlot[%, ImageSize → Small]
```

Out[169]=
```
{{-10, -29}, {-9, -26}, {-8, -23}, {-7, -20}, {-6, -17},
 {-5, -14}, {-4, -11}, {-3, -8}, {-2, -5}, {-1, -2}, {0, 1}, {1, 4}, {2, 7},
 {3, 10}, {4, 13}, {5, 16}, {6, 19}, {7, 22}, {8, 25}, {9, 28}, {10, 31}}
```

Out[170]=


- **5) Simplify and then fully simplify the solutions to exercises 6-8, 14-15 from section 13.**

- **6)**

In[141]:= $\texttt{Simplify}\left[-\frac{1}{-1+x}+\frac{x}{(-1+x)^2}\right]$

$\texttt{FullSimplify}\left[-\frac{1}{-1+x}+\frac{x}{(-1+x)^2}\right]$

Out[141]= $\frac{1}{(-1+x)^2}$

Out[142]= $\frac{1}{(-1+x)^2}$

- **7)**

In[143]:=
```
Simplify[Gamma[1 + n] PolyGamma[0, 1 + n]]
FullSimplify[Gamma[1 + n] PolyGamma[0, 1 + n]]
```

Out[143]= `Gamma[1 + n] PolyGamma[0, 1 + n]`

Out[144]= `Gamma[1 + n] PolyGamma[0, 1 + n]`

- **8)**

In[145]:= $\texttt{Simplify}\left[-\frac{1}{-1+x}\right]$

$\texttt{FullSimplify}\left[-\frac{1}{-1+x}\right]$

Out[145]= $\frac{1}{1-x}$

Out[146]= $\frac{1}{1-x}$

■ 14)

In[147]:= $$\text{Simplify}\left[\left\{\left\{u[x] \to -\frac{A\left(e^{\frac{\left(-B-\sqrt{4A+B^2-4AC}\right)x}{2A}} - e^{\frac{\left(-B+\sqrt{4A+B^2-4AC}\right)x}{2A}}\right)\Gamma}{\sqrt{4A+B^2-4AC}}\right\}\right\}\right]$$

$$\text{FullSimplify}\left[\left\{\left\{u[x] \to -\frac{A\left(e^{\frac{\left(-B-\sqrt{4A+B^2-4AC}\right)x}{2A}} - e^{\frac{\left(-B+\sqrt{4A+B^2-4AC}\right)x}{2A}}\right)\Gamma}{\sqrt{4A+B^2-4AC}}\right\}\right\}\right]$$

Out[147]= $$\left\{\left\{u[x] \to \frac{A\, e^{-\frac{\left(B+\sqrt{4A+B^2-4AC}\right)x}{2A}}\left(-1+e^{\frac{\sqrt{4A+B^2-4AC}\,x}{A}}\right)\Gamma}{\sqrt{B^2-4A(-1+C)}}\right\}\right\}$$

Out[148]= $$\left\{\left\{u[x] \to \frac{A\, e^{-\frac{\left(B+\sqrt{B^2-4A(-1+C)}\right)x}{2A}}\left(-1+e^{\frac{\sqrt{4A+B^2-4AC}\,x}{A}}\right)\Gamma}{\sqrt{B^2-4A(-1+C)}}\right\}\right\}$$

■ 15)

In[149]:= $$\text{Simplify}\left[\left\{\left\{\mu \to \frac{7\, e^{\frac{(-1+C)x}{A}+\frac{(-1+C)(At-Bx)}{AB}}}{e^{\frac{(-1+C)x}{A}+\frac{(-1+C)(At-Bx)}{AB}} - f\left[-\frac{At-Bx}{B}\right]}\right\}\right\}\right]$$

$$\text{FullSimplify}\left[\left\{\left\{\mu \to \frac{7\, e^{\frac{(-1+C)x}{A}+\frac{(-1+C)(At-Bx)}{AB}}}{e^{\frac{(-1+C)x}{A}+\frac{(-1+C)(At-Bx)}{AB}} - f\left[-\frac{At-Bx}{B}\right]}\right\}\right\}\right]$$

Out[149]= $$\left\{\left\{\mu \to \frac{7\, e^{\frac{(-1+C)t}{B}}}{e^{\frac{(-1+C)t}{B}} - f\left[-\frac{At}{B}+x\right]}\right\}\right\}$$

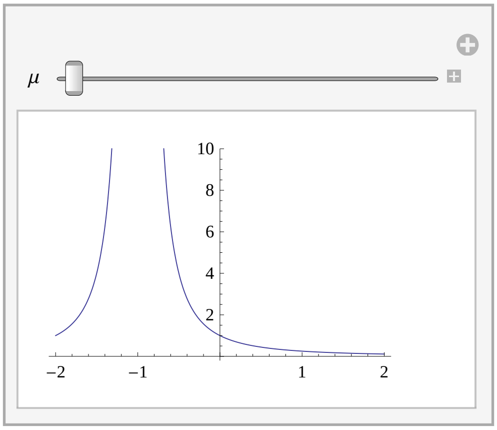
Out[150]= $$\left\{\left\{\mu \to \frac{1}{\frac{1}{7} - \frac{1}{7}\, e^{\frac{t-Ct}{B}}\, f\left[-\frac{At}{B}+x\right]}\right\}\right\}$$

- **6) For the solutions to each of the parts to the previous problem, create a scaling coefficient on some part of the solution. Then create a manipulatable plot for each solution and explore the di erent results.**

- **6)**

In[160]:=
```
Manipulate[
 Plot[1/(-1 + μ x)^2, {x, -2, 2}, PlotRange → {Automatic, 10}, ImageSize → Small],
 {μ, -1, 1}
]
```

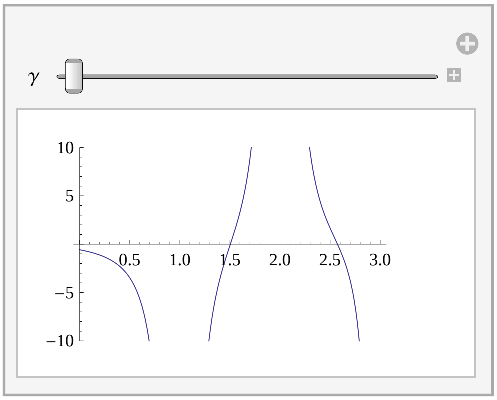
Out[160]=



- **7)**

In[161]:=
```
Manipulate[
 Plot[Gamma[1 + γ n] PolyGamma[0, 1 + γ n], {n, 0, 3},
  PlotRange → {Automatic, {-10, 10}}, ImageSize → Small],
 {γ, -1, 1}
]
```

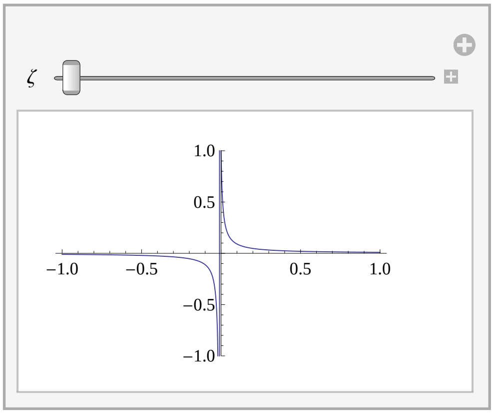
Out[161]=



■ 8)

In[162]:=
```
Manipulate[
 Plot[1/(1 - ξ x), {x, -1, 1}, PlotRange → Max[Table[1/(1 - ξ x), {x, -1, 1}]], ImageSize → Small],
 {ξ, -100, 100}
]
```

Out[162]=



- **14)**

In[163]:=

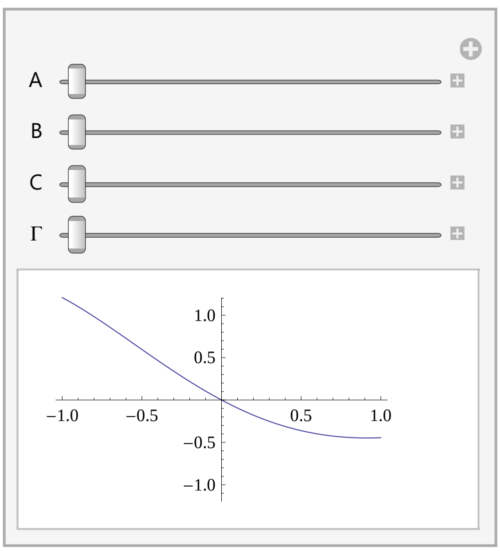
$$\mathtt{Manipulate}\Big[$$

$$\mathtt{Plot}\left[\frac{A\, e^{-\frac{\left(B+\sqrt{B^2-4\,A\,(-1+C)}\right)x}{2\,A}} \left(-1+e^{\frac{\sqrt{4\,A+B^2-4\,A\,C}\;x}{A}}\right)\Gamma}{\sqrt{B^2-4\,A\,(-1+C)}}, \{x,-1,1\}, \mathtt{PlotRange}\rightarrow\right.$$

$$\left.\mathtt{Max}\left[\mathtt{Table}\left[\mathtt{Abs}\left[\frac{A\, e^{-\frac{\left(B+\sqrt{B^2-4\,A\,(-1+C)}\right)x}{2\,A}} \left(-1+e^{\frac{\sqrt{4\,A+B^2-4\,A\,C}\;x}{A}}\right)\Gamma}{\sqrt{B^2-4\,A\,(-1+C)}}\right], \{x,-1,1\}\right]\right], \mathtt{ImageSize}\rightarrow\mathtt{Small}\right],$$

```
{A, -1, 1},
{B, -1, 1},
{C, -1, 1},
{Γ, -1, 1}
]
```

Out[163]=

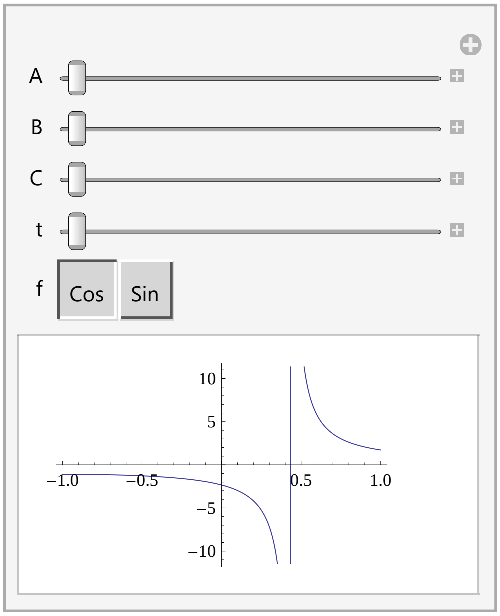


- **15)**

In[164]:=
```
Manipulate[
 Plot[1/(1/7 - 1/7 e^((t-C t)/B) f[-(A t)/B + x]), {x, -1, 1}, ImageSize → Small],
 {A, -1, 1},
 {B, -1, 1},
 {C, -1, 1},
 {t, -1, 1},
 {f, {Cos, Sin}}
]
```

$$\text{Plot}\left[\frac{1}{\frac{1}{7}-\frac{1}{7}\,e^{\frac{t-C\,t}{B}}\,f\left[-\frac{A\,t}{B}+x\right]},\ \{x,-1,1\},\ \text{ImageSize}\to\text{Small}\right]$$

Out[164]=



- **5) Make a GIF of a parabola that changes its curvature.**

In[156]:=
```
SetDirectory[NotebookDirectory[]]
Table[Plot[μ x^2, {x, -1, 1}, PlotRange → 1], {μ, -1, 1, .05}];
Export["Parabola.gif", %]
```

Out[156]= `U:\Science\Mathematica for the Constantly Busy\Solutions`

Out[158]= `Parabola.gif`