

CS 530: High-Performance Computing  
Seminar 1: Computational Physics

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# Chapter 1

## Introduction



## Chapter 2

# Mathematical Methods

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### 2.2 Fourier Analysis

#### 2.2.1 Fourier Series

#### 2.2.2 The Fourier Transform

### 2.3 Properties of Differential Equations

#### 2.3.1 Order

#### 2.3.2 Linearity

#### 2.3.3 Homogeneity

#### 2.3.4 Chaos & Entropy

Poincaré Sections

### 2.4 Ordinary Differential Equations

### 2.5 Partial Differential Equations

#### 2.5.1 Parabolic PDEs

The Heat Equation

#### 2.5.2 Hyperbolic PDEs

The Wave Equation

#### 2.5.3 Sturm-Liouville Theory

#### 2.5.4 Green's Functions

### 2.6 Differential Equations with Conditions

#### 2.6.1 Initial Conditions

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### 2.7 Systems of Differential Equations

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#### 3.1.3 Python

#### 3.1.4 Matlab & Mathematica

### 3.2 Finite-Differences

#### 3.2.1 Order

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### 3.3 Runge-Kutta Methods

### 3.4 Symplectic Integrators

### 3.5 Parareal - Parallel-in-Time Integration

### 3.6 The Fast Fourier Transform

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### 3.7 Finite-Element Methods

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# Chapter 4

## Physical Context

### 4.1 Orbital Dynamics

#### 4.1.1 The N-Body Problem

### 4.2 Fluid Dynamics

#### 4.2.1 The Lattice Boltzmann Method

### 4.3 Electrodynamics & Magnetohydrodynamics

#### 4.3.1 Fringing Electric Fields of Non-Ideal Capacitors

### 4.4 Many-Body Quantum Mechanics

#### 4.4.1 Quadrupole-Quadrupole Interactions in a BEC

### 4.5 Numerical Relativity

#### 4.5.1 Mercury's Perihelion Shift

Effects of Eccentricity

Gravitational Wave Chirp

#### 4.5.2 Gravitational Waves

### 4.6 Chaotic Systems

#### 4.6.1 Atmospheric Physics

#### 4.6.2 Forced Oscillators

### 4.7 Honorable Mentions

#### 4.7.1 Projectile Motion with Drag



## Chapter 5

# Numerical Analysis

- 5.1 Error
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