Computational Physics

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Why do we need computers to do physics?

An Illuminating Example - 2-Body Problem

$$\vec{F}_{ij} = \frac{G m_i m_j}{r_{ij}^2} \hat{r} \qquad \sum_j \vec{F}_{ij} = m_i \ddot{\vec{r}}_i$$

- 2 masses
- 1 unique force
- 2 equations of motion in 3 dimensions
- 6 coupled 2nd-order ordinary differential equations

An Illuminating Example - 3-Body Problem

$$\vec{F}_{ij} = \frac{G m_i m_j}{r_{ij}^2} \hat{r} \qquad\qquad \sum_j \vec{F}_{ij} = m_i \ddot{\vec{r}}_i$$

- 3 masses
- 3 unique forces
- 3 equations of motion in 3 dimensions
- 9 coupled 2nd-order ordinary differential equations

An Illuminating Example - “All”-Body Problem

$$\vec{F}_{ij} = \frac{G m_i m_j}{r_{ij}^2}\hat{r} \qquad\qquad \sum_j \vec{F}_{ij} = m_i \ddot{\vec{r}}_i$$

- 1.24 trillion masses
- 7.688×10^{23} unique forces
- 1.24 trillion equations of motion in 3 dimensions
- 3.72 trillion coupled 2nd-order ordinary differential equations

Cosmological N-Body Simulation[22]

Classical Mechanics

Remark

Given the position and velocity of and the forces acting on an object, the motion of that object is completely determined.

- Everday physics can be completed described by Newton's 2nd Law.

$$\underbrace{\sum_i \mathbf{F}_i = m\ddot{\mathbf{r}}}_{\text{Equation of Motion}} \tag{1}$$

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