



# OSCILLATIONS - REGULARLY REPEATING MOTION

→ PERIOD = the time the motion takes to repeat itself =  $T \dots [s]$

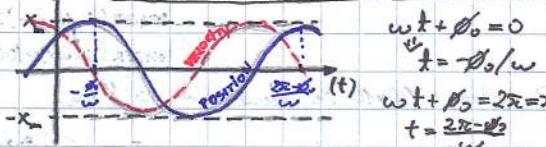
→ FREKVENCA = number of cycles per unit time =  $f = 1/T \dots [\text{Hz} = \text{s}^{-1}]$

$$x(t+T) = x(t)$$

$$\cos(\omega t) = \cos(\omega(t+T))$$

$$\cos(\omega t) = \cos(\omega t + 2\pi)$$

## SIMPLE HARMONIC MOTION (SHM)



$$x = x_m \cos(\omega t + \phi_0)$$

$$x(t) = x_m \cos(\omega t + \phi_0) \quad \omega T = 2\pi$$

$$N(t) = \dot{x}(t) = -\omega x_m \sin(\omega t + \phi_0) \quad N_m = \omega x_m$$

$$a(t) = \ddot{x}(t) = -\omega^2 x_m \cos(\omega t + \phi_0) \quad a_m = \omega^2 x_m$$

$$a(t) = -\omega^2 x(t) \Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0 \Rightarrow x(t) = A \cos(\omega t) + B \sin(\omega t)$$

$$\text{Now... } \frac{A \cos(\omega t) + B \sin(\omega t)}{\sqrt{A^2 + B^2}} \cdot \sqrt{A^2 + B^2} \dots \begin{cases} A \\ B \end{cases} \dots \frac{A}{\sqrt{A^2 + B^2}} = \cos \phi \text{ and } \frac{B}{\sqrt{A^2 + B^2}} = \sin \phi \Rightarrow x(t) = \sqrt{A^2 + B^2} (\cos \phi \cos \omega t + \sin \phi \sin \omega t)$$

$$\Rightarrow \cos \phi \cos \omega t + \sin \phi \sin \omega t = \cos(\omega t - \phi) \Rightarrow x(t) = \sqrt{A^2 + B^2} \cdot \cos(\omega t - \phi)$$

$$F = ma = -m \omega^2 x \quad (\text{for SHM})$$

$$\Rightarrow m \omega^2 = k \Rightarrow \omega = \sqrt{k/m} \rightarrow \text{inertia}$$

restoring force

For small displacements the force of spring is linear  $F = -kx$

$$T = 2\pi/\omega \Rightarrow T = 2\pi \sqrt{m/k}$$

spring const.

Hooke's Law

SHM conserves energy! ... force is a rate of change of potential energy

$$W = \frac{1}{2} k X^2 \Rightarrow U = \frac{1}{2} k x^2 = \frac{1}{2} k x_m^2 \cos^2(\omega t + \phi)$$

$$V_E = \frac{1}{2} m \omega^2 = \frac{1}{2} m \omega^2 x_m^2 \sin^2(\omega t + \phi) \Rightarrow U + V_E = \frac{1}{2} k x_m^2 + \frac{1}{2} m \omega^2 x_m^2 = \frac{1}{2} k x_m^2$$

TOTAL ENERGY IS CONSERVED

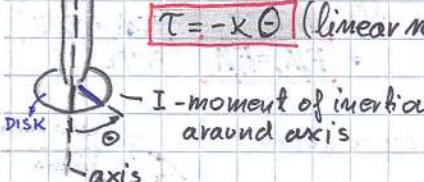
STRIP OF METAL

## TORSION PENDULUM

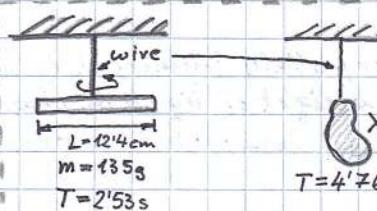
$$F = -k\theta = m\ddot{\theta}$$

$$T = -k\theta \quad (\text{linear response})$$

$$\ddot{\theta} = -\frac{k}{I} \theta = -\omega^2 \theta \Rightarrow \omega = \sqrt{\frac{k}{I}} ; T = 2\pi \sqrt{\frac{I}{k}}$$



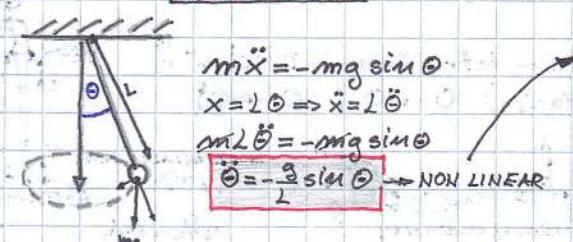
I - moment of inertia around axis



$$I_{\text{rod}} = \frac{1}{12} m L^2 = 1.73 \times 10^{-4} \text{ kg m}^2$$

$$k = \frac{4\pi^2 I_{\text{rod}}}{T^2} = \frac{4\pi^2 I_x}{T_x^2} \Rightarrow I_x = \frac{T_x^2}{4\pi^2} \cdot I_{\text{rod}} = 6.02 \times 10^{-4} \text{ kg m}^2$$

## PENDULUM



$$m\ddot{x} = -mg \sin \theta$$

$$x = L\theta \Rightarrow \ddot{x} = L\ddot{\theta}$$

$$mL\ddot{\theta} = -mg \sin \theta$$

$$\ddot{\theta} = -\frac{g}{L} \sin \theta \rightarrow \text{NON LINEAR}$$

FOR SMALL  $\theta \Rightarrow \sin \theta \sim \theta - \frac{\theta^3}{6} + \dots$

$$\ddot{\theta} = -\frac{g}{L} \theta \Rightarrow \omega = \sqrt{\frac{g}{L}}$$

PS: za natančno merjenja g, maje kot nitna nihalo, uporabimo palico!

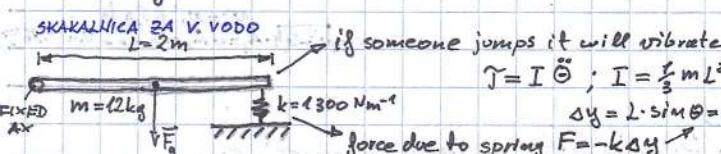
## PHYSICAL PENDULUM

$$\text{NAVAR: } T = -mgh \theta = I \ddot{\theta}$$



$$\ddot{\theta} = -\frac{mgh}{I} \theta$$

$$\omega = \sqrt{\frac{mgh}{I}} \quad T = \frac{2\pi}{\omega}$$



$$T = I \ddot{\theta} ; I = \frac{1}{3} m L^2 \quad \Rightarrow -kL^2 \ddot{\theta} = \frac{1}{3} m L^2 \ddot{\theta} \Rightarrow \ddot{\theta} = -\frac{3k}{m} \theta$$

$$\Delta y = L \sin \theta = L \cdot \theta$$

$$\omega = \sqrt{\frac{3k}{m}}$$

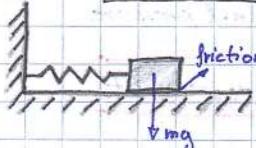
## DAMPED SHM

ASSUME FRICTION IS PROPORTIONAL TO VELOCITY

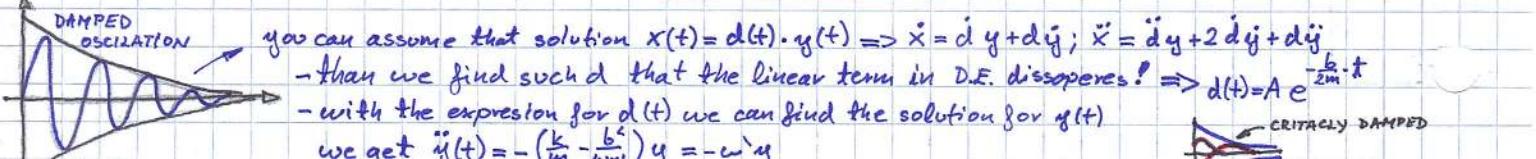
friction with coefficient b  $\Rightarrow F_f = -b v = -b \dot{x}$

$$F = m\ddot{x} = -kx - b\dot{x} \Rightarrow \ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\omega^2 = \omega_0^2 - \frac{b^2}{4m^2}$$



you can assume that solution  $x(t) = d(t) \cdot y(t) \Rightarrow \dot{x} = d \dot{y} + dy; \ddot{x} = d \ddot{y} + 2d\dot{y} + dy$   
 - then we find such d that the linear term in D.E. disappears!  $\Rightarrow d(t) = A e^{-\frac{b}{2m}t}$   
 - with the expression for d(t) we can find the solution for y(t)  
 we get  $y(t) = -\left(\frac{b}{m} - \frac{b^2}{4m^2}\right) u = -\omega u$



## APPLY EXTERNAL DRIVING FORCE

DRIVING FORCE  $F_e(t)$ ; IF THE DRIVING FORCE OSCILLATES  $\Rightarrow F_e = A_e \cos(w_e t)$

now we get  $m\ddot{x} = -b\dot{x} - kx + F_e(t) \Rightarrow \ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = \frac{F_e}{m}$

$F_e$  OSCILLATES  $\Rightarrow \ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = A_e \cos(w_e t)$  we make a guess:  $x(t) = B \cos(w_e t) + C \sin(w_e t)$

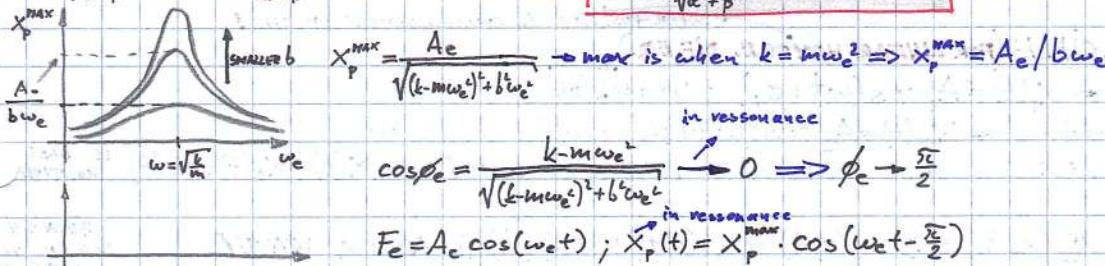
WE GET...  
 $\Rightarrow (-m w_e^2 B + b w_e^2 C + k B) \cos(w_e t) + (-m w_e^2 C - b w_e B + k C) \sin(w_e t) = ...$   
 $\dots = A_e \cos(w_e t)$  ... must be true for all values of  $t$ !  
 $t=0 \Rightarrow -m w_e^2 B + b w_e^2 C + k B = A_e \Rightarrow$  LET  $m w_e^2 - k = P \Rightarrow C = \frac{A_e b w_e}{b^2 w_e^2 + P^2}$  and  $B = \frac{-P}{b^2 w_e^2 + P^2} \cdot A$

$t = \pi/2 w_e \Rightarrow -m w_e^2 C - b w_e B + k C = 0 \Rightarrow B = \frac{(k - m w_e^2) \cdot C}{b w_e}$

The solutions to D.E. can also be written as:  $x(t) = x_{\text{HOM}}(t) + x_p(t) = (x_m) e^{-\frac{b}{2m}t} \cos(w_e t + \phi_0) + x_p(t)$

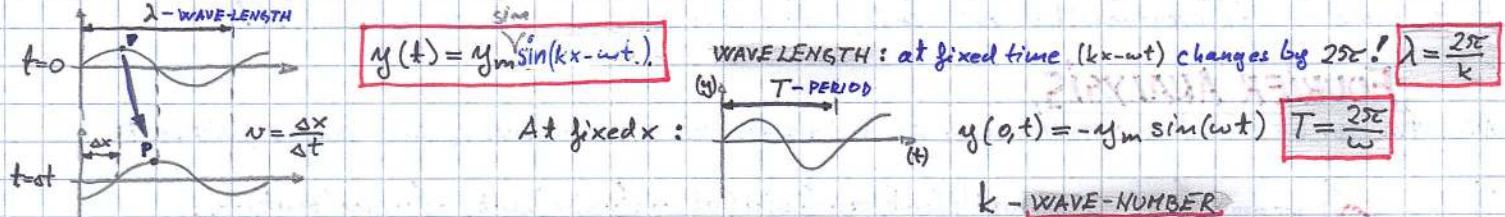
$$\begin{pmatrix} k - m w_e^2 & b w_e \\ -b w_e & k - m w_e^2 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} A_e \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} A \\ B \end{pmatrix} = \frac{1}{(k - m w_e^2)^2 + b^2 w_e^2} \begin{pmatrix} k - m w_e^2 & -b w_e \\ b w_e & k - m w_e^2 \end{pmatrix} \begin{pmatrix} A_e \\ 0 \end{pmatrix} \Rightarrow x_p(t) = \frac{A_e}{(k - m w_e^2)^2 + b^2 w_e^2} \left[ (k - m w_e^2) \cos(w_e t) + b w_e \sin(w_e t) \right]$$

Let  $\frac{\alpha}{\sqrt{\alpha^2 + \beta^2}} = \cos \phi_e$  ;  $\frac{\beta}{\sqrt{\alpha^2 + \beta^2}} = \sin \phi_e$  and we get  $x_p(t) = \frac{A_e}{\sqrt{\alpha^2 + \beta^2}} \cdot \cos(w_e t - \phi_e) \Rightarrow$  RESONANCE (when driving frequency  $w_e$  is close to oscill. freq.  $\sqrt{\frac{k}{m}}$   $\Rightarrow$  amplitudes are big)



## WAVES $\rightarrow$ LONGITUDINAL W. and TRANSVERSE W.

Ex. of LONGITUDINAL: - pressure waves (sound) ... Ex. of TRANSVERSE: - surface waves, string, drum, EM waves ...

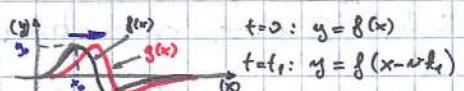


SPEED = ? - look at the point P on the wave (for point P is  $kx - \omega t = \pi/2$ )

$$v = \frac{\Delta x}{\Delta t} \quad kx = \frac{\pi}{2} + \omega t \Rightarrow x_p(t) = \frac{\omega t}{k} + \frac{\pi}{2k} \Rightarrow N_p = \dot{x}_p(t) = \frac{\omega}{k} \Rightarrow v = \lambda / T$$

RIGHT-MOVING WAVE  $y(x, t) = y_m \sin(kx - \omega t)$  ...  $v = \omega / k$

LEFT-MOVING WAVE  $y(x, t) = y_m \sin(kx + \omega t)$  ...  $v = -\omega / k$



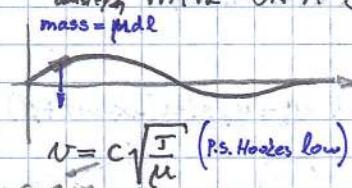
Wave = disturbance of regular shape moving in some medium!

$$g(x_0) = g(x_0 - vt_1) \Rightarrow g(x, t) = g(x - vt)$$

In general: left-moving wave with profile  $y(x)$  is  $y_L(x, t) = y(x + vt)$ !

- right-moving wave with profile  $y_R(x, t) = y(x - vt)$ !

## WAVE ON A STRING



- tension produces a restoring force (tension-T)  $T \dots [Nm^{-1} = kg s^{-2}]$

- it's like an oscillator

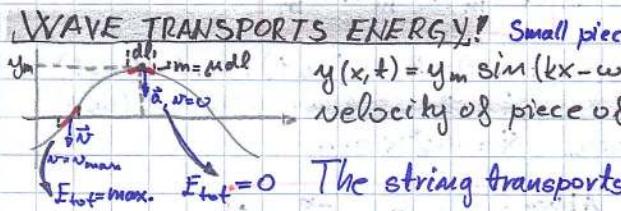
- mass density ...  $\mu = \text{mass/length} \Rightarrow \mu \dots [\text{kg m}^{-1}]$

Observer moves with the displacement on a string. This observer sees fixed wave front and a string, that is moving to the left with velocity = v!

$\frac{w_0}{R} = \frac{w_0}{R} - v \Rightarrow dl = 2\theta R$  There is no force along direction of motion, but there is vertical force (RESTORING) restoring force:  $F = -T \sin \theta \approx -2\theta R$

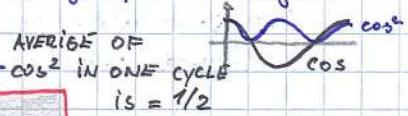
$$\Rightarrow F = ma \Rightarrow -2\pi \theta_0 = -2\mu \theta_0 R \cdot v^2 / R \Rightarrow N = \sqrt{5/\mu}$$

T-tension of the string  
μ - mass density

**WAVE TRANSPORTS ENERGY!** Small piece of string is harmonic with ang. fr. = ω and amplitude = y<sub>m</sub>!  


$$\text{Energy in string: } dK = \frac{1}{2} dm u^2 = \frac{1}{2} \mu dx \cdot \omega^2 y_m^2 \cos^2(kx - \omega t)$$

$$\text{Rate of change of } K: \frac{dK}{dt} = \frac{1}{2} \mu \nu \omega^2 y_m^2 \cos^2(kx - \omega t) \Rightarrow \text{Average: } \frac{\bar{dK}}{dt} = \frac{1}{4} \mu \nu \omega^2 y_m^2$$



We saw for SHM that  $\bar{K} = \bar{U}$ ! Same argument holds here! Average power transmitted:  $\frac{d\bar{K}}{dt} + \frac{d\bar{U}}{dt} = 2 \frac{d\bar{K}}{dt}$

Example: string - lin. dens.  $\rho = 525 \text{ g/m}$ ; tension  $T = 45 \text{ N}$ ; wave with  $\nu = 120 \text{ Hz}$  and  $y_m = 8.5 \text{ mm}$   
 Find average power transmitted  $P_{av}$  ...  $P_{av} = 100 \text{ W}$

$$P_{av} = \frac{1}{2} \mu \nu \omega^2 y_m^2$$

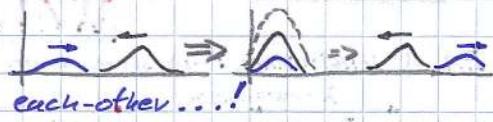
**SUPERPOSITION PRINCIPLE (FOR LINEAR HOMOGENEOUS DIFF. EQ.)**  
 Ex: SFM  $\Rightarrow \ddot{x} + \frac{k}{m} x = 0 \rightarrow x_1(t) = A_1 \cos(\omega t + \phi_1); \omega = \sqrt{k/m}$   
 but we can have another solution  $\rightarrow x_2(t) = A_2 \cos(\omega t + \phi_2)$ ; with same  $\omega$   
 $\Rightarrow$  THE SUM OF THE  $x_1(t) + x_2(t)$  IS ALSO SOLUTION TO DIFF. EQ. FOR SHM!  $\Rightarrow x(t) = x_1(t) + x_2(t)$  IS ALSO SOLUTION

FOR WAVES:  $y_1(x, t), y_2(x, t) \Rightarrow y(x, t) = y_1(x, t) + y_2(x, t)$  IS ALSO A WAVE!

The profile of sum is the sum of each of individual waves!

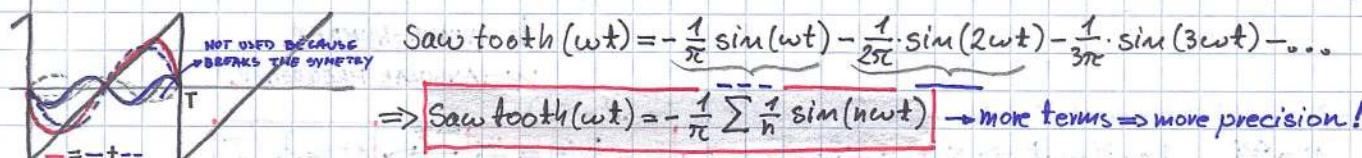
BECUSE THEY ARE IN THE SAME MEDIUM!

P.S.: FOR SHM the  $\omega$  of each solution should be fixed. WITH WAVES THE SPEED OF WAVE IS FIXED!

It is interesting when two waves go in opposite direction!   
 As a consequence of super-position principle, waves can pass through each-other...!

## FOURIER ANALYSIS

- ANY CURVE CAN BE DESCRIBED MATHEMATICALLY AS A SUPER-POSITION OF SINOSIDUAL (SIMPLE HARMONIC) CURVES!



## INTERFERENCE

We have 2 waves on a string (with the same amplitude);  $y_1(x, t) = y_m \sin(kx - \omega t)$

$$y_2(x, t) = y_m \sin(kx - \omega t + \phi)$$

$$y(x, t) = y_1 + y_2 = y_m [\sin(kx - \omega t) + \sin(kx - \omega t + \phi)] \quad \oplus \sin a + \sin b = 2 \sin \left[ \frac{a+b}{2} \right] \cdot \sin \left[ \frac{a-b}{2} \right]$$

$$\Rightarrow y(x, t) = 2y_m \sin(kx - \omega t + \phi/2) \cdot \cos(\phi/2) = [2y_m \cos(\phi/2)] \cdot \sin(kx - \omega t + \phi/2) \Rightarrow \text{if } \phi = 0, 2\pi, 4\pi, \dots \text{ Ampl.} = 2y_m$$

CONSTRUCTIVE INTERFERENCE:  $\phi = 0, 2\pi, 4\pi, \dots \Rightarrow$  Amplitude =  $2y_m$  (MAX. AMPLITUDE)

DESTRUCTIVE INTERFERENCE:  $\phi = \pi, 3\pi, \dots \Rightarrow$  Amplitude = 0 (WAVES CANCEL EACH OTHER)

We have INTERMEDIATE INTERFERENCE if  $\phi \neq n\pi$ !

Now consider 2 waves with different amplitudes!  $y_1 = y_1 e \sin(kx - \omega t)$  and  $y_2 = y_2 e \sin(kx - \omega t + \phi)$   
 $y_1 + y_2 = \sqrt{y_1^2 + y_2^2} \left[ \cos \left( \frac{y_2}{\sqrt{y_1^2 + y_2^2}} \right) \cdot \sin(kx - \omega t) + \sin \left( \frac{y_2}{\sqrt{y_1^2 + y_2^2}} \right) \cdot \sin(kx - \omega t + \phi) \right] = \dots$  (complicated...)

$$\text{do this: } y_1 + y_2 = \frac{1}{2i} \left[ y_1 (e^{i(kx - \omega t)} - e^{-i(kx - \omega t)}) + y_2 (e^{i(kx - \omega t + \phi)} + e^{-i(kx - \omega t + \phi)}) \right] = \frac{1}{2i} \left[ (y_1 + y_2 e^{i\phi}) \cdot e^{i(kx - \omega t)} - (y_1 + y_2 e^{-i\phi}) \cdot e^{-i(kx - \omega t)} \right]$$

$$a = y_1 + y_2 e^{i\phi} = \alpha e^{i\phi} \text{ where } \alpha = \sqrt{y_1^2 + y_2^2} = \sqrt{y_1^2 + y_2^2 + 2y_1 y_2 \cos\phi} = \text{-AMPLITUDE!}$$

## 2 WAVES TRAVELING IN OPPOSITE DIRECTIONS $\Rightarrow$ STANDING WAVES

we absorbed phase in  
the  $x'$  and  $t'$ ...

THEY HAVE SAME AMPLI.  $y_{1m}(x, t) = y_{1m} \sin(kx - \omega[t + \frac{L}{2\omega}] + \frac{\pi}{2}) \Rightarrow y_1(x, t) = y_{1m} \sin(kx' - \omega t')$   
 $y_{2m}(x, t) = y_{2m} \sin(kx + \omega[t + \frac{L}{2\omega}] + \frac{\pi}{2}) \Rightarrow y_2(x, t) = y_{2m} \sin(kx' + \omega t')$

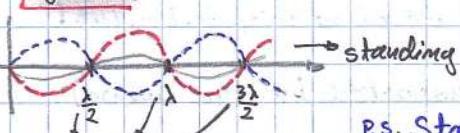
$\sin(x + \sin(p)) = 2 \sin(\frac{x+p}{2}) \cos(\frac{x-p}{2})$

STANDING WAVE

A POINT X UNDERGOES SHM WITH AMPLITUDE =

$$= 2y_m \cdot \sin(kx)$$

$\Rightarrow$  No energy is transmitted!

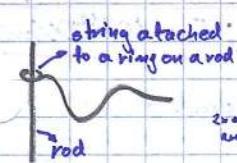
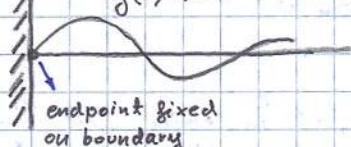


P.S. Standing wave - is the sum of two waves going in opposite directions!

NODES ... amplitude = 0  $\Rightarrow kx_N = n\pi \Rightarrow x_N = \frac{n\pi c}{k} = \frac{n\lambda}{2}$

ANTI-NODES ... amplitude = max.  $\Rightarrow k(x_{AN}) = (n + \frac{1}{2})\pi \Rightarrow x_{AN} = (n + \frac{1}{2})\frac{\lambda}{2}$

BOUNDARY CONDITION  $y(0, t) = 0$



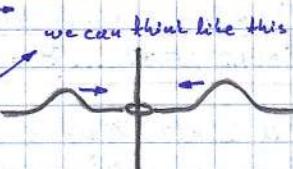
STRING FIXED ON BOTH END-POINTS

$y(0, t) = 0$  boundary conditions

$y(L, t) = 0$  boundary conditions

you can think about reflection  
of wave like this

at wall they cancel each other!  
at wall amplitude is always zero!



if end-point is free to move we have like  
two waves moving in opposite directions  
but with constructive interference!

NOTE:  
 $v = 100\text{ m/s}$   
 $\omega = 200\text{ rad/s}$   
 $n = 300\text{ Hz}$

We end with STANDING WAVE AGAIN

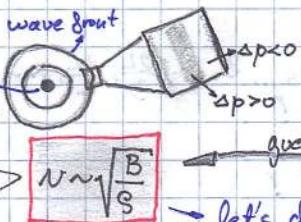
$$y(x, t) = [2y_m \sin(kx)] \cdot \cos(\omega t)$$

$$\Rightarrow \sin(kL) = 0 \Rightarrow kL = n\pi \Rightarrow \lambda = \frac{2L}{n}$$

$$n = \sqrt{\frac{J}{\mu}} = \frac{\omega}{k}$$

$$k = \frac{2\pi}{\lambda} = \frac{n\pi c}{L}; \omega = n \frac{\pi c}{L}$$

RESONANT FREQUENCIES - natural frequencies of a system! For waves on a string fixed between two boundaries the resonant frequencies form a harmonic series!

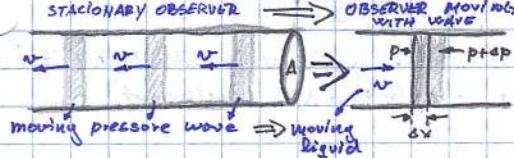


## LONGITUDINAL WAVES, (SOUND WAVES)

elasticity is related to quantity of  $\Delta p$  required  
to change volume  $\Delta V/V$ !  
inertia  $\propto G$

$$B = \frac{-\Delta p}{\Delta V/V} \quad \text{BULK MODULUS}$$

$\Rightarrow N \cdot \frac{\sqrt{B}}{S}$   $\rightarrow$  let's do it more rigorously...



$$\text{mass} = \rho \alpha x \cdot A; \text{force} = F = pA - (p_{\text{ref}})A = -\Delta p A$$

$$F = m \frac{\Delta v}{\Delta t} = \rho \alpha x A \frac{\Delta v}{\Delta t}$$

$$\frac{\Delta V}{\Delta t} = \Delta V/A \Rightarrow \frac{\Delta V}{V} = \frac{\Delta v \cdot A}{V} = \frac{\Delta v \cdot t}{\Delta x} = \frac{\Delta v}{V}$$

air 0°C 331 ms<sup>-1</sup>

air 20°C 343 ms<sup>-1</sup>

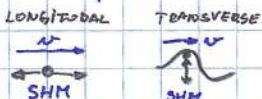
hydrogen 1284 ms<sup>-1</sup>

H<sub>2</sub>O 1482 ms<sup>-1</sup>

aluminium 6420 ms<sup>-1</sup>

$$\Rightarrow -\Delta p A = \rho \alpha x A \frac{\Delta v}{\Delta t} \Rightarrow -\frac{\Delta p}{\rho} = N \frac{\alpha x}{\Delta t} \frac{\Delta v}{V} \Rightarrow -\frac{\Delta p}{\rho} = N^2 \frac{\alpha v}{V} = N^2 \frac{\Delta V}{V} \Rightarrow N = \sqrt{\frac{\Delta p}{\rho \Delta V}} = \sqrt{\frac{B}{S}}$$

A particle undergoes SHM in the direction of wave-front about its equilibrium position!



$$S(x, t) = S_m \cos(kx - \omega t)$$

$$\Delta p = -B \frac{\Delta V}{V}; V = A \alpha x; \Delta V = A \Delta x \Rightarrow \Delta p = -B \frac{\Delta x}{\alpha x} = -B \frac{\partial S}{\partial x} \Rightarrow$$

$$\Rightarrow \Delta p = B k S_m \sin(kx - \omega t) = \Delta p_m \sin(kx - \omega t)$$

$$\Rightarrow \Delta p_m = B k S_m = (\nu^2 \rho \omega) S_m$$

max. sound for

$$\text{human ear} \Rightarrow S_m = \frac{\Delta p_m}{\nu^2 \rho \omega} = \text{depends on frequency! Let's take } f = 1\text{ kHz}; \nu = 343 \text{ m/s}; \rho = 1.2 \text{ kg/m}^3$$

$$\Delta p_m = 28 \text{ Pa}$$

$$\Rightarrow S_m \approx 10 \mu\text{m} \quad (\text{This already causes pain!})$$

The faintest sound a human ear can detect is  $\sim 28 \times 10^{-5} \text{ Pa} \Rightarrow S_m \sim 10^{-14} \text{ m} = 0.01 \mu\text{m}!$

# INTERFERENCE

S<sub>1</sub> S<sub>2</sub> LISTENER  $\Delta L = L_2 - L_1 \Rightarrow$  phase difference  $\phi = 2\pi \frac{\Delta L}{\lambda}$  (due to different paths that sound-waves make)

$\Rightarrow$  CONSTRUCTIVE:  $\phi = 2\pi m; m \in \mathbb{Z}$  DESTRUCTIVE:  $\phi = 2\pi(n + \frac{1}{2}); n \in \mathbb{Z}$

if we move the observer we get places with constr. int. and places with destr. int!

## INTENSITY - SOUND LEVEL (how much energy is transported by the wave)

$$I = \overline{P}/A \quad \text{intercepting area}$$

Kin. En. of particles

$$dK = \frac{1}{2} dm v_p^2 = \frac{1}{2} SA d\lambda s_m^2 \omega^2 \sin^2(kx - \omega t)$$

Source  $\rightarrow$  displacement of particles  $s = s_m \cos(kx - \omega t)$   $\Rightarrow \frac{ds}{dt} = \frac{1}{2} SA \dot{N} s_m^2 \omega^2 \sin^2(kx - \omega t)$

$N = -\omega s_m \sin(kx - \omega t)$   $\Rightarrow \overline{P}_k = \frac{dK}{dt} = \frac{1}{4} SA N s_m^2 \omega^2$  (power)  $\sin^2 \rightarrow \frac{1}{2}$

we know  $\overline{P}_k = \overline{P}_u \Rightarrow I = \frac{1}{2} S N \omega^2 s_m^2$

$$\Rightarrow I = \frac{\text{Power of source}}{\text{Area}} = \frac{P}{4\pi r^2} \sim \frac{1}{r^2}$$

### DECIBEL ... [dB]

car drum displacement ... max  $s_m \approx 10^{-5} \text{ m} \rightarrow$  min  $s_m \approx 10^{-11} \text{ m}$

Therefore is more useful to use a logarithmic scale!

$$\text{SOUND LEVEL} \dots \beta = 10 \text{ dB} \cdot \log(I/I_0); I_0 \approx 10^{-12} \text{ W/m}^2$$

$\Rightarrow$  lower limit of hearing ...  $\beta \approx 0 \text{ dB}$  ... if  $I_0 = 10^{-12} \text{ W/m}^2 \Rightarrow s_m = \frac{1}{\omega} \sqrt{\frac{2I_0}{S_N}}$   $\rightarrow$  depends on  $\omega$

our ear is most sensitive in 1kHz region  $\Rightarrow \omega = 2\pi \cdot 10^3 \text{ s}^{-1} \Rightarrow s_m \approx 10^{-10} \text{ m}$

electrodes  $\rightarrow$  Source is not a point, but a line  $\Rightarrow$  cylindrical wavefront

$= 10 \text{ m} \quad P = 1.6 \times 10^4 \text{ W} \rightarrow$  what is intensity 12 m away?  $\Rightarrow$  area  $= 2\pi \cdot 12 \text{ m} \cdot 10 \text{ m} \approx 750 \text{ m}^2$

spark  $\Rightarrow$  intensity  $I = P/A = 21.3 \text{ W/m}^2$

We can see that for big cylindrical source intensity falls like  $\frac{1}{r^2}$ ! power/length =  $P \Rightarrow I = \frac{P}{A} = \frac{PL}{2\pi r L} = \frac{P}{2\pi r}$

1-dim wave  $\Rightarrow$  constant area  $\Rightarrow I$  is constant!

## MUSICAL SOUND (HARMONIC)

string  $\rightarrow$  string  $\Rightarrow$  boundary condition ... fixed at both ends!

if fundamental frequency  $= f_0 \rightarrow f_0, 2f_0, 3f_0, 4f_0 \dots$

MUSICAL SCALE ... one octave  $= f_0 = 28 \text{ Hz}$  (frequency goes to twice the frequency)!

Possible wave-lengths ...  $\frac{2L}{1}, \frac{2L}{2}, \frac{2L}{3}, \frac{2L}{4}, \dots$

STANDING WAVES  
SOUND-WAVES in TUBES open open closed open closed closed

OPEN BOUNDARY  $\Rightarrow$  pressure is equal to atmospheric pressure  $\Rightarrow$  PRESSURE NODE  $\Rightarrow$  DISPLACEMENT anti-NODE!

pressure  $\rightarrow$  allowed frequency ...  $f_0, 2f_0, 3f_0 \dots$

CLOSED BOUNDARY  $\Rightarrow$  displacement = 0  $\Rightarrow$  DISPLACEMENT NODE, PRESSURE ANTI-NODE

displacement  $\rightarrow$  series of resonances,  $f_0 \rightarrow 3f_0 \rightarrow 5f_0 \rightarrow \dots$

1<sup>st</sup> harmonics      2<sup>nd</sup> harmonics      3<sup>rd</sup> harmonics

## HELMHOLTZ RESONATOR

The air in the bottle is undergoing ADIABATIC PROCESSES  $\Rightarrow \frac{\Delta P}{P_0} = -\gamma \frac{\Delta V}{V}; B = -\gamma P_0$

$m = \rho A L$   $B = \frac{-\Delta P}{\Delta V/V}$   $\ddot{x} = \frac{F}{m} = \frac{\Delta P A}{\rho A L V} = \frac{B A^2 x}{\rho L V} = \frac{B A}{\rho L} x \Rightarrow \omega = \sqrt{\frac{-BA}{\rho L V}} = \sqrt{\frac{\gamma P_0 A}{\rho L V}}$  HARMONIC RESONATOR

$F = \Delta P A$  force, that returns air in the neck back to original place  $\Delta V = A \cdot x$  change of volume is negative

air inside the bulb of bottle acts like spring

pressure      velocity

$$\frac{p}{p_0} = \frac{v}{v_0}$$

$$p = p_0 \Rightarrow \frac{dp}{dt} = 0$$

Resonance when  $Z_{in}(w)$  is minimum!!

ACOUSTIC IMPEDANCE:  $Z = p/u$

output impedance  $\rightarrow$  we need input impedance  $\Rightarrow$  Fourier transformation, we didn't do it!

Trompet is like a tube closed at one end  $\Rightarrow$  resonant frequencies:  $f_0 \rightarrow 3f_0 \rightarrow 5f_0 \rightarrow \dots$   
but in reality it has full harmonic series  $f_0 \rightarrow 2f_0 \rightarrow 3f_0 \rightarrow \dots$   
It is because a mouth-piece is like Helmholtz resonator!

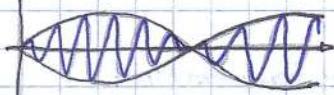


BEATS - variation in amplitude, due to interference of two waves with similar frequency

To simplify we take  
to waves with equal amplitude!

$$\Rightarrow S = S_1 + S_2 = S_m \cos(\omega_1 t) + S_m \cos(\omega_2 t) = 2S_m \cos\left(\frac{\omega_1 + \omega_2}{2} t\right) \cos\left(\frac{\omega_1 - \omega_2}{2} t\right)$$

$$= A(t) \cdot \cos\left(\frac{\omega_1 + \omega_2}{2} t\right); A(t) = 2S_m \cos\left(\frac{\omega_1 - \omega_2}{2} t\right) \rightarrow \text{time dependent amplitude!}$$



$$38 f = \frac{1}{2\pi c} \frac{\omega_1 - \omega_2}{2} < 20 \text{ Hz}, \text{ we don't hear it like beats, but feel a press. wave!}$$

frequency  $\geq$  press. ... max - max = max  $\rightarrow$  the balance between amplitude  $\rightarrow$

AMPLITUDE MODULATION:  $S \cdot \cos(\omega_{mod} t) \cdot \cos(\omega t)$  where  $\omega_{mod} \ll \omega$ !

if  $\omega_{mod} \sim \omega \Rightarrow$  RING MODULATION carrier freq. modulation freq.

FREQUENCY MODULATION: we are changing the frequency itself:  $\Rightarrow \cos([\omega + \lambda \cos(\omega_{mod} t)])$

DOPPLER EFFECT - moving source or moving detector

MOVING DETECTOR:



- because the detector is moving towards source, it will measure higher frequency additional number of wave-fronts detector intercept in time t!

$$f = \frac{v}{\lambda}; f_2 = \frac{v}{\lambda} + \frac{(v_d \cdot t)}{\lambda} \frac{1}{t} = \frac{v + v_d}{\lambda} \rightarrow \text{FOR DETECTOR MOVING TOWARDS THE SOURCE!}$$

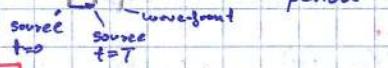
$$f_2 = \frac{v + v_d}{v} f$$

MOVING SOURCE:



- because of moving source the wave-fronts get compressed!

$$\lambda' = vT - N_s T$$

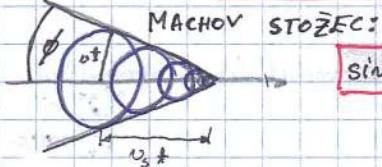


- SOURCE MOVING TOWARDS THE DETECTOR!

$$f_2 = \frac{v}{(v - v_s)} f$$

(+ sign in opposite case)

BOTH MOVING:  $f' = \left( \frac{v \pm v_s}{v \mp v_s} \right) f$  - the signs depend on the direction of moving of source and detector!



$$\sin \phi = v/v_s - \text{half-angle of a mach-cone!}$$

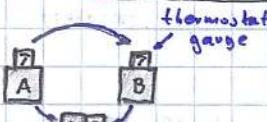
# THERMODYNAMICS

TEMPERATURE ... [K] - Kelvin

OK - ABSOLUTE ZERO

empty space... ~3K  
highest labor. ~ $10^8$ K  
sun (centre) ~ $10^7$ K  
sun (surface) ~ $10^4$ K  
boiling H<sub>2</sub>O ~273K  
lowest lab. temp. ~ $10^{-10}$ K

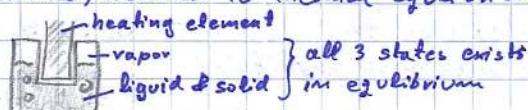
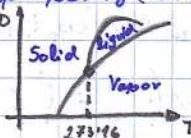
ZERO-TH LAW OF THERMODYNAMICS, ... Temperature  $\rightarrow$  Thermal equilibrium



if A and B are both in thermal equilibrium with a 3rd body C, then A and B are in thermal equilibrium with each other!

↳ this means that exists a property (TEMPERATURE) related to thermal equilibrium!

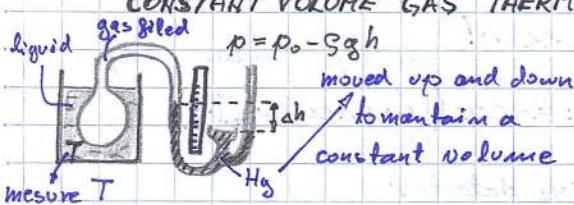
TRIPLE POINT OF WATER has 273'16 K!



} all 3 states exists  
} in equilibrium

THERMOMETER - standard thermometer... measuring properties that depend on temperature:

CONSTANT VOLUME GAS THERMOMETER



$$pV = mRT \Rightarrow T = c \cdot p \quad \left. \begin{array}{l} \text{constant} \\ T_3 = cP_3 \end{array} \right\} \Rightarrow T = T_3 \left( \frac{p}{p_3} \right)$$

3-pole point of H<sub>2</sub>O

calibrate for each

gas at  $T_3$  and then

measure  $T$  of boiling H<sub>2</sub>O

$T(K)$

373'4

373'2

373'16

373'0

20 60 100  $p(kPa)$

- resistivity of wire
- volume (liquid, gas, solid)
- length of rod
- pressure ...

N<sub>2</sub>

H<sub>2</sub>

He

20 60 100  $p(kPa)$

$\Rightarrow$  Temperature depend on pressure and gas type, but in the limit  $p \rightarrow 0$  all readings agree (independent of gas type used)  $T = (273'16 K) \cdot (\lim_{p \rightarrow 0} \frac{p}{p_3})$ !

Thermal Expansion of Solids



Brass

$T = T_0$

Steel

$T > T_0$

$T < T_0$

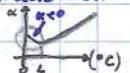
Brass changes length more rapidly than steel with changing temperature!

COEFFICIENT OF LINEAR EXPANSION

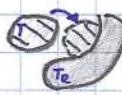
$$\frac{\Delta L}{L} = \alpha \Delta T \quad (\text{K or } ^\circ\text{C}) \quad \alpha (10^{-6}/\text{K}) \dots \text{ice } -51, \text{ lead } -29, \text{ diamond } 1'2 \dots$$

VOLUME:  $\frac{\Delta V}{V} = \beta \Delta T$  but we know  $V \sim L^3 \Rightarrow \frac{\Delta V}{V} = \frac{3L \Delta L}{L^3} = 3 \frac{\Delta L}{L} \Rightarrow \beta = 3\alpha$

Linear relation  $\frac{\Delta L}{L} = \alpha \Delta T$  is generally true... but not at phase changes! H<sub>2</sub>O above 4°C exhibits linear expansion but between 0°C and 4°C is the opposite!



TEMPERATURE  $\rightarrow$  HEAT



$T \neq T_R$  - when two bodies come in contact they are not in thermal equilibrium  $\Rightarrow$  eventually equilibrium is obtained

Equilibrium is obtained by a transfer of energy! (molecular oscillations ...)

In macroscopic view we say that HEAT is transferred between bodies!  $T > T_R \Rightarrow Q > 0$  (from system to surroundings)

$Q_{\text{heat}} [J]$  old unit was CALORIE ... 1 cal = 4186 J!

We can also change temperature of a body by doing work!  $\square \rightarrow \square$  - change of V with piston...

HEAT CAPACITY - HOW MUCH ENERGY IS NEEDED TO CHANGE  $T \rightarrow T + \Delta T$

SPECIFIC HEAT

$$Q = c m (T_f - T_i) \dots c \dots [J/kg \cdot K]$$

MOLAR SPECIFIC HEAT

$$Q = C_{\text{molar}} (T_f - T_i) \quad C_{\text{molar}} \dots [J/K]$$

	$C [J/kg \cdot K]$	$C_m [J/K]$
LEAD	128	26'5
COPPER	386	24'5
Al	900	24'4
H <sub>2</sub>	140	/
H <sub>2</sub> O	4200	/

HEAT OF TRANSFORMATION

SOLID  $\rightarrow$  LIQUID  $\rightarrow$  GAS

$L \dots [J/kg]$

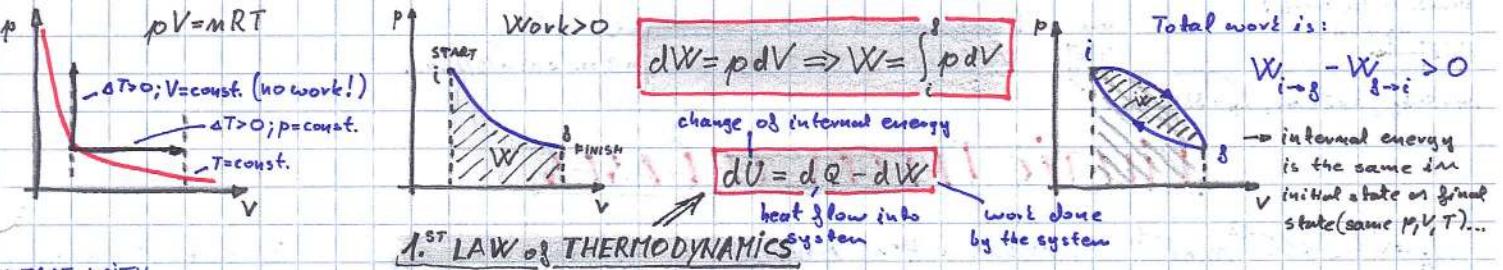
heat of fusion  $\rightarrow L_f$

$L_v \rightarrow$  heat of vaporisation

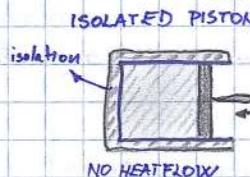
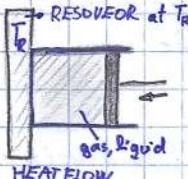
$$Q = L_f \cdot m$$

$$H_2O \dots L_f = 333 \text{ kJ/kg}, L_v = 2256 \text{ kJ/kg}$$

$T_{\text{boiling}} (K)$	$T_v (K)$	$melt T (K)$	$L_f (kJ/kg)$
20'3	455	H <sub>2</sub>	14
90'2	213	O <sub>2</sub>	55
630	296	Hg	234
373	2256	H <sub>2</sub> O	273
			333



IN CONTACT WITH



$$pV = mRT$$

System is described by  $p, V, T$

$i \rightarrow A$  const.  $p$  (colling)  $W=0$

$A \rightarrow g$  const.  $p$ ;  $dQ > 0$ ;  $\Delta W > 0$

HEAT FLOW INTO SYSTEM

$$W = \int_A^g pdV = \int_A^g Fdx$$

$$F = pA; dV = Adx$$

1<sup>ST</sup> LAW of THERMODYNAMICS ... (conservation of energy)  $dE_{int} = dQ - dW$  WORK DONE BY THE SYSTEM

1.) ADIABATIC PROCESS ( $dQ=0$ ):  $dE_{int} = -dW$

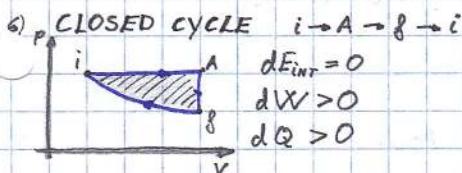
2.)  $\Delta V=0 \Rightarrow \Delta W=0 : dE_{int}=dQ$

3.)  $\Delta p=0 : \Delta W \neq 0$  and  $\Delta Q \neq 0$

4.)  $\Delta T=0 \Leftrightarrow \Delta E_{int}=0 : dQ=dW$ .

5.) FREE EXPANSION ( $dQ=0; \Delta W=0; \Delta E_{int}=0$ )

- isolated containers, all gas in one side, then open valve  
→ is not a line in p-V diagram (because of no equilibrium)



example: 1kg of H<sub>2</sub>O

- convert to steam at 100°C (1atm)

- initial volume =  $1 \times 10^{-3} \text{ m}^3$

- final volume =  $1.67 \times 10^{-3} \text{ m}^3$

Heat added =  $Q = L_v \cdot m = 2260 \text{ kJ}$ ;  $\Delta T=0$ ... only vapourisation

Change of  $E_{int}$   $\Rightarrow dE_{int} = dQ - dW = 2030 \text{ kJ}$  most of the added heat goes into breaking bonds of H<sub>2</sub>O mol..

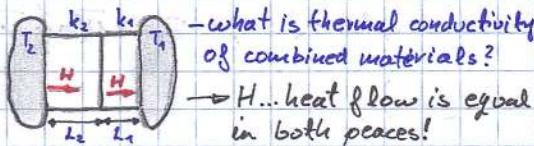
## HEAT TRANSFER MECHANISM

① CONDUCTION (ex. metal rod) → vibrations of atoms ... more vibrations transfer to less vibrating atoms

② RADIATION (also in vacuum)

$k$  - thermal conductivity

③ CONVECTION (liquids or gases)



- what is thermal conductivity of combined materials?

→  $H$ ... heat flow is equal in both places!

$$H = \frac{Q}{t} = \frac{kA(T_H - T_C)}{L}$$



THERMAL RESISTANCE

$$R = \frac{L}{k}$$

$$H = \frac{A(T_H - T_C)}{R}$$

$$\Rightarrow H = \frac{k_2 A}{L_2} (T_2 - T_B) = \frac{k_2 A}{L_2} (T_B - T_1) \Rightarrow T_B \left( \frac{k_1}{L_1} + \frac{k_2}{L_2} \right) = \frac{k_2}{L_2} T_2 + \frac{k_1}{L_1} T_1 \dots R = \frac{L}{k} \dots$$

$$H = \frac{A(T_H - T_C)}{\sum R_i}$$

$$R = R_1 + R_2$$

dobit isto formula kde za upomesti pri elektrici... tadi za "zpravidlo" verava o babilicitu  $\frac{1}{R_1} = \frac{1}{R_1} + \frac{1}{R_2} \dots$   
CONVECTION:

→ warmer liquids or gases at same pressure have greater volume ...  $\rho \propto V = m R \alpha T \dots \Rightarrow \rho \propto \frac{1}{V}$  is lower  
→ warmer parts of the gas rises! examples: - heating water, - inside stars, - weather...



## RADIATION

→ energy is transferred by the waves (EM) ... transport energy in a vacuum

absorbed EM-wave increases vibrational energy of atoms

Power transmitted:  $P = \sigma \epsilon A T^4$  emissivity:  $\epsilon$ ;  $0 \leq \epsilon \leq 1$

Stefan-Boltzmann constant:  $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$

ABSORPTION of RADIATION from exterior:

$$P = P_R - P_A = \sigma \epsilon A (T_1^4 - T_2^4)$$

IDEAL CASE ( $\epsilon=1$ ) ⇒ BLACK-BODY



example:

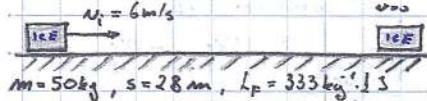
- air temperature  $\approx 6^\circ\text{C}$   
- clear night, no moon ... atmosphere absorbs energy like black-body ( $\epsilon=1$ ) with  $T=-23^\circ\text{C}$   
- once air inside container gets colder than outside air ( $6^\circ\text{C}$ ), because it gets denser, the out-side air can not get in... ⇒ temp. of air above H<sub>2</sub>O is less than  $6^\circ\text{C}$

4.5g H<sub>2</sub>O;  $A = 3 \text{ cm}^2$ ;  $d = 5 \text{ mm}$

... to cool  $6^\circ\text{C} \rightarrow 0^\circ\text{C} \dots Q_1 = c m \Delta T = -113 \text{ J} \Rightarrow T_f = T_0 - \Delta T = -6^\circ\text{C} \dots$  negative  $\Delta T$

... water-ice ...  $Q_2 = m L_f = -1493 \text{ J} \Rightarrow$  have to radiate 1612 J to produce all ice

How much ice melts?



kinetic en.  $\rightarrow$  work of friction  $\Rightarrow$  melting ice

$$Q = \frac{1}{2} m v_0^2 = \dots = 126 \text{ J} \Rightarrow \text{cm} L_F = Q \Rightarrow \text{cm} = Q/L_F = \dots$$

## KINETIC THEORY OF GASES

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}; \text{ mole} = \# \text{ of particles in 12 grams of C}_2\text{H}_6$$

By "particle" we mean molecules or atoms!

$$N \text{ particles} \rightarrow m = \frac{N}{N_A} = \# \text{ of moles; molar mass } M = m N_A; m - \text{mass of a particle}$$

IDEAL GASES :  $pV = nRT$   $R = 8.31 \text{ J/mol.K}$

Real gasses do not follow precisely  $pV = nRT$  (due to inter-molecular forces) ... accurate at low densities!

### WORK DONE BY A GAS

$P$

$T = \text{const.}$

$$W_{i \rightarrow f} = \int_{V_i}^{V_f} p dV = nRT \int_{V_i}^{V_f} \frac{1}{V} dV = nRT \ln\left(\frac{V_f}{V_i}\right) \rightarrow \text{ISOTHERMAL EXPANSION}$$

$$W_{i \rightarrow f} = nRT \cdot \ln\left(\frac{V_f}{V_i}\right)$$

$$\text{CONSTANT VOLUME PROCESS} \Rightarrow dV = 0 \Rightarrow W = 0$$

$$\text{CONSTANT PRESSURE PROCESS} \Rightarrow W = \int_{V_i}^{V_f} p dV = p(V_f - V_i) = p \Delta V$$

### PRESSURE, TEMPERATURE ~ SPEED (molecular)

Assume we have  $n$ -moles of some gas, in a volume  $V$  at some  $T$ !

$V$   $\rightarrow$  molecule of gas bounces from a wall  $\Rightarrow$  transfer of momentum

$\Delta G_x = 2mv_x$

(to wall)  $t = 2L/v_x$

Now, force in  $x$ -direction is  $F_x = \frac{\Delta G_x}{\Delta t} = \frac{2mv_x^2}{2L} = \frac{mv_x^2}{L}! \Rightarrow$  pressure on the wall  $= p = F/A$

$$p = F_x/L^2 = mv_x^2/V \dots \text{but all particles produce pressure } p = p_1 + p_2 + \dots = \sum p_i = \frac{m}{V} \cdot \sum v_x^2$$

we do not know all of the velocity for all particles  $\Rightarrow$  we introduce an average:

$$\Rightarrow \overline{v_x^2} = \frac{1}{N} \sum_{i=1}^N v_{ix}^2 \dots \text{now: } p = \frac{m N_A}{V} \cdot \overline{v_x^2} \dots N \text{ is in all directions} \Rightarrow \overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2} \Rightarrow \overline{v^2} = \frac{1}{3} \overline{v_x^2}$$

now we get:  $p = \frac{m N_A}{3V} \overline{v^2} = \frac{m N_A \cdot N_{rms}^2}{3V}; N_{rms} = \sqrt{\overline{v^2}}$  (root, mean, square)

$\Rightarrow$  we can measure  $N_{rms}$ :  $N_{rms} = \sqrt{\frac{3RT}{M}}$  P.S.:  $M$  - mass of one molecule

$$M = m \cdot N_A$$

**TRANSLATIONAL KINETIC ENERGY** :  $K = \frac{1}{2} m \overline{v^2} = \frac{1}{2} m v_{rms}^2 = \frac{3}{2} \frac{RTm}{M} = \frac{3}{2} \frac{RT}{N_A} = \frac{3}{2} kT = R$

$k$  ... Boltzmann constant ...  $k = R/N_A$

$$k = 1.38 \times 10^{-23} \text{ J/K} \dots \text{we can also write } pV = nRT \text{ in this form: } pV = mRT = N k T$$

# of particles  
 $N = n \cdot N_A$   
# of moles

### MEAN FREE PATH

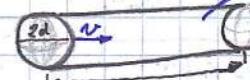
$b_i$  - distance traveled by  $i$ -th particle between two collisions

$\lambda$  - mean free path = average ( $b_i$ ) ...  $\lambda \sim \frac{1}{N} \cdot \langle b_i \rangle$  (cross-sectional area)

$$\lambda = \frac{1}{\sqrt{2} \pi d^2 N/V}$$

let's simplify by ... all gas particles are point-like, but colliding particle diameter =  $2d$

$b$  - impact parameter  
 $b < d = 2r$   
collision for  $b < d = 2r$



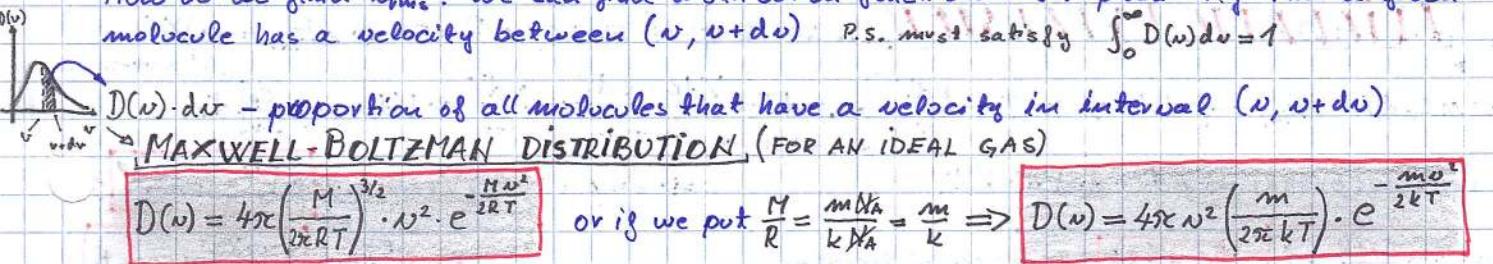
volume covered during  $\Delta t := N \cdot \Delta t \cdot \pi r^2$   
# of particles in volume =  $\frac{N}{V} \cdot N \cdot \Delta t \cdot \pi r^2$

$$\Rightarrow \# \text{ of collisions / unit time} = \frac{N}{V} N \cdot \Delta t \cdot \pi r^2$$

$$\Rightarrow \text{total distance traveled / # of collisions} = \lambda = \frac{N \cdot \Delta t}{\# \text{ of collisions} \cdot N/V} = \dots$$

$N_{rms}^2$  of particles in moving frame =  $\sqrt{2} N_{rms}^2$  in rest frame  $\Rightarrow \sqrt{2}$  in denominator of  $\lambda = 1/\sqrt{2} \pi d^2 N/V$   
average velocity in moving frame is bigger for a factor of  $\sqrt{2}$ !

How do we find  $N_{rms}$ ? We can find distribution function  $D(v)$  - probability that a given molecule has a velocity between  $(v, v+dv)$ . P.S. must satisfy  $\int_0^\infty D(v)dv = 1$



$\Rightarrow$  for small  $v$ :  $D(v) \sim v^2$ ; but for larger  $v$  the exponential term "wins"!

$$\text{Now we can calculate } N_{rms}: N_{rms}^2 = \int_0^\infty D(v) \cdot N^2 dv \sim \int_0^\infty v^4 e^{-\frac{mv^2}{2kT}} dv$$

Look at  $I(x) = \int_0^\infty e^{-xu^2} du \Rightarrow \frac{dI}{dx} = \int_0^\infty u^2 e^{-xu^2} du \dots$  we have to do integral  $\int_0^\infty e^{-xu^2} du$ ; put  $x = mv^2/2kT$

$$\Rightarrow \text{we get } \frac{1}{\sqrt{\pi}} \int_0^\infty e^{-xu^2} du \dots \text{we get } N_{rms}^2 = \frac{2kT}{\pi m}$$

## MOLAR SPECIFIC HEAT

heat is going into internal energy

$$\text{CONSTANT VOLUME: } Q = m C_v \Delta T$$

Let  $\bar{K}$  be average kin. en.  $\bar{K} = \frac{1}{2} m \bar{v}^2 = \frac{1}{2} m v_{rms}^2$   
and we know  $v_{rms}^2 = 3kT/m$

$$Q \rightarrow \begin{cases} V=\text{const.} \\ \Delta V=0 \Rightarrow \Delta W=0 \\ Q>0 \Rightarrow \Delta T>0 \Rightarrow \Delta \theta>0 \end{cases}$$

... microscopic level  $E_{int} = \text{kinetic energy}$

$E_{int}$  is only a function of  $T$ !

for mono-atomic gas

$$\bar{K} = \frac{3}{2} kT \Rightarrow E_{int} = m \cdot N_A \cdot \frac{3}{2} kT = \frac{3}{2} m R T$$

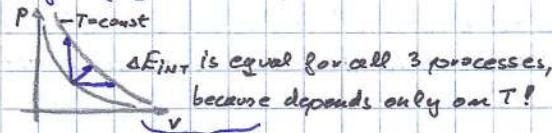
$$\Delta E_{int} = Q - W = m C_v \Delta T \Rightarrow C_v = \frac{\Delta E_{int}}{m \Delta T} = \frac{3}{2} R = 12.5 \text{ J/K} \cdot \text{mol}$$

This is just for mono-atomic gases ... only kinetic energy, no rotations!

P.S. For example monoatomic gas ( $\text{He} \sim 12.5$ ,  $\text{Ar} \sim 12.6$ ); diatomic gas ( $\text{N}_2 \sim 20.7$ ,  $\text{O}_2 \sim 20.8 \approx \frac{5}{2} R$ ) ... poly-atomic gases ( $\text{NH}_3 \sim 29.0$ ,  $\text{CO}_2 \sim 29.7 \dots$ )

$$E_{int} = m C_v T \Rightarrow \Delta E_{int} = m C_v \Delta T$$

$\Rightarrow$  for any process



$$\text{CONSTANT PRESSURE: } Q = m C_p T$$

$$pV = nRT; p = \text{const.}$$

$$\dots \Delta V = m R \Delta T = W!$$

$$\Delta E_{int} = Q - W = m C_p \Delta T - m R \Delta T = m C_v \Delta T \Rightarrow C_p = C_v + R$$

P.S. Each degree of freedom gives us a  $\frac{1}{2} kT \Rightarrow$  for mono-atomic only  $x-y-z \Rightarrow \frac{3}{2} kT$   
 $\Rightarrow$  di-atomic molecule (like  $\text{O}_2$ ) 3 from  $x-y-z$  + 2 from rotations  $\Rightarrow \frac{3}{2} kT + \frac{3}{2} kT = \frac{5}{2} kT$   
... poly-atomic molecules have no symmetry  $\Rightarrow$  3 rotations  $\Rightarrow \frac{6}{2} kT$

P.S.: Oscillation are another degree of freedom!

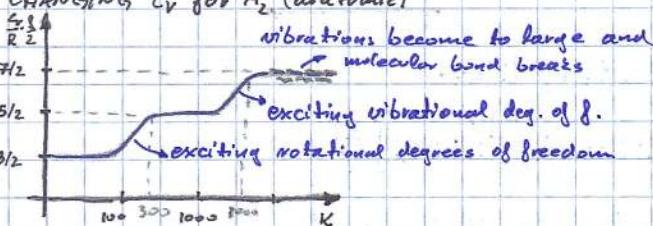
$$\Rightarrow C_v = \frac{1}{2} g R$$

number of degrees of freedom!

$$\Rightarrow E_{int} = \frac{1}{2} g \cdot m R T$$

... for one particle we have ...  
 $E = \frac{1}{2} g k T$

CHANGING  $C_v$  for  $\text{H}_2$  (diatomic)



$\Rightarrow$  There is some minimum amount required to excite rotational and vibrational degree of freedom  $\Rightarrow$  This is characteristic of quantum mechanics!

# ADIABATIC EXPANSION

... NO FLOW OF ENERGY:  $Q=0 \Rightarrow \Delta E_{int} = -\Delta W$

$$\Rightarrow m C_v dT = dE_{int} = -pdV \Rightarrow m dT = -\frac{pdV}{C_v}$$

$\left. \begin{aligned} M dT &= \frac{Vdp - pdV}{C_p} = -\frac{pdV}{C_p} \Rightarrow Vdp = -p \left[ \frac{C_p}{C_v} \right] dV \\ C_p &= C_v + R \end{aligned} \right\} \Rightarrow \ln p = -\gamma \log V + \text{const.} \Rightarrow pV^\gamma = \text{const.}$

$\gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v} > 1$ ;  $C_v = \frac{1}{2} g R$ ;  $g$ -degrees of freedom

npn. 1mol of  $O_2$  ... adiabatic expansion  $12L \rightarrow 13L$  --- 800  $O_2$  we have 3 translational + 2 rotational degrees of freedom  $\Rightarrow C_v = \frac{5}{2} R \Rightarrow \gamma = \frac{7}{5} \approx 1.4$

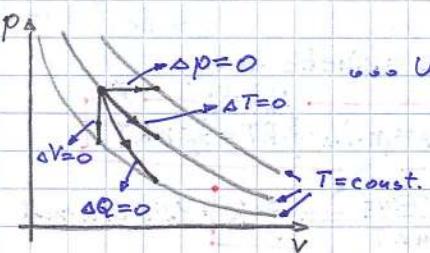
## FREE EXPANSION



can not be shown in p-V diagram ... because no equilibrium ...

$$Q=0 \text{ and } W=0 \Rightarrow \Delta E_{int}=0 \Rightarrow \Delta T=0$$

$$pV=nRT=\text{const.} \Rightarrow p_i V_i = p_f V_f$$



## ENTROPY ... 2<sup>ND</sup> LAW OF THERMO-DYNAMICS

PROCESSES HAVE DIRECTION ... heat flows from higher  $T$  to lower  $T$ ! ... IRREVERSIBLE PROCESS

... all processes have an implicit sense of direction ... we introduce entropy  $S$ !

IF IRREVERSIBLE PROCESS OCCURS IN A CLOSED SYSTEM, ENTROPY ALWAYS INCREASES!  $\Rightarrow \Delta S > 0$ !

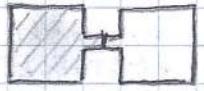
... if we have a reversible process, entropy can be not changed:  $\Delta S = 0$

ARROW OF TIME

2 WAYS TO DEFINE ENTROPY: - in terms of temperature change due to flow of heat

- counting possible microscopic arrangements of atoms and molecules

CLASSIC EXAMPLE OF IRREVERSIBLE PROCESS



$p_i, V_i, T_i$



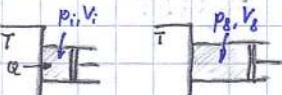
$p_f, V_f, T_f$

$\Delta S$  should be only a property of initial and final state!

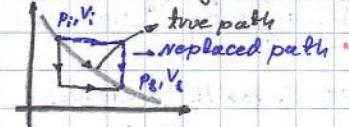
$$\Delta S = S_f - S_i = \int_i^f \frac{dQ}{T}$$

$$S_i = S_0 + S_{i0}; S_f = S_0 + S_{f0} \dots$$

For free expansion we replace the non-equilibrium process by a series of equilibrium process ... (general rule)



$$\Delta S = \frac{Q}{T} \text{ for isothermal process} \dots Q > 0 \Rightarrow \Delta S > 0$$



$$dQ = dE_{int} + dW; dE_{int} = m C_v dT; dW = pdV \Rightarrow dQ = m C_v dT + pdV \Rightarrow$$

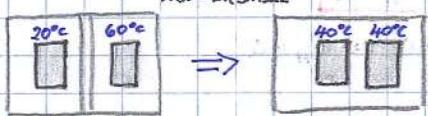
$$\Rightarrow \Delta S = \int_i^f \frac{dQ}{T} = \int_i^f \frac{pdV}{T} + \int_i^f \frac{m C_v dT}{T} \text{ if we use } \frac{p}{T} = \frac{nR}{V} \Rightarrow \Delta S = mR \int_i^f \frac{dV}{V} + nC_v \int_i^f \frac{dT}{T}$$

$$\Rightarrow \Delta S = mR \ln \left( \frac{V_f}{V_i} \right) + nC_v \cdot \ln \left( \frac{T_f}{T_i} \right) \Rightarrow \Delta S \text{ DEPENDS ONLY ON INITIAL AND FINAL STATE!}$$

2<sup>ND</sup> LAW OF THERMO-DYNAMICS: for a closed system  $\Delta S > 0$  (reversible process  $\Delta S = 0$ )

$$dS = \frac{dQ}{T} = m \left( R \frac{dV}{V} + C_v \frac{dT}{T} \right) = m \cdot d \left( R \ln V + C_v \ln T \right)$$

**IRREVERSABLE PROCESS:** calculate  $\Delta S$  by looking at a reversible process with same initial and final states.



→ Replace the process by a reversible process, we can change the temperature of the block slowly ...

for both blocks

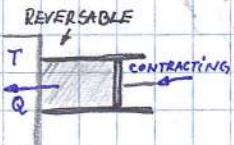
$$\Rightarrow \Delta S_{TOT} = MC \left( \log \frac{313}{333} + \log \frac{313}{333} \right) = 2.4 \text{ J/K}$$

$$dQ = mc dT; \Delta S_L = \int_{20}^{40} \frac{dQ}{T} = mc \int_{20}^{40} \frac{dT}{T} = mc \log \left( \frac{T_f}{T_i} \right) \xrightarrow[333K]{313K} \Delta S_L = 2.4 \text{ J/K}$$

for left block



$$\Delta S = \int \frac{dQ}{T} > 0$$



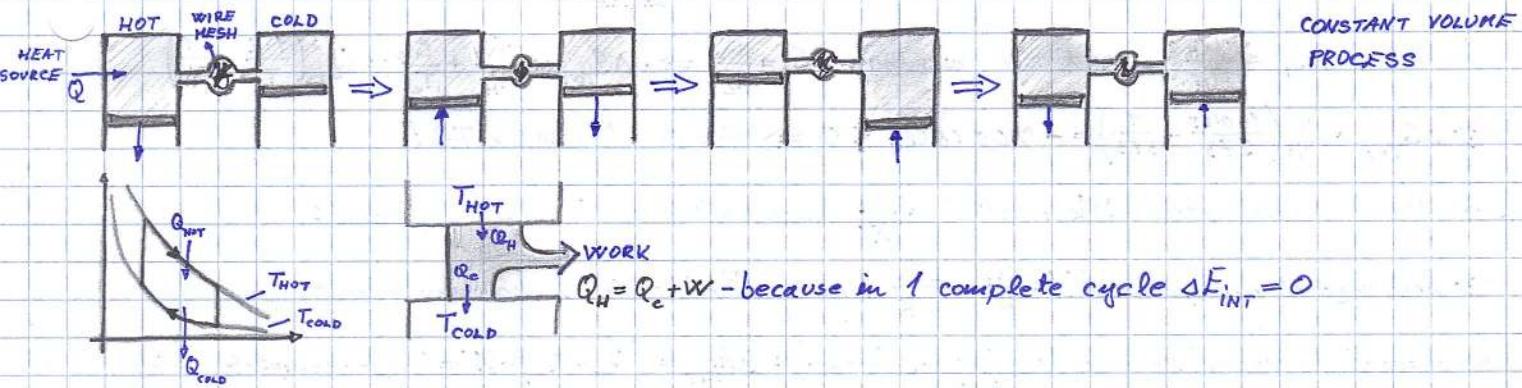
$$\Delta Q < 0 \Rightarrow \Delta S = \int \frac{dQ}{T} = -\frac{|Q|}{T}$$

$$\Delta S_{reservoir} = \frac{|Q|}{T}$$

⇒ for a closed system process, the entropy increases; for an irreversible process and remains constant for a reversible process!

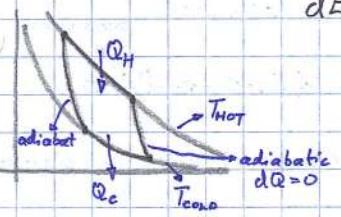
$$\Rightarrow \Delta S \geq 0$$

## ENGINES (STIRLING ENGINE)



$$\text{EFFICIENCY: } \epsilon = \frac{W}{Q_H} = \frac{Q_H - Q_c}{Q_H} = 1 - \frac{Q_c}{Q_H}$$

## CARNOT CYCLE



$$dE_{INT} = dQ - dW = TdS - dW \Rightarrow dW = TdS - dE_{INT}$$

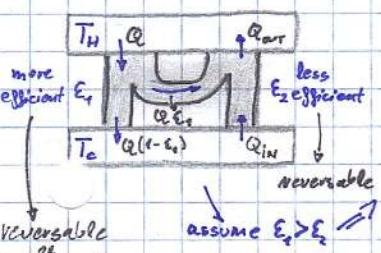
$$\Rightarrow \Delta V = \oint T dS \text{ because } \oint dE_{INT} = 0 \dots \text{but for a closed cycle}$$

is  $\Delta S = 0$ , because  $S$  is a function of state ...  $\Rightarrow \frac{Q_H}{T_H} = \frac{Q_c}{T_c}$  FOR A CARNOT CYCLE

$$\Rightarrow \epsilon = 1 - \frac{Q_c}{Q_H} = 1 - \frac{T_c}{T_H} = \frac{\Delta T}{T_H}$$

**CARNOT THEOREM:** All reversible engines have the same efficiency!  
(irreversible engines (real engines) are less efficient than a reversible engine)

## ENG: REVERSED ENGINE

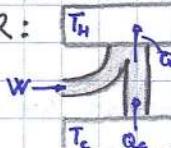


$$\epsilon_1 = 1 - \frac{Q_{c1}}{Q} \Rightarrow Q_{c1} = Q(1 - \epsilon_1); W = Q - Q_{c1} = Q \cdot \epsilon_1$$

$$\epsilon_2 = \frac{W}{Q_{out}} \Rightarrow Q_{out} = \frac{W}{\epsilon_2} = Q \frac{\epsilon_1}{\epsilon_2}$$

$$Q_{in} = Q \frac{\epsilon_1}{\epsilon_2} - Q \epsilon_1 = Q \epsilon_1 \left( \frac{1}{\epsilon_2} - 1 \right)$$

FLOW OF HEAT

**REFRIGERATOR:**  efficiency of refrigerator:  $k = \frac{|Q_c|}{W} = \frac{T_c}{T_H - T_c}$   $\Rightarrow$  more efficient for  $T_H - T_c$  small

**PERFECT REFRIGERATOR:**  $k \rightarrow \infty$ ;  $W \rightarrow 0 \Rightarrow Q_H = Q_c$

for the system (reversible) :  $\Delta S(\text{system}) = 0$

for the reservoirs :  $\Delta S = -\frac{Q}{T_c} + \frac{Q}{T_H} < 0$  because  $T_c < T_H$  }  $\Delta S < 0$ ! NO!

$\Rightarrow$  perfect refrigerator does not exist (violates 2<sup>nd</sup> law of thermo-dynamics)

$\Rightarrow$  ANY TWO REVERSIBLES HEAT ENGINES HAVE EQUAL EFFICIENCY =  $1 - \frac{T_c}{T_H}$   
and an irreversible heat engine cannot be more efficient than a reversible one!

$\Rightarrow$  IT IS IMPOSSIBLE TO BUILD A PERFECT ENGINE WITH  $\epsilon = 1$  (should be  $T_c = 0K$ )!

npr: inventor claims to build an engine of  $\epsilon = 75\%$  between boiling and freezing point of H<sub>2</sub>O!

$$\Rightarrow \epsilon_{\max} = 1 - \frac{T_c}{T_H} = 1 - \frac{273}{373} \sim 27\% \Rightarrow \text{NOT POSSIBLE}$$

npr: assume that the engine is approximately ideal:  $\Rightarrow \Delta S_{\text{ENGINE}} = 0$  ... reservoir on the

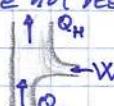
$$\text{hot side: } \Delta S_H = \frac{Q_H}{T_H} \dots \text{and cold reservoir: } \Delta S_c = \frac{Q_c}{T_c} \Rightarrow \Delta S_{\text{TOT}} = \frac{-Q_H}{T_H} + \frac{Q_c}{T_c} = Q_H \left( \frac{1-\epsilon}{T_c} - \frac{1}{T_H} \right)$$

$$\Rightarrow \Delta S_{\text{TOT}} = Q_H \left( \frac{1-0.75}{273} - \frac{1}{373} \right) = -0.0018 Q_H < 0 \Rightarrow \text{violates 2<sup>nd</sup> law of T-D!}$$

npr: An ideal refrigerator with  $k = 4.7 = \frac{Q_c}{W}$  and extracts  $Q_c = 250 \text{ J/cycle}$ :

$$\Rightarrow \text{work required per cycle: } W = \frac{Q_c}{k} = \frac{250}{4.7} \text{ J/cycle} = 53 \text{ J/cycle} \dots \text{how much heat/cycle is}$$

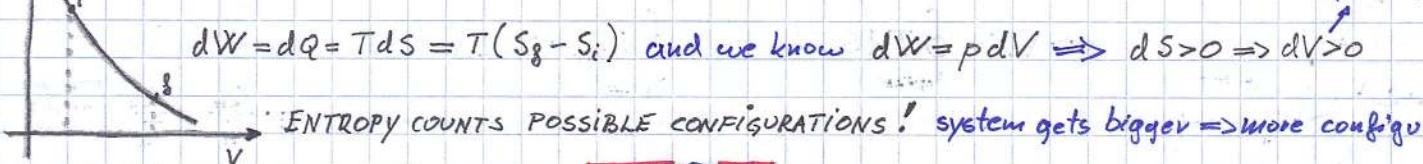
discharged to room (room is like hot reservoir):  $\Delta E_{\text{INT}} = (Q_H - Q_c) - W = 0$  (for complete cycle)

$$\Rightarrow Q_H = Q_c + W = 303 \text{ J/cycle}$$


## MICROSCOPIC DESCRIPTION OF ENTROPY ... STATISTICAL MECHANICS ...

consider iso-thermal expansion

$P$   $T = \text{const.} \Rightarrow pV = \text{const.} \Rightarrow$  work done  $W \neq 0 \Rightarrow Q \neq 0$  and  $W = Q$  (because  $\Delta E_{\text{INT}} = 0$ )



ENTROPY COUNTS POSSIBLE CONFIGURATIONS! system gets bigger  $\Rightarrow$  more configurations

$$N = \# \text{ of possible configurations} : S = k \cdot \log N \xrightarrow{\text{Boltzmann's constant}}$$

npr: 4 molecules in a box (identical molecules); look for possible states of left-right side of a box.



LLLL

LLLR / LLRL / LRLL / RLLL

LLRR / LRLR / LRRR / RLRR / RLRL / RRLL

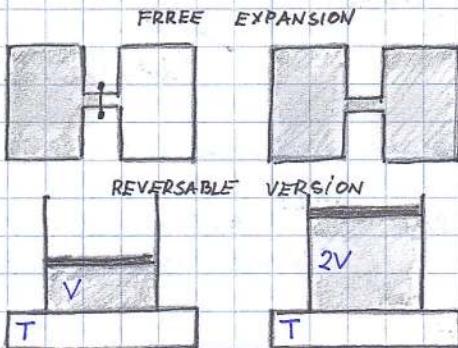
now it repeats in reverse order...

N	P
1	1/16
4	4/16
6	6/16
4	4/16
1	1/16

in general for N molecules:

$$\frac{(n_L + n_R)!}{n_L! n_R!} = \frac{m_o!}{(m_o - k)! k!} \Rightarrow \sum_{k=0}^{n_o} \binom{n_o}{k} = (1+1)^{n_o} = 2^{n_o}$$

Boltzmann's entropy  $S = k \cdot \log W$ ;  $W$  - # of possible configurations is for equilibrium system!



To calculate entropy change we replace this with reversible process (container on fixed T reservoir... slowly expands...) isothermal expansion:

$$Q = mRT \cdot \ln\left(\frac{V_f}{V_i}\right) = mRT \cdot \ln(2)$$

$$\Rightarrow \Delta S = \int \frac{dQ}{T} = \frac{1}{T} \cdot \Delta Q = mR \cdot \ln(2) \quad (\text{MACROSCOPIC VIEW})$$

Now for microscopic view...  $W_i = 1$  (all molecules on one side)

$$W_f = \frac{N!}{(N_1)!(N_2)!} \quad (\text{half of molecules on each side})$$

$\Rightarrow S_f = k \cdot \log(W_f) = 0$  and  $S_g = k \cdot \log(W_g)$  ... we should use STIRLING'S approximation for:

factorial:  $\log(N!) = N \cdot \log N - N$  for  $N \rightarrow \infty$  and  $N! \approx e^{-N} \cdot N^N$ , STIRLING'S FORMULA

$$\Rightarrow S_g = k \cdot (\log N! - 2 \cdot \log(N_2!)) \approx k(N \cdot \log N - N - 2 \cdot \frac{N}{2} \log \frac{N}{2} + 2 \cdot \frac{N}{2}) = kN \cdot \log 2 = mR \cdot \log 2 \checkmark$$

...  $S$  is largest for configuration with greatest multiplicity (most probable configuration)!

GIBBS ENTROPY  $S_g = -\sum p_i \cdot \log p_i$  (sum over all configurations) - in the limit for big numbers of particles this goes to Boltzmann's entropy!

npr: Box with  $N$  molecules... configuration A:  $N/2$  in Left,  $N/2$  in Right  
configuration B:  $3N/5$  in Left;  $2N/5$  in Right

let's take  $N=50$ :

$$\Rightarrow W_A = 50! / (25!)^2 = 1.26 \times 10^{14} \quad \Rightarrow f_A = W_A / 2^N = 11\% \quad \xrightarrow{\text{probability to be in this state...}}$$

$$W_B = 50! / 20! 30! = 4.7 \times 10^{13} \quad \Rightarrow f_B = W_B / 2^N = 4\%$$

Total number of configurations is  $2^N = 2^{50} = 1.13 \times 10^{15}$

For big  $N$  the binomial distribution becomes like Gauss distribution ... and for bigger  $N$  the peak becomes narrower!

we now go back to Maxwell-Boltzmann distribution for gases ...

$$f(\vec{v}) = \frac{n(\vec{v})}{n} = \text{probability that a particle has a velocity } \in (\vec{v}, \vec{v} + d\vec{v})$$

now for  $|\vec{v}| = v$ ;  $\vec{v} = v \cdot \hat{v}$  euotshi vector

$$P(v) = \int_{\text{SPHERE}} d\Omega \cdot f(\vec{v}) = \underbrace{4\pi v^2}_{\text{area } 4\pi r^2} \cdot f(\vec{v}) - \text{because } \vec{v} \text{ is symmetric, equal in all directions!}$$

$$\text{we have written } P(v) = 4\pi v^2 \cdot \left( \frac{m}{2\pi kT} \right)^{1/2} \cdot e^{-mv^2/2kT}$$



consider  $M(E) = \# \text{ of particles with energy between } (E, E+dE)$

$$\begin{aligned} & \text{elastic collision} \\ & R(1+2 \rightarrow 3+4) \sim M(E_1) \cdot M(E_2) \cdot C; C-\text{const.} \rightarrow \text{kinematics} \\ & \text{TIME REVERSE} \\ & R(3+4 \rightarrow 1+2) \sim M(E_3) \cdot M(E_4) \cdot C'; C'-\text{constant} \end{aligned}$$

MICRO-REVERSIBILITY ...  $\Rightarrow C = C'$  and  $R(1+2 \rightarrow 3+4) = R(3+4 \rightarrow 1+2)$  comes from assumption of equilibrium

$$\Rightarrow M(E_1) \cdot M(E_2) = M(E_3) \cdot M(E_4) + \text{conservation of EN. (it's elastic collision)} (E_1+E_2 = E_3+E_4)$$

$$\Rightarrow \log(M(E)) = A - BE \Rightarrow M(E) = M_0 \cdot e^{-BE} \dots \text{and we find } B = 1/kT!$$

### DISCRETE ENERGIES (M-B distribution)

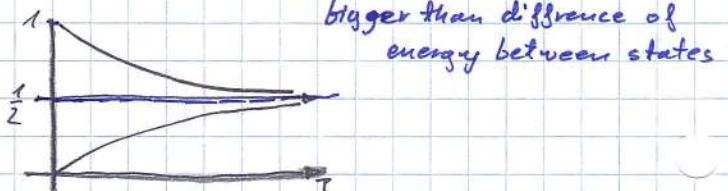
$$\begin{aligned} & \vec{B} \uparrow \\ & \begin{cases} \text{pointing along } \vec{B} \text{ field} \Rightarrow E_1 \\ \text{pointing reverse } \vec{B} \text{ field} \Rightarrow E_2 \end{cases} \quad E_2 > E_1 \quad \text{we have only two possible states } (E_1, E_2) \\ & \frac{\langle n_1 \rangle}{N} = \frac{e^{-E_1/kT}}{e^{-E_1/kT} + e^{-E_2/kT}} - \text{number... probability to be in } E_1\text{-state} \quad E_2 > E_1 \Rightarrow \langle n_1 \rangle > \langle n_2 \rangle \\ & \frac{\langle n_2 \rangle}{N} = \frac{e^{-E_2/kT}}{e^{-E_1/kT} + e^{-E_2/kT}} - \text{probability to be in } E_2\text{-state} \quad \text{lower energy is more probable} \end{aligned}$$

EXTREME:

$$T \rightarrow \infty \Rightarrow e^{-E_1/kT} \rightarrow 1 - \text{for big T we can not distinguish between / ... temp. energy is lot bigger than difference of energy between states}$$

$$T \rightarrow 0 \Rightarrow \frac{\langle n_1 \rangle}{N} = \frac{1}{1 + e^{(E_2 - E_1)/kT}} \rightarrow 1$$

$$\frac{\langle n_2 \rangle}{N} = \frac{e^{(E_2 - E_1)/kT}}{1 + e^{(E_2 - E_1)/kT}} \rightarrow 0$$



## FERMI-DIRAC DISTRIBUTION ... for fermions (spin = $\frac{1}{2}$ )

Quantum state is described by quantum numbers { $g_i$ }

Pauli exclusion principle ... can only have 1 particle of each quantum state { $g_i$ }

npv. for quantum state { $g_i$ } we have probability that particle is in this state =  $M_i$

ELASTIC COLLISION

$$\text{Rate } (1+2 \rightarrow 3+4) = C M_1 M_2 \underbrace{(1-u_1)(1-u_2)}_{\text{because of exclusion principle}} \quad \text{because of exclusion principle can not go in } u_3, u_4 \text{ state}$$

$$\text{Rate } (3+4 \rightarrow 1+2) = C M_3 M_4 (1-u_3)(1-u_4)$$

$\Rightarrow$  MICRO-REVERSEABILITY + EQUILIBRIUM

$$C = C' \quad R(1+2 \rightarrow 3+4) = R(3+4 \rightarrow 1+2)$$

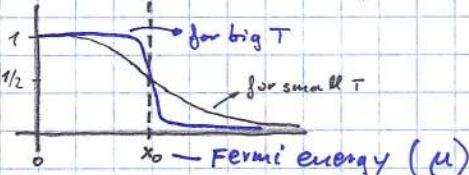
+ ENERGY CONSERVATION

$$E_1 + E_2 = E_3 + E_4$$

for electrons is 2 (2 states)

$$\text{Rewrite like: } \frac{M_1 M_2}{(1-u_1)(1-u_2)} = \frac{U_1 U_4}{(1-u_3)(1-u_4)} \quad \left. \begin{array}{l} \log \left( \frac{u_1}{u_3} \right) = \beta(\mu - E) \\ \end{array} \right\} \Rightarrow M = e^{\beta(\mu - E)(1-u)} \Rightarrow M = \frac{1}{1 + e^{\beta(E-\mu)}}$$

$$\text{for large } T : M \rightarrow e^{-E/kT} \Rightarrow \beta = \frac{1}{kT}$$



$\Rightarrow$  for electrons

$$M(E) = \frac{2}{1 + e^{(E-E_F)/kT}} \quad \text{Fermi energy}$$

## BOSE-EINSTEIN DISTRIBUTION (related to black-body radiation) ... gas of photon's

probability (photon  $\rightarrow$  state  $u$ )  $\sim (1+u)$

$$R(1+2 \rightarrow 3+4) = C M_1 M_2 (1+u_1)(1+u_2) \dots$$

going through the same as before we get

$$\Rightarrow \log \left( \frac{u}{1+u} \right) = \beta(\mu - E) \Rightarrow M = (1+u) e^{\beta(\mu - E)}$$

$$\Rightarrow M = \frac{1}{e^{\beta(E-\mu)} - 1} \dots \text{large } E \rightarrow e^{-\beta E} \Rightarrow \beta = \frac{1}{kT}$$

in this case must be  $\mu < 0$  ... for photon - no conservation of particle number and  $\mu \rightarrow 0$

# of photons with energy  $E$

$$\text{BLACK-BODY} \rightarrow N(E)dE \sim \frac{E^2}{e^{E/kT} - 1}$$

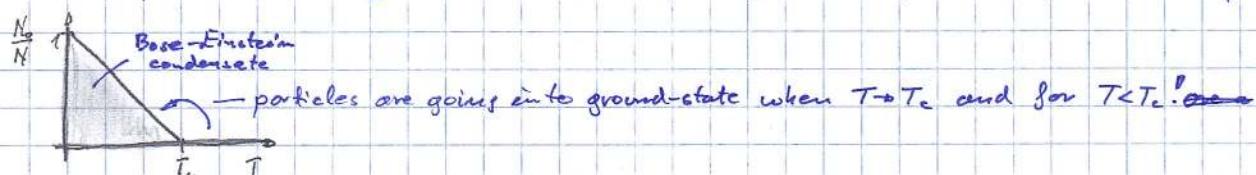
BUT IF THE # OF PARTICLES IS CONSERVED

$$\hookrightarrow N = \frac{1}{4\pi r^2} \left( \frac{2m}{\hbar} \right)^{3/2} \int_0^\infty \frac{\sqrt{E} dE}{e^{E/kT} - 1} \quad \text{because } \mu < 0 \Rightarrow (E\mu) \rightarrow 0$$

... small  $T \rightarrow 0 \Rightarrow$  also  $(E\mu) \rightarrow 0$

... there is some critical  $T$  for which  $\mu = 0 \dots T_c$  (ground state)

what happens when  $T < T_c \Rightarrow$  we can not keep  $n$  fixed we put all of the extra particles in  $E=0$  state



npv. 1 liter of liquid helium of  $T < 2.18K = T_c$ , mass density is  $\rho \approx 0.145 \text{ kg m}^{-3} \Rightarrow 10^{23} \text{ atoms}$   
we have  $\sim 10\%$  of atoms in ground state  
but if  $T \approx 3K > T_c$  we have just few atoms in each energy state