AN INTRODUCTION TO ERROR ANALYSIS 2ND EDITION by J.R. TAYLOR

ERROR ANALYSIS ... MERILNE TEHNIKE ...

P1 (28/3/14)

THE PHYSICAL QUANTITY is a property of a phenomenon, body or substance ...
... which has a magnitude which can be expressed with a number and a reference ... mass $m = \boxed{15}\,\boxed{g}$ - reference
number

- Uncertanty of a mesurment is a fundamental part of a mesurment (unavoidable)!

- Mesurment without an uncertenty is useles!

Smo merili višino vrat ...
1) 213'2 cm
2) 214 cm
3) 210'9 cm
4) 211'5 cm
5) 210'7 cm

... source of error ...
- error of definition (en je meril zunanjo višino en notranjo)
- operator error (napaka tistega ki je meril)
- instrument precision (kaj instrument omogoča)
- ambient condition (noise – šum ...)
- instrument calibration

Error always consists of two parts: 1) RANDOM ERROR
2) SYSTEMATIC ERROR (te je težko analizirat ... npr. s štoparico zamud zalasnitve vedno zamudna)

$l = l_1 + l_2$; $l_1 = \bar{l}_1 \pm \delta l_1$, $l_2 = \bar{l}_2 \pm \delta l_2$ } npr $l_1 = 1000 \pm 1\,mm$, $l_2 = 778 \pm 1\,mm$ $l = 1778 \pm 2\,mm$ ... $\Rightarrow \delta l = \delta l_1 + \delta l_2$

... napaka vsote je vsota napak ...

$\boxed{q = a + b \Rightarrow \delta q = \delta a + \delta b}$ → it is not excacly true ... it is over-estimated ... $\boxed{q = a \pm b \Rightarrow \delta q = \delta a + \delta b}$

KONEC P1 (28/3/14)

P2 (31/3/14)

... poglejmo sedaj kako je z napako produkta $q = a \cdot b$ ...!

npr.

$M = \rho \cdot V$
$\rho = 6'2 \pm 0'2\,g/cm^3$
$V = 41 \pm 1\,cm^3$

$M_{MAX} = 6'4 \times 42\,g = 268'8\,g$
$M_{MIN} = 6'0 \times 40\,g = 240\,g$
} $M_{BEST} = 6'2 \times 41\,g = 254'2\,g$ $\Rightarrow M = (254'2 \pm 14'4)\,g$

This is usualy writen as $\underbrace{255}_{\text{ne bolj natančno kot napaka}} \pm \underbrace{15}_{\text{ena ali dve štev. mesti}}\,g$!

PRODUKT:

$q = a \cdot b \Rightarrow q_{max} = (a+\delta a)(b+\delta b) = ab + a\delta b + b\delta a + \underline{\delta a \delta b}$ ... we can neglect second order term!

$= ab + a\delta b + b\delta a$

$q_{min} = (a-\delta a)(b-\delta b) = ab - a\delta b - b\delta a + \delta a \delta b$

$= ab - a\delta b - b\delta a$ → we can see that $\boxed{q = \overbrace{a \cdot b}^{q_{BEST}} \pm \overbrace{(a\delta b + b\delta a)}^{\delta q}}$ $/ : a \cdot b$

$\Rightarrow \frac{\delta q}{q} = \frac{\cancel{a}\delta b}{b} + \frac{\cancel{b}\delta a}{a}$ $\boxed{\frac{\delta q}{q} = \frac{\delta a}{a} + \frac{\delta b}{b}}$

KVOCIENT:

npr. merimo hitrost vozička: $v = \frac{\Delta s}{\Delta t}$; $\Delta s = 1584 \pm 2\,mm$, $\Delta t = 1'5 \pm 0'1\,s$ $\Rightarrow v_{max} = 1133\,mm/s$; $v_{min} = 989\,mm/s$

... $v_{BEST} = 1056\,mm/s$ $\Rightarrow$ $v = v_{BEST} \pm \delta v = 1056 \pm 77 = \underline{1060 \pm 70}\,mm/s$

$$q = \frac{a}{b} \Rightarrow q_{max} = \frac{a+\delta a}{b-\delta b} \cdot \frac{b+\delta b}{b+\delta b} = \frac{ab + a\delta b + b\delta a + \cancel{\delta a \delta b}}{b^2 - \cancel{\delta b^2}} \simeq \frac{ab + a\delta b + b\delta a}{b^2} = \frac{a}{b} + \frac{\delta a}{b} + \frac{a\delta b}{b^2}$$

$$q_{min} = \frac{a-\delta a}{b+\delta b} \cdot \frac{b-\delta b}{b-\delta b} = \frac{ab - a\delta b - b\delta a + \cancel{\delta a \delta b}}{b^2 - \cancel{\delta b^2}} = \frac{a}{b} - \frac{\delta a}{b} - \frac{a\delta b}{b^2}$$

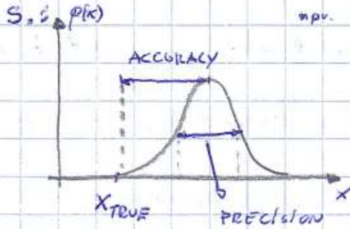
$$\Rightarrow \boxed{q = \frac{a}{b} \pm \left(\frac{\delta a}{b} + \frac{a\delta b}{b^2}\right) = q \pm \delta q} \; / \cdot \frac{b}{a} \Rightarrow \boxed{\frac{\delta q}{q} = \frac{\delta a}{a} + \frac{\delta b}{b}}$$

→ ZA RELATIVNO NAPAKO DOBIMO ISTO ENAČBO KOT PRI PRODUKTU!

PRODUKT S KONSTANTO: $g = k \cdot a \Rightarrow \boxed{\delta g = k \cdot \delta a}$ ($k = const.$) $\boxed{\frac{\delta g}{g} = \frac{\delta a}{a}}$ • LAHKO GLEDAMO TUDI KOT PRODUKT DVEH KOLIČIN, KONSTANTA IMA PAČ NAPAKO

To je primerno za merjenje ... npr. z merilom smo izmerili debelino 1000 strani v knjigi ... 1000 strani = 46 mm ± 1 mm ⇒ 1 list = 46 μm ± 1 μm!

POTENCE: $y = x^n \Rightarrow \boxed{\frac{\delta g}{g} = n \frac{\delta x}{x}}$

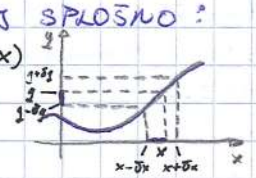
P.S.: 

npr. izmeriš 1'21 V
error 0'005 V

npr: PROSTI PAD ... merjenje g ...

$$g = \frac{2h}{t^2}; \quad h = h_0 \pm \delta h, \quad t = t_0 \pm \delta t \quad \Rightarrow \frac{\delta g}{g} = \frac{\delta h}{h} + 2\frac{\delta t}{t}$$

SEDAJ BOLJ SPLOŠNO:

$g = g(x)$ 

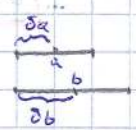
– v približku (ko je δx „majhen") lahko privzamemo, da je ~ linearna ⇒ uporabimo odvod ...

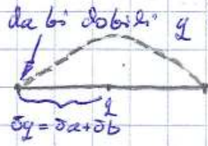
$$\boxed{\delta g = \left|\frac{dg}{dx}\right| \cdot \delta x} \quad \text{npr.: } g = x^n \Rightarrow \delta g = n \cdot x^{n-1} \cdot \delta x = n\, x^n \cdot \frac{\delta x}{\cancel{x}}$$

ČE IMAMO FUNKCIJO VEČ SPREMENLJIVK ... DELAŠ S PARCIALNIMI ODVODI ...

$$g = g(x,y) \qquad g_{max} = g(x+\underset{\delta x}{v}, y+\underset{\delta y}{v}) = g(x,y) + \frac{\partial g}{\partial x}\cdot \delta x + \frac{\partial g}{\partial y}\cdot \delta y$$

$$\Rightarrow \boxed{\delta g = \left|\frac{\partial g}{\partial x}\right| \delta x + \left|\frac{\partial g}{\partial y}\right| \cdot \delta y}$$ – SPLOŠNO PRAVILO!

Sedaj imamo $g = a + b$ 



da bi dobili g tam, bi morali biti a in b hkrati minimalna ... ... to je manj verjetno kot da bi dobili povprečje ...

(ps. predpostavili smo napako a, b neodvisne in naključne po intervalu)

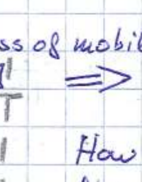
⇒ resnična napaka je nekoliko manjša! $\delta g = \sqrt{(\delta a)^2 + (\delta b)^2}$ če je $g = a + b$

SPLOŠNO PREIDE V: $\boxed{\delta g = \sqrt{\left|\frac{\partial g}{\partial x}\right|^2 \delta x^2 + \left|\frac{\partial g}{\partial y}\right|^2 \delta y^2}}$ in $\boxed{\left(\frac{\delta g}{g}\right)^2 = \left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2}$ toda pazi ⇒

⇒ PS: v $\boxed{g = x^n \Rightarrow \frac{\delta g}{g} = n\frac{\delta x}{x}}$ SE NE MODIFICIRA V $\cancel{\frac{\delta g}{g} = \sqrt{n}\frac{\delta x}{x}}$ ker meritve niso neodvisne!

KONEC P2 (31/3/14)

P3 (04/03/14)

npr. 

$$n = \frac{\sin(i)}{\sin(v)} \Rightarrow \frac{\delta n}{n} = \sqrt{\left(\frac{\delta \sin(i)}{\sin(i)}\right)^2 + \left(\frac{\delta \sin(v)}{\sin(v)}\right)^2}$$

$$\delta \sin(i) = \cos(i)\,\delta i$$

$$\Rightarrow \frac{\delta n}{n} = \sqrt{\left(\frac{\cos(i)}{\sin(i)}\delta i\right)^2 + \left(\frac{\cos(v)}{\sin(v)}\delta v\right)^2} = \sqrt{ctg^2(i)\,\delta i^2 + ctg^2(v)\,\delta v^2}$$

PS. ni treba vzet najbolj splošnega pravila ... lahko po krajšincah ...

NOW ... we can colect many mesorments to make random error smoler ...

DATASET: $x_1, x_2, \ldots, x_N$ ... than best guess is: $\boxed{\overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i}$

$$\boxed{VAR(x) = \frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})^2 = \sigma^2 \qquad \sigma = \sqrt{VAR(x)}}$$

npr. mass of mobile phone

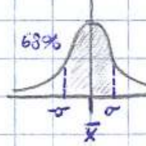
⇒ $\overline{m} = 77g$ ...

$m[g]$	$m_i - \overline{m}$	VAR(x) $(m_i - \overline{m})^2$
77	0	0
78	+1	1
77	0	0
75	−2	4
76	−1	1
77	0	0
78	+1	1

How do we find the error ... we could look at the biggest and smallest mesurment ⇒ error is smaller than

$\Rightarrow VAR(m) = 1'2\, g^2 \Rightarrow \sigma(m) = 1'1 g$

PS. FROM DATASET YOU CAN CALCULATE THE AVERIGE $\overline{x}$ AND STANDARD DEVIATION σ!



σ means that mesurment (true value) is with 68% in the interval $\overline{x}\pm\sigma$!
(if error is random)

IN BOOKS YOU FIND FORMULA: $\boxed{\sigma=\sqrt{\frac{1}{N-1}\sum(x_i-\overline{x})^2}}$ → for big N there is almost no difference

σ je karakteristika meritve ... se ne spremeni če namesto 100 narediš 1000 meritev ... seveda pa si bolj "siguren" če narediš več meritev

NOW. we have dataset $x_1, x_2, \ldots, x_n \Rightarrow \overline{x}=\frac{1}{N}(x_1+x_2+\ldots)$ and $\sigma=\sqrt{\ldots}$ (sigma je nedoločenost posamezne meritve) ... koliko je nedoločenost končnega rezultata $\sigma_{\overline{x}}$? (oziroma sedaj s $\delta\overline{x}$)

$$\delta\overline{x}=\sqrt{\left(\frac{\partial\overline{x}}{\partial x_1}\delta x_1\right)^2+\ldots}=\sqrt{\left(\frac{1}{N}\sigma\right)^2+\ldots}=\sqrt{N\left(\frac{1}{N}\sigma\right)^2}=\frac{\sigma}{\sqrt{N}} \Rightarrow \boxed{\sigma_{\overline{x}}=\frac{\sigma}{\sqrt{N}}}$$

PS. σ-napaka posamezne meritve ; $\sigma_{\overline{x}}$ - napaka povprečja meritev (ta je odvisna od N)

npr... pri meritvah masse mobitela smo dobili $\sigma=1'1g$ ... $\Rightarrow \sigma_{\overline{x}}=\frac{\sigma}{\sqrt{10}}=0'35\,g$!

HOW TO REPRESENT A DATASET (npr. meritve mase mobitela od prej ...)

i	m_i
1	77
2	78
3	77
4	75
5	76
6	77
7	78
8	77
9	79
10	76

→ not usefull for big dataset ... namesto tega lahko izraziš s frekvenco ponovitve posamezne meritve:

m_k	75	76	77	78	79
n_k	1	2	4	2	1

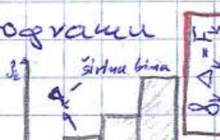
... v tem primeru je $\overline{x}=\frac{1}{N}\sum_k m_k x_k$!

→ vidimo da je naravni način za predstavitev histogram ...
(namesto št. ponovitev lahko uporabiš NORMALIZIRANO FREKVENCO)

F_k 0'1 0'2 0'4 0'2 0'1 $\Rightarrow \overline{x}=\sum_k F_k x_k$; $\sum F_k=1$

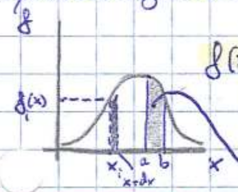
→ ČE imaš podatke porazdeljene "bolj" zvezno, uporabiš na histogramu intervale (BEAN HISTOGRAM) ...

BIN #	1	2	3	...
BIN OF.	$x<75$	$75<x<76$	$76<x<77$	...
F_k	0	3	4	



Ponavadi je pravilo, da naj bo ploščina bina sorazmerna frekvenci (ne višina bina)

When the number of data $N\to\infty$, than the histogram beans become thiner and it is aproching LIMITING DISTRIBUTION! npr. Binomial, gaussian ...



$f(x)dx$ - probability for a mesurment to fall in the interval $(x, x+dx)$!
(pri histogramih je bilo $F_k=f_k\cdot\Delta_k$ ... sedaj $\Delta_k\to 0 \Rightarrow f(x)dx$)

$\boxed{\int_a^b f(x)dx}$ - probability for a mesorment to be in interval (a,b)!

$$\boxed{\int_{-\infty}^{\infty} f(x)dx=1} \qquad \boxed{\overline{x}=\int_{-\infty}^{+\infty} x\cdot f(x)\,dx} \qquad \boxed{\sigma^2=\int_{-\infty}^{\infty}(x-\overline{x})^2 f(x)\,dx}$$

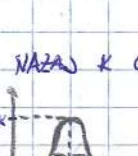
$$\boxed{G_{\overline{x},\sigma}(x)=\underbrace{k}_{\text{normal. const.}}\cdot e^{-\frac{(x-\overline{x})^2}{2\sigma^2}}}$$

- GAUSS DISTRIBUTION ... describes distribution of errors if they are random, small and independent!

PS. before we saw $\sigma^2=\int_{-\infty}^{\infty}(x-\overline{x})^2 f(x)dx=\int_{-\infty}^{\infty}(x^2+\overline{x}^2-2x\overline{x})f(x)dx=\int_{-\infty}^{\infty}x^2f(x)dx+\int_{-\infty}^{\infty}\overline{x}^2f(x)dx-2\int_{-\infty}^{\infty}x\overline{x}f(x)dx=$

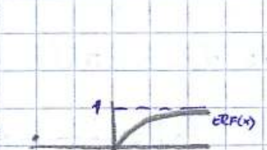
$$=\overline{x^2}+(\overline{x})^2-2\,\overline{x}\cdot\overline{x}=\overline{x^2}-\overline{x}^2 \Rightarrow \boxed{\sigma^2=\langle x^2\rangle-\langle x\rangle^2}$$

NAZAJ K GAUSS:



hočemo tak k, da je $\int_{-\infty}^{\infty} G_{\overline{x},\sigma}(x)\,dx=1$ ...

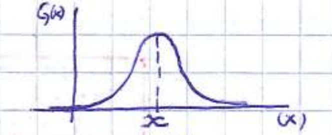
$k\int_{-\infty}^{+\infty} e^{-\frac{(x-\overline{x})^2}{2\sigma^2}}dx=1$ ... we set $\overline{x}=0$ to be more "comfortable" ...



$\Rightarrow k\int_{-\infty}^{+\infty} e^{-x^2/2\sigma^2}dx=1$ we put $z=\frac{x}{\sigma}$ ... $\Rightarrow k\sigma\int_{-\infty}^{+\infty} e^{-z^2/2}dz$

PS. $ERF(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \Rightarrow \int e^{-az^2}dz=\frac{\sqrt{\pi}}{2\sqrt{a}}ERF(x\sqrt{a})$

$$G_{\mathcal{X},\sigma}(x)=\frac{1}{\sigma\sqrt{2\pi}}\,e^{-\frac{(x-\mathcal{X})^2}{2\sigma^2}}$$



Gaussovo distribucijo dobimo če so napake MAJHNE, NAKLJUČNE in NEODVISNE!

$$\overline{x}=\int_{-\infty}^{\infty} x\,G(x)\,dx \quad \text{ali} \quad \overline{(x-\bar{x})^2}=\int_{-\infty}^{\infty}(x-\bar{x})^2\,G(x)\,dx \quad \ldots$$

1) $$\bar{x}=\int_{-\infty}^{\infty} x\cdot\frac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{(x-\mathcal{X})^2}{2\sigma^2}}\,dx = \ldots \begin{matrix} y=x-\mathcal{X} \\ dy=dx\end{matrix} \ldots = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}(y+\mathcal{X})\,e^{-\frac{y^2}{2\sigma^2}}\,dy =$$

$$=\frac{1}{\sigma\sqrt{2\pi}}\left[\underbrace{\int_{-\infty}^{\infty} y\,e^{-\frac{y^2}{2\sigma^2}}\,dy}_{=0 \ (\text{povprečje } y \text{ je } 0)} + \mathcal{X}\int_{-\infty}^{\infty} e^{-\frac{y^2}{2\sigma^2}}\,dy\right] \quad (+) \text{ vemo } \boxed{\int e^{-ax^2}dx=\frac{\sqrt{\pi}}{2\sqrt{a}}\,\mathrm{ERF}(x\sqrt{a})}$$

$$=\frac{1}{\sigma\sqrt{2\pi}}\cdot\mathcal{X}\cdot\sigma\sqrt{2\pi}=\underline{\underline{\mathcal{X}}}\ \checkmark \quad \text{POVPREČJE}$$

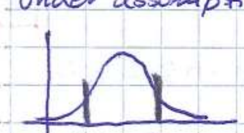
2) $$\overline{(x-\bar{x})^2}=\int_{-\infty}^{\infty}(x-\bar{x})^2\frac{1}{\sigma\sqrt{2\pi}}\,e^{-\frac{(x-\mathcal{X})^2}{2\sigma^2}}\,dx = \begin{matrix} y=x-\mathcal{X} \\ dy=dx\end{matrix} \ldots = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} y^2\,e^{-\frac{y^2}{2\sigma^2}}\,dy = \ldots \quad \frac{y}{\sigma}=z;\ \sigma dz=dy \quad =$$

$$=\frac{\sigma^2}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(z)\left(z\,e^{-z^2/2}\right)dz \quad (+) \text{ vemo } \boxed{\int x\,e^{-ax^2}dx=-\frac{1}{2a}\,e^{-ax^2}} \ \ldots \text{PER PARTES} \ldots$$

$$=\frac{\sigma^2}{\sqrt{2\pi}}\left[-z\,e^{-z^2/2}+\int e^{-z^2/2}dz\right]_{-\infty}^{\infty}=\frac{\sigma^2}{\sqrt{2\pi}}\sqrt{2\pi}=\underline{\sigma^2}\ \checkmark \quad \text{STAND. DEVIACIJA}$$

We have data-set $[x_1, x_2, \ldots, x_N]$ ... we can extract $\bar{x}, \sigma_x, \sigma_{\bar{x}}$!

under assumption that gaussian describes best our dataset, which are the best values for $\mathcal{X}, \sigma$?



$$P(x_1)\propto\frac{1}{\sigma}\,e^{-\frac{(x_1-\mathcal{X})^2}{2\sigma^2}};\quad P(x_2)\propto\frac{1}{\sigma}\,e^{-\frac{(x_2-\mathcal{X})^2}{2\sigma^2}};\ \ldots$$

$$\Rightarrow P(x_1,x_2\ldots x_N)\propto\frac{1}{\sigma^N}\,e^{-\frac{\sum(x_i-\mathcal{X})^2}{2\sigma^2}}$$ ... we must find $\mathcal{X}$ and σ

that maximize the probability $P(x_1, x_2, \ldots, x_N)$!

<u>for $\mathcal{X}$</u>: to max. $P(\ldots)$ we must minimize the exponent $\frac{\sum(x_i-\mathcal{X})^2}{2\sigma^2} \Rightarrow$

$$\Rightarrow \frac{\partial}{\partial\mathcal{X}}\frac{\sum(x_i-\mathcal{X})^2}{2\sigma^2}=\frac{1}{2\sigma^2}\cdot 2\cdot(-1)\cdot\sum(x_i-\mathcal{X})=0$$ $\Rightarrow$ we must find

such $\mathcal{X}$ that $\sum(x_i-\mathcal{X})=0 \Rightarrow$ $\boxed{\mathcal{X} \text{ is the AVERAGE of } [x_1, x_2, \ldots, x_N]}$!

<u>for σ</u>: to max. $P(\ldots)$ we put

$$\frac{\partial}{\partial\sigma}\left[\sigma^{-N}e^{-\frac{\sum(x-\mathcal{X})^2}{2\sigma^2}}\right]=-N\sigma^{-N-1}e^{-\frac{\sum(\,)^2}{2\sigma^2}}+\sigma^{-N}e^{-\frac{\sum(\,)^2}{2\sigma^2}}\cdot\frac{2\sum(\,)^2}{2\sigma^3}=0$$

always (+)

$$\Rightarrow -N\sigma^{-N-1}+\sigma^{-N-3}\sum(x-\mathcal{X})^2=0 \Rightarrow \sigma^{-N-1}\left[-N+\sigma^{-2}\sum(x-\mathcal{X})^2\right]=0$$

the solution $\sigma=0$ is TRIVIAL $\Rightarrow \sigma^{-2}\sum(x-\mathcal{X})^2=N \Rightarrow$ $\boxed{\sigma^2=\frac{\sum(x-\mathcal{X})^2}{N}}$

We demonstrated that for data-set $[x_1 \ldots x_N]$ the best gaussian description

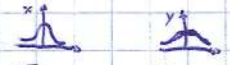
$$G_{\mathcal{X},\sigma}(x)=\frac{1}{\sqrt{2\pi}\,\sigma}\cdot e^{-\frac{(x-\mathcal{X})^2}{2\sigma^2}}$$ with $\mathcal{X}=\bar{x}$ and $\sigma=\sigma_x$!

-Now let $q=x+A$ (we know that $\sigma_q=\sigma_x$): $P(x)\propto e^{-\frac{(x-\mathcal{X})^2}{2\sigma^2}}$ and $P(q)\propto e^{-\frac{(q-A+\mathcal{X})^2}{2\sigma^2}}$

$\Rightarrow \bar{q}=\bar{x}+A$ (+) sigma is the same ... $\sigma_q=\sigma_x$

2) Now let $q = Bx$: $P(x) \propto e^{-\frac{(x-\overline{x})^2}{2\sigma^2}} \Rightarrow P(q) \propto e^{-\frac{(\frac{q}{B}-\overline{x})^2}{2\sigma^2}} = e^{-\frac{(q-B\overline{x})^2}{2B^2\sigma^2}} \Rightarrow$

$\Rightarrow \overline{q} = B\overline{x}$ and $\sigma_q = B\sigma_x$



3) Now let $s = x+y$: for simplicity we first let $\overline{x}=0$ and $\overline{y}=0$; $P(s) = P(x+y) \propto \int_{-\infty}^{\infty} P(x)P(s-x)\,dx$

$= \int_{-\infty}^{\infty} e^{-\frac{1}{2}\frac{x^2}{\sigma_x^2}} \cdot e^{-\frac{1}{2}\frac{(s-x)^2}{\sigma_x^2}} dx = \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2} + \frac{(s-x)^2}{\sigma_x^2}\right)\right] dx$ ... we must use the

identity $\frac{x^2}{A} + \frac{y^2}{B} = \frac{(x+y)^2}{A+B} + \frac{(Bx-Ay)^2}{AB(A+B)}$ ... $= \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\left(\frac{(\cancel{x}+s-\cancel{x})^2}{\sigma_x^2+\sigma_x^2} + \frac{(\sigma_x^2 x - \sigma_x^2(s-x))^2}{\sigma_x^2\sigma_x^2(\sigma_x^2+\sigma_x^2)}\right)\right] dx =$

ni več funk. x => gre pred integral

$= e^{-\frac{s^2}{2(\sigma_x^2+\sigma_x^2)}} \int_{-\infty}^{\infty} e^{(\;)} dx$ — just a number $= p$ $\Rightarrow \boxed{P(s) \propto e^{-\frac{s^2}{2(2\sigma_x^2)}}} \Rightarrow \overline{s} = 0$ and $\sigma_s = \sqrt{\sigma_x^2+\sigma_x^2}$

general $s = x+y = \underbrace{(x-\overline{x})}_{} + \underbrace{(y-\overline{y})}_{} + (\overline{x}+\overline{y})$ → just a number $\Rightarrow$ we are back to the case with $\overline{x}=0$ and $\overline{y}=0$!

centred over 0

$\Rightarrow \overline{s} = \overline{x}+\overline{y}$ and $\sigma_s = \sqrt{\sigma_x^2+\sigma_y^2}$

4) $q = q(x,y) = \underbrace{q(\overline{x},\overline{y})}_{\text{NUMBER}} + \underbrace{\frac{\partial q}{\partial x}}_{\text{CONSTANT}}\underbrace{(x-\overline{x})}_{\text{CENTER AT 0}} + \underbrace{\frac{\partial q}{\partial y}}_{\text{CONSTANT}}\underbrace{(y-\overline{y})}_{\text{CENTRED AT 0}}$ ... only if errors are small $(x-\overline{x}) \sim 0$, $(y-\overline{y}) \sim 0$

we aproximate with linear function ! => we use previous results ...

$\boxed{\overline{q} = q(\overline{x},\overline{y})}$; $\boxed{\sigma_q = \sqrt{\left(\frac{\partial q}{\partial x}\sigma_x\right)^2 + \left(\frac{\partial q}{\partial y}\sigma_y\right)^2}}$! → this is why $\boxed{\sigma_{\overline{x}} = \sigma_x/\sqrt{N}}$

by more complicated calculation we get the uncertenty on σ_x : $\boxed{\sigma_{\sigma_x} = \sigma_x/\sqrt{2(N-1)}}$

Now we want to extend our discusion to variables that do NOT have INDEPENDENT ERRORS !

$q_i = q(x_i,y_i)$ ... errors are still small => we can aproximate $q_i(x,y) = q(\overline{x},\overline{y}) + \frac{\partial q}{\partial x}(x_i-\overline{x}) + \frac{\partial q}{\partial y}(y_i-\overline{y})$

now - average q : $\overline{q} = \frac{1}{N}\sum_i q_i = \frac{1}{N}\sum_i\left[q(\overline{x},\overline{y}) + \frac{\partial q}{\partial x}(x_i-\overline{x}) + \frac{\partial q}{\partial y}(y_i-\overline{y})\right] =$

$= \frac{1}{N}\cdot N\cdot q(\overline{x},\overline{y}) + \frac{1}{N}\frac{\partial q}{\partial x}\cancel{\sum_i(x_i-\overline{x})} + \frac{1}{N}\frac{\partial q}{\partial y}\cancel{\sum_i(y_i-\overline{y})} = \boxed{q(\overline{x},\overline{y})}$ WE HAVE SAME RESULT AS FOR GAUSSIAN DIS. $\Rightarrow \boxed{\overline{q} = q(\overline{x},\overline{y})}$

what about σ_q :

$\sigma_q^2 = \frac{1}{N}\sum(q_i-\overline{q})^2 = \frac{1}{N}\sum\left[\cancel{q(\overline{x},\overline{y})} + \frac{\partial q}{\partial x}(x_i-\overline{x}) + \frac{\partial q}{\partial y}(y_i-\overline{y}) - \cancel{q(\overline{x},\overline{y})}\right]^2 =$

$= \frac{1}{N}\sum\left[\left(\frac{\partial q}{\partial x}(x_i-\overline{x})\right)^2 + \left(\frac{\partial q}{\partial y}(y_i-\overline{y})\right)^2 + 2\frac{\partial q}{\partial x}\cdot\frac{\partial q}{\partial y}(x_i-\overline{x})(y_i-\overline{y})\right] =$

$= \left(\frac{\partial q}{\partial x}\right)^2\sigma_x^2 + \left(\frac{\partial q}{\partial y}\right)^2\sigma_y^2 + \underbrace{\frac{2}{N}\frac{\partial q}{\partial x}\frac{\partial q}{\partial y}\sum(x_i-\overline{x})(y_i-\overline{y})}_{\text{This is THE NEW TERM: KOVARIANCE}}$

KOVARIANCE : $\boxed{\sigma_{xy} = \frac{1}{N}\sum_i(x_i-\overline{x})(y_i-\overline{y})}$ → if x and y are INDEPENDENT $\Rightarrow \sigma_{xy} = 0$! $\lim_{N\to\infty}\sigma_{xy} = 0$

npr : if the errors of x and y are linearly dependent $\Rightarrow \frac{(x_i-\overline{x})}{\sigma_x} = \frac{(y_i-\overline{y})}{\sigma_y}$ we put this into equation ...

WHEN LINEARLY RELATED (WORST CASE)

$\boxed{\sigma_{xy}} = \frac{1}{N}\sum(x_i-\overline{x})^2\frac{\sigma_y}{\sigma_x} = \frac{\sigma_y\cdot\sigma_x^2}{\sigma_x} = \boxed{\sigma_x\cdot\sigma_y}$ ... what happens to σ_q^2 ?

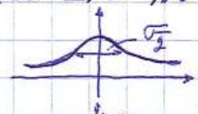
$\boxed{\sigma_q^2} = \left[\frac{\partial q}{\partial x}\sigma_x\right]^2 + \left[\frac{\partial q}{\partial y}\sigma_y\right]^2 + 2\frac{\partial q}{\partial x}\frac{\partial q}{\partial y}\sigma_x\sigma_y = \boxed{\left[\frac{\partial q}{\partial x}\sigma_x + \frac{\partial q}{\partial y}\sigma_y\right]^2}$ → Primerjaj z enačbo za neodvisne napake ...

Kovariance obeys SCWARZ inequality :

TO SUMMARIZE : $\boxed{\sigma_g \leq \left(\frac{\partial g}{\partial x}\cdot\sigma_x\right) + \left(\frac{\partial g}{\partial y}\cdot\sigma_y\right)}$ - zaradi $\sigma_g^2 = (g_x)^2\sigma_x^2 + (g_y)^2\sigma_y^2 + 2(g_x)(g_y)\sigma_x\sigma_y = (g_x\sigma_x + g_y\sigma_y)^2$ ...
- edino absolutna vrednost bi lahko bla ...

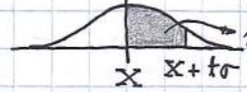
NOW LET'S SEE HOW WE CAN USE ALL THIS RESULTS IN PRACTICE:

in physics npr. we get: $\boxed{g_{BEST} \pm \sigma_{\bar{g}}}$ where $\sigma_{\bar{g}} = \sigma_g/\sqrt{N}$



if expected (theoretical) value is close to experimental ⇒ we can say that model is good, if not ⇒ model not good ... but how to determine more exactly?

DISCREPANCY: $\boxed{\frac{|g_{BEST} - g_{EXP.}|}{\sigma_{\bar{g}}} = t}$



(± 1×σ → ~68% probability)

how to determine the limit is up to us ... USUALY IS TAKEN $t = 1'8$ ⇒ $\pm 1'8\sigma$ → ~93% TRUE

DISCREPANCY GIVES US THE CONFIDENCE TO OUR MODEL! (LEVEL OF CONFIDENCE)

npr. merili smo g (prosti pad) → dobili smo $g_{EXP.}$ in $\sigma_{\bar{g}}$ (tudi sigma je "experimental")
⇒ izračunaš DISCREPANCY in vidiš ...

Second use of uncertenty is when to reject a data (DATA REJECTION)

[dataset] → $g_{BEST} \pm \sigma_{\bar{g}}$ ... BUT CAN HAPPEN THAT SOME RESULTS FROM DATA-SET ARE VERY FAR AWAY FROM g_{BEST}!

When can we reject some data → depends also on number of mesurments!
(not depends only on σ)

npr [3'8, 3'5, 3'9, 3'9, 3'4, 1'8] → $\bar{x} = 3'4$ and $\sigma_x = 0'85$

discrepancy: $\boxed{\frac{|x_{BAD} - \bar{x}|}{\sigma_x \text{ - sedaj je the } \sigma_x \text{ ne } \sigma_{\bar{x}}} = t = 2}$ ⇒ tabelu vidiš 4% verjet.

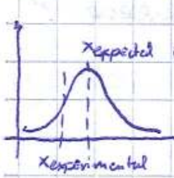
4% verjetnost – naredili pa smo samo 6 meritev!

$4\% \times N = 4\% \times 6 \cong 0'3$ → we can reject it

IF THE NUMBER OF EXPECTET MESURMENTS OF SUCH ERROR is ≤ 0'5 ⇒ REJECT DATA

15/04/14

PONOVITEV: data-set $[x_1, \dots x_N]$ → x_{best} = averige ; $\delta x = \sigma_{\bar{x}}$ 1×σ - 68% confidence; 2×σ - 98% confidence
comparison: $|x_{experiment} - x_{expected}| \sim 0$



we make an experiment → data-set → $\bar{x}, \sigma_{\bar{x}}$ hypotesis: ① Distribution of mesures is centred around expected value $x_{expected}$!
② The uncertenty is corectly evaluated!

discrepancy: $t = \frac{|x_{expected} - x_{experim}|}{\sigma_{\bar{x}}}$ npr = 1 ⇒

⇒ napaka je 1×σ ⇒ še vedno je 32% verjetnost da dobimo slabšo meritev (DISCREPENCY is UN-SIGNIFICANT)

Če je $t \geq 2'5$ (DISCREPANCY is SIGNIFICANT) ⇒ nekaj ni kul ⇒ hipoteza ① ali ② ne drži ali napaka meritve...

[$t \leq 1'8$ (UN-SIGNIFICANT) $1'8 < t < 2'5$ (UNCONCLUSIVE EXPERIMENT) $t \geq 2'5$ (SIGNIFICANT)]

DATA REJECTON:

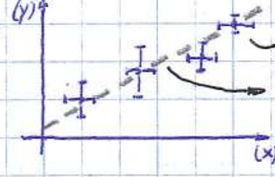
- data-set $[x_1, \dots x_N]$ → suspected mesurment x_{BAD} → $n = N \cdot$ Probab. (to mesure X out of $t_{susp.}\sigma$)

SCHOVENET CRITERIUM : if $n \leq 0'5$ we can reject the mesurment!

HOW TO MESURE IF TWO QUANTATIES ARE LINEARLY CORELATED?

two data-set $[x_1, \dots x_N]$ and $[y_1, \dots y_N]$

① MAKE GRAPH



→ data with error-bars
→ a line that connets almost all points

z grafa lahko oceniš ali obstaja povezava ali ne!

npr. prosti pad $h = \frac{1}{2} g t^2$ → narišeš graf (h) v odvisnosti od (t^2)!

CORRELATION COEFFICIENT: $\boxed{r = \dfrac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum(x_i-\bar{x})^2}\sqrt{\sum(y_i-\bar{y})^2}}}$ similar to $\frac{\sigma_{xy}}{\sigma_x \sigma_y}$ - but this was to describe errors DON'T CONFUSE!

npr. spring and mass - we do not expect the masses to be similar ...

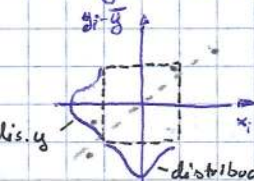
IF WE APPLY THE SCHWARZ INEQUALITY: $\sum(x_i-\bar{x})(y_i-\bar{y}) \leq \sqrt{\sum(x_i-\bar{x})^2} \cdot \sqrt{\sum(y_i-\bar{y})^2} \Rightarrow \boxed{-1 \leq r \leq 1}$

if two variables are perfectly linear related $\Rightarrow (x_i-\bar{x}) = B(y_i-\bar{y})$?

$$\Rightarrow r = \frac{B\sum(y_i-\bar{y})^2}{\sqrt{B^2(y_i-\bar{y})^2}\sqrt{(y_i-\bar{y})^2}} = 1$$

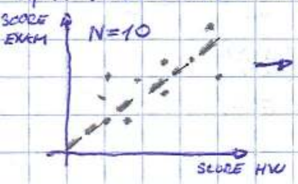
⇒ $r=1$ če sta perfektno linearno povezani!
$r=-1$ če sta perfektno obratno sorazmerni!
$r=0$ če sta povsem nepovezani

P.S. if N is not very big this tool is not very usefull (N mora biti vsaj 10)!



- če je večina točk v ① in ③ kvadratu ⇒ $r>0$
- če je večina točk v ② in ④ kvadratu ⇒ $r<0$
- če je enakomerno povsod kvadratih ⇒ $r \sim 0$

npr. 10 students → povezava med oceno IZPIT in oceno HOMEWORK



→ DOBIMO $r = 0.8$ → POIŠČEMO V TABELI → 0.5% that variables are un-correlated
TABELA POVE VERJETNOST PRI DANEM N in r DA STA NEPOVEZANI

- opišemo napake x in y z vektorjem (N-dim) $a_i = (x_i-\bar{x})$ and $b_i = (y_i - \bar{y})$

TAKO ZAPIŠEMO: KOVARIANCE: $\sigma_{xy} = \sum a_i b_i = \vec{a}\cdot\vec{b}$
SIGMA: $\sqrt{\sum a_i^2} = |\vec{a}|$

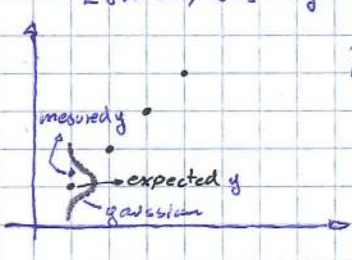
C-S: $\vec{a}\cdot\vec{b} \leq |\vec{a}|\cdot|\vec{b}|$ ⇒ $\boxed{r = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}} (= \cos\phi)$

r DOES NOT TELL US HOW THE VARIABLES ARE RELATED / ONLY IF THEY ARE CORRELATED!

How to find the relation?

npr. $y = A + Bx$ (HYPOTESIS) samo za x za y: $\sigma_y \neq 0$
$[x_1, \dots, x_N]$ naj bo $\sigma_x = 0$ (zaradi poenostavitve)
$[y_1, \dots, y_N]$ σ_y

BY HYPOTESIS ($y = A+Bx$): EXPECTED VALUE FOR y_i →
$y_1^E = A + Bx_1$
$y_2^E = A + Bx_2$
$\vdots$
$y_N^E = A + Bx_N$



$$P(y_1) \propto \frac{1}{\sigma_y} e^{-\frac{(y_1-A-Bx_1)^2}{2\sigma_y^2}}$$
$\vdots$

napaka meritve σ_y ne $\sigma_{\bar{y}}$!

$$\Rightarrow P(y_1 \dots y_N) \propto \frac{1}{\sigma_y^N} e^{-\frac{\sum(y_i-A-Bx_i)^2}{2\sigma_y^2}}$$

$$\boxed{P(y_1 \dots y_N) \propto \frac{1}{\sigma_y^N} e^{-\frac{\chi^2}{2}}; \quad \chi^2 = \sum_i (y_i - \underbrace{A - Bx_i}_{\text{UNKNOWN}})^2 / \sigma_y^2}$$

A and B that maximize $P(\dots)$ are those who minimize χ^2!

SYSTEM
$$\Rightarrow \begin{cases} \dfrac{\partial \chi^2}{\partial A} = 0 = \dfrac{-1}{\sigma_y^2} \cdot 2\sum(y_i - A - Bx_i) = 0 \Rightarrow \boxed{\sum y_i - NA - B\sum x_i = 0} \\ \dfrac{\partial \chi^2}{\partial B} = 0 = \dots \Rightarrow \boxed{\sum x_i y_i - A\sum x_i - B\sum x_i^2 = 0} \end{cases}$$

D.N. REŠI SISTEM NA A in B!

LEAST-SQUARE FITTING

mpr. coefficient of a spring ...

Δl_1	Δl_2	Δl_3
m_1	m_2	m_3

↳ mass has much less uncertenty than Δl_i ($\sigma_x = 0$; $\sigma_y \neq 0$)

$\Rightarrow$ error on Δl_i should be gaussian distributed around best value

$\Rightarrow$ Probability $(y_i) \propto \frac{1}{\sigma_y^N} e^{-\chi^2/2}$; $\chi^2 = \frac{1}{\sigma_y^2}\sum(y_i - A - Bx_i)^2 \to$ we have to minimize this!

$\Rightarrow$ we have two partial derivative $= 0 \to$ system of two equations for A and B!

$$\begin{cases} A = (\sum y_i - B\sum x_i)/N \\ \sum x_i y_i - A\sum x_i - B\sum x_i^2 = 0 \end{cases} \Rightarrow \sum xy - \frac{\sum y - B\sum x}{N}\sum x - B\sum x^2 = 0 \Rightarrow \ldots\ B = \frac{N\sum x_i y_i - \sum x_i \sum y_i}{\underbrace{-(\sum x_i)^2 + N\sum x_i^2}_{=\Delta}}$$

$$A = \frac{\Delta\sum y - (N\sum xy - \sum x\sum y)\sum x}{N\Delta} = \ldots = \boxed{A = \frac{\sum x_i^2 \sum y_i - \sum x_i y_i \sum x_i}{\Delta}\ ;\ B = \frac{N\sum x_i y_i - \sum x_i \sum y_i}{\Delta}}$$

- we can also find which σ_y makes Probability (data-set) maximum!

$$\Rightarrow \frac{\partial Prob(y_i)}{\partial \sigma_y} = \ldots = -N\sigma_y^{-N-1}\cdot e^{-\frac{\sum()^2}{2\sigma_y^2}} + \sigma_y^{-N}\cdot e^{-\frac{\sum()^2}{2\sigma_y^2}}\cdot\sum()^2\cdot\sigma_y^{-3} = \left[-N\sigma_y^{-N-1} + \sum()^2\cdot\sigma_y^{-N-3}\right]\cdot e^{(\ldots)} = 0$$

$$\Rightarrow N\sigma_y^{-N-1} = \sigma_y^{-N-3}\cdot\sum(\ldots)^2 \Rightarrow \sigma_y^2 = \sum(\ldots)^2/N \Rightarrow \boxed{\sigma_y = \sqrt{\sum(y_i - A - Bx_i)^2/N}}$$

THE CORRECT EXPRESION WOULD BE WITH (N-2)!

What is the UNCERTEINTY OF A and B?

$A = \frac{\sum x^2\sum y - \sum x\sum xy}{\Delta}$; $\sigma_x = 0$, $\sigma_y \neq 0$ $\to$ A is a function of x, y $\to$ we use the formula for the propagation of errors ...

because $\sigma_x = 0$ is A only function of y!

$$\Rightarrow \sigma_A^2 = \left(\frac{\partial A}{\partial y_1}\cdot\sigma_y\right)^2 + \left(\frac{\partial A}{\partial y_2}\cdot\sigma_y\right)^2 + \ldots = \left(\frac{\sigma_y}{\Delta}\right)^2\cdot\left[\left(\sum x^2 - \sum x\, x_1\right)^2 + \left(\sum x^2 - \sum x\, x_2\right)^2 + \ldots\right] =$$

$$= \left(\frac{\sigma_y}{\Delta}\right)^2\cdot\left[\left((\sum x^2)^2 + (\sum x)^2 x_1^2 - 2\sum x^2\sum x\, x_1\right) + \ldots\right] = \left(\frac{\sigma_y}{\Delta}\right)^2\left[N(\sum x^2)^2 + (\sum x)^2\sum x^2 - 2\sum x^2(\sum x)^2\right]$$

$$= \left(\frac{\sigma_y}{\Delta}\right)^2\cdot\left[N(\sum x^2)^2 - \sum x^2(\sum x)^2\right] \Rightarrow \boxed{\sigma_A = \frac{\sigma_y}{\Delta}\sqrt{\Delta\cdot\sum x^2}}$$

$\Delta = N\sum x^2 - (\sum x)^2$

UNCERTENTY FOR A and B!

Similarly we do for $\sigma_B = \ldots \Rightarrow$ $\boxed{\sigma_B = \sigma_y\sqrt{N/\Delta}}$

P.S.:

But if we have that $\boxed{y = A + Bx + Cx^2}$ we have $\chi^2 = \frac{1}{\sigma_y^2}\sum_i(y_i - A - Bx_i - Cx_i^2)^2$

so we get 3 equations $\frac{\partial\chi^2}{\partial A} = 0$, $\frac{\partial\chi^2}{\partial B} = 0$, $\frac{\partial\chi^2}{\partial C} = 0$ for 3 unknowns A, B, C! SOLVE SYSTEM!

Sometimes we can reduce more complicated things to linear:

mpr.

$y = Ae^{Bx} \to \ln y = \ln A + Bx \to z = (\ln A) + Bx = A' + Bx \to$ now apply previous rules!

$\to$ but we must watch for uncertenty: $\delta z = \frac{dz}{dy}\cdot\sigma_y = \frac{1}{y}\cdot\sigma_y \to$ we should do <u>WEIGHTED LIN. FITTING!</u>

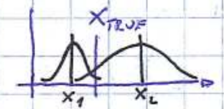
(different aparatus)

We are mesuring g : 1. experiment $\rightarrow g_1 \pm \sigma_{g1}$; 2. experiment $\rightarrow g_2 \pm \sigma_{g2}$

How to merge the 2 results and get better result ?
(npr. mesuring g → 1. with free-fall ; 2. with pendulum!)

① we look if the 2 answers are compatible! $|g_1 - g_2| > \sigma_{g1} + \sigma_{g2}$ no cross-section / bad experiment!

if 2 answers are compatible...
we make assumptions : – NO SYSTEMATIC ERROR
– ALL ERRORS ARE RANDOM
– MESURMENTS ARE NORMAL DISTRIB. ARAUND A TRUE VALUE X



How do we now calculate TRUE VALUE:
–we look at the $\mathrm{ProB}_{X_{TRUE}}(x_1)$ and $\mathrm{Prob}_{X_{TRUE}}(x_2)$!

$$\left.\begin{aligned}\mathrm{Prob}_X(x_1) &\propto \frac{1}{\sigma_1} e^{-\frac{(x_1-X)^2}{2\sigma_1^2}}\\ \mathrm{Prob}_X(x_2) &\propto \frac{1}{\sigma_2} e^{-\frac{(x_2-X)^2}{2\sigma_2^2}}\end{aligned}\right\} \Rightarrow \mathrm{Prob}_X(x_1,x_2) \propto \frac{1}{\sigma_1}\cdot e^{-\frac{(x_1-\overline{X})^2}{2\sigma_1^2}} \cdot \frac{1}{\sigma_2}\cdot e^{-\frac{(x_2-\overline{X})^2}{2\sigma_2^2}}$$

NOW WE FIND WICH $\overline{X}$ (TRUE VALUE) MAXIMIZEZ THIS!

$$\mathrm{Prob}(x_1 x_2) \propto \frac{1}{\sigma_1 \sigma_2} \cdot e^{-\chi^2/2} \; ; \; \boxed{\chi^2 = \left(\frac{x_1 - \overline{X}}{\sigma_1}\right)^2 + \left(\frac{x_2 - \overline{X}}{\sigma_2}\right)^2}$$

→ TO MAX Prob(..) we MINIMIZE χ^2!

$$\Longrightarrow \frac{\partial \chi^2}{\partial \overline{X}} = -2\left(\frac{x_1 - \overline{X}}{\sigma_1^2}\right) - 2\left(\frac{x_2 - \overline{X}}{\sigma_2^2}\right) = 0 \Longrightarrow \frac{\sigma_2^2 x_1 - \sigma_2^2 \overline{X} - \sigma_1^2 x_2 - \sigma_1^2 \overline{X}}{\sigma_1^2 \sigma_2^2} = 0$$

$\Longrightarrow$ we get TRUE VALUE : $\boxed{\overline{X} = \frac{\sigma_2^2 X_1 + \sigma_1^2 X_2}{\sigma_1^2 + \sigma_2^2}} \longrightarrow \boxed{\overline{X} = \frac{\omega_1 x_1 + \omega_2 x_2}{\omega_1 + \omega_2} \; ; \; \omega_i = \frac{1}{\sigma_i^2}}$ WEIGHTED AVERIGE! –compere w. center of mass in 1-D!

HOW DO WE GET UNCERTENTY OF $\overline{X}$? (propogation of error – LIKE ALWAYS)

$$\sigma_{\overline{x}} = \sqrt{\left(\frac{\partial \overline{x}}{\partial x_1}\sigma_1\right)^2 + \left(\frac{\partial \overline{x}}{\partial x_2}\sigma_2\right)^2} = \sqrt{\left(\frac{\omega_1}{\sum \omega_i}\sigma_1\right)^2 + \left(\frac{\omega_2}{\sum \omega_i}\sigma_2\right)^2} = \frac{1}{\sum \omega_i}\sqrt{(\omega_1\sigma_1)^2 + (\omega_2\sigma_2)^2 + \dots} =$$

$$= \frac{1}{\sum \omega_i}\sqrt{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} + \dots} = \frac{1}{\sum \omega_i}\sqrt{\sum \omega_i} = \boxed{\sigma_{\overline{x}} = \sqrt{1/\sum \omega_i}}$$

npr.
3 students mesure resistance R:
#1 : $R_1 = 11 \pm 1 \Rightarrow \omega_1 = 1$
#2 : $R_2 = 12 \pm 1 \Rightarrow \omega_2 = 1$
#3 : $R_3 = 10 \pm 3 \Rightarrow \omega_3 = 1/9$

we see that the #3 mesurment will be much less important...

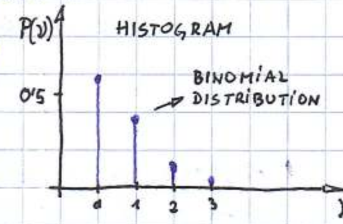
WE CALCULATE ... → $\overline{R} = 11'4 \pm 0'7$

There are some experiments where the distribution of errors is NOT Gaussian!

npr. 3 kocke - what is the probability to get 3× AS (AS = 1)

ν = number of observed ① $P(\nu=1) \rightarrow$ As $\bar{A}_s$ $\bar{A}_s$ } lahko prvi met ① druge NE
$\bar{A}_s$ As $\bar{A}_s$ } lahko 2. met ① —ıı—
$\bar{A}_s$ $\bar{A}_s$ As } lahko 3. met ① —ıı—
→ trije ugodni izidi!

$$A\bar{A}\bar{A} \rightarrow \frac{1}{6}\cdot\frac{5}{6}\cdot\frac{5}{6} = 0'116$$
$$\bar{A}A\bar{A} \rightarrow \frac{5}{6}\cdot\frac{1}{6}\cdot\frac{5}{6} = 0'116$$
$$\bar{A}\bar{A}A \rightarrow \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} = 0'116$$
$$\Rightarrow P(\nu=1) = 3\times 0'116 = \underline{0'347}$$



$$P(\nu=2) = \underbrace{\boxed{3}}_{\binom{n}{\nu}} \times \underbrace{\boxed{\left(\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{5}{6}\right)}}_{p^{\nu}(1-p)^{n-\nu}} = 3\times 0'023 = \underline{0'069}$$
↳ tudi tokrat je ugoden izid mogoče narediti na 3 načine (1.&2. / 2.&3. / 1.&3.)

$$P(\nu=3) = \frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6} = \underline{0'5\%} \quad \text{and} \quad P(\nu=0) = \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6} = \underline{57'9\%}$$

$$\boxed{B_{n,p}(\nu) = \binom{n}{\nu} p^{\nu} q^{n-\nu}} \quad q = 1-p \quad \rightarrow \text{BINOMIAL DISTRIBUTION}$$
n poskusov — verjetnost ugodnega izida

kako upravičimo binomski koeficient ... npr 5 kart v škatli ... 2 povlečemo .. koliko različnih parov lahko povlečemo (vrstni red ni važen AB = BA) ⇒ 5×4/2 = 10 parov
npr: n kart → ν izvlečeno ⇒ $n(n-1)\ldots(n-\nu+1)$ načinov $= n!/(n-\nu)!$
sedaj, če vrstni red ni pomemben, moramo deliti še z številom permutacij ν elementov $= \nu!$
⇒ $n!/\nu!(n-\nu)!$ - kar je ravno binomski koeficient!

averige : remember $\boxed{\bar{x} = \frac{1}{N}\sum \underbrace{n_k}_{\text{frekvenca ponovitve}} x_k}$ ⟹ če normaliziramo frekvenco $F_k = \frac{n_k}{N}$ ⇒ $\boxed{\bar{x} = \sum F_k x_k}$

$\boxed{\bar{\nu} = \sum_{\nu=0}^{n} \nu \cdot B_{n,p}(\nu)}$ → averige $\bar{\nu}$ can NOT BE a whole number... $\bar{\nu}$ CAN BE DIFERENT FROM MOST PROBABLE VALUE!

$$\frac{\partial}{\partial p}\sum\binom{n}{\nu}p^{\nu}q^{n-\nu} = \frac{\partial}{\partial p}(p+q)^n = n(p+q)^{n-1}$$

$$\frac{d}{dp}\left[\binom{n}{0}p^0q^n + \binom{n}{1}p^1q^{n-1} + \binom{n}{2}p^2q^{n-2} + \ldots\right] = \left[0 + \binom{n}{1}q^{n-1} + \binom{n}{2}\cdot 2pq^{n-2} + \ldots\right] = \sum_{\nu=1}^{n}\binom{n}{\nu}\cdot\nu\cdot p^{\nu-1}q^{n-\nu}$$

$$\Rightarrow n(\underbrace{p+q}_{1})^{n-1} = \sum_{\nu=1}^{n}\nu\cdot\binom{n}{\nu}p^{\nu-1}q^{n-\nu} \;/\cdot p \Rightarrow np = \sum_{\nu=1}^{n}\nu p^{\nu}q^{n-\nu}\cdot\binom{n}{\nu}$$
→ lahko spustimo vsoto do 0, ker je člen pomnožen z ν (ki je tam = 0) ⇒ ne spremeni nič ⇒ $np = \sum_{\nu=0}^{n}\nu\binom{n}{\nu}p^{\nu}q^{n-\nu} = \bar{\nu}$ $\boxed{\bar{\nu} = np}$

WE CAN ALSO CALCULATE STANDARD DEVIATION:
$$\sigma^2 = \sum_{\nu=0}^{n}(\nu-\bar{\nu})^2\cdot B_{n,p}(\nu) = \ldots \quad \boxed{\sigma_{\nu} = \sqrt{np(1-p)}}$$

COMPARISON:

$G_{x,\sigma}(x)$	$B_{n,p}(\nu)$
COUNTINOUS	DISCRETE
SYMETRIC	NOT-SYMETRIC
$\bar{x}$ = MOST PROBABLE	$\bar{\nu} \neq$ MOST PROBABE

D.N. Try plotting on same graph gaussian and binomial with $\bar{x} = np$; $\sigma = \sqrt{np(1-p)}$!
When $n \to \infty$ BINOMIAL aproaches GAUSSIAN!

npr. 36 kart črne ena stran / rdeče druga → vržeš na mizo → verjetnost 23 črnih = ?

$$\left.\begin{matrix} n = 36 \\ p = \frac{1}{2} \\ \nu = 23 \end{matrix}\right\} \Rightarrow B_{36,\frac{1}{2}}(23) = \binom{36}{23}\left(\frac{1}{2}\right)^{23}\left(\frac{1}{2}\right)^{13} = \ldots = \underline{3'36\%}$$

Gauss približek: $G_{18,3}(23) = \underline{3'32\%}$ → zelo podobno pravi vrednosti npr. volitve, kandidat A in B, A trdi da 55% ljudi bo volilo zanj. B preiskus in vpraša 100 ljudi in trdi da 50 vol za A. Koliko je verjetnost (če je trditev A 55% pravilna) da dobimo ta rezultat? (ta ali manj)

08/05/2014

ponovitev: $B_{n,p}(\nu) := \binom{n}{\nu} p^{\nu}(1-p)^{n-\nu}$

- je pokazal v excelu primerjavo med binomsko in Gauss porazdelitvijo
  za $n \to \infty$ gre binomska $\to$ Gauss!

$$\boxed{B_{n,p}(\nu) \xrightarrow{n\to\infty} G_{np,\sqrt{np(1-p)}}(x)}$$

How to model a situation of a mesure with random and independent errors?

Hypotesis: there exists a true value (x ali $\bar{x}$)

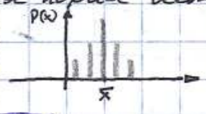
- each of mesures is subject to n sources of error (size ε, prob $p = 1/2$)

ν = how many times the error is $\oplus$; $\nu = 0 \to n$

$$X = \bar{x} + \nu\varepsilon - (n-\nu)\varepsilon = \bar{x} + \nu\varepsilon - n\varepsilon + \nu\varepsilon = \bar{X} + \boxed{\varepsilon(2\nu - n)}$$ $\to$ ta del je med $-\varepsilon n$ in εn

- predpostavili smo da so vse napake velike ε in so samo lahko $\oplus$ ali $\ominus$ $\Rightarrow$ binomska porazdelitev

$\Rightarrow X = \bar{x} + \varepsilon(2\nu - n) \Rightarrow$



$\Rightarrow$ what is standard deviation $\to$ propagation of error $\Rightarrow$

$\Rightarrow \sigma_x = \frac{dx}{d\nu}\cdot\sigma_\nu = 2\varepsilon\sqrt{np(1-p)}$; $p = \frac{1}{2} \Rightarrow \boxed{\sigma_x = \varepsilon\sqrt{n}}$; sedaj $n \to \infty$ and $\varepsilon \to 0$ tako da $\sigma_x = \text{const.}$

... $n \to \infty \Rightarrow$ gre proti Gauss ; $\varepsilon \to 0 \Rightarrow$ postaja vedno bolj "zvezna"!

$\to$ sedaj gremo nazaj k primeru z vrha strani (kandidata A in B, A trdi da 55% ...)

A, B ; A say that 55% vote for him

$n = 100$, $\nu = 50$

$B_{100,0'55}(50) = 4'8\%$ ... 50 ali manj $\Rightarrow B_{100,0'55}(\nu \le 50) = \sum_{0}^{\nu=50}\binom{n}{\nu}p^{\nu}(1-p)^{n-\nu}$ približek

$\Rightarrow$ v Gaussu $\Rightarrow \bar{x} = np = 55$; $\sigma_x = \sqrt{np(1-p)} = 5 \Rightarrow$ med 55 in 50 je $1\sigma_x$ razlike $\Rightarrow$ 16%

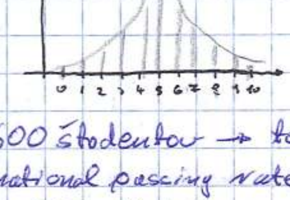
... drugi intervju (600 ljudi $\to$ 300 voli za A)

$\bar{x} = 330$; $\sigma = 12'2 \Rightarrow$ med 330 and 300 je $2'46\times\sigma$ razlike (DISCREPANCY) $\Rightarrow$ možnost za tak ali slabši rezultat je samo $0'7\%$ $\Rightarrow$ trditev A ni resnična!

npr. proizvajalec voska za smuči $\Rightarrow$ preizkusijo 10 parov smuči (vosek na eno smučko, na drugo ne)

naredijo tekme : voskane smuči zmagajo 8

HYPOTESIS : vosek nima učinka $\Rightarrow p = 1/2$; $\Rightarrow np = 5$; $\sqrt{np(1-p)} = 1'1 \Rightarrow 2'73\sigma$



$B_{10,0'5}(8) + B_{10,0'5}(9) + B_{10,0'5}(10) = \underline{5'5\%}$ - to je bolj natančno kot Gauss približek

$\to$ zelo verjetno je hipoteza narobe

$\Rightarrow$ vosek je učinkovit!

npr. 600 študentov $\to$ test $\to$ 420 passed the test

national passing rate is 60% (can the school claim that their students are better prepared)

$n = 600$; $\nu = 420$

$$\left.\begin{array}{l} p = 0'6 \Rightarrow np = 360 \\ \sqrt{np(1-p)} = 12 \end{array}\right\} \Rightarrow \frac{420-360}{12} = 5$$ - DISCREPANCY (5σ odstopanje) $\Rightarrow$ trditev šole je resnična!

POISSONOVA PORAZDELITEV

$$\boxed{P_\mu(\nu) = e^{-\mu}\frac{\mu^\nu}{\nu!}}$$

μ - število pričakovanih dogodkov na nekem intervalu (npr. 5 potresov/dan; 16 gob/hektar...)
ν - število dogodkov, ki so se zgodili v tistem intervalu

BINOMSKA → POISSON when n is big and p is small!

KAKŠNO JE POVPREČJE za Poisson? $\bar{\nu} = \mu$

$$\bar{\nu} = \sum_{\nu=0}^{\infty} \nu \cdot P_\mu(\nu) = \sum_{\nu=0}^{\infty} \nu\, e^{-\mu}\frac{\mu^\nu}{\nu!} = \sum_{\nu=1}^{\infty} e^{-\mu}\frac{\mu^\nu}{(\nu-1)!} = \mu e^{-\mu}\sum_{\nu=1}^{\infty}\frac{\mu^{\nu-1}}{(\nu-1)!} = \mu e^{-\mu}\overbrace{\left(1+\mu+\frac{\mu^2}{2!}+\frac{\mu^3}{3!}+\dots\right)}^{\text{TAYLOR ZA } e^{\mu}} = \boxed{\mu}$$

⇒ averige of sucsesses $\bar{\nu} = \mu$! $\boxed{\bar{\nu} = \mu}$ POVPREČJE!

Je pokazal primerjavo med BINOMSKO in POISSON za velik n/majhen p: $\bar{\nu} = \mu = np$

P 13/05/2014

Kolikšen je standard deviation za Poissonovo porazdelitev?

$$\sigma_\nu^2 = \sum_{\nu=0}^{\infty}(\nu-\bar{\nu})^2 \cdot e^{-\mu}\frac{\mu^\nu}{\nu!}$$ ⊕ remember that $\boxed{\sigma_x^2 = \overline{x^2} - \bar{x}^2} \Rightarrow \sigma_\nu^2 = \overline{\nu^2} - \bar{\nu}^2 = \overline{\nu^2} - \mu^2$!

tako da rabimo izračunat samo še $\overline{\nu^2}$! Spomnimo se pravila ...

$$\overline{\nu^2} = \sum_{\nu=0}^{\infty}\nu^2 \cdot P_\mu(\nu) = \sum_{\nu=0}^{\infty}\nu^2 \cdot e^{-\mu}\frac{\mu^\nu}{\nu!}$$ → ⊕ (1.) $\sum_{\nu=0}^{\infty} P_\mu(\nu) = 1$ (je normalizirano)

$$\Rightarrow \frac{\partial}{\partial\mu}\left[\sum_{\nu=0}^{\infty} P_\mu(\nu)\right] = 0 \Rightarrow$$

$$\Rightarrow \frac{\partial}{\partial\mu}\left[\sum_{\nu=0}^{\infty} e^{-\mu}\frac{\mu^\nu}{\nu!}\right] = \underbrace{\sum_{\nu=0}^{\infty}(-1)\, e^{-\mu}\frac{\mu^\nu}{\nu!}}_{=-1} + \sum_{\nu=0}^{\infty}\nu\, e^{-\mu}\frac{\mu^{\nu-1}}{\nu!} = 0 \Rightarrow$$

$$\Rightarrow \boxed{\sum^{\infty}\nu\, e^{-\mu}\frac{\mu^{\nu-1}}{\nu!} = 1}$$ - sedaj odvajamo še 1x

$$\Rightarrow \frac{\partial}{\partial\mu}\left[\sum^{\infty}\nu\, e^{-\mu}\frac{\mu^{\nu-1}}{\nu!}\right] = 0 \dots = \frac{\partial}{\partial\mu}\left[\sum e^{-\mu}\frac{\mu^{\nu-1}}{(\nu-1)!}\right] = \sum(-1)e^{-\mu}\frac{\mu^{\nu-1}}{(\nu-1)!}\frac{\mu}{\mu} + \sum e^{-\mu}(\nu-1)\frac{\mu^{\nu-2}}{(\nu-1)!}\cdot\frac{\mu^2}{\mu^2}\cdot\frac{\nu}{\nu}$$

$$= \sum\frac{(-1)}{\mu}\cancel{e^{-\mu}}\cdot\nu\frac{\mu^\nu}{\nu!} + \sum\cancel{e^{-\mu}}\frac{(\nu-1)\nu}{\mu^2}\frac{\mu^\nu}{\nu!} = 0 \Rightarrow \sum\frac{(-1)}{\mu}\cdot\nu\frac{\mu^\nu}{\nu!} + \sum\frac{\nu^2}{\mu^2}\frac{\mu^\nu}{\nu!} - \sum\frac{\nu}{\mu^2}\frac{\mu^\nu}{\nu!} = 0$$

$$\Rightarrow \frac{1}{\mu^2}\sum\nu^2\frac{\mu^\nu}{\nu!} = \left(\frac{1}{\mu^2}+\frac{1}{\mu}\right)\sum\nu\frac{\mu^\nu}{\nu!} = \frac{1}{\mu^2}+\frac{1}{\mu} \Rightarrow$$ ... nek sem fali ... v glavnem na koncu

nekje smo falili, bomo naslednjič pogledali, končni

je $\sum\nu^2\frac{\mu^2}{\nu!} = \mu+\mu^2 \Rightarrow$ ne vem kako ... $\overline{\nu^2} = \sum\nu^2 e^{-\mu}\frac{\mu^\nu}{\nu!} = \mu+\mu^2 \Rightarrow$ rezultat je prav.

$$\Rightarrow \sigma_\nu^2 = \overline{\nu^2} - \bar{\nu}^2 = \mu+\mu^2-\mu^2 = \mu \Rightarrow \boxed{\sigma_\nu = \sqrt{\mu}}$$ STANDARD DEVIATION!

npr. RADIOAKTIVNI RAZPAD ... merimo razpade ... dobimo data-set $[\nu_1 \dots \nu_N]$ ... which μ is best?

$$\text{Prob}(\nu_i) = e^{-\mu}\frac{\mu^{\nu_i}}{\nu_i!} \dots \Rightarrow \text{Prob}(\nu_1,\nu_2,\dots,\nu_N) = e^{-N\mu}\cdot\frac{\mu^{\sum\nu_i}}{\nu_1!\dots\nu_N!}$$ → which μ maximises probability?

⇒ da dobimo μ odvajamo: $$\frac{\partial}{\partial\mu}\text{Prob}(\nu_1\dots\nu_N) = -Ne^{-N\mu}\cdot\frac{\mu^{\sum\nu_i}}{\nu_1!\dots\nu_N!} + e^{-N\mu}\frac{\sum\nu\cdot\mu^{\sum\nu-1}}{\nu_1!\nu_2!\dots\nu_N!} = 0$$

$$\Rightarrow \sum\nu_i\mu^{\sum\nu_i-1} = N\mu^{\sum\nu_i} \Rightarrow \mu = \frac{1}{N}\sum_i\nu_i \qquad \boxed{\mu = \frac{1}{N}\sum_i\nu_i}$$ → acording to the principle of maximum lickehood ... je enaka enačba kot za povprečni μ ... vendar ne mešat ...?

REMARKS about P.D.: - for N Big and p small BINOM. → POISSON.

- when $\mu \to \infty$ $\underline{P_\mu(\nu) \sim G_{\mu,\sqrt{\mu}}(\nu)}$ POISSON → GAUSS ko je μ VELIK!

P.S. rate of events $R = \frac{\nu}{T}$ → v fiziki nas ponavadi zanima rate of ocurences!

example: sample of radioactive Torium emits α-particles at $R = 1'5\,min^{-1}$.

① we observe 2min ... how many ocurancies we expect?

$T = 2min \Rightarrow \mu = 3$ pričakujemo 3 α-delce

② kolikšna je verjetnost da dobimo res 3?

$$P_3(3) = e^{-3}\frac{3^3}{3!} \approx 0'22 = 22\%$$

③ what is probability to have $\mu = 0$ in $T = 2$ min?

$$P_3(0) = e^{-3}\frac{3^0}{0!} = 0'05 = 5\%! \; ; \; P_3(1) \approx 15\% \; ; \; P_3(2) = 22\% \; ; \ldots$$

What is probability to have 5 or more counts?

$$P_3(x \geq 5) = 1 - \left(P_3(0) + P_3(1) + \ldots + P_3(4)\right) \approx \boxed{19\%}$$

for BIG numbers we use Gaussian aproximation!

example: we use the same Torium sample ($R = 1'5\,min^{-1}$) but we mesure for 30min!
We observe 49 events!
He does know the R (what is best guess for decay rate R)

$$R_{BEST} = \frac{49}{30\,min} = \boxed{1'6 \pm 0'2} \quad \left(\text{use } \mu_{30\,min} = 49 \pm \sqrt{49} = 49 \pm 7 \text{ and } R = \frac{\mu}{T}\right)$$

VAJA 11.19: 7075 counts in $T = 120\,min$ (suspicious pipe / we want to know if radioactive)
background mesurment: 3015 counts in 60min

$$\left.\begin{aligned} \nu_{PIPE} &= 7075 \text{ in } T = 120\,min \\ \nu_{BRG} &= 3015 \text{ in } T = 60\,min \end{aligned}\right\} R_{PIPE-BRG} = ?$$

$$R_{PIPE} = \frac{7075 \pm \sqrt{7075}}{120} = 59'0 \pm 0'7\,min^{-1} \; ; \; R_{BRG} = \frac{3015 \pm \sqrt{3015}}{60} = 50'2 \pm 0'9\,min^{-1}$$

sumljivo... mislim da je 1'1

$\Rightarrow R_{PIPE-BRG} = 8'8 \pm 1'3\,min^{-1} \Rightarrow$ IS CONTAMINATED (because we can calculate the probability $P_{2 \times BACKGROUND}(PIPE) = P_{6030}(7075)$ = very small)

VAJA

Radioactive rock - we want to check if it is realy radioactive

$\nu_{ROCK} = 225$ counts in $T = 10\,min \Rightarrow R_{ROCK-BRG} = ?$

$\nu_{BRG} = 90$ counts in $T = 6\,min$

~~$R_{ROCK} = 0'375 \pm 0'025\,hour^{-1} \approx 0'38 \pm 0'03\,hour^{-1}$~~
~~$R_{BRG} = 0'25 \pm 0'03\,hour^{-1}$~~ ~~$R_{ROCK-BRG} = 0'125 \pm$~~

$$P_{150}(225) = e^{-150}\frac{150^{225}}{225!} \approx$$

$$\sim G_{150,\sqrt{150}}(225) = G_{150,12'25}(225)$$

$\Rightarrow$ discrepancy ~~6'15~~ $\Rightarrow$ ~~Prob ~0%~~
približno 5 $\Rightarrow$ Prob ~ 0%

$R_{ROCK} = 1350 \pm 90\,hour^{-1}$

$R_{BRG} = 900 \pm 95\,hour^{-1}$

$$R_{ROCK-BRG} = \boxed{450 \pm 130 \text{ counts/hour}} \quad (= \sqrt{90^2 + 95^2})$$

20/05/2014

Zadnjič je prišlo do zmede pri standardni deviaciji Poisson dis.

$P_\mu(\nu) = e^{-\mu}\frac{\mu^\nu}{\nu!}$ ... hočemo σ_ν!

for and data-set is true $\boxed{\sigma_\nu^2 = \overline{\nu^2} - \bar{\nu}^2} \Rightarrow$ for Poison $\sigma_\nu^2 = \boxed{\overline{\nu^2}} - \mu^2$ (we need this)

We start from normalization condition: $\boxed{\sum_{\nu=0}^{\infty} e^{-\mu}\frac{\mu^\nu}{\nu!} = 1} \to$ odvajamo $\frac{d}{d\mu}[\,]$

$\frac{d}{d\mu}\sum() \Rightarrow \sum e^{-\mu}\frac{\mu^{\nu-1}}{(\nu-1)!} = 1$

$\sum_\nu (-1)\nu\frac{e^{-\mu}}{\mu}\frac{\mu^{\nu-1}\mu}{(\nu-1)!\,\nu} + \sum_\nu \frac{e^{-\mu}}{\mu^2}(\nu-1)\frac{\mu^{\nu-2}\mu^2}{(\nu-1)!\cdot\nu} = \sum(-1)\frac{e^{-\mu}}{\mu}\nu\frac{\mu^\nu}{\nu!} + \sum\frac{\nu^2}{\mu^2}e^{-\mu}\frac{\mu^\nu}{\nu!} - \sum\frac{e^{-\mu}}{\mu^2}\nu\frac{\mu^\nu}{\nu!}$

$= -\frac{1}{\mu}\bar{\nu} + \frac{1}{\mu^2}\overline{\nu^2} - \frac{1}{\mu^2}\bar{\nu} = 0 \Rightarrow \boxed{\overline{\nu^2} = \bar{\nu}(1+\mu) = \mu(1+\mu) = \mu + \mu^2}$

Tako dobimo da $\sigma_\nu^2 = \overline{\nu^2} - \bar{\nu}^2 = \mu + \mu^2 - \mu^2 = \mu \Rightarrow \boxed{\sigma_\nu = \sqrt{\mu}}$ (naredi še 1x sam doma)

HI-SQUARE TEST.

$G_{\bar{x},\sigma}(x) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}$ $\quad B_{n,p}(\nu) = \binom{n}{\nu}p^\nu(1-p)^{n-\nu}$ $\quad P_\mu(\nu) = e^{-\mu}\frac{\mu^\nu}{\nu!}$

We need a mathematical tool to verify the hypotesis that some data-set fits with some distribution. $\Rightarrow$ χ^2-TEST

npr.
merili smo 40x neko stvar (del fotokopije lista iz knjige)... we want to verify that is Gauss. dis.
data-set $\to (\bar{x}, \sigma) \to$ histogram (4 bins) $\to$ ... glej liste.

How many bins to select for histogram? (Rule 1.: expected ocurances of meaorment should be around 5 (minimum))

(Rule 2: minimum number of bins $n \geq 4$)

$\Rightarrow$ iz rule1 in rule2 $\Rightarrow$ minimum 20 meritev za χ^2-test (manj nima smisla)

npr. 200x vržeš kocko (kolikokrat 6 dobiš)
$\to$ ... nisem poslušal.

χ^2 smo že prej videli! (def.: $\chi^2 = \frac{(Obs_k - Exp_k)^2}{\sigma^2}$; least-square $\Rightarrow$ minimize $\chi^2 = \sum\frac{(x_i - A - Bx_i)^2}{\sigma_y^2}$

or with wheighted overages: $\chi^2 = \left(\frac{x_1-\bar{x}}{\sigma_1}\right)^2 + \left(\frac{x_2-\bar{x}}{\sigma_2}\right)^2 \to$ which $\bar{x}, \sigma$ - minimizes

Generaly we can not solve the minimazation proces (for complex or complicated...)

In reality it's practical to use REDUCED $\chi^2 \Rightarrow \boxed{\chi^2 \leq n}$

$\chi^2 \leq$ (degree of freedom for my analysis $\to d = n - c =$ number of ~~measurements~~ bins $-$ N° constrains

ker prej smo naredili tabelo

exp_k	obs_k	$\frac{(e-o)^2}{e}$

TOBA EXP_k ni neodvisna od OBS_k (ker smo Obs_k uporabili za $\bar{x}, \sigma$ in N)

št. informacij, ki smo jih dobili iz data-set
(npr. $N, \bar{x}, \sigma \Rightarrow c = 3$)
$\Rightarrow$ za primer iz lista je $d = 4 - 3 = 1$)

DEF.: REDUCED χ^2: $\boxed{\tilde{\chi}^2 = \frac{\chi^2}{d}}$ expected to be ~ 1

npr: kocka 240x ⇒

1	2	3	4	5	6	
20	46	35	45	42	52	– observed O_k
1/6	1/6	1/6	1/6	1/6	1/6	– P_k
40	40	40	40	40	40	– E_k
10	0'9	0'625	0'625	0'1	3'6	– $\frac{(O_k-E_k)^2}{E_k}$

degree of freedom = 6 − 1 = 5

$\Rightarrow \Sigma = 15'9 \Rightarrow \boxed{\tilde{\chi}^2 \approx 3}$ → 1% to have this or more ⇓ KOCKA JE OBTEŽENA

npr. 3 kocke 400x vržem → število 6-stk zapišem

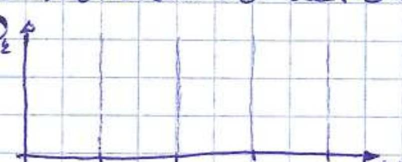
0x 6
1x 6
2 ali 3x 6

3 bins / use Binomial distrib $B_{3,1/6}$ (0 ali 1 ali (2,3)) / tu primer $d = \underset{\text{bins}}{3} - \underset{\text{400 experiments}}{1} = \boxed{2}$?

students wants to verify the exponential law for radioactive activity

τ – we don't know (mean life) (Hypotesis na τ)

T – interval (measurement from $t=0 \to t=T \to t=2T \dots$)



HYPOTESIS: $N(t) = N_0 e^{-t/\tau}$

$E_k = ?$ → end of $(k-1)$ bin we have $N_0 e^{-(k-1)T/\tau}$
→ end of (k) bin we have $N_0 e^{-kT/\tau}$ $\Bigg\} \Rightarrow$ EXPECTED! $\boxed{E_k = N_0 \cdot \left(e^{-T/\tau} - 1\right) \cdot e^{-kT/\tau}}$!

from measurement we get table:

bin #	DESCRIPTION OF BIN	O_k	E_k
1	$0 < t < T$	528	515'2
2	$T < t < 2T$	180	189'5
3	$2T < t < 3T$	71	69'7
4	$3T < t < 4T$	20	25'6
5	$4T < t$	16	14'9
		$\Sigma = 815$	

$d = 5$ bins $- 1$ inf $= 4$ (N_0)

we get: $\Rightarrow \tilde{\chi}^2 - 0'53$

22/05/2014

Ex 12.13 of the book: radiactive decay with 3 min⁻¹ ⇒ Poison $P_\mu(\nu) = e^{-\mu}\frac{\mu^\nu}{\nu!}$ with $\mu = 3$!
100 meritev (koliko events v 1 min) ⇒ tabela data-set

Ex 12.6

2 kocki vržemo 360x → score of each throw is mesured → are dices loaded ?

TOTAL	# of events	verjetnost	E_k = 360·verjetnost
2	6	1/36	10
3	14	2/36	20
4	23	3/36	30
5	35	4/36	40
6	57	5/36	50
7	50	6/36	60
8	44	5/36	50
9	49	4/36	40
10	39	3/36	30
11	37	2/36	20
12	16	1/36	10

$\Rightarrow \chi^2 = 19'8$; d = 11 bins − 1 (# of trials)

$\Rightarrow \tilde{\chi}^2 = \chi^2/d = 1'98$

⇒ 2'9% verjetnost za slabši rezultat ⇒ KOCKE SO VERJETNO OBTEŽENE !

P.S. IZPIT: → short report about free-fall (data-set → $\bar{t}, \sigma_t$ → $\tilde{\chi}^2$ ⊕ error of g ($\bar{g} \pm \sigma_g$))
⊕ final result of from all 4 height and pendulum ⇒ $g_{BEST} \pm \square$
compare with expected value ! (in PDF)
→ writen exam (5/6/2014)
→ oral exam (few days later)

We want to veryfy that is Gauss distribution !

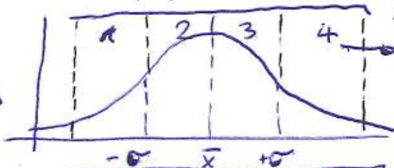
Table 12.1. Measured values of x (in cm).

731	772	771	681	722	688	653	757	733	742
739	780	709	676	760	748	672	687	766	645
678	748	689	810	805	778	764	753	709	675
698	770	754	830	725	710	738	638	787	712

} 40 meritev

we can calculate average, std. ... => $\overline{X} = 730'1\,cm$; $\sigma = \sqrt{\frac{\sum(x-\bar{x})}{39}} = 46'8\,cm$

=> we put data in a histogram =>



→ 4 bins
- maj kot $\bar{x}-\sigma$
- med $\bar{x}-\sigma$ in $\bar{x}$
- med $\bar{x}$ in $\bar{x}+\sigma$
- več kot $\bar{x}+\sigma$

16%	34%	34%	16%

=> we can find the probability for a mesurment to fall in specific bin

=> after 40 mesurments we expect | 6'4 | 13'6 | 13'6 | 6'4 | - EXPECTED

we see from data set that | 8 | 10 | 16 | 6 | - TRUE

=> ! expected values are poisson diss (counting experiment) =>

=> σ (of bin) = $\sqrt{\text{expected}}$ => σ → $\boxed{\sqrt{6'4} / \sqrt{13'6} / \sqrt{13'6} / \sqrt{6'4}}$

We can verify the discrepancy in each bin and find the probability...

$\Rightarrow$ data set discrepancy $\boxed{\quad}$

razlika $\boxed{1'6 \mid -3'6 \mid 2'4 \mid -0'4}$ we look at

$$\frac{(\text{observed} - \text{expected})^2}{\text{expected}} \sim 1$$

$\sigma^2 = (\sqrt{\text{expec}})^2$.

pričakujemo, da bo reda 1

$$\Rightarrow \chi^2 = \sum_k \frac{(obs_k - exp_k)^2}{exp_k} \sim n$$

in this case we get 1'8 !

ps. degrees of freedom = # bins − # constrains = $4 - 3 = \boxed{1}$

št podatkov, ki smo jih morali uporabiti iz data-set!