

# Show the proposed algorithm has the claimed running time by making all the computer verification explicit

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## ABSTRACT

This project will carry out the process of theoretical reproduction in the field of algorithms. This project will focus on the field of #2SAT. It will design a complete computerized validation process for that paper against the theories from previous important papers by prominent contributors in the field. Since there are few papers in the field that explains the reasoning process in detail, a large number of calculations and inferences are embedded in theorems. Therefore, sharing codes that reproduce the reasoning process is more challenging. This is not only a validation of the work of previous eminent workers in the field but also lays the foundation for later readers and learners in the field to avoid the appearance of repetition due to a large and cumbersome computational process. This is why it is essential to produce a complete computer verification.

This project will simulate the workflow of an oracle machine through computer code, replicate the recursive logic of the paper, and accurately reduce a large number of branches in the paper to a solvable level through recursive algorithms. Verify the correctness of the thesis results by classifying the branch results in each case to determine if they are consistent with the original thesis results.

On hardware systems, with the development of computer computing power, the amount of computing that was previously not readily available has become very easy. Thus the recursive calculations also become more reproducible than when the original paper was published. This also provides the hardware basis for the conduct of this project.

For the evaluation of the validation of the results, since this project is a factual validation of existing theories, the results are compared according to the original thesis. If the results are entirely consistent, it is proved that the results are accurate in the original hypothesis, thus also proving the correctness of the original theory. If the results are not entirely consistent, then both the accuracy of the results of this project and the accuracy of the original theory need to be checked.

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## Author Keywords

#SAT; #2SAT; graph theory; complexity theory

## INTRODUCTION

First, the project will design a computer validation process of Magnus Wahlström's work in 2004.[3] This project will focus on the #2-SAT area of the algorithm domain, which will be covered in detail later in this introduction.

Most algorithm designs are algorithm designs for decision problems. For example, to find a solution that makes a Boolean formula satisfying. By finding a satisfying answer to a Boolean expression, we mean that given an arbitrary Boolean expression, such as  $A \vee B$ , one of the solutions that can make its result to be true if A is true, and B is true. This is the SAT question in the algorithmic field. SAT is the first issue that was demonstrated to be NP-complete.[1] As we all know, P-class is a fundamental complexity class that is verifiable by a deterministic Turing machine. However, NP is a generalization of P, which the lesson of choice problems decidable by a non-deterministic Turing machine that runs in polynomial time. A decisive question that is NP-complete means that it is complete for NP, which means that any question that is NP can be reduced to it in polynomial time.

Let us go back to the SAT problem. Going further, we will not only be content to find out if we can satisfy a particular Boolean expression, but we are trying to find out exactly how many solutions can satisfy that expression. This is the #SAT question, brought up by Valiant in 1979.[5] Valiant, meanwhile, raised the issue that this is a #P-complete.

To find the final solution to a complete Boolean expression, we split out each of the propositional variables. Each propositional variable can contain either true or false. We define a literal to denote both a propositional variable  $x$  and its negation  $\neg x$ . A disjunction of literals is defined as a clause. And a conjunctive normal form, short for CNF, is a conjunction of clauses.

So we can represent some special case SAT questions, such as if each clause contains at most 2 literals, then we call this formula a 2-SAT formula. A more general representation is that if each clause contains at most no more than  $k$  literals

in a hypothetical CNF, then we call it a kSAT formula ( $k > 0$ ). The #2-SAT question of concern for this project can then be expressed as: how many possible solutions are there to make the formula satisfy a maximum of 2 literals per clause in any given proposed formula. Take for example the following.

$$(x_0 \vee x_1) \wedge (x_1 \vee x_2) \wedge (x_2 \vee x_3) \wedge (x_4)$$

Since we cannot give specific conclusions for all Boolean expressions, and indeed it is impossible, in this research area, we usually design a computational model to calculate the time complexity  $O$  of this model method. The time complexity usually describes the running time of an algorithm in a worst-case scenario. Computational models get different results in many branches, and we usually verify the worst time complexity to determine the time complexity of this algorithm.

In this project, the #2SAT algorithm is explored from the initial upper bound of  $O(2^n)$ , as proposed by Dubois, Zhang, Littman, and Dahllöf[2] et al. The scheme is continuously optimized to propose  $O(1.3247^n)$ , until this paper we forced uses the computer-verified work of Magnus Wahlström, who accelerated the algorithmic model of #2SAT to  $O(1.2561^n)$  in 2004[3].

However, research in this field is purely theoretical, and the literature and papers are full of mathematical expressions and model reasoning. Papers in this field do not usually provide a specific computer code reproduction process for the given model, a process that is often hidden in the deductions of formulas, and it is difficult for later readers to rely solely on this paper to reproduce the full process of deductions. Moreover, as the theoretical research progresses, the inability to replicate the work of previously distinguished practitioners will have a very significant impact on subsequent research.

A survey of previous research in the field found that theory proponents tend to present only descriptions of algorithmic models, which in turn mostly use recursive algorithms, a method of solving problems by repeatedly decomposing them into subproblems of the same kind. The advantage of recursive algorithms is that they often effectively divide significant branching problems into small branching problems, and then solve them by targeting each branching problem that can be effectively focused on. However, recursive algorithms also have an irreparable disadvantage, if the recursive algorithm is simulated manually and the results are calculated manually, this leads to a considerable amount of computation, making it difficult or even impossible for later readers to verify the correctness of the reasoning process.

With the refinement of computer science and programming languages, we now can use more sophisticated language tools to refine the verification recursion problem. Moreover, with the increase in computer computing power in recent years, running large-scale recursion is no longer difficult. So both the theoretical basis and the hardware facilities were prepared for this project.

So an experimental replication of the reasoning process in this area of research will be very necessary. This is not only an experimental corroboration of the important theories of previous distinguished contributors but also an important reference for future continuing researchers in the field.

The project will be conducted based on the reading and validation of the thesis, which involves the validation of the different branches of Wahlström's work. (See Gantt chart). The initial design will be organized into a brief thesis validation report in the form of an algorithmic code design ensemble and proof draft, followed by specific code writing and validation.

As for the software required, python and related computing packages will be selected for this project because of the simplicity and ease of writing python. Due to the special nature of this project, the project does not require the operational efficiency of a complete project, only the verification of results, and therefore python has the advantage over c and java. The latest version of 3.8.2 will be chosen because the project will provide as much as possible a reasonable interpretation of previous outstanding work for future researchers in the field, so choosing the latest version of python will avoid creating a gap for future readers. As for the hardware part, since this project is a reproduction of a theoretical research example, there are no special hardware equipment requirements, just a computer that can run python.

A trial prototype will be made in June (see Gantt chart). In order to test our hypothesis, we will conduct quantitative and qualitative evaluations. Since the proof inference contains numerous branches, a defined structure will be designed for each branch to obtain the final result. We also hope to obtain predicted results from the computational model by randomly generating Boolean expressions to assess the validity of the computationally validated model that we produced.

The evaluation criteria will be determined by the degree of branching that completes the validation. Since the papers that we need to compute validation have numerous branches, we need to validate them item by item. If the results of the paper we verify are all correct, then the results of the calculations in all branches should be the same as predicted by the process of theoretical reasoning. If the results are different through computer code validation, then we need to discuss whether there was a problem with our step-by-step approach or with the original theoretical work.

The evaluation criteria will be determined by the degree of branching that completes the validation. Since the paper we need to verify numerous branches computationally, we need to verify the reasonableness of each branch item by item, comparing the results of the computational branches with the results of the original paper. If the results of the paper we verify are all correct, then the results of the calculations in all branches should be the same as predicted by the process of theoretical reasoning. If the results are different through computer code validation, then we need to discuss whether there was a problem with our step-by-step approach or with the original theoretical work.

## LITERATURE REVIEW

Algorithms on counting problems had continued to evolve since the 1860s when Ryser proposed the first counting algorithm[4], and Ryser pioneered the counting problem algorithm by proposing a time complexity of  $A$  for counting perfectly matched numbers in a binary graph. In the late 1870s, Valiant concluded that the complexity of the counting problem makes it a #P class problem and can be statute as a #P problem, so it is a #P-complete problem[5].

Further, in the work of Magnus Wahlstrom et al. in 2004, they designed a computational model that reduced the temporal complexity of #2SAT to  $O(1.2561^n)$  [3]., which is a great achievement. In this mathematical model, they designed a set of weights to measure the impact of each branch in the CNF on the final result. Under the influence of such a weighting model, the CNF can be continuously reduced by the recursive algorithm, thus speeding up the algorithm. This weighting model allows us to split a constraint graph into its dual connected components. Among other things, this provides a way to remove variables that appear only once in the formula during the polynomial time. On the other hand, the model can condense formula complexity into a single value containing the number of variables and variable clauses, which is more reflective of the complexity of a CNF than the original way of expressing formula complexity in numbers only, without regard to clause weights.

Since the main process of this project was to replicate the process of this paper and to simulate the validation with a computer program, our first task was to investigate what the technical line of this paper was. First let's introduce the computational tools and computational models he uses.

First of all, as introduced in the introduction above, the fundamental conceptual element of the field in which the project is located is literal, which contains either a variable  $x$  or its negation  $\neg x$ . A clause is the disjunction of literals, and then the conjunction of multiple clauses forms the conjunction normal form, abbreviated to CNF. If each clause in a CNF contains up to  $k$  characters, we can call this CNF the  $k$ -SAT formula. For example.

$$\begin{aligned} 2\text{-SAT: } & (A \vee B) \wedge (B \vee C) \\ 3\text{-SAT: } & (A \vee B \vee C) \wedge (B \vee C \vee D) \end{aligned}$$

In this paper, we define the degree  $d(x)$  of a variable  $x$  in formula  $F$  as the number of clauses in  $F$  containing  $x$ . e.g.: in  $(A \vee B) \wedge (B \vee C)$   $d(B)=2$ . And the maximum degree of any variable in  $F$  is  $d(F)$ , the number is  $n_d(F)$ .

In this way, we get a method to measure formula complexity:

$$m(F) = \sum_{x \in \text{Var}(F)} d(x)$$

We define a model  $M$  for  $F$ , a set  $L$  of all literals in  $F$ , and a weight vector  $w$ :

$$W(M) = \sum_{\{l \in L | l \text{ is true in } M\}} w(l)$$

Be the same, we get a cardinality vector  $c$  for  $F$ :

$$C(M) = \prod_{\{l \in L | l \text{ is true in } M\}} c(l)$$

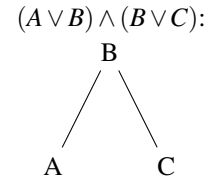
In this way, we get a weighted model of  $F$  for #2-SAT(#3-SAT):

$$\#2\text{SAT}_w(F, c, w) = (\sum_{M \in S} C(M), W(M'))$$

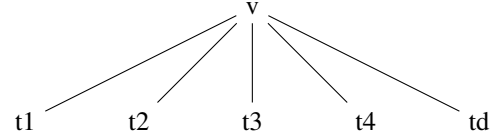
At the same time, Magnus Wahlström introduced the concept of some graphs to illustrate the relationship between branches and branches better. Graphs are another way we look at Boolean expressions. We define a constraint graph of a formula:

$$\{(a, b) | a \text{ and } b \text{ occur together in at least one clause}\}$$

Connect: A formula  $F$  is connected iff in the corresponding constraint graph, there is a path from each variable to every other. As follow is a tree for



Next, after the model style is built, we start calculating the temporal complexity of the branch.



Think of a tree of formula. There is a node  $v$  which has  $d$  branches children. Each branch is labeled by recursion complexity positive real number  $(t_1, t_2, \dots, t_d)$ , the branching number is the positive real-valued solution of

$$\sum_{i=1}^d x^{-t_i} = 1$$

We defined the branching number of tuple  $(t_1, t_2, \dots, t_d)$  is  $\tau(t_1, t_2, \dots, t_d)$  ie.  $\tau(4, 4) \rightarrow x = \sqrt[4]{2} \approx 1.1892$

In order to avoid the result deviation caused by the simplified formula, the prof formula is introduced, and the objective formula is simplified on the basis of increasing the weight.

1. If  $F$  contains an empty clause then  $F := (\emptyset)$ ,  $c := 0$  and  $w := 0$
2. If there is a clause  $(1 \vee \dots)$ , then it is removed. If any variable  $a$  thereby gets removed then there are three cases:
  - (a) If  $w(a) = w(\neg a)$  then  $c := c \cdot (c(a) + c(\neg a))$ ;  $w := w + w(a)$
  - (b) If  $w(a) < w(\neg a)$  then  $c := c \cdot c(\neg a)$ ;  $w := w + w(\neg a)$
  - (c) If  $w(a) > w(\neg a)$  then  $c := c \cdot c(a)$ ;  $w := w + w(a)$
3. If there is a clause  $(0 \vee \dots)$ , then 0 is removed from it.
4. If there is a clause  $(a)$ , then it is removed and  $c := c \cdot c(a)$ ,  $w := w + w(a)$ . If  $a$  still appears in  $F$  then  $F := F[a = 1]$

5. If there are two clauses  $x = (a \vee b \vee a')$ ,  $y = (a \vee b)$  then remove  $x$ . If the variable  $a'$  thereby gets removed then handle it as in case 2.

After some steps of simplification and proof we get Lemma 1:

[Lemma 1] Let  $(F', c, w) = Prop(F, c, w)$   
and  $(c', w') = \#2SAT_w(F', c, w)$ .  
Then  $\#2SAT_w(F, c, w) = (c * c', w + w')$ .

In order to simplify the formula  $F$ , we have  $F_1, F_2$ , such that  $Var(F_1) \cap Var(F_2) = \{v\}$ .

1. Let  $(c_t, w_t) = \#2SAT_w(F_1[v=1], \mathbf{c}, \mathbf{w})$  and  $(c_f, w_f) = \#2SAT_w(F_1[v=0], \mathbf{c}, \mathbf{w})$
2. Modify  $\mathbf{c}$  and  $\mathbf{w}$  so that  $c(v) \leftarrow c_t \cdot c(v)$ ,  $c(\neg v) \leftarrow c_f \cdot c(\neg v)$ ,  $w(v) \leftarrow w_t + w(v)$  and  $w(\neg v) \leftarrow w_f + w(\neg v)$
3. Finally, return  $\#2SAT_w(F_2, \mathbf{c}, \mathbf{w})$

This procedure is to remove  $F_1$  by multiplier reduction. And we get Lemma 2:

[Lemma 2] Applying multiplier reduction does not change the return value of  $\#2SAT_w(F, c, w)$ .

According to Lemma 1 and Lemma 2, we can naturally get Lemma 3:

[Lemma 3] The result of recursively branching on the variable  $v$  in the formula  $F$  equals  $\#2SAT_w(F, c, w)$ .

Since  $d(F)$  is discrete, it is discussed in three cases:

1. the main function:  $d(F) > 5$
2. help function1  $4 \leq d(F) \leq 5$
3. help function2  $d(F) \leq 3$

At First, we need to solve the  $C_3(F, c, w)$ . Algorithm  $C_3(F, \mathbf{c}, \mathbf{w})$  as follow:

1. Case 1: If  $F$  contains no clauses, return  $(1, 0)$ . If  $F$  contains an empty clause, return  $(0, 0)$ .
2. Case 2: If  $F$  is not connected, return  $(c, w)$  where  $c = \prod_{i=0}^j c_i$ ,  $w = \sum_{i=0}^j w_i$  and  $(c_i, w_i) = C(F_i, \mathbf{c}, \mathbf{w})$  for the connected components  $F_0, \dots, F_j$
3. Case 3: If multiplier reduction applies, apply it, removing the part with lowest  $n_3(F)$  value.
4. Case 4: If  $d(F) = 3$ , pick a variable  $x$ ,  $d(x) = 3$ , with as many neighbours of degree 3 as possible, and recursively branch on it. Otherwise, recursively branch on any variable.

By calling the loop recursively, we could get Lemma 5.

[Lemma 5]  $C_3(F, c, w)$  runs in  $O(ploy(n) * \tau(4, 4)^{n_3(F)})$  time, where  $p(n)$  is a polynomial in  $n$ .

In the same way, we can get algorithm  $C_5(F, \mathbf{c}, \mathbf{w})$

1. Case 1: If  $F$  contains no clauses, return  $(1, 0)$ . If  $F$  contains an empty clause, return  $(0, 0)$ .

2. Case 2: If  $F$  is not connected, return  $(c, w)$  where  $c = \prod_{i=0}^j c_i$ ,  $w = \sum_{i=0}^j w_i$  and  $(c_i, w_i) = C(F_i, \mathbf{c}, \mathbf{w})$  for the connected components  $F_0, \dots, F_j$
3. Case 3: If  $d(F) < 4$ , return  $C_3(F, \mathbf{c}, \mathbf{w})$
4. Case 4: If multiplier reduction applies, apply it, removing the part with lowest  $f(F)$  value.
5. Case 5: Pick a variable  $x$  of maximum degree such that  $S(x)$  is maximized. (a) If  $N(x)$  is connected to the rest of the graph through only 2 external vertices  $y, z$  such that  $d(y) \geq d(z)$  then branch on  $y$ . (b) Otherwise, branch on  $x$ .  $S(x) = \sum_{y \in N(x)} d(y)$

From the quotient of the complexity of the formula and the maximum number of branches, it can be inferred that the larger the quotient, the greater the time complexity of the worst case.

So discuss the worst case of C5 in the value range of linear function  $f(n, m)$ , where  $n = n(F)$  and  $m = m(F)$ .

We need a sequence of worst cases as the  $m/n$  quotient increases, and with each worst case we associate a linear function  $f_i(n, m) = a_i n + b_i m$ .

$$f(n, m) = f_i(n, m) \text{ if } k_i < m/n \leq k_{i+1}, \quad 0 \leq i \leq 9$$

$$f_i(n, m) = \chi_i n + (m - k_i n) b_i, \quad 0 \leq i \leq 9$$

$$\chi_0 = 0$$

$$\chi_i = \chi_{i-1} + (k_i - k_{i-1}) b_{i-1}, \quad 1 \leq i \leq 10$$

$$a_i = \chi_i - k_i b_i$$

$T_{ab} \ k_i, \chi_i$ and running times			
$i$	$k_i$	$\chi_i$	Running time
0	0	0	$O(1)$
1	2	0	$O(poly(n))$
2	3	1	$O(1.1892^n)$
3	3.5	1.1340	$O(1.2172^n)$
4	3.75	1.1914	$O(1.2294^n)$
5	4	1.2410	$O(1.2400^n)$
6	$4 + 4/29$	1.2536	$O(1.2427^n)$
7	$4 + 4/9$	1.2788	$O(1.2481^n)$
8	$4 + 4/7$	1.2881	$O(1.2481^n)$
9	4.8	1.3033	$O(1.2501^n)$
10	5	1.3154	$O(1.2534^n)$

We have got  $a_i, b_i, k_i$ . We need a lemma that allows us to make a connection between the value of  $m(F)/n(F)$  and worst-case branchings.

[Lemma 6] Let  $F$  be a non-empty formula such that  $m(F)/n(F) = k$ , and define  $\alpha(x)$  and  $\beta(x)$  such that

$$\alpha(x) = d(x) + |\{y \in N(x) | d(y) < k\}|$$

$$\beta(x) = 1 + \sum_{\{y \in N(x) | d(y) < k\}} 1/d(y)$$

There exists some variable  $x \in Var(F)$  with  $d(x) \geq k$  such that  $\alpha(x)/\beta(x) \geq k$

We will need to prove that the worst-case branching number in each section is  $\tau(4, 4)$ . For different ranges of  $m/n$  values, we need to show that when  $m/n \leq 5$ , the time complexity is less than  $O(1.2561^n)$ . So we broke down a number of situations.

$b_i$  and  $a_i$  parameters

$i$	$b_i$ , definitions	$b_i$	$a_i$
0	0	0	0
1	1	1	-2
2	$\tau(1 + 5b_2, 5 + 5b_2) = \tau(4, 4)$	0.2680	0.1961
3	$\tau(\chi_3 + 4.5b_3, 5\chi_3 + 4.5b_3) = \tau(4, 4)$	0.2295	0.3308
4	$\tau(\chi_4 + 4.25b_4, 5\chi_4 + 5.25b_4) = \tau(4, 4)$	0.1987	0.4461
5	$\tau(\chi_5 + 6b_5, 6\chi_5 + 2b_5) = \tau(4, 4)$	0.0914	0.8755
6	$\tau(\chi_6 + (5 + 25/29)b_6, 6\chi_6 + (3 + 5/29)b_6) = \tau(4, 4)$	0.0821	0.9139
7	$\tau(\chi_7 + (5 + 5/9)b_7, 6\chi_7 + (3 + 1/3)b_7) = \tau(4, 4)$	0.0736	0.9517
8	$\tau(\chi_8 + (5 + 3/7)b_8, 6\chi_8 + (4 + 4/7)b_8) = \tau(4, 4)$	0.0665	0.9841
9	$\tau(\chi_9 + 5.2b_9, 6\chi_9 + 5.2b_9) = \tau(4, 4)$	0.0602	1.0143

1. Case:  $m/n \leq 2$
2. Case:  $m/n \in [2, 3]$
3. Case:  $d(F) = 5$  which gets a branching number less than  $\tau(4, 4)$ .
4. Case:  $m/n \in [3, 3.5], d(F) = 4$
5. Case:  $m/n \in [3.5, 3.75], d(F) = 4$
6. Case:  $m/n \in [3.75, 4], d(F) = 4$ .
7. Case:  $m/n \in [4, 4 + 4/29]$
8. Case:  $m/n \in [4 + 4/29, 4 + 4/9]$
9. Case:  $m/n \in [4 + 4/9, 4 + 4/7]$
10. Case:  $m/n \in [4 + 4/7, 4.8]$
11. Case:  $m/n \in [4.8, 5]$

After all of the above branches have been recursively calculated, we can deduce the conclusion.

[Theorem 11]  $C(F, \mathbf{c}, \mathbf{w})$  runs in  $O(1.2561^n)$  time.

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The heading of a section should be in Helvetica or Arial 9-point bold, all in capitals. Sections should *not* be numbered.

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Place figures and tables at the top or bottom of the appropriate column or columns, on the same page as the relevant text (see Figure 1). A figure or table may extend across both columns to a maximum width of 17.78 cm (7 in.).

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and notes may use color figures, which are included in the page limit; the figures must be usable when printed in black-and-white in the proceedings.

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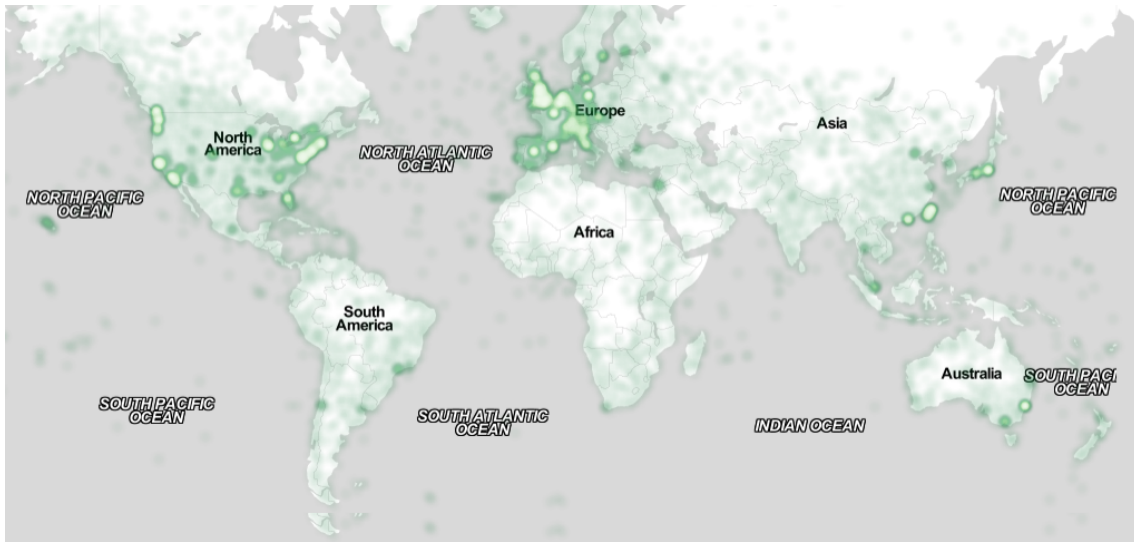


Figure 2. In this image, the map maximizes use of space. You can make figures as wide as you need, up to a maximum of the full width of both columns. Note that L<sup>A</sup>T<sub>E</sub>X tends to render large figures on a dedicated page. Image: © ⓘ ayman on Flickr.

advice and examples regarding gender and other personal attributes [?]. Be particularly aware of considerations around writing about people with disabilities.

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## CONCLUSION

It is important that you write for the SIGCHI audience. Please read previous years’ proceedings to understand the writing style and conventions that successful authors have used. It is particularly important that you state clearly what you have done, not merely what you plan to do, and explain how your work is different from previously published work, i.e., the unique contribution that your work makes to the field. Please consider what the reader will learn from your submission, and how they will find your work useful. If you write with these questions in mind, your work is more likely to be successful, both in being accepted into the conference, and in influencing the work of our field.

## ACKNOWLEDGMENTS

Sample text: We thank all the volunteers, and all publications support and staff, who wrote and provided helpful comments on previous versions of this document. Authors 1, 2, and 3 gratefully acknowledge the grant from NSF (#1234–2012–ABC). *This whole paragraph is just an example.*

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