I want to compare the eigenvalues of a linear system compared to the same linear system smoothed by a linear kernel

Let x_t evolve according to linear dynamics

$x_t = Ax_{t-1}$

Convolve x_t with filter h such that

$h = \begin{bmatrix} \frac{8}{15} \\ \frac{4}{15} \\ \frac{2}{15} \\ \frac{1}{15} \end{bmatrix}$

$x_t * h = \frac{8}{15}x_t + \frac{4}{15}x_{t-1} + \frac{2}{15}x_{t-2} + \frac{1}{15}x_{t-3}$

$x_t * h = \frac{8}{15}Ax_{t-1} + \frac{4}{15}Ax_{t-2} + \frac{2}{15}Ax_{t-3} + \frac{1}{15}Ax_{t-4}$

$x_t * h$ can be described as an AR(4) process with a delay embedding of x_t

$\begin{bmatrix} x_t \\ x_{t-1} \\ x_{t-2} \\ x_{t-3} \end{bmatrix} = \begin{bmatrix} \frac{8}{15}A & \frac{4}{15}A & \frac{2}{15}A & \frac{1}{15}A \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \\ x_{t-3} \\ x_{t-4} \end{bmatrix}$

Compare the eigenvalues of x_t and $x_t * h$

Eigenvalues of an AR(4) model (I derived this in lds_eigenvalues.lyx) where W_i is the weights for each lag i

$\left|\sum_{i=1}^{g} \frac{1}{\lambda^{i-1}} W_i - \lambda I\right| = 0$

Eigenvalues for the original system are the eigenvalues of A

$|A - \lambda I| = 0$

The eigenvalues for the smoothed system are the solutions of

$\left|\frac{8}{15}A + \frac{4}{15\lambda}A + \frac{2}{15\lambda^2}A + \frac{1}{15\lambda^3}A - \lambda I\right| = 0$

$\left|\left(\frac{8}{15} + \frac{4}{15\lambda} + \frac{2}{15\lambda^2} + \frac{1}{15\lambda^3}\right)A - \lambda I\right| = 0$

$\left|\left(\frac{8\lambda^3}{15\lambda^3} + \frac{4\lambda^2}{15\lambda^3} + \frac{2\lambda}{15\lambda^3} + \frac{1}{15\lambda^3}\right)A - \lambda I\right| = 0$

$\left|\frac{8\lambda^3+4\lambda^2+2\lambda+1}{15\lambda^3}A - \lambda I\right| = 0$

$\left|\frac{(2\lambda+1)(4\lambda^2+1)}{15\lambda^3}A - \lambda I\right| = 0$