

Model

$$x_t = Ax_{t-1} + Bu_t + b + w_t$$

$$y_t = Cx_t + Du_t + d + v_t$$

$$w_t \sim \mathcal{N}(0, \Sigma_w)$$

$$v_t \sim \mathcal{N}(0, \Sigma_v)$$

Constraints

C is the identity

D is the 0 matrix

B is diagonal

When you want to consider multiple time points in the past, append older values of x to the state vector
i.e. for 2 lags in the past for dynamics and 3 lags for inputs:

$$\hat{x}_t = \hat{A}\hat{x}_{t-1} + \hat{B}\hat{u}_t$$

$$\hat{x}_t = \begin{bmatrix} x_t \\ x_{t-1} \end{bmatrix}$$

$$\hat{A} = \begin{bmatrix} A_1 & A_2 \\ I & 0 \end{bmatrix}$$

$$\hat{B} = \begin{bmatrix} B_1 & B_2 & B_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\hat{u}_t = \begin{bmatrix} u_t \\ u_{t-1} \\ u_{t-2} \end{bmatrix}$$

$$\begin{bmatrix} x_t \\ x_{t-1} \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ I & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \end{bmatrix} + \begin{bmatrix} B_1 & B_2 & B_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_t \\ u_{t-1} \\ u_{t-2} \end{bmatrix}$$