Including multiple lags in a linear dynamical system (LDS)

Standard LDS:

$$x_t = Ax_{t-1} + Bu_t + w_t \tag{1}$$

$$y_t = Cx_t + v_t \tag{2}$$

$$w_t \sim \mathcal{N}(0, Q), v_t \sim \mathcal{N}(0, R) \tag{3}$$

Where at time t, x_t is the state vector, u_t is the input vector, w_t is the vector of added noise.

Now consider an LDS which integrates 3 previous time points in the past of x and 2 time points in the past from the inputs u

$$x_t = A_1 x_{t-1} + A_2 x_{t-2} + A_3 x_{t-3} + B_1 u_t + B_2 u_{t-1} + w_t \tag{4}$$

$$y_t = Cx_t + v_t \tag{5}$$

$$w_t \sim \mathcal{N}(0, Q), v_t \sim \mathcal{N}(0, R) \tag{6}$$

You can rewrite this LDS in the form of the standard LDS, with an expanded input vector $\bar{u}$ and latent space $\bar{x}$

$$\bar{x}_t = \bar{A}\bar{x}_{t-1} + \bar{B}\bar{u}_t + w_t \tag{7}$$

$$\bar{y}_t = \bar{C}\bar{x}_t + v_t \tag{8}$$

$$w_t \sim \mathcal{N}(0, \bar{Q}), v_t \sim \mathcal{N}(0, \bar{R}) \tag{9}$$

Where the parameters have block structure

$$\bar{x}_t = \begin{bmatrix} x_t \\ x_{t-1} \\ x_{t-2} \end{bmatrix}, \bar{u}_t = \begin{bmatrix} u_t \\ u_{t-1} \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} A_1 & A_2 & A_3 \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix}, \bar{B} = \begin{bmatrix} B_1 & B_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \bar{C} = \begin{bmatrix} C & 0 & 0 \end{bmatrix}$$

$$\bar{Q} = \begin{bmatrix} Q & 0 & 0 \\ 0 & \iota I & 0 \\ 0 & 0 & \iota I \end{bmatrix}, \bar{R} = \begin{bmatrix} R & 0 & 0 \\ 0 & \iota I & 0 \\ 0 & 0 & \iota I \end{bmatrix}$$

With ι some very small number to keep the covariances invertible