The results of CAV can not directly extend to NRA theory

```
In[ ]:= verifyResult[precondition_, invariant_, guardcond_, loopcond_,
  postcondition_, f_, vars_, domain_] := Module[{ce1, ce2, ce3},
  ce1 = FindInstance[Not[Implies[domain && precondition, invariant]], vars];
  ce2 = Table[FindInstance[Not[Implies[domain && invariant &&
         guardcond ≤ 0 && And @@ (LessEqual[#, 0] & /@ Flatten[{loopcond[[i]]}]),
        invariantnext = invariant /. Table[vars[[j]] → f[[i]][[j]], {j, 1, Length[vars]}]]],
     vars], {i, 1, Length[loopcond]}];
  ce3 = FindInstance[
    Not[Implies[domain && invariant && guardcond > 0, postcondition]], vars];
  Print[ce1, ce2, ce3];
  If[Length[Flatten[Join[ce1, ce2, ce3]]] == 0,
   Print["Verified."],
   Print["Numerical Error Deteced."]
  ];
 ]
```

cohendiv

```
In[ ]:= vars2 = {vx, vy, vq, vr, vd};
domain2 = And @@ Table[vars2[[i]]^2 - 50^2 ≤ 0, {i, 1, Length[vars2]}];
guardcond2 = -1;
loopcond2 = {-vr + vy};
f2 = {{vy, vd, vr - vy * vd, vq + vd, vx}};
precond2 = vr - vx ≤ 0 && -vr + vx ≤ 0 && vq ≤ 0 && -vq ≤ 0;

postcond2 = 1 ≥ 0;

inv2 = vx == vq * vy + vr;
verifyResult[precond2, inv2, guardcond2, loopcond2, postcond2, f2, vars2, domain2];
```

$$\{\}\left\{\left\{\left\{vx \to -\frac{2}{21}, vy \to -\frac{1}{7}, vq \to -\frac{2}{15}, vr \to -\frac{4}{35}, vd \to \frac{2}{105}\right\}\right\}\right\}\{\}$$

```
Numerical Error Deteced.
```

fermat2

In[ ]:=

```
vars6 = {rr, u, v, r, n};
domain6 = And @@ Table[vars6[[i]]^2 - 50^2 ≤ 0, {i, 1, Length[vars6]}];
guardcond6 = -1;
loopcond6 = {-r + 1, r};
f6 = {{rr, u, 2 + v, r - v, n}, {rr, 2 + u, v, r + u, n}};
precond6 = -1 - 2 rr + u ≤ 0 && 1 + 2 rr - u ≤ 0 && -1 + v ≤ 0 &&
    1 - v ≤ 0 && n + r - rr^2 ≤ 0 && -n - r + rr^2 ≤ 0 && n ≥ (rr - 1)^2 + 1 && n ≤ r^2;

postcond6 = 1 ≥ 0;

inv6 = 4 n == u^2 - v^2 - 2 * u + 2 * v
verifyResult[precond6, inv6, guardcond6, loopcond6, postcond6, f6, vars6, domain6];
```

Out[ ]=

$$4\,n == -2\,u + u^2 + 2\,v - v^2$$

$$\left\{\left\{rr \to -\frac{1}{2},\ u \to 0,\ v \to 1,\ r \to -\frac{15}{4},\ n \to 4\right\}\right\}\left\{\left\{\left\{rr \to -50,\ u \to -\frac{92}{25},\ v \to -\frac{138}{25},\ r \to 1,\ n \to -\frac{644}{125}\right\}\right\},\right.$$
$$\left.\left\{\left\{rr \to -50,\ u \to -\frac{92}{25},\ v \to -\frac{138}{25},\ r \to -50,\ n \to -\frac{644}{125}\right\}\right\}\right\}\{\}$$

```
Numerical Error Deteced.
```

lcm

In[ ]:=

```
vars12 = {a, b, x, y, u, v, c};
domain12 = And @@ Table[vars12[[i]]^2 - 100^2 ≤ 0, {i, 1, Length[vars12]}];
guardcond12 ={{-x + y}, {x - y}};
loopcond12 = {-1};
f12 = {{a, b, x - y, y, u, u + v, c}, {a, b, x, -x + y, u + v, v, c}};
precond12 = -a + x ≤ 0 && a - x ≤ 0 && -b + y ≤ 0 && b - y ≤ 0 &&
    -b + u ≤ 0 && b - u ≤ 0 && v ≤ 0 && -v ≤ 0 && -a b + c ≤ 0 && a b - c ≤ 0;

postcond12 =- c + u x + v y ≤ 0 && c - u x - v y ≤ 0;

inv12 = u * y + v * y == a * b;
verifyResult[precond12, inv12,
  guardcond12, loopcond12, postcond12, f12, vars12, domain12];
```

```
{{a → -11, b → -5, x → -11, y → -5, u → -5, v → 0, c → 55}}
 {{{a → 0, b → 0, x → -1, y → -1, u → -1, v → 1, c → -100}}}{}
Numerical Error Deteced.
```

ex_sqrt

In[ ]:=
```
vars18 = {su, t, n, a};
domain18 =
  su ≥ -100 && su ≤ 100 && t ≥ -100 && t ≤ 100 && a ≥ -100 && a ≤ 100 && n ≥ 0 && n ≤ 100;
guardcond18 ={-1};
loopcond18 = {su - n};
f18 = {{su + t + 2, t + 2, n, a + 1}};
precond18 = a ≤ 0 && -a ≤ 0 && su - 1 ≤ 0 && 1 - su ≤ 0 && t - 1 ≤ 0 && 1 - t ≤ 0 && n ≥ 0;

postcond18 = a^2 - n ≤ 0 && n - (a + 1)^2 ≤ 0;

inv181 = su == (a + 1) * (a + 1);


verifyResult[precond18, inv181,
  guardcond18, loopcond18, postcond18, f18, vars18, domain18];
```

```
{}{{{su → 0, t → -100, n → 0, a → -1}}}{}
```

```
Numerical Error Deteced.
```