Exercise set #2

Game Theory 2023/24

Exercise 1 It is the discount sales season, and Lou (L) wants to go shopping. He thinks it is best to wait until the last three days of the discount sales, because prices are cheapest. On day 1, Lou asks Karen (K) to go with him. If Karen says yes (Y), they go shopping and the game ends. If Karen says no (N), Lou can either give up (G) or request (R) again the following day. On day 2, the same happens. On day 3, if K still says N, the game ends as well and does not go into a further day. If in the end K and L do not go shopping, both of their payoffs are 0. If they do, their payoffs are computed as $u_K = d$ and $u_L = 5 - 2d$, respectively, where d is the day in which they go. All of this information about the game is common knowledge among the players.

1. Represent the game in its extensive form.
2. Find the SPE of this game.
3. Find *one* NE that is not subgame-perfect.

Exercise 2 Ashley and Brook live together. During the winter break they contemplate giving each other a nice gift (G) for Christmas or not (N). They know each other's preferences so they are able to buy a gift for 10 euros that is worth like 100 euros for the other. They make this decision independently and without telling each other. After Christmas, they also consider whether to celebrate New Year's eve downtown (D) or stay home (H). This means that receiving a gift gives utility of 100 to the receiver that is moved by the gesture, but also -10 to the utility of the buyer. Not giving gifts implies no variation of the utility for both.

For the New Year's eve celebration, they decide independently of each other in a coordination-game fashion. Staying home has utility of 0 for both. Going downtown has utility of 50. However, spending New Year's eve apart from each other has utility of -100 for both. The total payoff of the players is the sum of the partial payoffs in each stage with a discount factor of δ for the second stage.

1. Write down the normal form of both stages of the multi-stage game.
2. Find a trivial subgame-perfect equilibrium of the game where the players just play a Nash equilibrium in all stages, without any strategic connection.
3. Is there a strategically connected SPE of the whole game where Ashley and Brook give gifts to each other? If so, show the minimum required discount factor value $\delta_{\min}$ for that to hold.

Exercise 3 Carl (C) and Diana (D) are two university students. Every night they go to the department library, but they do not coordinate or plan any action together. Upon their arrival, they independently decide whether to: (S) study or (M) watch some movies on their laptop. If they both study, they both get utility 10. The individual benefit from watching a movie is instead 15 for C and 18 for D. However, if they both choose M, their individual benefit is halved (since they have half the connection speed). Also, trying studying while somebody else is playing a movie breaks the concentration, so $u_C(\text{S,M}) = u_D(\text{M,S}) = 0$. Call $\mathbb{G}$ this game, and consider it in a repeated version $\mathbb{G}(T)$. Individual payoffs are summed with discount factor δ.

1. Find the Nash equilibria of $\mathbb{G}(3)$, for $\delta = 1$
2. What values of δ allow for sustaining a Nash equilibrium of $\mathbb{G}(\infty)$ via a "Grim Trigger" strategy where each player ends up in always choosing S?
3. Consider an *extended* game where a punishment strategy P is also available to both players. When either player P, payoffs are -10 for *both* players (that would correspond, e.g., to do something really stupid in the library and get the library permanently closed). Call this game $\mathbb{G}'$. If you see a SPE of $\mathbb{G}'(2)$ where players may play S, state at which round do they play it, and what value of δ do you need to obtain it.

Exercise 4 Consider the following game in normal form. Players are denoted as 1 and 2 and their strategy sets are $\mathcal{S}_1 = \{A, B, C\}$ and $\mathcal{S}_2 = \{X, Y\}$. The payoff matrix is as follows:

1 \ 2	X	Y
A	9, -9	-5, 5
B	-2, 2	7, -7
C	8, -8	-1, 1

1. Describe what kind of game is that and name a specific solution concept you might use to find its Nash equilibria.
2. What is the support within $\mathcal{S}_1$ of the mixed strategy played at the Nash equilibrium by 1?
3. Find the mixed strategies played by 1 and 2 at the Nash equilibrium.