University of Padova – prof. Leonardo Badia
Game Theory exam – January 28, 2016

Exercise 1 Agatha (A) and Bruno (B) are two siblings. They own a model train set (T) and they usually play together with it. Their respective utilities when doing so is $u_A(\mathtt{T},\mathtt{T}) = u_B(\mathtt{T},\mathtt{T}) = 3$. Their mom (M) buys them a new toy (N), which can be either a dollhouse (D) or a set of small soldier figurines (S). The two kids must independently decide whether to keep playing with T or the new toy N. All the three players (A, B, M) involved decide their action simultaneously and unbeknownst to each other. The action set of the kids includes just T or N; the mom instead decides what new toy to buy. If the kids end up in not playing together, *all* the players (also M) get utility equal to 0. If they play together with the new toy, they get a positive utility that for A is equal to 6 and 1 for D and S, respectively; for B, the values are instead 1 and 4, respectively. The utility of M is the lump sum of the utility of the two kids.

1. Write down the normal form of this game (likely, you will need a 3D matrix: you can write two matrices instead, one per each move of player M).
2. Find all Nash equilibria of this game in pure strategies.
3. Find all additional Nash equilibria of this game in mixed strategies.

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Exercise 2 Carl (C) and Diana (D) are two university students that have found that the department library is unoccupied overnight. It is a really good place to study and has a very fast Internet connection. So, they go there every night, but they do not coordinate or plan any action together. Upon their arrival every night, they independently decide whether: (S) study or (M) watch some movies on their laptop. If they both study, they both get utility 10. The individual benefit from watching a movie is instead 15 for C and 18 for D. However, if they both choose M, their individual benefit is halved (since they have half the connection speed). Also, trying studying while somebody else is playing a movie breaks the concentration, so $u_C(\mathtt{S},\mathtt{M}) = u_D(\mathtt{M},\mathtt{S}) = 0$ (C is written as the first player). Call $\mathbb{G}$ this game, and consider it in a repeated version $\mathbb{G}(T)$, where $\mathbb{G}$ is played every night for T nights. Individual payoffs are cumulated with discount factor δ. Finally, consider an *extended* game where a punishment strategy P is also available to both players. When either player chooses P, payoffs are -10 for *both* players (that would correspond, e.g., to do something really stupid in the library and get the library permanently closed). Call this game $\mathbb{G}'$. Note: despite P being weakly dominated, (P,P) is a NE for $\mathbb{G}'$.

1. Find the Nash equilibria of $\mathbb{G}(3)$, for $\delta = 1$.
2. What values of δ allow for sustaining a Nash equilibrium of $\mathbb{G}(\infty)$ via a "Grim Trigger" strategy where each player ends up in always choosing S?
3. If you see an SPE of $\mathbb{G}'(2)$ where players may play S, state at which round do they play it, and what value of δ do you need to obtain it.

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Exercise 3 Jane's bank account contains 10000 euros. She can keep (K) the whole sum in the bank account, or buy a utility car (C) that costs 8000 euros. The bank B is also a player: they observe Jane's decision and after that can either (L) leave Jane's money in the account, or (S) buy some subprime bonds for Jane, regardless of Jane's desire. In case (S), they invest every euro that Jane has left in the account (either 2000 or 10000). After one year, Jane's account is increased with some interest $\mathcal{G}$. If the bank played L, $\mathcal{G}$ is equal to 5% of the initial amount. In case they played S, $\mathcal{G}$ depends on whether the bank is a "good" bank or a "bad" bank. Jane estimates the probability of "good" as p and this is a common prior. A good bank playing S gives Jane her money back, plus $\mathcal{G} = 15\%$ of yearly interest rate. A bad bank steals every euro that Jane put in the bank account ($\mathcal{G} = -100\%$). The final payoff are computed after one year as:
for Jane: the grand total of the euros in the account (i.e., the initial value increased by $\mathcal{G}$); plus the value of the car, if she owns one: a one-year old car is worth 5000 euros.
for the bank: if $\mathcal{G}$ is positive, then their payoff is $\mathcal{G}$; otherwise it is $-\mathcal{G}/5$.

1. Represent this Bayesian game with J and B in extensive form.
2. Represent this Bayesian game in normal form, with a type-agent representation of player B (i.e., its strategies are n-tuples of actions, one per type).
3. Find the Bayesian equilibria of the game if $p = 0.2$ and discuss whether they are (subgame) perfect.