Question 1

$$(x_1, x_2, x_3, x_4) = (0.7, 1.2, 1.1, 2)$$
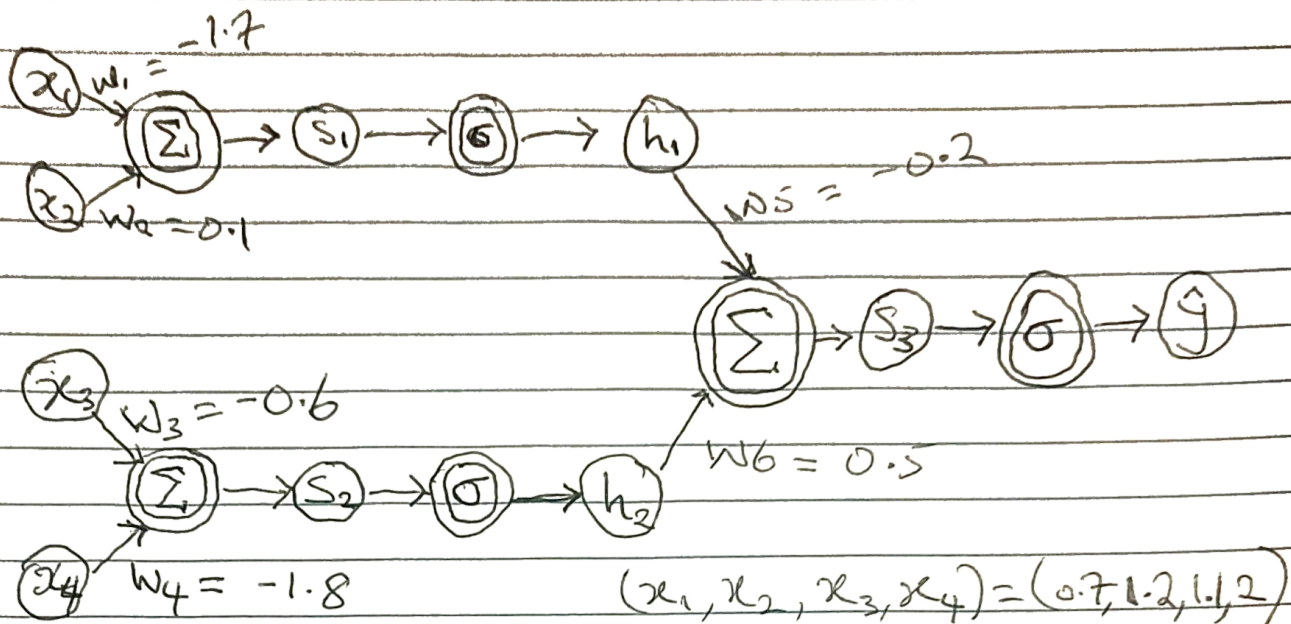
First we apply the weights to our inputs to compute S_1 and S_2

$$S_1 = x_1 W_1 + x_2 W_2$$
$$= (0.7 \times -1.7) + (1.2 \times 0.1)$$
$$= -1.19 + 0.12$$
$$S_1 = -1.07$$

$$S_2 = x_3 W_3 + x_4 W_4$$
$$= (1.1 \times -0.6) + (2 \times -1.8)$$
$$= -0.66 + (-3.6)$$
$$S_2 = -4.26$$

Next solve for h_1 and h_2

$$h_1 = \frac{1}{1 + e^{-W_1 x_1 - W_2 x_2}}$$

$$h_1 = \frac{1}{1 + e^{-(-1.19) - 0.12}} = \frac{1}{1 + e^{1.07}}$$

$$h_1 = \frac{1}{1 + 2.915} = \frac{1}{3.915}$$

$$h_1 = 0.2554$$

$$h_2 = \frac{1}{1 + e^{-w_3 x_3 - w_4 x_4}}$$

$$h_2 = \frac{1}{1 + e^{-(-0.66) - (-3.6)}}$$

$$h_2 = \frac{1}{1 + e^{0.66 + 3.6}} = \frac{1}{1 + e^{4.26}}$$

$$h_2 = \frac{1}{1 + 70.81} = \frac{1}{71.81}$$

$$h_2 = 0.0139$$

Next we compute S_3

$$S_3 = h_1 w_5 + h_2 w_6$$
$$= 0.2554 \times (-0.2) + (0.0139 \times 0.5)$$
$$= -0.05108 + 0.00695$$
$$S_3 = -0.04413$$

Finally we can compute y output

$$y = \frac{1}{1 + e^{-w_5 h_1 - w_6 h_2}}$$

$$y = \frac{1}{1 + e^{-(-0.05108) - 0.00695}}$$

$$y = \frac{1}{1 + e^{0.04413}} = \frac{1}{1 + 1.0451}$$

$$\hat{y} = \frac{1}{2.0451} = 0.48897$$

To compute $\dfrac{dL}{dw_1}$ using back propagation

$$\frac{dL}{dw_1} = \frac{dL}{d\hat{y}} \times \frac{d\hat{y}}{ds_3} \times \frac{ds_3}{dh_1} \times \frac{dh_1}{ds_1} \times \frac{ds_1}{dw_1}$$

first we compute all the derivatives above
$$L = \|\hat{y} - y\|^2$$

1 $$\frac{dL}{d\hat{y}} = 2(\hat{y} - y) = 2(0.48897 - 0.5)$$

$$= 2(-0.01103)$$

$$\frac{dL}{d\hat{y}} = -0.02206$$

2 $$\frac{d\hat{y}}{ds_3} = (1 - \sigma(s_3))\, \sigma(s_3) \qquad \text{Since } \hat{y} = \sigma(s_3)$$

$$\frac{d\hat{y}}{ds_3} = [1 - 0.48897] \times 0.48897$$

$$\frac{d\hat{y}}{ds_3} = 0.24987$$

3   $S_3 = h_1 W_5 + h_2 W_6$

$\dfrac{dS_3}{dh_1} = W_5 = -0.2$

4   $h_1 = \sigma(s_1)$ and $\dfrac{dh_1}{ds_1} = [1 - \sigma(s_1)]\,\sigma(s_1)$

$\dfrac{dh_1}{ds_1} = [1 - 0.2554] \times 0.2554$

$\dfrac{dh_1}{ds_1} = 0.19017$

5   $S_1 = x_1 W_1 + x_2 W_2$

$\dfrac{dS_1}{dW_1} = x_1 = 0.7$

$\dfrac{dL}{dW_1} = \dfrac{dL}{d\hat{y}} \times \dfrac{d\hat{y}}{dS_3} \times \dfrac{dS_3}{dh_1} \times \dfrac{dh_1}{ds_1} \times \dfrac{dS_1}{dW_1}$

$\dfrac{dL}{dW_1} = (-0.02206) \times 0.24987 \times (-0.2) \times 0.19017 \times 0.7$

$\dfrac{dL}{dW_1} = 0.000146$