(I.E on probability) proof by induction

Sunday, August 10, 2025 12:15 PM

IDEA: to prove this, use disjoint partitions, and only difficulty I ha
recognition part in last

So, I will be using these result from lecture-3

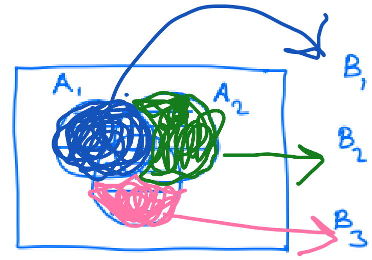
Let $B_1 = A_1$,

$$B_i = A_i \setminus \left(\bigcup_{j=1}^{i-1} A_j \right).$$

Claim 1. $B_1, B_2, \ldots$ are disjoint, i.e. $B_i \cap B_{i'} = \phi$, $i \neq i'$.

Claim 2. $\bigcup_{i=1}^{n} A_i = \bigcup_{i=1}^{n} B_i$, $n \in \mathbb{N}$, and

$$\bigcup_{i=1}^{\infty} A_i = \bigcup_{i=1}^{\infty} B_i.$$



As you can see, these are disjoint and simultaneously t
Original A1,A2,A3

More generally if $A_1, A_2, \ldots$ are events then

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{1 \le i \le n} P(A_i) - \sum_{1 \le i < j \le n} P(A_i \cap A_j) + \sum_{1 \le i < j < k \le n} P(A_i \cap A_j \cap A_k)$$

$$- \cdots + (-1)^{n+1} P(A_1 \cap A_2 \cap \cdots \cap A_n).$$

This is called the inclusion-exclusion principle, proof follows by induction.

Here we go

PROOF BY INDUCTION: (strong 💪 induction)

BASE STEP:

n=1 (too trivial)

n=2 (see this)

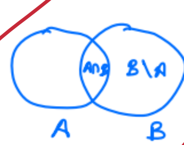
$$\because B = B \setminus A + A \cap B$$

$$\therefore P(B \setminus A) = P(B) - P(A \cap B)$$

$$P(A \cup B) = P(A \cup (B \setminus A))$$

$$= P(A) + P(B \setminus A)$$

$$= P(A) + P(B) - P(A \cap B).$$



Inductive Step:- for any k>2, assume the hypothesis is true

∴

That is:

$$P(A_1 \cup \cdots \cup A_k) = \text{expanded inclusion–exclusion up to } k$$

Now, for k=k+1;

$$B_{k+1} := A_{k+1} \setminus \bigcup_{i=1}^{k} A_i = A_{k+1} \cap \bigcap_{i=1}^{k} A_i^c.$$

k+1

Using k+1th atom to $\bigcup_{i=1} A_i$ like this

$$\bigcup_{i=1}^{k+1} A_i = \Big(\bigcup_{i=1}^{k} A_i\Big) \sqcup B_{k+1}$$

Disjoint

$$P\Big(\bigcup_{i=1}^{k+1} A_i\Big) = P\Big(\bigcup_{i=1}^{k} A_i\Big) + P(B_{k+1}).$$

Axiom (c)

— (i)

Find this

We can write.(see defintion of Bi above)

$$P(B_{k+1}) = P(A_{k+1}) - P\Big(A_{k+1} \cap \bigcup_{i=1}^{k} A_k\Big) \quad \text{— (ii)}$$

Also, this is used but why tf??

$$A_{k+1} \cap \bigcup_{i=1}^{k} A_i = \bigcup_{i=1}^{k} (A_{k+1} \cap A_i).$$

(distributive law, must have studied in discrete structures)

Therefore,

$$P(B_{k+1}) = P(A_{k+1}) - P\Big(\bigcup_{i=1}^{k} (A_{k+1} \cap A_i)\Big)$$

(now after that I had trouble so, I used LLM :)
Now, use strong induction on

so:

$$P\left(\bigcup_{i=1}^{k}(A_{k+1}\cap A_i)\right) = [P(A_{k+1}\cap A_1) + P(A_{k+1}\cap A_2) + \cdots + P(A_{k+1}\cap A_k)]$$
$$- [P(A_{k+1}\cap A_1\cap A_2) + P(A_{k+1}\cap A_1\cap A_3) + \cdots]$$
$$+ [P(A_{k+1}\cap A_1\cap A_2\cap A_3) + \cdots]$$
$$- \cdots + (-1)^{k+1}P(A_{k+1}\cap A_1\cap A_2\cap\cdots\cap A_k)$$

Now, the equation I should be like this

$$P\left(\bigcup_{i=1}^{k+1}A_i\right) = P\left(\bigcup_{i=1}^{k}A_i\right) + P(A_{k+1}) - P\left(\bigcup_{i=1}^{k}(A_{k+1}\cap A_i)\right)$$

- [ ] (Expand using inductive hypothesis)

Put these terms together and you will notice, that the proof for k+1 is complete