CLASS-1

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3 Approaches to probability:

I)Classical Approach:

$$P(E)=\frac{\text{number of favourable outcomes to E}}{\text{Total number of possible outcomes}}$$

Why it sucks??

Let's suppose scenario 1:

We throw two dice, what's the probability for (3,4)

It's 1/36 (because here it's equally likely)

Because there are 6* 6 possible outcomes and 3,4 is one of them

P(sum=7)??

According to classical approach:-

∵Possible sums are 2,3,4,5,6,7,8,9,10,11,12

∴ probability is 1/11 but it's wrong

So something is wrong here??

What is it?

Red Die

Green Die

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Well, we are assuming something inherently tha are likely to occur which is not at all true (refer above image for clarity)

Issues:- things are not equally likely to happen, as we assume it to be

II)Frequency approach:-

Perform experiment n times

∴ $P(E)=\frac{nE}{N}$ (number of times e occurs)

Now time to find out its flaws:-

i)Event may never occur even tho it has a chance
Eg)let's say we threw a coin 4 times to check out its probability of getting head, but we get TTTT
∴ heads occurred 0 times, hence P(head)=0, which is not at all true

So harshit, let's try this experiment tending to infinity times to ensure all outcomes,
Hmm Interesting idea, now F

$$P(E) = \lim_{n \to \infty} \frac{n(E)}{n}$$

Now this also has issues:
i) How tf, someone performs experiment infinite times.
And how can we guarantee that the P(e) converges to what we usually expected

Eg)We rolled a die and we get probabilities like this

Let's simulate fair random die rolls:

Trial n	Outcome	Count of 6s $n(6)$	Frequency $P(6) = \frac{n(6)}{n}$
1	2	0	0.000
2	3	0	0.000
3	1	0	0.000
4	4	0	0.000
5	5	0	0.000
6	3	0	0.000
7	4	0	0.000
8	2	0	0.000
9	5	0	0.000
10	1	0	0.000

You roll the die **10 times**, and by bad luck, **you neve**[illegible] **a 6**.

Even tho it is supposed to be 1/6, how tf can we guarantee it will converge to 1/6 at infinity, because now it is 0....

(but for Intuition: we can say that it will converge at a constant)
III)Axiomatic approach:-
Axioms:
Probability space is a tuple consisting of these three things:-

$$(\Omega, \mathcal{F}, P)$$

Sample space, event space, probability law

And event space is a subset of sample space

Now time for **SET THEORY**:-

$$A \setminus B = \{x \in A \mid x \notin B\} \quad (-)$$
$$A \cup B = \{x \in A \text{ or } x \in B\} \quad (+)$$

So analogically (idk if it is the correct word or not),

Like 4-3+3=4
∴ (A\B)UB=A (it's wrong bro, it is only true when B is in A)
Correct ans is, (A\B)UB=AUB

(Revise set theory bro or you are cooked)
Reference: Sk mappa (chapter-1)

-->**Countably infinite set:-**(has bijection with natural numbers)

We can say a particular set is countably infinite if its elements can be listed like this:-
S=(x1,x2,x3...xn..) [indexing in accordance with natural numbers)

Ex: Q∩[0,1] is countably infinite

Proof:- Q∩[0,1]={p/q∈Q | 0≤p≤q,gcd(p,q)=1, p,q∈N}

We build a list like this:

When **q = 1**: → 0/1, 1/1
When **q = 2**: → 0/2, 1/2, 2/2
When **q = 3**: → 0/3, 1/3, 2/3, 3/3
When **q = 4**: → 0/4, 1/4, 2/4, 3/4, 4/4
And so on...

For each q, list all p from 0 to q, and only include the fraction p/q if gcd(p, q) = 1.

Now arrange all such reduced fractions in a list like:

0/1, 1/1, 1/2, 1/3, 2/3, 1/4, 3/4, ...

This is a sequence — each item has a specific position (1st, 2nd, 3rd, etc.)., I.e we can index it with natural numbers

Hence, it is countably infinite

(kind of informally written proof but it works)

-->**<u>Uncountably infinite set:-</u>**

⇒ **Uncountably infinite**

(∄ a bijection b/w ℕ ⇒S and cardinality of S > cardinality of ℕ)

OR

∃ injection from ℕ → S

Exercise: Prove that ⟨0,1⟩^∞ is uncountably infinite

(**Cantor's Diagonalizable Argument**)

$$
\begin{array}{ll}
s_1 & = 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\ldots \\
s_2 & = 1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\ldots \\
s_3 & = 0\,1\,0\,1\,0\,1\,0\,1\,0\,1\,0\ldots \\
s_4 & = 1\,0\,1\,0\,1\,0\,1\,0\,1\,0\,1\ldots \\
s_5 & = 1\,1\,0\,1\,0\,1\,1\,0\,1\,0\,1\ldots \\
s_6 & = 0\,0\,1\,1\,0\,1\,1\,0\,1\,1\,0\ldots \\
s_7 & = 1\,0\,0\,0\,1\,0\,0\,0\,1\,0\,0\ldots \\
s_8 & = 0\,0\,1\,1\,0\,0\,1\,1\,0\,0\,1\ldots \\
s_9 & = 1\,1\,0\,0\,1\,1\,0\,0\,1\,1\,0\ldots \\
s_{10} & = 1\,1\,0\,1\,1\,1\,0\,0\,1\,0\,1\ldots \\
s_{11} & = 1\,1\,0\,1\,0\,1\,0\,0\,1\,0\,0\ldots \\
\vdots & \quad \vdots\,\vdots\,\vdots\,\vdots\,\vdots\,\vdots\,\vdots\,\vdots\,\vdots\,\vdots\,\vdots\,\ddots
\end{array}
$$

$$
s = 1\,0\,1\,1\,1\,0\,1\,0\,0\,1\,1\ldots
$$

Assume: this set is countably infinite:-

→ Now, take **one digit from each string**, going **diagonally** (like first digit from first string, second digit from second string, and so on).

→ Flip each of these digits (i.e., take their complement — change 0 to 1 or 1 to 0).

→ This gives a **new string** that is **different from every string in the original list**, because it differs by at least **one digit** (the diagonal one) from each.

Hence, by contradiction we have proved .

De Morgan's Law and Infinite Unions

De Morgan's Law

$$\left(\bigcup_{i=1}^{\infty} A_i\right)^c = \bigcap_{i=1}^{\infty} A_i^c$$

De Morgan's Law (see its proof) ✓

Where: $A_i \in \mathbb{N}$, i.e., $A_1, A_2, \ldots$

Infinite Union of Sets

$$\bigcup_{i=1}^{\infty} A_i = \{x : x \in A_i \text{ for some } i \in \mathbb{N}\}$$

$$\bigcap_{i=1}^{\infty} A_i = \{x : x \in A_i \text{ for all } i \in \mathbb{N}\}$$

Example

Let

$$A_n = \left(0, 2 - \frac{1}{n}\right), \quad n \in \mathbb{N}$$

Example:1

$$\bigcup_{n=1}^{\infty} A_n = (0, 2)$$

Find n such that $2 - \varepsilon \in A_n$ for sufficiently small $\varepsilon > 0$. (I mean I forgot RA shit but it doesnt matter in later classes so, ignore this pls)

Example 2:

$$\bigcap_{n=1}^{\infty} A_n = (0, 2)$$

$$2.0001 \notin \bigcap_{n=1}^{\infty} A_n$$

$$\left(\bigcup_{n=1}^{\infty} A_n\right)^c \subseteq \bigcap_{n=1}^{\infty} A_n^c$$

(Can use induction to prove, or basic logic?) → will do later

Define the sets:

$$B_n = \{n, n+1, n+2, \ldots\}, \quad \text{for } n \in \mathbb{N}$$

Now consider the infinite intersection:

$$\bigcap_{n=1}^{\infty} B_n$$

We ask: Is this set non-empty?