# Evaluating Noise Resilience in Quantum Machine Learning

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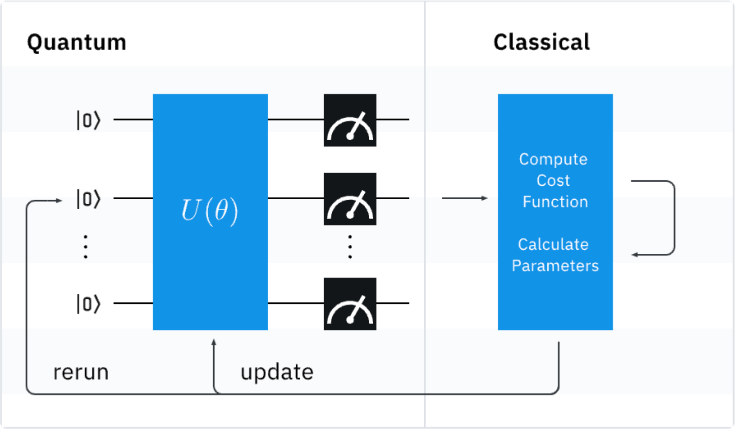
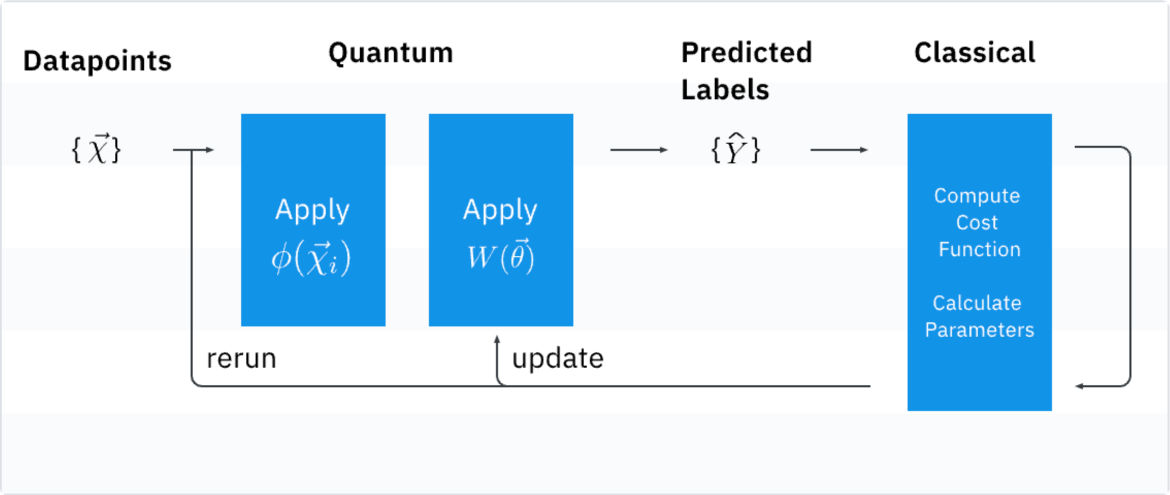
# Overview of VQAs?

- VQAs are a class of quantum algorithms based on hybrid quantum-classical approach.
- Involves parameterized quantum circuits (also Ansatz).
- Uses classical optimization techniques to minimize a certain cost function.
  - Typically a gradient descent after evaluating cost function from the measurement
- Near term algorithms (NISQ) and are resilient to noise [1-4]
  - Optimized parameters can mitigate the effects of noise
  - Compensates rotation errors
  - Robust to readout noise



# Examples of VQAs

- Variational Quantum Eigensolver (VQE):
  - Designed to find the lowest eigenvalue (ground state energy) of a Hamiltonian
  - Typically used in quantum chemistry
- Quantum Approximate Optimization Algorithm (QAOA):
  - Solving combinatorial optimization problems
- Quantum Machine Learning (QML) (Focused)
  - Perform machine learning tasks using quantum circuits
  - Supervised learning with Variational Quantum Classifiers



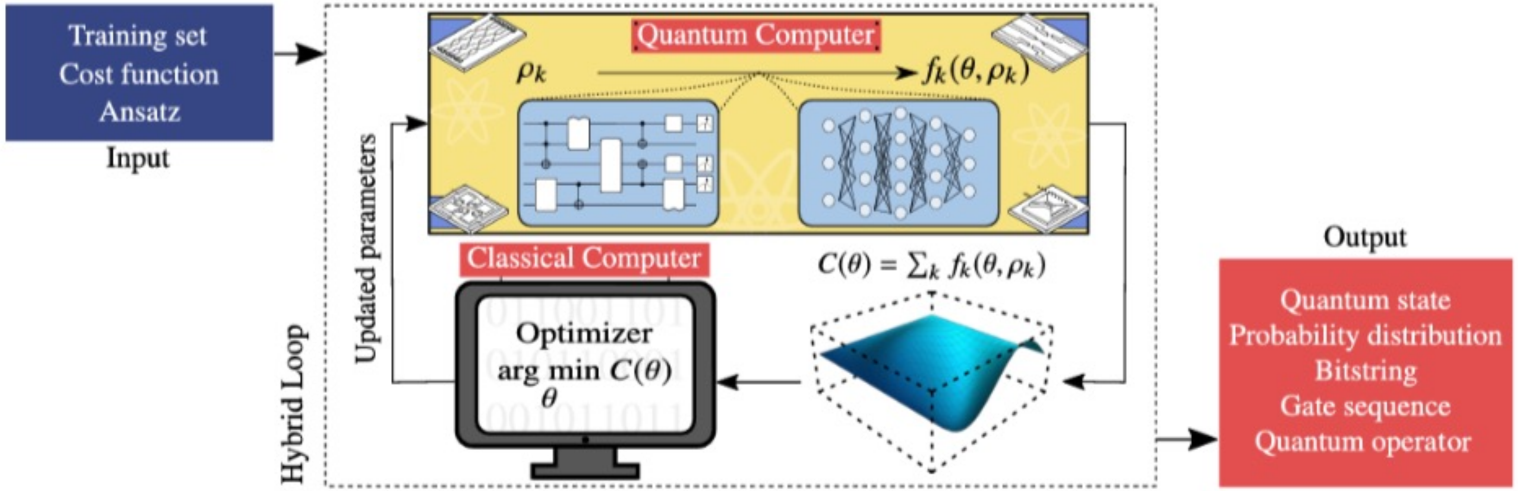

*Source: https://learn.qiskit.org/course/machine-learning/variational-classification*

# Components of VQA

- Feature Map (Data Loading)
- Parameterized quantum circuit (Ansatz)
- Classical Optimizer (Gradient Descent)
- Cost Function [1]

$$U(\boldsymbol{\theta}) = U_L(\boldsymbol{\theta}_L) \cdots U_2(\boldsymbol{\theta}_2) U_1(\boldsymbol{\theta}_1)$$

$$C(\boldsymbol{\theta}) = \sum_k f_k\left(\mathrm{Tr}[O_k U(\boldsymbol{\theta}) \rho_k U^\dagger(\boldsymbol{\theta})]\right)$$



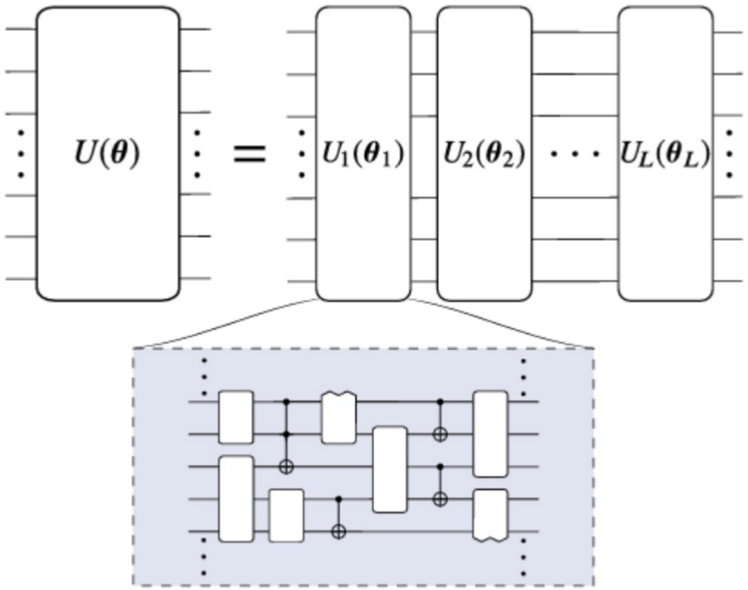

source: https://arxiv.org/pdf/2012.09265.pdf

# Data Encoding & Ansatz

- Data Encoding
  - Basis Encoding $|\mathcal{X}\rangle = \frac{1}{\sqrt{M}} \sum_{m=1}^{M} |x^m\rangle$
  - Amplitude Encoding $|\psi_x\rangle = \sum_{i=1}^{N} x_i |i\rangle$
  - Angle Encoding
    - encodes features into the rotation angles of qubits

$$|x\rangle = \bigotimes_{i=1}^{N} \cos(x_i)|0\rangle + \sin(x_i)|1\rangle$$

- Ansatz
  - Gates are represented with parameters $\theta$
  - $\rho(\theta) = U(\theta_l) \ldots U(\theta_1) U(\theta_0)$




Ansatz

*source: https://arxiv.org/pdf/2012.09265.pdf*

# Gradients & Optimization

- Analytical Gradient

$$\nabla_j f(\theta) = \frac{[f(\theta + \mathrm{s}e_j) - f(\theta - \mathrm{s}e_j)]}{2\sin(s)}$$

- Gradient Descent

$$\theta^t = \theta^{t-1} - \eta\, \nabla f(\theta)$$

- Newton's Optimizer
- Higher Order Gradient Descent with the the Hessian [5]
- Scales $n^2$

$$\theta^t = \theta^{t-1} - \eta\, H^{-1} \nabla f(\theta)$$

$$\begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdot\ \cdot & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdot\ \cdot & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \cdot & \cdot & \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot & \cdot \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdot\ \cdot & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

# Noise Models Evaluated

Kraus Operators $\sum_j K_j \rho K_j^\dagger$

- Depolarizing Noise
  - Common in many quantum computing architectures
  - Randomly flips or depolarizes qubit states

$$K_0 = \sqrt{1 - \frac{3\lambda}{4}} I, K_1 = \sqrt{\frac{\lambda}{4}} X, K_2 = \sqrt{\frac{\lambda}{4}} Y, K_3 = \sqrt{\frac{\lambda}{4}} Z$$

- Amplitude Damping Noise
  - Qubit loses energy to its environment, damping its amplitude
  - Models decay and loss of coherence in quantum systems
  - Represented with parameter $\gamma = 1 - e^{-t/T_1}$ (t = circuit time, T1 = relaxation time)

$$K_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}, \qquad K_1 = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}.$$

- Measurement Noise

- Noise from Fake IBM Backend Snapshots

# Simulations

Go through the code…

# Conclusions & Future work

## Conclusions

- Simulations mostly agree with existing work in the literature [1-4]
- Accuracies drop according to the noise models within some delta.
- Need more work on this kind of noise simulations and experiments to generalize.

## Further work:

- Simulations with error mitigation strategies
- Explore the potential of hybrid quantum-classical algorithms that leverage classical computing resources to be more resilient to quantum noise.
- Develop noise models tailored to specific quantum applications to enhance the efficiency of noise mitigation techniques.

# References

[1] Cerezo, M., Arrasmith, A., Babbush, R., Benjamin, S. C., Endo, S., Fujii, K., McClean, J. R., Mitarai, K., Yuan, X., Cincio, Ł., & Coles, P. J. (2021). Variational quantum algorithms. *Nature Reviews Physics*, *3*(9), 625–644. https://doi.org/10.1038/s42254-021-00348-9

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[3] McClean, J. R., Romero, J., Babbush, R., & Aspuru-Guzik, A. (2016). The theory of variational hybrid quantum-classical algorithms. *New Journal of Physics*, *18*(2), 023023. https://doi.org/10.1088/1367-2630/18/2/023023

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[5] Mari, A., Bromley, T. R., & Killoran, N. (n.d.). *Estimating the gradient and higher-order derivatives on quantum hardware*. Physical Review. https://doi.org/10.1103/physreva.103.012405