Dylan's MATH211 Notes

Week 1

Lecture 1: *Parametric Intro*

Multivariable calculus is about extending the core topics from Calc I and II to cases with many variables. Historically this originated in physics, but now this necessity shows up all over the place. For example, machine learning makes usage of this very frequently.

Basic Notation :

$$[a, b] \leftarrow \text{ all real numbers } x \text{ s.t. } a \leq x \leq b$$

$$(a, b) \leftarrow \text{ all real numbers } x \text{ s.t. } a < x < b$$

$$\mathbb{R} \leftarrow \text{ all real numbers } x$$

$$\mathbb{R}^2 \leftarrow \text{ all ordered pairs of real numbers}$$

$$\mathbb{R}^n \leftarrow \text{ all ordered collections of real numbers}$$

The intuitive idea that this is describing a Cartesian coordinate set in n-dimensional space.

Function Notation :

$$\begin{aligned} f : A &\longrightarrow B \\ x &\longmapsto f(x) \end{aligned}$$

$$f : \text{ Name of the function}$$

$$A : \text{ domain: the set of all possible inputs}$$

$$B : \text{ codomain : The set of available outputs}$$

$$x : \text{ A given input in A}$$

$$f(x) : \text{Whatever f(x) is}$$

Examples :

$$\begin{aligned} \sin : \mathbb{R} &\longrightarrow \mathbb{R} \\ \theta &\longmapsto (\sin(\theta)) \end{aligned}$$

$$\begin{aligned} f : [0,1] &\longrightarrow \mathbb{R} \\ x &\longmapsto \frac{x^3+5}{x-2} \end{aligned}$$

$$\begin{aligned} g : \{3,4,5,6,\dots\} &\longrightarrow \mathbb{R} \\ n &\longmapsto \begin{cases} 0 \text{ if there are no positive whole numbers s.t } a^n + b^n + c^n \\ 1 \text{ if there are} \end{cases} \end{aligned}$$

Our primary focus as a class revolves around functions $\mathbb{R}^n \to \mathbb{R}^m$

Parametric Equations : A helpful way of writing down function $f : \mathbb{R} \to \mathbb{R}^n (n = 2$ for today.)

$$x(t) = \text{ function of } t \leftarrow (\mathbb{R} \to \mathbb{R})$$

$$y(t) = \text{ function of } t \leftarrow (\mathbb{R} \to \mathbb{R})$$

$$t \mapsto f(x)$$

$x(t)$ describes how x changes as we vary t, $y(t)$ does the same for y.

How does this relate to function notation? Graphic $x(t), y(t)$ as we have done is the same as defining a function

$$\begin{aligned} f : \mathbb{R} &\longrightarrow \mathbb{R}^2 \\ t &\longmapsto (x(t), y(t)) \end{aligned}$$

For another example :

$$\begin{aligned} f : \mathbb{R} &\longrightarrow \mathbb{R}^2 \\ t &\longmapsto (\cos(t), \sin(t)) \end{aligned}$$

$$x(t) = \cos(t)$$

$$y(t) = \sin(t)$$

This will just chart out the unit circle, unable to draw these right now because of a dead pen battery.

$$x(t) = e^{-t}\cos(t)$$

$$y(t) = e^{-t}\sin(t)$$

This is almost like a circle, but because of the dampening factor, it will spiral inwards!

Lecture 2: *Parametric Equations*

Vectors :

So why do we need vectors in multi? Vectors will allow us to break larger problems down into smaller, much more manageable pieces. For our purposes a vector will be a point in $\mathbb{R}^n$ where we visualize it as an arrow from the origin. Taking the example of a standard vector :

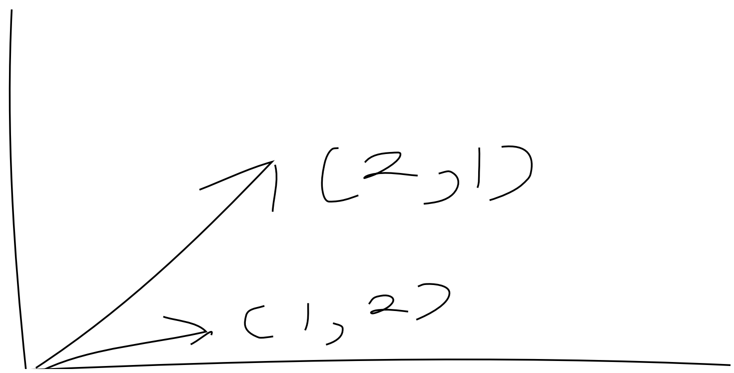

Figure 1: SimpleVectorEx

Warning : Many authors allow vectors that start at points other than the origin. But this doesn't really change anything or do much other than increase complexity.

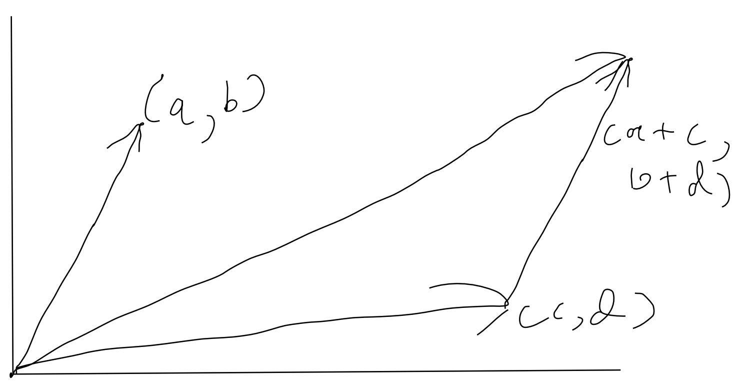

Figure 2: Sept6thAlgebraicVectors

So, to add $\vec{v}+\vec{w}$, we just add the corresponding coordinates :

$$\vec{v} = (v_1, v_2, v_3, \ldots, v_n)$$

$$\vec{w} = (w_1, w_2, w_3, \ldots, w_n)$$

$$\vec{v}+\vec{w} = (v_1+w_1, v_2+w_2, v_3+w_3, \ldots, v_n+w_n)$$

Consider the following problem :
A wheel is rolling across the x-axis at time $t = 0$, it is resting at the origin. Come up with an parametric equation, $x(t) =?$ and $y(t) =$? that describes the position of a point on the whell at time t, which was at the origin at time $t = 0$?

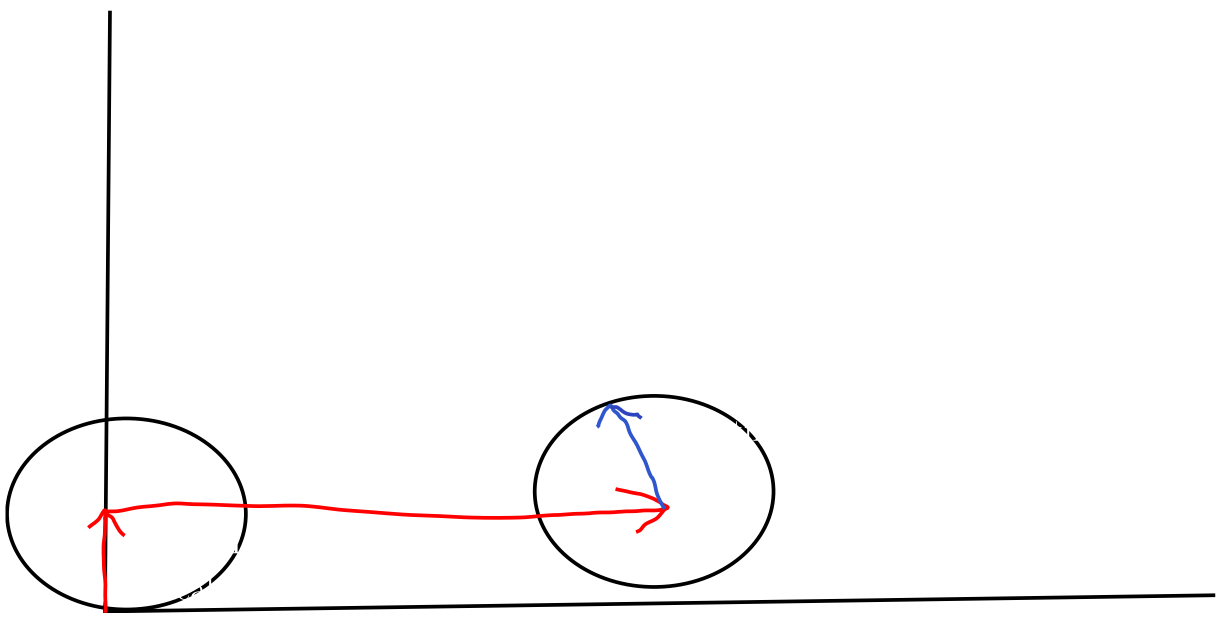

Figure 3: Circle Question

The vertical displacement vector is quite simple, it simply is a $(0, c)$ where c represents the radius of the "ball"

The horizontal displacement vector is also pretty simple, we know the circle will make 1 full revolution once it has traveled $2\pi r$. With our disk of radius 1, we know that the disk will move 2 π units, in 2π time. This means our horizontal vector will be :

$$(t, 0)$$

The spinning vector can be a few things, we know that these two will create a unit circle.

$$x(t) = \cos(t)$$

$$y(t) = \sin(t)$$

However, the ball is rotating in a way such that it's rolling forwards, this means that the terms must be negative, and we have to shift our starting point from being horizontal to facing straight down.

$$x(t) = \cos\left(t - \frac{\pi}{2}\right)$$
$$y(t) = -\sin\left(t - \frac{\pi}{2}\right)$$

Adding everything together :

$$(0, 1) + (t, 0) + \left(\cos\left(t - \frac{\pi}{2}\right), -\sin\left(t - \frac{\pi}{2}\right)\right)$$

$$\left(t + \cos\left(t - \frac{\pi}{2}\right), 1 + \sin\left(t - \frac{\pi}{2}\right)\right)$$

Or more readably

$$x(t) = \left(t + \cos\left(t - \frac{\pi}{2}\right)\right.$$
$$y(t) = 1 + \sin\left(t - \frac{\pi}{2}\right)$$
