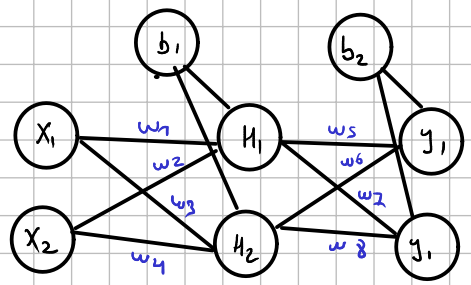


Activation function:

$$f(x) = \frac{1}{1+e^{-x}}$$

EJEMPLO

$(X_1\ X_2) = (0.05 \quad 0,10)$

$\begin{pmatrix} w_1 & w_3 \\ w_2 & w_4 \end{pmatrix} = \begin{pmatrix} 0,15 & 0,25 \\ 0.2 & 0.3 \end{pmatrix}$

$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} 0,35 \\ 0,60 \end{pmatrix}$

TARGET VALUE

$T_1 = 0.01$

$T_2 = 0,99$

$(X_1\ X_2)_{1\times2} \begin{pmatrix} w_1 & w_3 \\ w_2 & w_4 \end{pmatrix}_{2\times2} = (h_1\ h_2)_{1\times2}$

$H_1 = X_1 * w_1 + X_2 * w_2 + b_1$

$= 0.05 * 0.15 + 0.1 * 0.2 + 0.35$

$H_1 = 0.3775$

$out_{H_1} = \frac{1}{1+e^{-H_1}} = \frac{1}{1+e^{-0,3775}} = 0,593269992$

$H_2 = X_1 * w_3 + X_2 * w_4 + b_1$

$= 0.05 * 0,25 + 0.1 * 93 + 0,35$

$= 0,3925$

$out_{H_2} = \frac{1}{1+e^{-H_2}} = \frac{1}{1+e^{-0,3925}} = 0,596884378$

$(out_{H_1}\ out_{H_2}) = (0,593269992 \quad 0,596884378)$

$\begin{pmatrix} w_5 & w_7 \\ w_6 & w_8 \end{pmatrix} = \begin{pmatrix} 0,4 & 0,5 \\ 0,45 & 0,55 \end{pmatrix}$

$(out_{H_1}\ out_{H_2})_{1\times2} \begin{pmatrix} w_5 & w_7 \\ w_6 & w_8 \end{pmatrix}_{2\times2} = (y_1\ y_2)_{1\times2}$

$y_1 = out_{H_1} * w_5 + out_{H_2} * w_6 + b_2$

$= 0,593269992 * 0,4 + 0,596884378 * 0,45 + 0,6$

$= 1.105905967$

$out_{y_1} = \frac{1}{0+e^{-1.105905967}} = 0,751365507$

$(out_{y_1}\ out_{y_2}) = (0,751365507 \quad 0,772928465)$

$y_2 = out_{H_1} * w_7 + out_{H_2} * w_8 + b_2$

$= 0.593269992 * 0,5 + 0,596884378 * 0,55 + 0,6$

$= 1.224921395$

$out_{y_2} = \frac{1}{1+e^{-y_2}} = 0,772928465$

Calculo del error total:

$$E_{total} = \sum \frac{1}{2}(target - output)^2$$

$$= \frac{1}{2}(T_1 - out_{y_1})^2 + \frac{1}{2}(T_2 - out_{y_2})^2$$

$$= \frac{1}{2}(0.01 - 0,751365507)^2 + \frac{1}{2}(0,99 - 0,772928465)^2$$

(E_1 = $\frac{1}{2}(0.01 - 0,751365507)^2$, E_2 = $\frac{1}{2}(0,99 - 0,772928465)^2$)

$= 0,274811083 + 0,023560026$

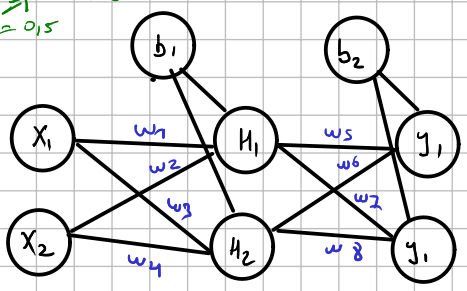
$= 0.2983711092702163$

$E_1 = \frac{1}{2}(T_1 - out_{y_1})^2$

$E_2 = \frac{1}{2}(T_2 - out_{y_2})^2$

Backward Pass : To update weights.

η: LEARNING RATE $= 0,5$ ← TUNING PARAMETER



Updating w_5 :

$$w_5 = w_5 - \eta \left(\frac{\partial E_{TOTAL}}{\partial w_5} \right)$$

$$\frac{\partial E_{TOTAL}}{\partial w_5} = \frac{\partial E_{TOTAL}}{\partial out_{y_1}} * \frac{\partial out_{y_1}}{\partial y_1} * \frac{\partial y_1}{\partial w_5}$$

* $E_{TOTAL} = \frac{1}{2}(T_1 - out_{y_1})^2 + \frac{1}{2}(T_2 - out_{y_2})^2$

$$\frac{\partial E_{TOTAL}}{\partial out_{y_1}} = \frac{1}{2}(2)(T_1 - out_{y_1})^1 * (-1) + 0$$

$$= -(T_1 - out_{y_1})$$

$$= -(0,01 - 0,75136507)$$

$$\frac{\partial E_{TOTAL}}{\partial out_{y_1}} = 0,74136507$$

* $out_{y_1} = \frac{1}{1 + e^{-y_1}}$

$$\frac{\partial out_{y_1}}{\partial y_1} = \left[(1 + e^{-y_1})^{-1} \right]'$$

$$= -1 (1 + e^{-y_1})^{-2} * [e^{-y_1}]'$$

$$= \frac{-1}{(1 + e^{-y_1})^2} * e^{-y_1} * (-1)$$

$$= \frac{e^{-y_1}}{(1 + e^{-y_1})^2}$$

$$= \frac{1}{1 + e^{-y_1}} * \frac{e^{-y_1}}{1 + e^{-y_1}}$$

$$= out_{y_1} * \left(1 - \frac{1}{1 + e^{-y_1}} \right)$$

$$= out_{y_1} * (1 - out_{y_1})$$

$$= 0,75136507 * (1 - 0,75136507)$$

$$\frac{\partial out_{y_1}}{\partial y_1} = 0,18681560$$

* $y_1 = out_{H_1} * w_5 + out_{H_2} * w_6 + b_2$

$$\frac{\partial y_1}{\partial w_5} = out_{H_1} + 0 + 0$$

$$\frac{\partial y_1}{\partial w_5} = 0,59326992$$

Luego,

$$\frac{\partial E_{TOTAL}}{\partial w_5} = \frac{\partial E_{TOTAL}}{\partial out_{y_1}} * \frac{\partial out_{y_1}}{\partial y_1} * \frac{\partial y_1}{\partial w_5}$$

$$= 0,74136507 * 0,18681560 * 0,59326992$$

$$= 0,08216704$$

Finalmente,

$$w_5 = w_5 - \eta \left(\frac{\partial E_{TOTAL}}{\partial w_5} \right)$$

$$= 0,4 - 0,5 (0,08216704)$$

$$w_5 = 0,35891648$$