

Activation function :

$$f(x) = \frac{1}{1 + e^{-x}}$$

Ejemplo

$$(x_1 \ x_2) = (0.05 \ 0.10)$$

$$\begin{pmatrix} w_1 & w_3 \\ w_2 & w_4 \end{pmatrix} = \begin{pmatrix} 0.15 & 0.25 \\ 0.2 & 0.3 \end{pmatrix}$$

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} 0.35 \\ 0.60 \end{pmatrix}$$

Target value

$$T_1 = 0.0$$

$$T_2 = 0.99$$

$$\begin{pmatrix} x_1 & x_2 \end{pmatrix}_{1 \times 2} \begin{pmatrix} w_1 & w_3 \\ w_2 & w_4 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} h_1 & h_2 \end{pmatrix}_{1 \times 2}$$

$$h_1 = x_1 \cdot w_1 + x_2 \cdot w_2 + b_1$$

$$= 0.05 \cdot 0.15 + 0.1 \cdot 0.2 + 0.35$$

$$h_1 = 0.3775$$

$$\text{act} h_1 = \frac{1}{1 + e^{-h_1}} = \frac{1}{1 + e^{-0.3775}} = 0.593269992$$

$$h_2 = x_1 \cdot w_3 + x_2 \cdot w_4 + b_1$$

$$= 0.05 \cdot 0.25 + 0.1 \cdot 0.3 + 0.35$$

$$= 0.3925$$

$$\text{act} h_2 = \frac{1}{1 + e^{-h_2}} = \frac{1}{1 + e^{-0.3925}} = 0.596884378$$

$$(\text{act} h_1, \text{act} h_2) = (0.593269992 \ 0.596884378)$$

$$\begin{pmatrix} w_5 & w_7 \\ w_6 & w_8 \end{pmatrix} = \begin{pmatrix} 0.4 & 0.5 \\ 0.45 & 0.55 \end{pmatrix}$$

$$\begin{pmatrix} \text{act} h_1 & \text{act} h_2 \end{pmatrix}_{1 \times 2} \begin{pmatrix} w_5 & w_7 \\ w_6 & w_8 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} y_1 & y_2 \end{pmatrix}_{1 \times 2}$$

$$y_1 = \text{act} h_1 \cdot w_5 + \text{act} h_2 \cdot w_6 + b_2$$

$$= 0.593269992 \cdot 0.4 + 0.596884378 \cdot 0.45 + 0.6$$

$$= 1.105905967$$

$$\text{out} y_1 = \frac{1}{1 + e^{-y_1}} = 0.75136507$$

$$(\text{out} y_1, \text{out} y_2) = (0.75136507 \ 0.7729284645)$$

$$y_2 = \text{act} h_1 \cdot w_7 + \text{act} h_2 \cdot w_8 + b_2$$

$$= 0.593269992 \cdot 0.5 + 0.596884378 \cdot 0.55 + 0.6$$

$$= 1.2249213995$$

$$\text{out} y_2 = \frac{1}{1 + e^{-y_2}} = 0.7729284645$$

Calculo del error total :

$$E_{\text{total}} = \sum \frac{1}{2} (\text{target} - \text{output})^2$$

$$= \frac{1}{2} (T_1 - \text{out} y_1)^2 + \frac{1}{2} (T_2 - \text{out} y_2)^2$$

$$= \frac{1}{2} (0.0 - 0.75136507)^2 + \frac{1}{2} (0.99 - 0.7729284645)^2$$

$$= 0.2748110835 + 0.0235600297$$

$$= 0.2983711092702163$$

$$E_1 = \frac{1}{2} (T_1 - \text{out} y_1)^2$$

$$E_2 = \frac{1}{2} (T_2 - \text{out} y_2)^2$$

Backward Pass : To update weights.

$\eta$ : LEARNING RATE  $\rightarrow$  tuning parameter  $= 0,5$

Updating  $w_5$  :

$$w_5 = w_5 - \eta \left( \frac{\partial E_{total}}{\partial w_5} \right)$$

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out_{y1}} * \frac{\partial out_{y1}}{\partial y_1} * \frac{\partial y_1}{\partial w_5}$$

$$* E_{total} = \frac{1}{2} (T_1 - out_{y1})^2 + \frac{1}{2} (T_2 - out_{y2})^2$$

$$\frac{\partial E_{total}}{\partial out_{y1}} = \frac{1}{2} (2) (T_1 - out_{y1}) * (-1) + 0$$

$$= -(T_1 - out_{y1})$$

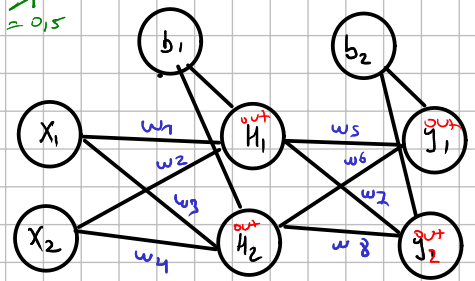
$$= -(0,01 - 0,75136507)$$

$$\frac{\partial E_{total}}{\partial out_{y1}} = 0,74136507$$

$$* y_1 = out_{H1} * w_5 + out_{H2} * w_6 + b_2$$

$$\frac{\partial y_1}{\partial w_5} = out_{H1} + 0 + 0$$

$$\frac{\partial y_1}{\partial w_5} = 0,593269992$$



$$* out_{y1} = \frac{1}{1 + e^{-y_1}}$$

$$\frac{\partial out_{y1}}{\partial y_1} = \left[ (1 + e^{-y_1})^{-1} \right]^1$$

$$= -1 (1 + e^{-y_1})^{-2} * [e^{-y_1}]^1$$

$$= \frac{-1}{(1 + e^{-y_1})^2} * e^{-y_1} * (-1)$$

$$= \frac{e^{-y_1}}{(1 + e^{-y_1})^2}$$

$$= \frac{1}{1 + e^{-y_1}} * \frac{e^{-y_1}}{1 + e^{-y_1}}$$

$$= out_{y1} * \left( 1 - \frac{1}{1 + e^{-y_1}} \right)$$

$$= out_{y1} * (1 - out_{y1})$$

$$= 0,75136507 * (1 - 0,75136507)$$

$$\frac{\partial out_{y1}}{\partial y_1} = 0,1868156015$$

Let's go,

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out_{y1}} * \frac{\partial out_{y1}}{\partial y_1} * \frac{\partial y_1}{\partial w_5}$$

$$= 0,74136507 * 0,1868156015 * 0,593269992$$

$$= 0,0821670404$$

finally we,

$$w_5 = w_5 - \eta \left( \frac{\partial E_{total}}{\partial w_5} \right)$$

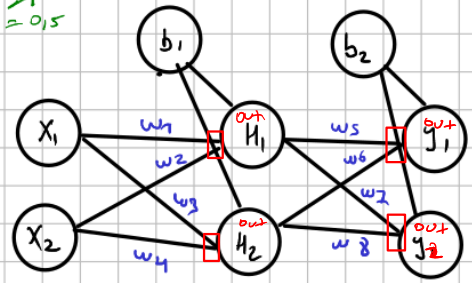
$$= 0,4 - 0,5 (0,0821670404)$$

$$w_5 = 0,3589164798$$

UPDATING  $w_6$ :

$$w_6 = w_6 - \eta \left( \frac{\partial E_{total}}{\partial w_6} \right)$$

$\eta$ : LEARNING RATE  $\rightarrow$  TUNING PARAMETER  $\approx 0,5$



$$* E_{total} = \frac{1}{2} (T_1 - out_{y_1})^2 + \frac{1}{2} (T_2 - out_{y_2})^2$$

$$\begin{aligned} \frac{\partial E_{total}}{\partial out_{y_1}} &= \frac{1}{2} (T_1 - out_{y_1}) (-1) + 0 \\ &= (-1) (T_1 - out_{y_1}) \\ &= (-1) (0,01 - 0,75136507) \end{aligned}$$

$$\frac{\partial E_{total}}{\partial out_{y_1}} = 0,74136507$$

$$* out_{y_1} = \frac{1}{1 + e^{-y_1}}$$

calculado en update  $w_5$

$$\begin{aligned} \frac{\partial out_{y_1}}{\partial y_1} &= out_{y_1} (1 - out_{y_1}) \\ &= 0,75136507 (1 - 0,75136507) \end{aligned}$$

$$\frac{\partial out_{y_1}}{\partial y_1} = 0,18681560158$$

$$* y_1 = out_{h_1} * w_5 + out_{h_2} * w_6 + b_2$$

$$\frac{\partial y_1}{\partial w_6} = 0 + out_{h_2} + 0$$

$$= out_{h_2}$$

$$\frac{\partial y_1}{\partial w_6} = 0,596884378$$

Luego

$$\begin{aligned} \frac{\partial E_{total}}{\partial w_6} &= \frac{\partial E_{total}}{\partial out_{y_1}} * \frac{\partial out_{y_1}}{\partial y_1} * \frac{\partial y_1}{\partial w_6} \\ &= 0,74136507 * 0,18681560158 * 0,596884378 \\ &= 0,0826676277 \end{aligned}$$

Luego

$$\begin{aligned} w_6 &= w_6 - \eta \left( \frac{\partial E_{total}}{\partial w_6} \right) \\ &= 0,45 - 0,5 (0,0826676277) \\ w_6 &= 0,40866618615 \end{aligned}$$