Simple Linear Regression

Michael Noonan

January 11, 2021

Biol 520C: Statistical modelling for biological data

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Housekeeping

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- You also have digital access to the textbooks I mentioned for free through the UBCO Library:

  https://muse-jhu-edu.ezproxy.library.ubc.ca/book/33194

  https://link-springer-com.ezproxy.library.ubc.ca/book/10.1007%2F978-0-387-87458-6

  https://www-degruyter-com.ezproxy.library.ubc.ca/princetonup/view/title/563354

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- Practical 01 is up on the course website, and will be unlocked on Canvas after today's lecture. It is due before the start of the lecture next Tuesday.

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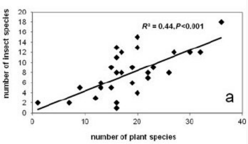
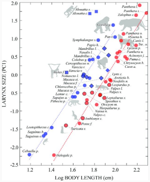
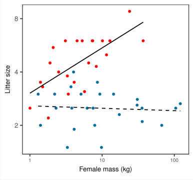


Chmura et al. 2013



Bowling et al. 2020



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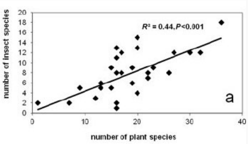

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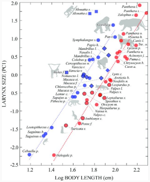

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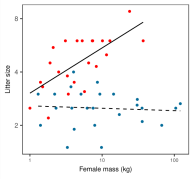

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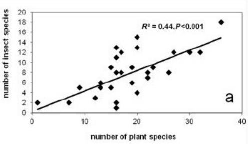

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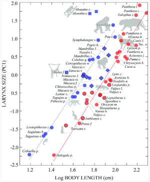

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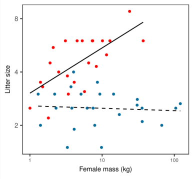

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Our verbal hypothesis in this case is ‘X is proportional to Y’. But looking at the data isn’t enough. So how do we approach the problem statistically?

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The method itself isn’t simple and there’s a lot going on under the hood.

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Data is of the form:

$$d = \{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$$

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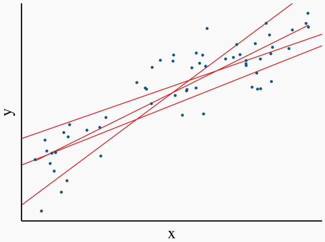
(i.e., what line do you draw through the data?)



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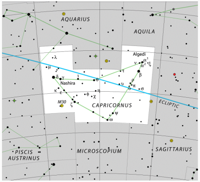

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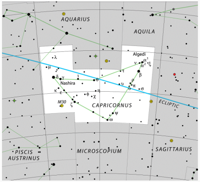

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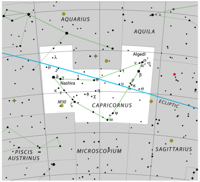

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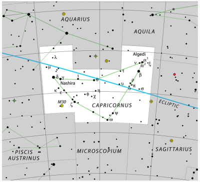

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Source: Wikipedia

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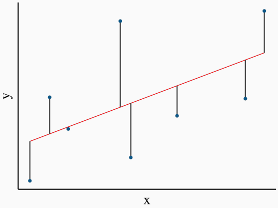
We want to find the value for β_0 and β_1 that minimizes this quantity.



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One solution is to calculate $\sum_{i=1}^{n}(y_i - (\beta_0 + x_i\beta_1))^2$ for all values of β_0 and β_1 between $-\infty$ and ∞.

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But who wants to do that?

Matrix Algebra Review

Matrices

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'Ordinary' numbers can be thought of as a 1×1 matrices, or scalars (e.g., $D = 7$).

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Matrix addition has many of the same properties as normal addition.

$$A + B = B + A$$

$$A + (B + C) = (A + B) + C$$

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If a matrix is its own transpose, then that matrix is said to be symmetric, e.g.:

$$A = \begin{pmatrix} 1 & -5 & 4 \\ -5 & 7 & 3 \\ 4 & 3 & 3 \end{pmatrix} = A' = A^T$$

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$$\begin{pmatrix} 21 + 18 - 48 & -9 - 27 - 16 \\ 14 - 8 + 18 & -6 + 12 + 6 \end{pmatrix} = \begin{pmatrix} -9 & -52 \\ 24 & 12 \end{pmatrix}$$

Matrix Properties

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An identity matrix is a square matrix where every diagonal entry is 1 and all the other entries are 0

$$I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

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The trace of a $n \times n$ matrix is the sum of all the diagonal entries. In other words, for $n \times n$ matrix $trace(A) = tr(A) = \sum_{i=1}^{n} a_{i,i}$

$$tr(I) = tr \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 1 + 1 + 1 = 3$$

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Inverting matrices requires a complicated algorithm, so we usually rely on computers to perform the calculations (e.g. the `solve()` function in `R`).

Linear regression and matrix notation

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Given our dataset $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ we can re-write x, y, and our regression parameters as matrices:

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Linear regression and matrix notation

Why the column of 1s in $\mathbf{x}$?

Linear regression and matrix notation

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So our original regression problem in matrix notation is:

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{pmatrix} \cdot \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}$$

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After some derivations we won't go over, we obtain a formula for the least squares estimates of the parameters:

$$\beta = (\mathrm{x}^\mathsf{T}\mathrm{x})^{-1}\mathrm{x}^\mathsf{T}\mathrm{y}$$

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So instead of plugging in all of the possible values of β_0 and β_1 between $-\infty$ and ∞ to obtain our parameter estimates, all we have to do is a matrix calculation.

Assumptions of linear regression

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Just because a specific estimator makes assumptions that aren't met by real data, this doesn't mean that the relationship doesn't exists or that the estimator is useless, but it does tell you that your estimator can be improved.

Linear regression assumptions

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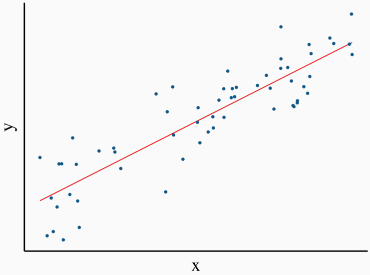

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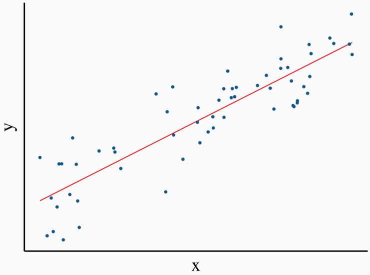

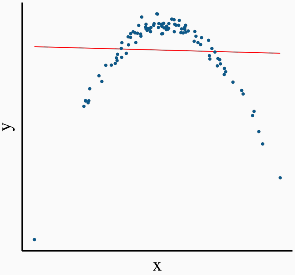

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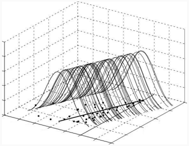

Source: Zuur et al. (2009)

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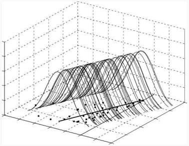

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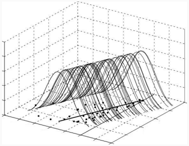

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In practice, we usually don't have many repeat measures of a specific x value, so checking for this usually means pooling all of the residuals and checking for normality. Normality of pooled residuals is reassuring, but does not necessarily mean the population is normally distributed.

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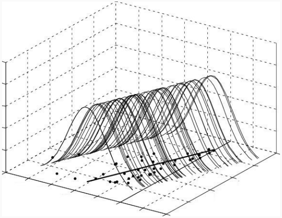

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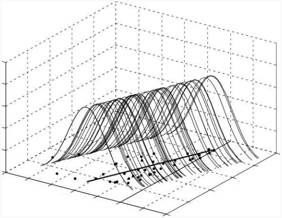

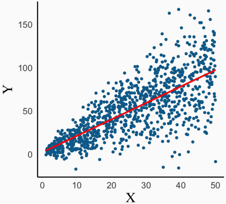

What will heterogeneity do to your estimates?

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Situations where x is accompanied by a meaningful amount measurement error can break this assumption.

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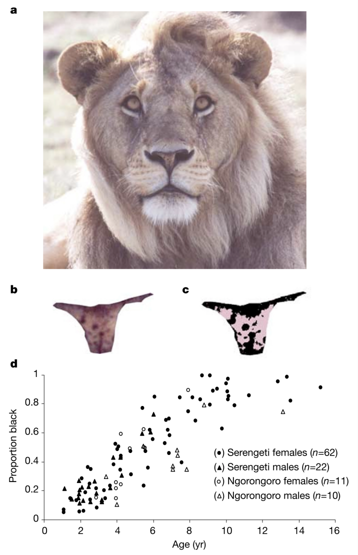
The standard deviation is given by: $\sigma = \sqrt{\frac{\sum(x_i - \mu)^2}{n}}$, what effect does breaking the assumption of independence have?

Linear regression example

The Problem: In lion populations, the sustainable application of trophy hunting is often used as a way of maintaining stable populations while generating valuable funds to support conservation efforts. If you hunt older lions that are past their reproductive prime, the impact on the population is negligible, but if you hunt lions that are too young, there is a risk of the population destabilising. Whitman et al. (2004) looked at whether there was a relationship between how black a male lion's nose was and its age.



Lion noses: the data

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proportionBlack	Age
0.21	1.1
0.14	1.5
0.11	1.9
0.13	2.2
0.12	2.6
0.13	3.2
0.12	3.2
0.18	2.9
0.23	2.4
0.22	2.1
0.2	1.9
0.17	1.9
0.15	1.9
0.27	1.9
0.26	2.8
0.21	3.6
0.3	4.3
0.42	3.8
0.43	4.2
0.59	5.4
0.6	5.8
0.72	6
0.29	3.4
0.1	4
0.48	7.3
0.44	7.3
0.34	7.8
0.37	7.1
0.34	7.1
0.74	13.1
0.79	8.8
0.51	5.4

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0.27	1.9
0.26	2.8
0.21	3.6
0.3	4.3
0.42	3.8
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0.29	3.4
0.1	4
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0.34	7.8
0.37	7.1
0.34	7.1
0.74	13.1
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0.51	5.4

The regression problem in matrix notation is:

$$\begin{pmatrix} 0.21 \\ 0.14 \\ \vdots \\ 0.51 \end{pmatrix} = \begin{pmatrix} 1 & 1.1 \\ 1 & 1.5 \\ \vdots & \vdots \\ 1 & 5.4 \end{pmatrix} \cdot \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}$$

Lion noses: Estimating the parameters

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Calculating our parameters using $\beta = (\mathrm{x}^{\mathsf{T}}\mathrm{x})^{-1}\mathrm{x}^{\mathsf{T}}\mathrm{y}$ can easily done in R:

Lion noses: Estimating the parameters

Calculating our parameters using $\beta = (\mathrm{x}^{\mathsf{T}}\mathrm{x})^{-1}\mathrm{x}^{\mathsf{T}}\mathrm{y}$ can easily done in R:

```
data <- read.csv("LionNoses.csv")

x <- matrix(c(rep(1, nrow(data)),data$ageInYears),
            nrow = nrow(data), ncol = 2)
y <- matrix(data$proportionBlack,
            nrow = nrow(data), ncol = 1)

xtx <- t(x) %*% x
xtx.inv <- solve(xtx)
xty <- t(x) %*% y

beta <- xtx.inv %*% xty
```

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which gives us $\beta_0 = 0.06969626$ and $\beta_1 = 0.05859115$

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We can also do this the easy way by using the `lm()` function:

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lm(proportionBlack ~ ageInYears, data = data)
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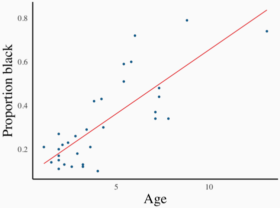
```

```
Call:
lm(formula = proportionBlack ~ ageInYears, data = data)

Residuals:
     Min       1Q   Median       3Q      Max
-0.20406 -0.07758 -0.01750  0.07913  0.29876

Coefficients:
            Estimate Std. Error t value Pr(>t)
(Intercept) 0.069696   0.041956   1.661    0.107
ageInYears  0.058591   0.008307   7.053 7.68e-08 ***
---
Signif. codes:  0 ?***? 0.001 ?**? 0.01 ?*? 0.05 ?.? 0.1 ? ? 1

Residual standard error: 0.1238 on 30 degrees of freedom
Multiple R-squared:  0.6238,  Adjusted R-squared:  0.6113
F-statistic: 49.75 on 1 and 30 DF,  p-value: 7.677e-08
```



Least Squares and Stochasticity

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We'll continue along this train of thought next lecture.