Model Selection and Model Averaging

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February 7, 2021

Biol 520C: Statistical modelling for biological data

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Housekeeping

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- Rubric and time slots for the first talk are posted on Canvas.

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- If you're still struggling with R have a look at Prof. Schluter's R tips page for a similar course run on the Vancouver campus. https://www.zoology.ubc.ca/ schluter/R/

The ΔAIC Grey Zone

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   Sometimes IC approaches result in a clear winner, other times...

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FIT <- gls(num_sp ~ latitude + elevation, data = data, method = "ML")

dredge(FIT)

Global model call: gls(model = num_sp ~ latitude + elevation, data = data, method = "ML")
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Model selection table
  (Intrc)     elvtn   lattd df   logLik AICc delta weight
4  11.120 -0.001373 -0.2018  4   -4.059 18.5  0.00  0.922
2   2.489 -0.001613          3   -8.672 24.7  6.21  0.041
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Models ranked by AICc(x)
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Which parameters would you include in your model?

A — latitude and elevation.

B — Only elevation.

C — Only latitude.

D — Only the intercept.

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49	-103.000					+	0.05326	4	-10370.46	20748.9	0.00	0.202
34	-104.200	0.25270					0.05385	5	-10369.64	20749.3	0.36	0.168
50	-104.200	0.25270				+	0.05385	5	-10369.64	20749.3	0.36	0.168
1	4.208							3	-10372.99	20752.0	3.07	0.043
17	4.208					+		3	-10372.99	20752.0	3.07	0.043
2	4.212	-0.05585						4	-10372.23	20752.5	3.55	0.034
18	4.212	-0.05585				+		4	-10372.23	20752.5	3.55	0.034
37	-51.480			-0.028750			0.05640	5	-10372.58	20755.2	6.24	0.009

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D — A, B, and C are good options.

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Burnham & Anderson (2002)

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Lower AIC values are better (all else being equal), but what's so special about ΔAIC <2?

The ΔAIC $= 2$ Threshold

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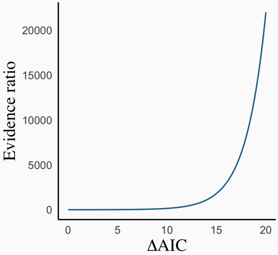

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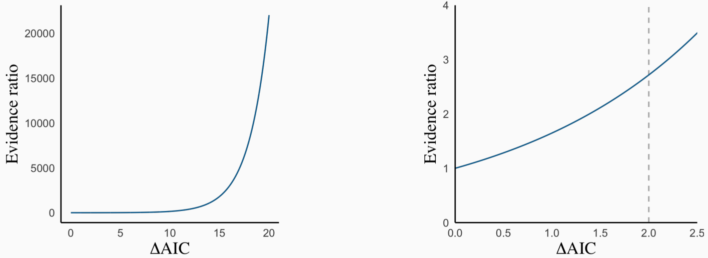

With ΔAIC $\lesssim$ 1.38 the evidence $<$ 2, with ΔAIC $\lesssim$ 2, evidence $<$ 3.

So in the ΔAIC $<$ 2 regime models are only $\sim$twice as likely as the AIC 'best' model.

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What happens if two models have the $\sim$ the same likelihood and only differ by one parameter?

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Because the penalty term is 2K, models with the $\sim$ the same likelihood that only differ by one parameter will, by definition, have a $\Delta\mathrm{AIC}$ of 2.

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So this sheds some more light on the $\Delta\mathrm{AIC}$ of 2 threshold and explains why in practice you'll find models that differ from the 'best fit' model by only one parameter within the $\Delta\mathrm{AIC}$ of 2 threshold.

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Var 1	Var 2	Var 3	Var 4	Var 5	AIC	ΔAIC	Weight
	-0.539 (± 0.244)		**-0.602** (± 0.190)		1789.73	0.00	0.26
	-0.674 (± 0.336)		**-0.609** (± 0.192)	0.173 (± 0.295)	1791.38	1.65	0.12
	-0.544 (± 0.245)	0.003 (± 0.008)	**-0.566** (± 0.214)		1791.60	1.86	0.10
-0.090 (± 0.333)	**-0.541** (± 0.244)		**-0.574** (± 0.217)		1791.66	1.93	0.10
			-0.641 (± 0.201)		1792.23	2.50	0.08
-0.070 (± 0.335)	**-0.670** (± 0.336)		**-0.586** (± 0.220)	0.167 (± 0.296)	1793.34	3.61	0.04
			-0.622 (± 0.198)	-0.212 (± 0.222)	1793.34	3.61	0.04
	-0.662 (± 0.344)	0.001 (± 0.008)	**-0.591** (± -0.591)	0.155 (± 0.316)	1793.35	3.62	0.04

Source: Mark Brewer

Second best model is $\mathcal{M}_1 + \mathrm{Var}_5$, third best model is $\mathcal{M}_1 + \mathrm{Var}_3$, fourth best is $\mathcal{M}_1 + \mathrm{Var}_1$, all within the $\Delta\mathrm{AIC}$ of 2 threshold.

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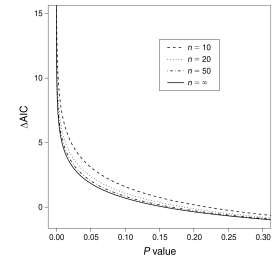

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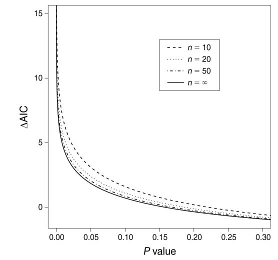

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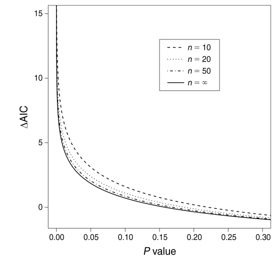

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There's a one-to-one relationship between ΔAIC and p-values e.g., a ΔAIC of 2 when models differ by 1 parameter corresponds to a p-value of $\sim$0.047 (but a p-value of $\sim$0.015 when they differ by 10 parameters).

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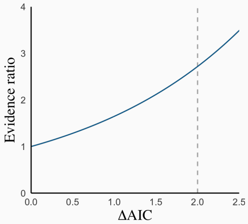

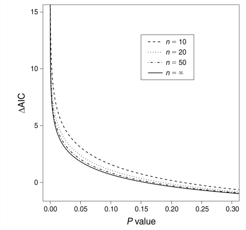

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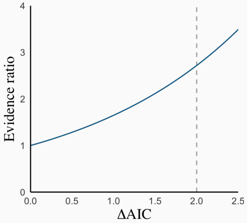

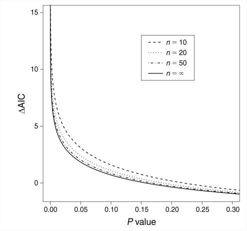

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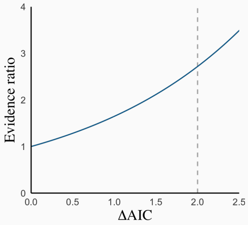
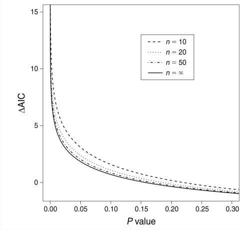
Putting the pieces together:

ΔAIC = 2 corresponds to an evidence ratio of ∼2.7

When two models have identical likelihoods, they can not have ΔAIC <2 (caps our capacity to distinguish models).

A ΔAIC of 2 with a differences of 1 parameter corresponds to a *p*-value of ∼0.047, meaning the complexity is a significant improvement.





(Murtaugh, 2014)

ΔAIC of 2

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But are we any closer to knowing what to do when we have a number of top contenders (i.e., ΔAIC <2)?

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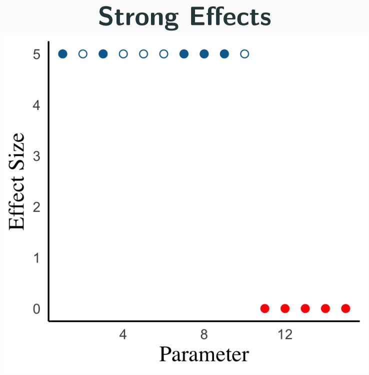
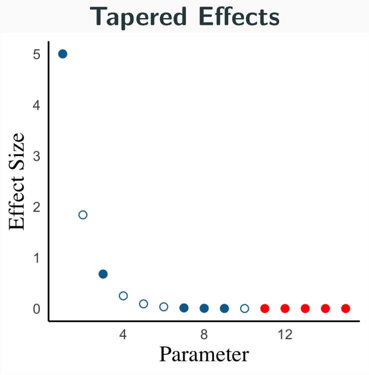
It's well known in the statistics literature that AIC has a tendency to overfit, but what does that look like?

Simulation Study

Simulation Study





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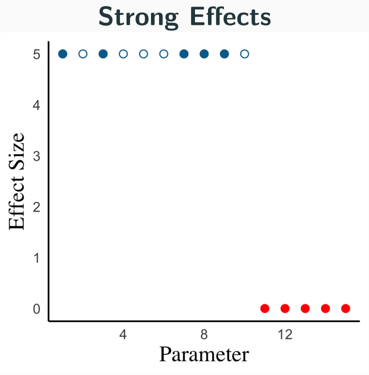
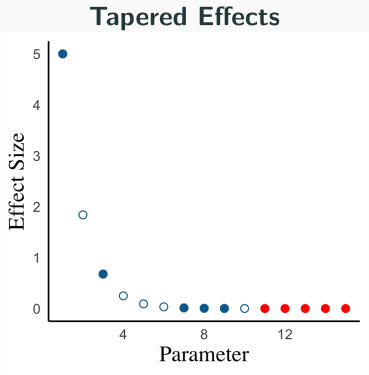
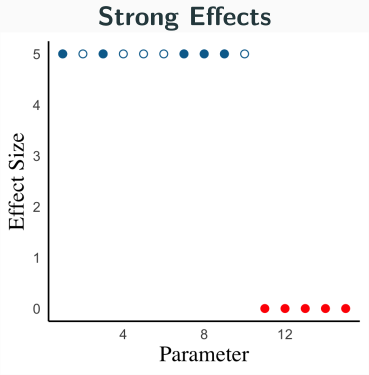
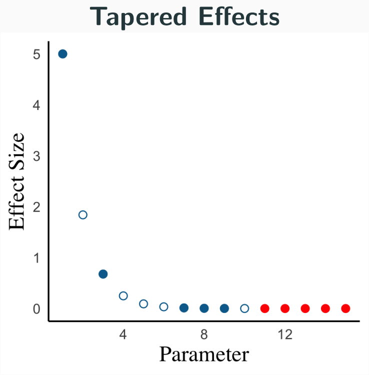




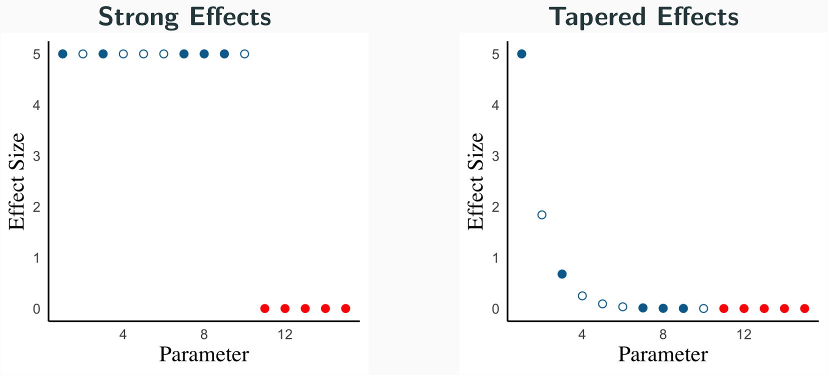
Randomly used 5 of them





Randomly used 5 of them, and added 5 other ‘noise’ parameters.

Simulation Study



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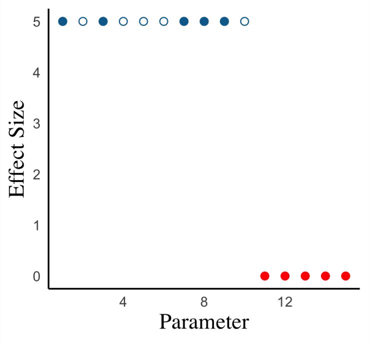
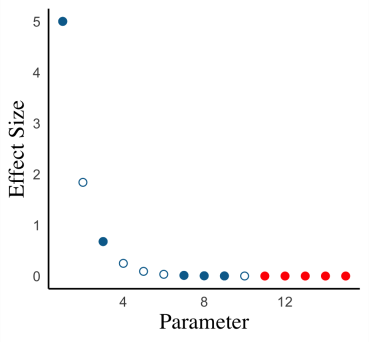
Fit models

Simulation Study

Strong Effects



Tapered Effects



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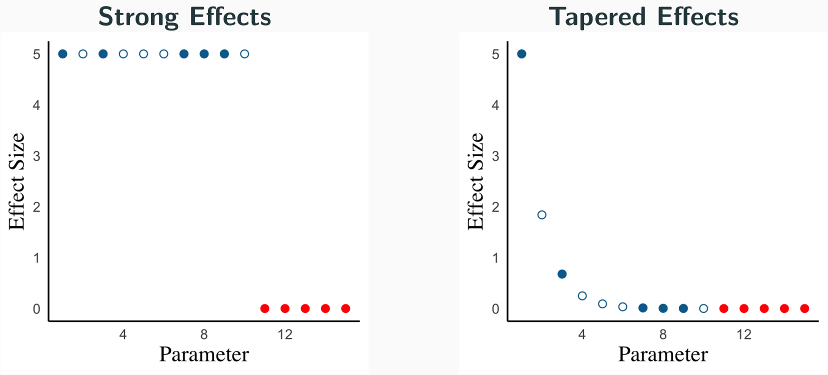
Fit models, selected the best by AIC and AICc

Simulation Study



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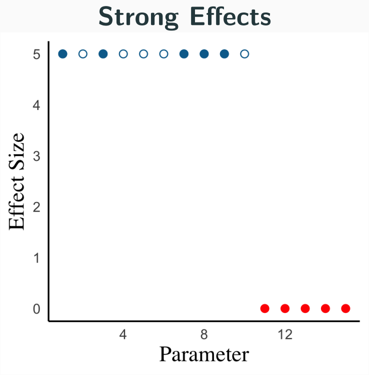
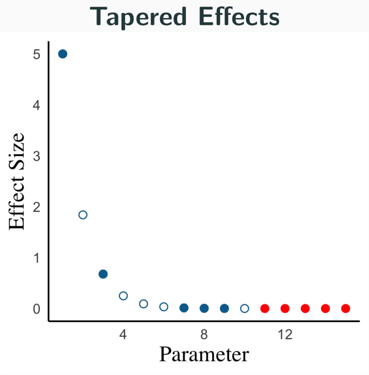
Simulation Study





Randomly used 5 of them, and added 5 other 'noise' parameters.

Fit models, selected the best by AIC and AICc, compared which parameters were selected based on the true system, repeated this 1000s of times

Strong Effects Results

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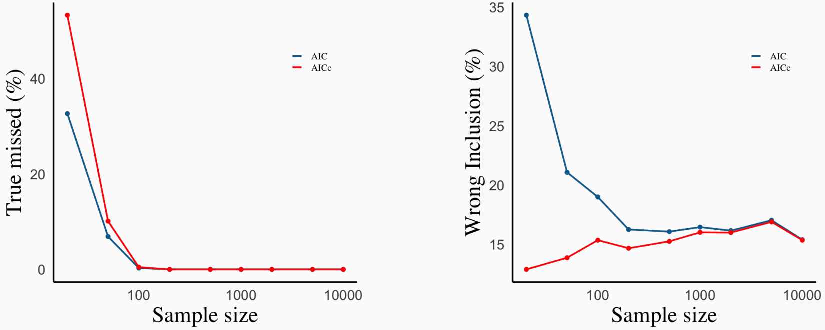

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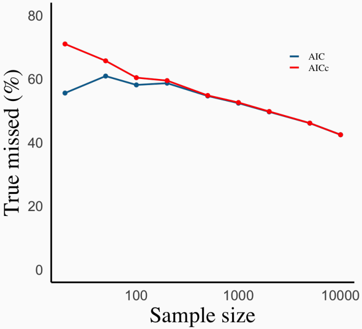
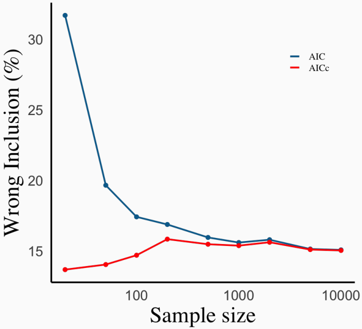
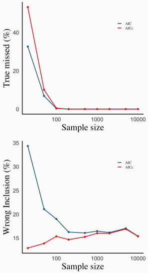




AIC/AICc performance

THE UNIVERSITY OF BRITISH COLUMBIA
Okanagan Campus

Strong Effects



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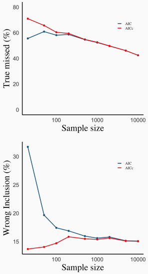
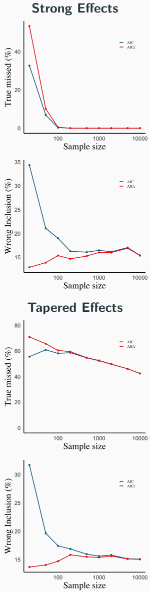


For systems with strong effect sizes, both AIC and AICc identified true parameters well for high *n*.



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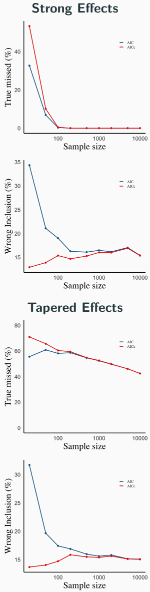

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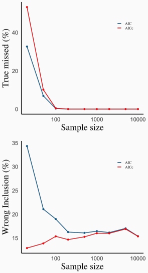

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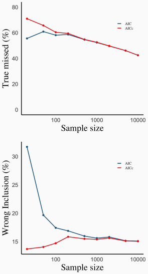

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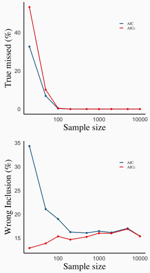

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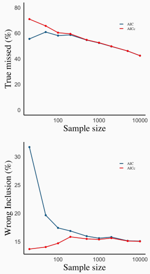

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For both systems AIC and AICc consistently identified noise parameters as being important.

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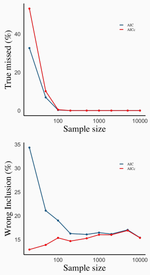

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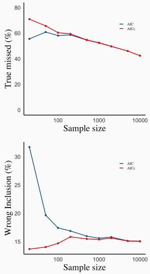

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4. Perform model averaging.

Model Averaging

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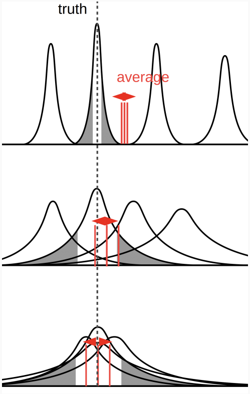

(Dormann *et al.*, 2018)

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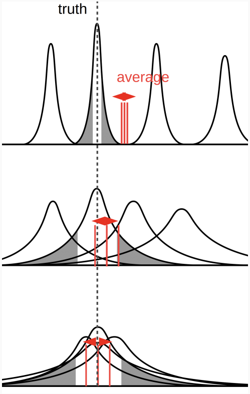
Averaging parameter values from different models, with biases in either way, should cancel out and reduce bias in the average.



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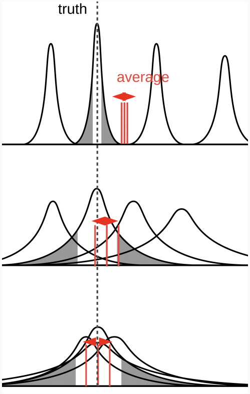
Model averaging cont.



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Variance between models is also important!
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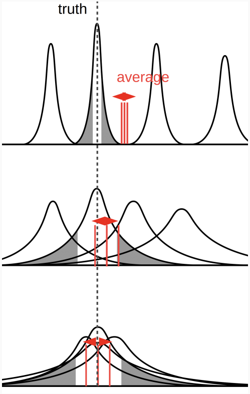

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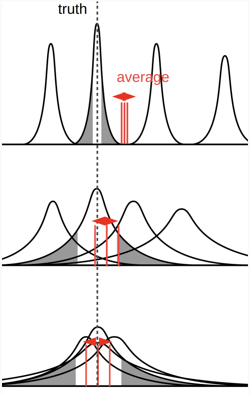

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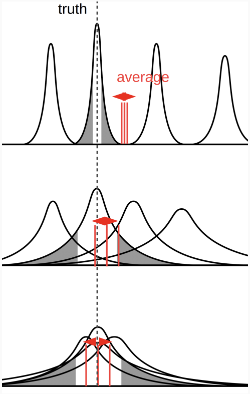

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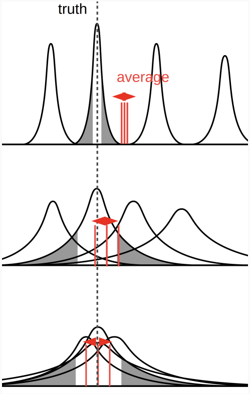
In other words, you only then I get the full benefits of model averaging when models have very different parameter estimates.



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The question is how do we assign model weights?

Model weights

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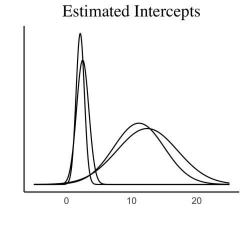
$$w_i = \Pr(\mathcal{M}_i|\mathrm{data}) = \frac{\ell_i}{\sum_{i=1}^{N} \ell_i} \qquad \text{where} \qquad \sum_{i=1}^{N} \ell_i = 1$$

Model averaging in R

Model averaging in R


```
library(nlme)
library(MuMIn)

data <- read.csv("Ant_Richness.csv")

FIT <- gls(num_sp ~ latitude + elevation, data = data, method = "ML", na.action = na.fail)

FITS <- dredge(FIT)

FITS

Global model call: gls(model = num_sp ~ latitude + elevation, data = data, method = "ML",
    na.action = na.fail)
```

```
---
Model selection table
  (Intrc)     elvtn   lattd df  logLik AICc delta weight
4  11.120 -0.001373 -0.2018  4  -4.059 18.5  0.00  0.922
2   2.489 -0.001613          3  -8.672 24.7  6.21  0.041
3  12.390           -0.2388  3  -8.857 25.0  6.58  0.034
1   2.114                    2 -13.224 31.1 12.61  0.002
Models ranked by AICc(x)
```

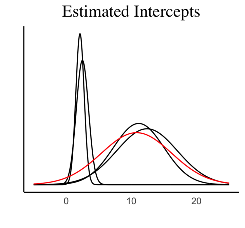


Model averaging in R cont.


```
AVG.FIT <- model.avg(FITS)
```

Model averaging in R cont.


```
AVG.FIT <- model.avg(FITS)

summary(AVG.FIT)

Component models:
       df  logLik   AICc  delta  weight
12      4   -4.06  18.47   0.00    0.92
1       3   -8.67  24.68   6.21    0.04
2       3   -8.86  25.05   6.58    0.03
(Null)  2  -13.22  31.08  12.61    0.00

Term codes:
elevation   latitude
        1          2

Model-averaged coefficients:
(full average)
              Estimate Std. Error Adjusted SE z value Pr(>|z|)
(Intercept) 10.7869966  3.2388986   3.3934619   3.179  0.00148 **
elevation   -0.0013333  0.0004967   0.0005211   2.558  0.01051 *
latitude    -0.1943997  0.0758035   0.0794094   2.448  0.01436 *

(conditional average)
              Estimate Std. Error Adjusted SE z value Pr(>|z|)
(Intercept) 10.7869966  3.2388986   3.3934619   3.179  0.00148 **
elevation   -0.0013832  0.0004323   0.0004612   2.999  0.00271 **
latitude    -0.2031600  0.0650028   0.0693561   2.929  0.00340 **
```



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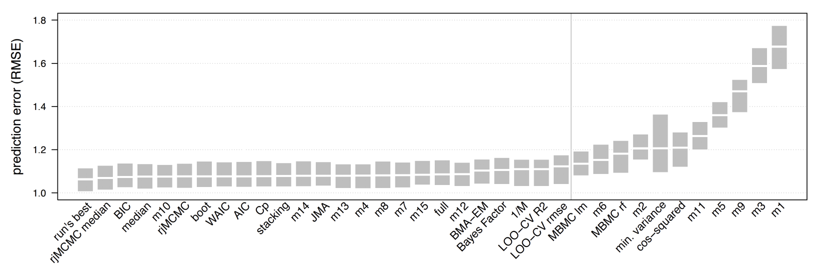

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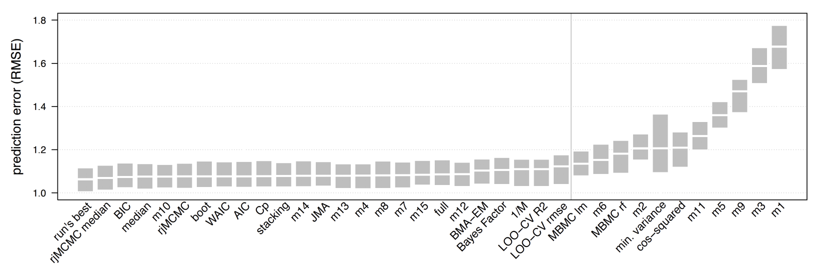

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"We found little in our results to justify the dominance of AIC-based model averaging. And model-averaging did not necessarily outperform single models."

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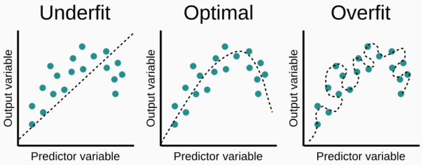
Costs include extra work/computation time, the fact that it does not always work, and that confidence intervals and p-values are difficult to provide.

Model Selection and Averaging Recap

Our goal when building models is to identify the fit that optimally balances over- and under-fitting.



Model selection and averaging

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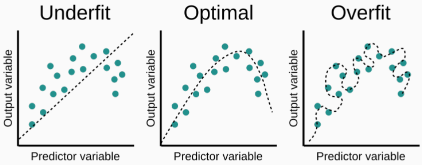

In practice, there is no perfect solution for doing this and how you proceed is part science part art.

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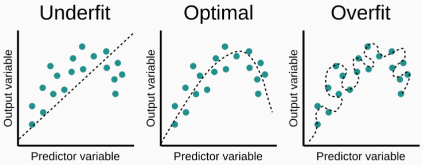

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Know your data

Model selection and averaging

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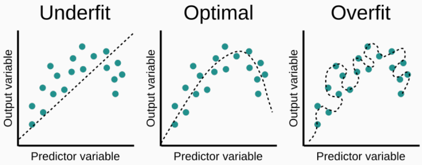

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Know your data, keep your research question in mind

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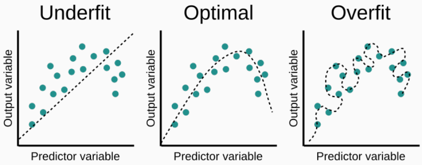

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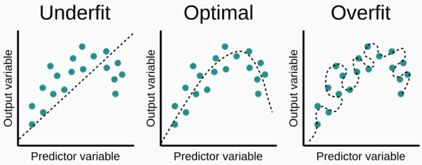

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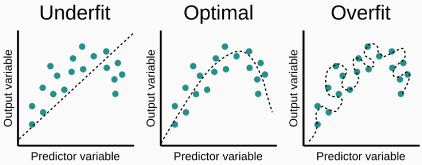

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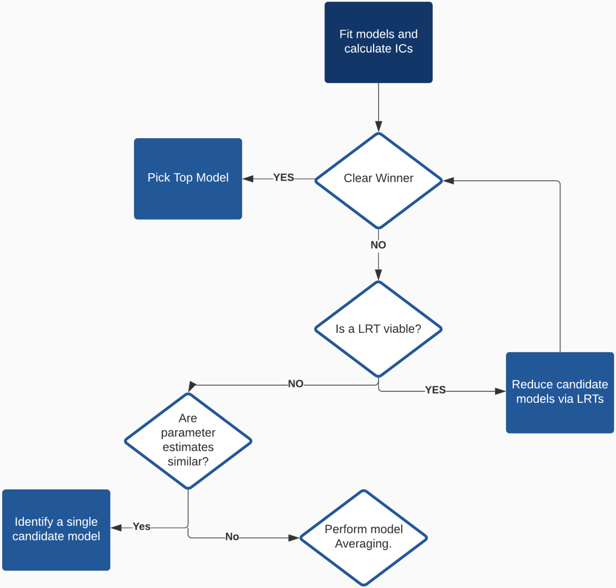
Know your data, keep your research question in mind, proceed cautiously, check model assumptions, and always check model quality/performance.

Pragmatic workflow



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