

Assignment

Random Variables

1. The random variable X has the probability density function:

$$f(x) = 3x^k \quad ; \quad 0 \leq x \leq 1$$

Find the value of ' k ', $E(x)$, $V(x)$, Cumulative distribution function & $P\left[\frac{1}{4} \leq x \leq \frac{3}{4}\right]$

2. The random variable X has the probability density function :

$$f(x) = a + bx^2 \quad ; \quad 0 \leq x \leq 1$$

Find

i. The value of ' a ' & ' b ' such that $E(x) = 3/4$

ii. Cumulative distribution function iii. $P\left[\frac{1}{4} \leq x \leq \frac{3}{4}\right]$

3. The probabilities of various numbers of failures in a mechanical test are as follows

Pr [0 failures] = 0.21, Pr [1 failure] = 0.43, Pr [2 failures] = 0.28, Pr [3 failures] = 0.08,
Pr [more than 3 failures] = 0.

- i. What is the expected number of failures that is, the mathematical expectation of the number of failures?
ii. What is the standard deviation of failures?

4. Roulette wheel is divided into 6 sectors of unequal area marked with the no's 1, 2, 3, 4, 5, & 6. The wheel is spun & X is the random variable the number on which the wheel stops. The probability of X is as follows:

x	1	2	3	4	5	6
$P(X=x)$	1/16	3/16	k	1/4	3/16	1/16

Find the value of ' k ' and find (i) $E(3x-5)$ (ii) $E(6x^2 + 6x - 10)$

5. The density function of the time Z in minutes between calls to an electrical supply store is given by

$$f(z) = \frac{1}{10} \exp(-z/10) \quad ; \quad z > 0$$

- i. What is the mean time between calls?
ii. What is the variance in the time between calls?
iii. What is the probability that the time between calls exceeds the mean?
6. An attendant at a car wash is paid according to the number of cars that pass through. Suppose the probabilities are 1/12, 1/12, 1/4, 1/4, 1/6, and 1/6, respectively, that the attendant receives \$7, \$9, \$11, \$13, \$15, or \$17 between 4:00 P.M. and 5:00 P.M. on any sunny Friday. Find the attendant's expected earnings and the standard deviation for this particular period.
7. Suppose that the probabilities are 0.4, 0.3, 0.2, and 0.1, respectively, that 0, 1, 2, or 3 power failures will strike a certain subdivision in any given year. Find the mean and variance of the random variable X representing the number of power failures striking this subdivision.

8. The random variable N has PMF

$$P_N(n) = \begin{cases} c \left(\frac{1}{2}\right)^n & n = 0, 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

- i. What is the value of constant c ?
ii. What is $P[N < 2]$?
iii. Calculate Expected value and standard deviation of N ?

Probability Distribution

9. A manufacturing company uses an acceptance scheme on production items before they are shipped. The plan is a two-stage one. Boxes of 25 are readied for shipment and a sample of 3 is tested for defectives. If any defectives are found, the entire box is sent back for 100% screening. If no defectives are found, the box is shipped. What is the probability that a box containing 3 defectives will be shipped? What is the probability that a box containing only 1 defective will be sent back for screening?
10. In an NBA (National Basketball Association) championship series, the team who wins four games out of seven will be the winner. Suppose that team A has probability 0.55 of winning over team B and both teams A and B face each other in the championship games. What is the probability that team A will win the series?
11. An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed, with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability of light bulbs that have an average life of less than 775 hours. Find the probability that a random sample of 16 bulbs will have an average life of less than 775 hours.
12. In certain industrial facility accidents occur infrequently. It is known that the probability of an accident on any given day is 0.005 and accidents are independent of each other. What is the probability that in any given period of 400 days there will be an accident on one day? What is the probability that there are at most three days with an accident?
13. A company generally purchases large lots of a certain kind of electronic device. A method is used that rejects a lot if two or more defective units are found in a random sample of 100 units. What is the probability of rejecting a lot that is 1% defective?
14. In airport luggage screening it is known that 3% of people screened have questionable objects in their luggage. What is the probability that a string of 15 people pass through successfully before an individual is caught with a questionable object?
15. The amount of time that a drive-through bank teller spends on a customer is a random variable with a mean $\mu = 3.2$ minutes and a standard deviation $\sigma = 1.6$ minutes. If a random sample of 64 customers is observed, find the probability that their mean time at the teller's counter is At most 2.7 minutes? More than 3.5 minutes? At least 3.2 minutes but less than 3.4 minutes?
16. A company is interested in evaluating its current inspection procedure on shipments of 50 identical items. The procedure is to take a sample of 5 and pass the shipment if no more than 2 are found to be defective. What proportion of 20% defective shipments will be accepted?
17. Ten is the average number of oil tankers arriving each day at a certain port city. The facilities at the port can handle at most 15 tankers per day. What is the probability that on a given day tankers have to be turned away?
18. The probability that a person, living in a certain city, owns a dog is estimated to be 0.3. Find the probability that the tenth person randomly interviewed in that city is the fifth one to own a dog?
19. Statistics released by the National Highway Traffic Safety Administration and the National Safety Council show that on an average weekend night, 1 out of every 10 drivers on the road is drunk. If 400 drivers are randomly checked next Saturday night, what is the probability that the number of drunk drivers will be Less than 32? More than 49? At least 35 but less than 47?
20. The finished inside diameter of a piston ring is normally distributed with a mean of 10 centimeters and a standard deviation of 0.03 centimeter. What proportion of rings will have inside diameters exceeding 10.075 centimeters? What is the probability that a piston ring will have an inside diameter between 9.97 and 10.03 centimeters? Below what value of inside diameter will 15% of the piston rings fall?

21. Printed circuit cards are placed in a functional test after being populated with semiconductor chips. A lot contains 140 cards, and 20 are selected without replacement for functional testing. If 5 cards are defective, what is the probability that at least 1 defective card appears in the sample?
22. An electronic switching device occasionally malfunctions and may need to be replaced. It is known that the device is satisfactory if it makes, on the average, no more than 0.20 errors per hour. A particular 5-hour period is chosen as a "test" on the device. If no more than 1 error occurs, the device is considered satisfactory. What is the probability that a satisfactory device will be considered satisfactory?
23. An electronics firm claims that the proportion of defective units of a certain process is 25%. A buyer has a standard procedure of inspecting 15 units selected randomly from a large lot. On a particular occasion, the buyer found 3 items defective. What is the probability of this occurrence, given that the claim of 25% defective is correct?
24. The probability is 0.6 that a calibration of a transducer in an electronic instrument conforms to specifications for the measurement system. Assume the calibration attempts are independent. What is the probability that at most three calibration attempts are required to meet the specifications for the measurement system?
25. The average life of a bread making machine is 7 years, with a standard deviation of 1 year. Assuming that the lives of these machines follow approximately a normal distribution. Find the probability that the mean life of a random sample of 9 such machines falls between 6.4 & 7.2 years? Find the value of 'k' to the right of which 14.69% of the means computed from a random sample of size 9 would fall?
26. A soft drink machine is regulated so that it discharges an average of 200 milliliters per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 milliliters. What is the probability that a cup contains more than 224 milliliters? Below what value do we get the smallest 25% of the drinks?
27. A manufacturer uses electrical fuses in an electronic system. The fuses are purchased in large lots & tested sequentially until the defective fuse is observed. Assumed that the lot contains 10% defective fuses. What is the probability that 1st defective fuse is found in 5th test? 3rd defective fuse is found in 10th test?
28. An engineering development laboratory conducted an experiment to investigate the life characteristics of a new solar heating panel, designed to have a useful life with probability ' $p=0.90$ '. A randomly of 20 such panels was selected & the useful life of each was recorded. What is the probability that exactly 18 will have a useful life? At most 18 will have a useful life?
29. An experiment is conducted to select a suitable catalyst for the commercial production ethylenediamine (EDA), a product used in soaps, suppose a chemical engineer randomly selects 3 catalysts for testing from a group of 10 catalysts, 6 of which have low acidity & 4 have high acidity. What is the probability that exactly 2 lower acidic catalysts are selected? Exactly no highly acidic catalyst is selected?
30. A local drugstore owner knows that, on average, 100 people per hour stop by his store. Find the probability that in a given 3-minute period nobody enters the store? Find the probability that in a given 3-minute period more than 5 people enter the store.

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