Solution to Problem Sheet 2

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Solve for problem no. 1

Given

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \tag{1}$$

From L.H.S.

$$
\begin{aligned}
\left[\gamma^\kappa\gamma^\lambda, \gamma^\mu\gamma^\nu\right] &= \gamma^\kappa\left[\gamma^\lambda, \gamma^\mu\gamma^\nu\right] + [\gamma^\kappa, \gamma^\mu\gamma^\nu]\gamma^\lambda \\
&= \gamma^\kappa\left(\gamma^\lambda\gamma^\mu\gamma^\nu - \gamma^\mu\gamma^\nu\gamma^\lambda\right) + (\gamma^\kappa\gamma^\mu\gamma^\nu - \gamma^\mu\gamma^\nu\gamma^\kappa)\gamma^\lambda \\
&= \gamma^\kappa\left(\left(\gamma^\lambda\gamma^\mu + \gamma^\mu\gamma^\lambda\right)\gamma^\nu - \gamma^\mu\left(\gamma^\lambda\gamma^\nu + \gamma^\nu\gamma^\lambda\right) + ((\gamma^\kappa\gamma^\mu + \gamma^\mu\gamma^\kappa)\gamma^\nu - \gamma^\mu(\gamma^\nu\gamma^\kappa + \gamma^\kappa\gamma^\nu))\gamma^\lambda\right) \\
&= \gamma^\kappa\left(\left(\{\gamma^\lambda, \gamma^\mu\}\gamma^\nu - \gamma^\mu\{\gamma^\lambda, \gamma^\nu\}\right) + (\{\gamma^\kappa, \gamma^\mu\}\gamma^\nu - \gamma^\mu\{\gamma^\nu, \gamma^\kappa\})\gamma^\lambda\right) \\
\left[\gamma^\kappa\gamma^\lambda, \gamma^\mu\gamma^\nu\right] &= 2\eta^{\lambda\mu}\gamma^\kappa\gamma^\nu - 2\eta^{\lambda\nu}\gamma^\kappa\gamma\mu + 2\eta^{\kappa\mu}\gamma^\nu\gamma^\lambda - 2\eta^{\nu\kappa}\gamma^\mu\gamma^\lambda \qquad \text{(Showed)}
\end{aligned}
$$

$$
\begin{aligned}
(a)\ Tr(\gamma^\mu) &= Tr(\gamma^\mu\gamma_5\gamma_5) && ([\text{because } (\gamma_5)^2 = 1]) \\
&= -Tr(\gamma_5\gamma^\mu\gamma_5) = -Tr(\gamma_5\gamma^\mu\gamma_5) \\
\implies Tr(\gamma^\mu\gamma_5\gamma_5) &= -Tr(\gamma_5\gamma^\mu\gamma_5) = Tr(\gamma_5\gamma^\mu\gamma_5) \\
Tr(\gamma^\mu\gamma_5\gamma_5) &= 0 \\
\implies Tr(\gamma^\mu) &= 0
\end{aligned}
$$

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$$(b)\ Tr(\gamma^{\mu}\gamma^{\nu}) = \frac{1}{2}\left(Tr(\gamma^{\mu}\gamma^{\nu}) + Tr(\gamma^{\mu}\gamma^{\nu})\right)$$

$$= \frac{1}{2}\left(Tr(\gamma^{\mu}\gamma^{\nu}) + Tr(\gamma^{\nu}\gamma^{\mu})\right)$$

$$= \frac{1}{2}\left(Tr(\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu})\right)$$

$$= \frac{1}{2}Tr\,\{\gamma^{\mu},\gamma^{\nu}\}$$

$$= \frac{1}{2}Tr2\eta^{\mu\nu}$$

$$= \frac{1}{2}2\eta^{\mu\nu}Tr(\mathbb{1})$$

$$= 4\eta^{\mu\nu}$$

$$(d)\ (\gamma^{5})^{2} = i\gamma_0\gamma_1\gamma_2\gamma_3 \cdot i\gamma_0\gamma_1\gamma_2\gamma_3$$

$$= -(-1)\gamma_0\gamma_0\gamma_1\gamma_2\gamma_3\gamma_1\gamma_2\gamma_3$$

$$= \gamma_1\gamma_2\gamma_3\gamma_1\gamma_2\gamma_3$$

$$= \gamma_1\gamma_1\gamma_2\gamma_3\gamma_2\gamma_3 \qquad\qquad\qquad = -\gamma_2\gamma_3\gamma_2\gamma_3$$

$$= \gamma_2\gamma_2\gamma_3\gamma_3$$

$$= 1$$

$$(e)\ Tr(\gamma^{5}) = Tr(\gamma^{5}\gamma^{0}\gamma^{0}) \qquad\qquad\qquad ((\gamma^{0})^{2} = 1)$$

$$= -Tr(\gamma^{0}\gamma^{5}\gamma0) = Tr(\gamma^{0}\gamma^{5}\gamma0) = 0$$

Solution to problem 2

Given,

$$\psi(\vec{x}) = \sum_{s=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}}\left[b_{\vec{p}}^s u^s(\vec{p})e^{i\vec{p}\cdot\vec{x}} + c_{\vec{p}}^{s\dagger} v^s(\vec{p})e^{-i\vec{p}\cdot\vec{x}}\right] \qquad (2)$$

And

$$\psi^\dagger(\vec{x}) = \sum_{s=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left[b_{\vec{p}}^{s\dagger} u^{s\dagger}(\vec{p}) e^{-i\vec{p}\cdot\vec{x}} + c_{\vec{p}}^{s} v^{s\dagger}(\vec{p}) e^{i\vec{p}\cdot\vec{x}} \right] \tag{3}$$

Now

$$\left\{ \psi_\alpha(\vec{x}), \psi_\beta^\dagger(\vec{y}) \right\}$$

$$= \left\{ \sum_{\alpha=1}^{4}\sum_{s=1}^{2} \int \frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left[b_{\vec{p}}^{s} u_\alpha^{s}(\vec{p}) e^{i\vec{p}\cdot\vec{x}} + c_{\vec{p}}^{s\dagger} v_\alpha^{s}(\vec{p}) e^{-i\vec{p}\cdot\vec{x}} \right], \right.$$

$$\left. \sum_{\beta=1}^{4}\sum_{r=1}^{2} \int \frac{d^3p'}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p'}}}} \left[b_{\vec{p'}}^{r\dagger} u_\beta^{r\dagger}(\vec{p'}) e^{-i\vec{p'}\cdot\vec{y}} + c_{\vec{p'}}^{r} v_\beta^{r\dagger}(\vec{p'}) e^{i\vec{p'}\cdot\vec{y}} \right] \right\}$$

$$= \sum_{\alpha=1}^{4}\sum_{\beta=1}^{4}\sum_{s=1}^{2}\sum_{r=1}^{2} \int \frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \frac{1}{\sqrt{2E_{\vec{p'}}}} \left[b_{\vec{p}}^{s} u_\alpha^{s}(\vec{p}) b_{\vec{p'}}^{r\dagger} u_\beta^{r\dagger}(\vec{p'}) e^{i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}} + b_{\vec{p}}^{s} u_\alpha^{s}(\vec{p}) c_{\vec{p'}}^{r} v_\beta^{r\dagger}(\vec{p'}) e^{i\vec{p}\cdot\vec{x}+i\vec{p'}\cdot\vec{y}} \right.$$

$$+ c_{\vec{p}}^{s\dagger} v_\alpha^{s}(\vec{p}) b_{\vec{p'}}^{r\dagger} u_\beta^{r\dagger}(\vec{p'}) e^{-i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}} + c_{\vec{p}}^{s\dagger} v_\alpha^{s}(\vec{p}) c_{\vec{p'}}^{r} v_\beta^{r\dagger}(\vec{p'}) e^{-i\vec{p}\cdot\vec{x}+i\vec{p'}\cdot\vec{y}} + b_{\vec{p'}}^{r\dagger} u_\beta^{r\dagger}(\vec{p'}) b_{\vec{p}}^{s} u_\alpha^{s}(\vec{p}) e^{i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}}$$

$$\left. + c_{\vec{p'}}^{r} v_\beta^{r\dagger}(\vec{p'}) b_{\vec{p}}^{s} u_\alpha^{s}(\vec{p}) e^{i\vec{p}\cdot\vec{x}+i\vec{p'}\cdot\vec{y}} + b_{\vec{p'}}^{r\dagger} u_\beta^{r\dagger}(\vec{p'}) c_{\vec{p}}^{s\dagger} v_\alpha^{s}(\vec{p}) e^{-i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}} + c_{\vec{p}}^{s\dagger} v_\alpha^{s}(\vec{p}) c_{\vec{p'}}^{r} v_\beta^{r\dagger}(\vec{p'}) e^{-i\vec{p}\cdot\vec{x}+i\vec{p'}\cdot\vec{y}} \right]$$

$$= \sum_{\alpha=1}^{4}\sum_{\beta=1}^{4}\sum_{s=1}^{2}\sum_{r=1}^{2} \int \frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \frac{1}{\sqrt{2E_{\vec{p'}}}} \left[\{b_{\vec{p}}^{s}, b_{\vec{p'}}^{r\dagger}\} u_\alpha^{s}(\vec{p}) u_\beta^{r\dagger}(\vec{p'}) e^{i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}} \right.$$

$$+ \{b_{\vec{p}}^{s}, c_{\vec{p'}}^{r\dagger}\} u_\alpha^{s}(\vec{p}) v_\beta^{r\dagger}(\vec{p'}) e^{i\vec{p}\cdot\vec{x}+i\vec{p'}\cdot\vec{y}} + \{c_{\vec{p}}^{s}, b_{\vec{p'}}^{r\dagger}\} v_\alpha^{s}(\vec{p}) u_\beta^{r\dagger}(\vec{p'}) e^{-i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}}$$

$$\left. + \{c_{\vec{p}}^{s}, c_{\vec{p'}}^{r\dagger}\} v_\alpha^{s}(\vec{p}) v_\beta^{r\dagger}(\vec{p'}) e^{-i\vec{p}\cdot\vec{x}+i\vec{p'}\cdot\vec{y}} \right] \tag{4}$$

Given

$$\{b_{\vec{p}}^{s}, b_{\vec{p'}}^{r}\} = \{c_{\vec{p}}^{s}, c_{\vec{p'}}^{r}\} = \{c_{\vec{p}}^{s}, b_{\vec{p'}}^{r\dagger}\} = \{c_{\vec{p}}^{s}, b_{\vec{p'}}^{r}\} = \ldots = 0 \tag{5}$$

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$$\{b_{\vec{p}}^{s}, b_{\vec{p'}}^{r\dagger}\} = \{c_{\vec{p}}^{s}, c_{\vec{p'}}^{r\dagger}\} = (2\pi)^3 \delta^{rs} \delta^{(3)}(\vec{p} - \vec{p'}) \qquad (6)$$

Therefore eqn(4) will be

$$\{\psi_\alpha(\vec{x}), \psi_\beta^\dagger(\vec{y})\} =$$

$$\sum_{\alpha=1}^{4} \sum_{\beta=1}^{4} \sum_{s=1}^{2} \sum_{r=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \frac{1}{\sqrt{2E_{\vec{p'}}}} \Big[(2\pi)^3 \delta^{rs} \delta^{(3)}(\vec{p} - \vec{p'}) u_\alpha^s(\vec{p}) u_\beta^{r\dagger}(\vec{p'}) e^{i\vec{p}\cdot\vec{x} - i\vec{p'}\cdot\vec{y}}$$

$$+ (2\pi)^3 \delta^{rs} \delta^{(3)}(\vec{p} - \vec{p'}) v_\alpha^s(\vec{p}) v_\beta^{r\dagger}(\vec{p'}) e^{-i\vec{p}\cdot\vec{x} + i\vec{p'}\cdot\vec{y}} \Big]$$

$$= \sum_{\alpha=1}^{4} \sum_{\beta=1}^{4} \int \frac{d^3p}{(2\pi)^6} \frac{1}{2E_{\vec{p}}} \Big[(2\pi)^3 u_\alpha(\vec{p}) u_\beta^\dagger(\vec{p}) e^{i\vec{p}\cdot(\vec{x}-\vec{y})} + (2\pi)^3 v_\alpha(\vec{p}) v_\beta^\dagger(\vec{p}) e^{-i\vec{p}\cdot(\vec{x}-\vec{y})} \Big]$$

$$= \sum_{\alpha=1}^{4} \sum_{\beta=1}^{4} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} \Big[u_\alpha(\vec{p}) \bar{u}_\beta^\dagger \gamma_{\alpha\beta}^0(\vec{p}) e^{i\vec{p}\cdot(\vec{x}-\vec{y})} + v_\alpha(\vec{p}) \bar{v}_\beta^\dagger \gamma_{\alpha\beta}^0(\vec{p}) e^{-i\vec{p}\cdot(\vec{x}-\vec{y})} \Big]$$

$$= \sum_{\alpha=1}^{4} \sum_{\beta=1}^{4} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} \Big[[(\gamma \cdot p + m)\gamma^0]_{\alpha\beta} e^{i\vec{p}\cdot(\vec{x}-\vec{y})} + [(\gamma \cdot p - m)\gamma^0]_{\alpha\beta} e^{-i\vec{p}\cdot(\vec{x}-\vec{y})} \Big]$$

$$= \sum_{\alpha=1}^{4} \sum_{\beta=1}^{4} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} \Big[[(\gamma^0 E_p - \gamma \cdot p + m)\gamma^0]_{\alpha\beta} e^{i\vec{p}\cdot(\vec{x}-\vec{y})} + [(\gamma^0 E_p - \gamma \cdot p - m)\gamma^0]_{\alpha\beta} e^{-i\vec{p}\cdot(\vec{x}-\vec{y})} \Big]$$

beacause of antiparticle second half of the eqn wil be $p \to -p$

$$= \sum_{\alpha=1}^{4} \sum_{\beta=1}^{4} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} \Big[[(\gamma^0 E_p - \gamma \cdot p + m)\gamma^0]_{\alpha\beta} + [(\gamma^0 E_p + \gamma \cdot p - m)\gamma^0]_{\alpha\beta} \Big] e^{i\vec{p}\cdot(\vec{x}-\vec{y})}$$

$$= \sum_{\alpha=1}^{4} \sum_{\beta=1}^{4} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} \Big[[\gamma^0 E_p \gamma^0]_{\alpha\beta} + [\gamma^0 E_p \gamma^0]_{\alpha\beta} \Big] e^{i\vec{p}\cdot(\vec{x}-\vec{y})}$$

$$= \int \frac{d^3p}{(2\pi)^3} \delta_{\alpha\beta} e^{i\vec{p}\cdot(\vec{x}-\vec{y})}$$

$$\therefore \{\psi_\alpha(\vec{x}), \psi_\beta^\dagger(\vec{y})\} = \delta_{\alpha\beta} \delta^{(3)}(\vec{x} - \vec{y})$$

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$$\left\{ \psi_\alpha(\vec{x}), \psi_\beta(\vec{y}) \right\} =$$

$$= \left\{ \sum_{\alpha=1}^{4} \sum_{s=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left[b_{\vec{p}}^s u_\alpha^s(\vec{p}) e^{i\vec{p}\cdot\vec{x}} + c_{\vec{p}}^{s\dagger} v_\alpha^s(\vec{p}) e^{-i\vec{p}\cdot\vec{x}} \right] , \right.$$

$$\left. \sum_{\beta=1}^{4} \sum_{r=1}^{2} \int \frac{d^3p'}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p'}}}} \left[b_{\vec{p'}}^r u_\beta^r(\vec{p'}) e^{i\vec{p'}\cdot\vec{y}} + c_{\vec{p'}}^{r\dagger} v_\beta^r(\vec{p'}) e^{-i\vec{p'}\cdot\vec{y}} \right] \right\}$$

$$= \sum_{\alpha=1}^{4} \sum_{\beta=1}^{4} \sum_{s=1}^{2} \sum_{r=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \frac{1}{\sqrt{2E_{\vec{p'}}}} \left[\left\{ b_{\vec{p}}^s, b_{\vec{p'}}^r \right\} u_\alpha^s(\vec{p}) u_\beta^r(\vec{p'}) e^{i\vec{p}\cdot\vec{x}+i\vec{p'}\cdot\vec{y}} \right.$$

$$+ \left\{ b_{\vec{p}}^s, c_{\vec{p'}}^{r\dagger} \right\} u_\alpha^s(\vec{p}) v_\beta^r(\vec{p'}) e^{i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}} + \left\{ c_{\vec{p}}^{s\dagger}, b_{\vec{p'}}^r \right\} v_\alpha^s(\vec{p}) u_\beta^r(\vec{p'}) e^{i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}}$$

$$+ \left. \left\{ c_{\vec{p}}^s, c_{\vec{p'}}^r \right\} v_\alpha^s(\vec{p}) v_\beta^r(\vec{p'}) e^{i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}} \right]$$

because of eqn(5) every term vanishes.
Therefore

$$\left\{ \psi_\alpha(\vec{x}), \psi_\beta(\vec{y}) \right\} = 0 \tag{7}$$

And

$$\left\{ \psi_\alpha^\dagger(\vec{x}), \psi_\beta^\dagger(\vec{y}) \right\} =$$

$$= \left\{ \sum_{\alpha=1}^{4} \sum_{s=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left[b_{\vec{p}}^{s\dagger} u_\alpha^{s\dagger}(\vec{p}) e^{-i\vec{p}\cdot\vec{x}} + c_{\vec{p}}^s v_\alpha^{s\dagger}(\vec{p}) e^{-i\vec{p}\cdot\vec{x}} \right] , \right.$$

$$\left. \sum_{\beta=1}^{4} \sum_{r=1}^{2} \int \frac{d^3p'}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p'}}}} \left[b_{\vec{p'}}^{r\dagger} u_\beta^{r\dagger}(\vec{p'}) e^{-i\vec{p'}\cdot\vec{y}} + c_{\vec{p'}}^r v_\beta^{r\dagger}(\vec{p'}) e^{i\vec{p'}\cdot\vec{y}} \right] \right\}$$

$$= \sum_{\alpha=1}^{4} \sum_{\beta=1}^{4} \sum_{s=1}^{2} \sum_{r=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \frac{1}{\sqrt{2E_{\vec{p'}}}} \left[\left\{ b_{\vec{p}}^{s\dagger}, b_{\vec{p'}}^{r\dagger} \right\} u_\alpha^{s\dagger}(\vec{p}) u_\beta^{r\dagger}(\vec{p'}) e^{-i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}} \right.$$

$$+ \left\{ b_{\vec{p}}^{s\dagger}, c_{\vec{p'}}^r \right\} u_\alpha^{s\dagger}(\vec{p}) v_\beta^{r\dagger}(\vec{p'}) e^{-i\vec{p}\cdot\vec{x}+i\vec{p'}\cdot\vec{y}} + \left\{ c_{\vec{p}}^s, b_{\vec{p'}}^{r\dagger} \right\} v_\alpha^{s\dagger}(\vec{p}) u_\beta^{r\dagger}(\vec{p'}) e^{i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}}$$

$$+ \left. \left\{ c_{\vec{p}}^s, c_{\vec{p'}}^r \right\} v_\alpha^{s\dagger}(\vec{p}) v_\beta^{r\dagger}(\vec{p'}) e^{i\vec{p}\cdot\vec{x}+i\vec{p'}\cdot\vec{y}} \right]$$

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Also for eqn(5)

$$\left\{ \psi_\alpha^\dagger(\vec{x}), \psi_\beta^\dagger(\vec{y}) \right\} = 0 \tag{8}$$

Now

$$H =$$

$$\int d^3x\, \overline{\psi}(-i\gamma^i\partial_i + m)\psi$$

$$= \int \frac{d^3x}{(2\pi)^6} \sum_{s,s'=1}^{2} \int \frac{d^3p'}{\sqrt{2E\vec{p'}}} \frac{d^3p}{\sqrt{2E\vec{p}}} (\overline{u}^s(\vec{p})b_{\vec{p}}^s e^{i\vec{p}\cdot\vec{x}} + c_{\vec{p}}^{s\dagger}\overline{v}^s(\vec{p})e^{i\vec{p}\cdot\vec{x}})(-i\gamma^i\partial_i + m)$$

$$\times (b_{\vec{p'}}^{s'\dagger}u^{s'\dagger}(\vec{p'})e^{i\vec{p'}\cdot\vec{x}} + c_{\vec{p'}}^{s'}v^{s'\dagger}(\vec{p'})e^{-i\vec{p'}\cdot\vec{x}})$$

$$= \int \frac{d^3x}{(2\pi)^6} \sum_{s,s'=1}^{2} \int \frac{d^3p'}{\sqrt{2E\vec{p'}}} \frac{d^3p}{\sqrt{2E\vec{p}}} (\overline{u}^s(\vec{p})b_{\vec{p}}^s e^{i\vec{p}\cdot\vec{x}} + c_{\vec{p}}^{s\dagger}\overline{v}^s(\vec{p})e^{i\vec{p}\cdot\vec{x}})$$

$$\times (e^{i\vec{p'}\cdot\vec{x}}b_{\vec{p'}}^{s'\dagger}(\gamma \cdot p' + m)u^{s'\dagger}(\vec{p'}) + e^{-i\vec{p'}\cdot\vec{x}}c_{\vec{p'}}^{s'\dagger}(\gamma \cdot p' - m)v^{s'\dagger}(\vec{p'}))$$

We know

$$(\not{p} - m)u^s(\vec{p}) = (\gamma^0 p^0 - \gamma \cdot p - m)u^s(\vec{p}) = 0$$
$$(\not{p} + m)v^s(\vec{p}) = (\gamma^0 p^0 - \gamma \cdot p + m)v^s(\vec{p}) = 0$$

Therefore

$$= \int \frac{d^3x}{(2\pi)^6} \sum_{s,s'=1}^{2} \int \frac{d^3p'}{\sqrt{2E_{\vec{p'}}}} \frac{d^3p}{\sqrt{2E_{\vec{p}}}} p^0 (\bar{u}^s(\vec{p}) b^s_{\vec{p}} e^{i\vec{p}\cdot\vec{x}} + c^{s\dagger}_{\vec{p}} \bar{v}^s(\vec{p}) e^{i\vec{p}\cdot\vec{x}})$$

$$\times \gamma^0 (e^{i\vec{p'}\cdot\vec{x}} b^{s'\dagger}_{\vec{p'}} u^{s'\dagger}(\vec{p'}) - e^{-i\vec{p'}\cdot\vec{x}} c^{s'}_{\vec{p'}} v^{s'\dagger}(\vec{p'}))$$

$$= \int \frac{d^3x}{(2\pi)^6} \sum_{s,s'=1}^{2} \int \frac{d^3p'}{\sqrt{2E_{\vec{p'}}}} \frac{d^3p}{\sqrt{2E_{\vec{p}}}} p^0 (\bar{u}^s(\vec{p}) \gamma^0 u^{s'}(\vec{p'}) b^{s'}_{\vec{p'}} b^{s\dagger}_{\vec{p}} \delta^3(p-p') - c^{s'\dagger}_{\vec{p'}} c^s_{\vec{p}} \bar{v}^s(\vec{p}) \gamma^0 v^{s'}(\vec{p'})) \delta^3(p-p')$$

$$- \delta^3(p+p')(b^{s\dagger}_{\vec{p}} c^{s'\dagger}_{\vec{p'}} \bar{u}^s(\vec{p}) \gamma^0 v^{s'}(\vec{p'}) - \bar{v}^s(\vec{p}) \gamma^0 u^{s'}(\vec{p'}) b^{s'}_{\vec{p'}} c^{s'}_{\vec{p'}})$$

$$= \sum_{s,s'=1}^{2} \int d^3p (u^{s\dagger}(\vec{p}) u^{s'}(\vec{p}) b^{s'}_{\vec{p}} b^{s\dagger}_{\vec{p}} + c^{s\dagger}_{\vec{p}} c^s_{\vec{p}} v^{s\dagger}(\vec{p}) v^{s'}(\vec{p}) + b^{s\dagger}_{-\vec{p}} c^{s'\dagger}_{-\vec{p}} u^{s\dagger}(-\vec{p}) v^{s'}(-\vec{p}) + v^{s\dagger}(-\vec{p}) u^{s'}(-\vec{p}) b^{s'}_{-\vec{p}} c^{s'}_{-\vec{p}})$$

$$= \sum_{s,s'=1}^{2} \int d^3p \delta_{ss'} E_{\vec{p}} (b^{s'\dagger}_{\vec{p}} b^s_{\vec{p}} - c^{s'}_{\vec{p}} c^{s\dagger}_{\vec{p}})$$

$$= \sum_{s,s'=1}^{2} \int d^3p E_{\vec{p}} (b^{s\dagger}_{\vec{p}} b^s_{\vec{p}} - c^s_{\vec{p}} c^{s\dagger}_{\vec{p}})$$

for normal ordering in fermionic field

$$: c^s_{\vec{p}} c^{s\dagger}_{\vec{p}} := -c^{s\dagger}_{\vec{p}} c^s_{\vec{p}} \tag{9}$$

Therefore

$$H = \sum_{s,s'=1}^{2} \int d^3p E_{\vec{p}} (b^{s\dagger}_{\vec{p}} b^s_{\vec{p}} c^{s\dagger}_{\vec{p}} c^s_{\vec{p}}) \tag{10}$$

Solve for problem no. 3

$$i\left[S^{\rho\sigma},\gamma^{\mu}\right] = i\frac{i}{4}\left[\left[\gamma^{\rho},\gamma^{\sigma}\right]\gamma^{\mu}\right]$$

$$= -\frac{1}{4}\left(\left[\gamma^{\rho},\gamma^{\sigma}\right]\gamma^{\mu} - \gamma^{\mu}\left[\gamma^{\rho},\gamma^{\sigma}\right]\right)$$

$$= -\frac{1}{4}\left(\gamma^{\rho}\gamma^{\sigma}\gamma^{\mu} - \gamma^{\sigma}\gamma^{\rho}\gamma^{\mu} - \gamma^{\mu}\gamma^{\rho}\gamma^{\sigma} + \gamma^{\mu}\gamma^{\sigma}\gamma^{\rho}\right)$$

$$= -\frac{1}{4}\left(\gamma^{\rho}(2\eta^{\sigma\mu} - \gamma^{\mu}\gamma^{\sigma}) - \gamma^{\sigma}(2\eta^{\rho\mu} - \gamma^{\mu}\gamma^{\rho}) + (2\eta^{\mu\sigma} - \gamma^{\sigma}\gamma^{\mu})\gamma^{\rho} - (2\eta^{\mu\rho} - \gamma^{\rho}\gamma^{\mu})\gamma^{\sigma}\right)$$

$$= -\frac{1}{4}\left(2\eta^{\sigma\mu}\gamma^{\rho} - 2\eta^{\rho\mu}\gamma^{\sigma} + 2\eta^{\mu\sigma}\gamma^{\rho} - 2\eta^{\mu\rho}\gamma^{\sigma}\right)$$

$$= \eta^{\rho\mu}\gamma^{\sigma} - \eta^{\sigma\mu}\gamma^{\rho}$$

the Dirac Lagrangian is

$$\mathcal{L} = \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi$$

Under Lorentz transformation it wil be

$$\mathcal{L} = \overline{\psi'}(i\gamma^{\mu}\partial'_{\mu} - m)\psi'$$
$$= \overline{\psi}L^{-1}(\Lambda)(i\gamma^{\mu}\partial_{\mu} - m)\psi L(\Lambda)$$
$$= i\overline{\psi}L^{-1}(\Lambda)\gamma^{\mu}L(\Lambda)\partial_{\mu}\psi - m\overline{\psi}L^{-1}(\Lambda)L(\Lambda)\psi \qquad = \overline{\psi}\Lambda^{\mu}_{\nu}i\gamma^{\nu}\partial_{\mu}\psi - m\overline{\psi}\psi$$

where

$$\Lambda^{\mu}_{\nu} = \overline{\psi}\gamma^{\mu}\psi = j^{\mu}$$

which is a current four vector transform or invariant under lorentz transformation.

$$\partial_{\mu}j^{\mu} = 0$$

L.H.S

$$[M^{\mu\nu}, M^{\rho\sigma}] =$$

$$[S^{\mu\nu}, S^{\rho\sigma}]$$

$$= \frac{i}{4}\frac{i}{4}([[\gamma^\mu, \gamma^{nu}], [\gamma^\rho, \gamma^\sigma]])$$

$$= \frac{i}{4}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma - \gamma^\mu\gamma^\nu\gamma^\sigma\gamma^\rho - \gamma^\nu\gamma^\mu\gamma^\rho\gamma^\sigma + \gamma^\mu\gamma^\nu\gamma^\sigma\gamma^\rho$$

$$- \gamma^\rho\gamma^\sigma\gamma^\mu\gamma^\nu + \gamma^\rho\gamma^\sigma\gamma^\nu\gamma^\mu - \gamma^\sigma\gamma^\rho\gamma^\mu\gamma^\nu + \gamma^\sigma\gamma^\rho\gamma^\nu\gamma^\mu$$

$$= \frac{i}{4}[\gamma^\mu(2\eta^{\nu\rho} - \gamma^\rho\gamma^\nu)\gamma^\sigma - \gamma^\sigma(2\eta^{\nu\rho} - \gamma^\nu\gamma^\rho)\gamma^\mu - \gamma^\nu(2\eta^{\mu\rho} - \gamma^\rho\gamma^\mu)\gamma^\sigma + \gamma^\sigma(2\eta^{\mu\rho} - \gamma^\mu\gamma^\rho)\gamma^\nu$$

$$+ \gamma^\rho(2\eta^{\sigma\nu} - \gamma^\nu\gamma^\sigma)\gamma^\mu - \gamma^\mu(2\eta^{\nu\sigma} - \gamma^\rho\gamma^\nu)\gamma^\sigma - \gamma^\mu(2\eta^{\nu\rho} - \gamma^\rho\gamma^\nu)\gamma^\sigma - \gamma^\mu(2\eta^{\nu\rho} - \gamma^\rho\gamma^\nu)\gamma^\sigma + \gamma^\mu(2\eta^{\nu\rho}$$

Given,

$$\psi(\vec{x}) = \sum_{s=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left[b^s_{\vec{p}} u^s(\vec{p})e^{i\vec{p}\cdot\vec{x}} + c^{s\dagger}_{\vec{p}} v^s(\vec{p})e^{-i\vec{p}\cdot\vec{x}} \right] \qquad (11)$$

And

$$\psi^\dagger(\vec{x}) = \sum_{s=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left[b^{s\dagger}_{\vec{p}} u^{s\dagger}(\vec{p})e^{-i\vec{p}\cdot\vec{x}} + c^s_{\vec{p}} v^{s\dagger}(\vec{p})e^{i\vec{p}\cdot\vec{x}} \right] \qquad (12)$$

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Now

$$\left\{ \psi_\alpha(\vec{x}), \psi_\beta^\dagger(\vec{y}) \right\}$$

$$= \left\{ \sum_{\alpha=1}^{4} \sum_{s=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left[b_{\vec{p}}^s u_\alpha^s(\vec{p}) e^{i\vec{p}\cdot\vec{x}} + c_{\vec{p}}^{s\dagger} v_\alpha^s(\vec{p}) e^{-i\vec{p}\cdot\vec{x}} \right], \right.$$

$$\left. \sum_{\beta=1}^{4} \sum_{r=1}^{2} \int \frac{d^3p'}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p'}}}} \left[b_{\vec{p'}}^{r\dagger} u_\beta^{r\dagger}(\vec{p'}) e^{-i\vec{p'}\cdot\vec{y}} + c_{\vec{p'}}^r v_\beta^{r\dagger}(\vec{p'}) e^{i\vec{p'}\cdot\vec{y}} \right] \right\}$$

$$= \sum_{\alpha=1}^{4} \sum_{\beta=1}^{4} \sum_{s=1}^{2} \sum_{r=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \frac{1}{\sqrt{2E_{\vec{p'}}}} \left[b_{\vec{p}}^s u_\alpha^s(\vec{p}) b_{\vec{p'}}^{r\dagger} u_\beta^{r\dagger}(\vec{p'}) e^{i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}} + b_{\vec{p}}^s u_\alpha^s(\vec{p}) c_{\vec{p'}}^r v_\beta^{r\dagger}(\vec{p'}) e^{i\vec{p}\cdot\vec{x}+i\vec{p'}\cdot\vec{y}} \right.$$

$$+ c_{\vec{p}}^{s\dagger} v_\alpha^s(\vec{p}) b_{\vec{p'}}^{r\dagger} u_\beta^{r\dagger}(\vec{p'}) e^{-i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}} + c_{\vec{p}}^{s\dagger} v_\alpha^s(\vec{p}) c_{\vec{p'}}^r v_\beta^{r\dagger}(\vec{p'}) e^{-i\vec{p}\cdot\vec{x}+i\vec{p'}\cdot\vec{y}} + b_{\vec{p'}}^{r\dagger} u_\beta^{r\dagger}(\vec{p'}) b_{\vec{p}}^s u_\alpha^s(\vec{p}) e^{i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}}$$

$$+ c_{\vec{p'}}^r v_\beta^{r\dagger}(\vec{p'}) b_{\vec{p}}^s u_\alpha^s(\vec{p}) e^{i\vec{p}\cdot\vec{x}+i\vec{p'}\cdot\vec{y}} + b_{\vec{p'}}^{r\dagger} u_\beta^{r\dagger}(\vec{p'}) c_{\vec{p}}^{s\dagger} v_\alpha^s(\vec{p}) e^{-i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}} + c_{\vec{p}}^{s\dagger} v_\alpha^s(\vec{p}) c_{\vec{p'}}^r v_\beta^{r\dagger}(\vec{p'}) e^{-i\vec{p}\cdot\vec{x}+i\vec{p'}\cdot\vec{y}} \right]$$

$$= \sum_{\alpha=1}^{4} \sum_{\beta=1}^{4} \sum_{s=1}^{2} \sum_{r=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \frac{1}{\sqrt{2E_{\vec{p'}}}} \left[\left\{ b_{\vec{p}}^s, b_{\vec{p'}}^{r\dagger} \right\} u_\alpha^s(\vec{p}) u_\beta^{r\dagger}(\vec{p'}) e^{i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}} \right.$$

$$+ \left\{ b_{\vec{p}}^s, c_{\vec{p'}}^{r\dagger} \right\} u_\alpha^s(\vec{p}) v_\beta^{r\dagger}(\vec{p'}) e^{i\vec{p}\cdot\vec{x}+i\vec{p'}\cdot\vec{y}} + \left\{ c_{\vec{p}}^s, b_{\vec{p'}}^{r\dagger} \right\} v_\alpha^s(\vec{p}) u_\beta^{r\dagger}(\vec{p'}) e^{-i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}}$$

$$\left. + \left\{ c_{\vec{p}}^s, c_{\vec{p'}}^{r\dagger} \right\} v_\alpha^s(\vec{p}) v_\beta^{r\dagger}(\vec{p'}) e^{-i\vec{p}\cdot\vec{x}+i\vec{p'}\cdot\vec{y}} \right]$$

$$(13)$$

Given

$$\left\{ b_{\vec{p}}^s, b_{\vec{p'}}^r \right\} = \left\{ c_{\vec{p}}^s, c_{\vec{p'}}^r \right\} = \left\{ c_{\vec{p}}^s, b_{\vec{p'}}^{r\dagger} \right\} = \left\{ c_{\vec{p}}^s, b_{\vec{p'}}^r \right\} = \ldots\ldots = 0 \tag{14}$$

$$\left\{ b_{\vec{p}}^s, b_{\vec{p'}}^{r\dagger} \right\} = \left\{ c_{\vec{p}}^s, c_{\vec{p'}}^{r\dagger} \right\} = (2\pi)^3 \delta^{rs} \delta^{(3)}(\vec{p} - \vec{p'}) \tag{15}$$

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Therefore eqn(14) will be

$$\left\{\psi_\alpha(\vec{x}), \psi_\beta^\dagger(\vec{y})\right\} =$$

$$\sum_{\alpha=1}^{4}\sum_{\beta=1}^{4}\sum_{s=1}^{2}\sum_{r=1}^{2}\int \frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{p}}}}\frac{1}{\sqrt{2E_{\vec{p'}}}}\left[(2\pi)^3\delta^{rs}\delta^{(3)}(\vec{p}-\vec{p'})u_\alpha^s(\vec{p})u_\beta^{r\dagger}(\vec{p'})e^{i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}}\right.$$

$$\left. + (2\pi)^3\delta^{rs}\delta^{(3)}(\vec{p}-\vec{p'})v_\alpha^s(\vec{p})v_\beta^{r\dagger}(\vec{p'})e^{-i\vec{p}\cdot\vec{x}+i\vec{p'}\cdot\vec{y}}\right]$$

$$=\sum_{\alpha=1}^{4}\sum_{\beta=1}^{4}\int \frac{d^3p}{(2\pi)^6}\frac{1}{2E_{\vec{p}}}\left[(2\pi)^3 u_\alpha(\vec{p})u_\beta^\dagger(\vec{p})e^{i\vec{p}\cdot(\vec{x}-\vec{y})} + (2\pi)^3 v_\alpha(\vec{p})v_\beta^\dagger(\vec{p})e^{-i\vec{p}\cdot(\vec{x}-\vec{y})}\right]$$

$$=\sum_{\alpha=1}^{4}\sum_{\beta=1}^{4}\int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\left[u_\alpha(\vec{p})\bar{u}_\beta^\dagger\gamma_{\alpha\beta}^0(\vec{p})e^{i\vec{p}\cdot(\vec{x}-\vec{y})} + v_\alpha(\vec{p})\bar{v}_\beta^\dagger\gamma_{\alpha\beta}^0(\vec{p})e^{-i\vec{p}\cdot(\vec{x}-\vec{y})}\right]$$

$$=\sum_{\alpha=1}^{4}\sum_{\beta=1}^{4}\int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\left[[(\gamma\cdot p+m)\gamma^0]_{\alpha\beta}e^{i\vec{p}\cdot(\vec{x}-\vec{y})} + [(\gamma\cdot p-m)\gamma^0]_{\alpha\beta}e^{-i\vec{p}\cdot(\vec{x}-\vec{y})}\right]$$

$$=\sum_{\alpha=1}^{4}\sum_{\beta=1}^{4}\int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\left[[(\gamma^0 E_p-\gamma\cdot p+m)\gamma^0]_{\alpha\beta}e^{i\vec{p}\cdot(\vec{x}-\vec{y})} + [(\gamma^0 E_p-\gamma\cdot p-m)\gamma^0]_{\alpha\beta}e^{-i\vec{p}\cdot(\vec{x}-\vec{y})}\right]$$

beacause of antiparticle second half of the eqn wil be $p\rightarrow -p$

$$=\sum_{\alpha=1}^{4}\sum_{\beta=1}^{4}\int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\left[[(\gamma^0 E_p-\gamma\cdot p+m)\gamma^0]_{\alpha\beta} + [(\gamma^0 E_p+\gamma\cdot p-m)\gamma^0]_{\alpha\beta}\right]e^{i\vec{p}\cdot(\vec{x}-\vec{y})}$$

$$=\sum_{\alpha=1}^{4}\sum_{\beta=1}^{4}\int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\left[[\gamma^0 E_p\gamma^0]_{\alpha\beta} + [\gamma^0 E_p\gamma^0]_{\alpha\beta}\right]e^{i\vec{p}\cdot(\vec{x}-\vec{y})}$$

$$=\int \frac{d^3p}{(2\pi)^3}\delta_{\alpha\beta}e^{i\vec{p}\cdot(\vec{x}-\vec{y})}$$

$$\therefore \left\{\psi_\alpha(\vec{x}), \psi_\beta^\dagger(\vec{y})\right\} = \delta_{\alpha\beta}\delta^{(3)}(\vec{x}-\vec{y})$$

$$\left\{ \psi_\alpha(\vec{x}), \psi_\beta(\vec{y}) \right\} =$$

$$= \left\{ \sum_{\alpha=1}^{4} \sum_{s=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left[b_{\vec{p}}^s u_\alpha^s(\vec{p}) e^{i\vec{p}\cdot\vec{x}} + c_{\vec{p}}^{s\dagger} v_\alpha^s(\vec{p}) e^{-i\vec{p}\cdot\vec{x}} \right], \right.$$

$$\left. \sum_{\beta=1}^{4} \sum_{r=1}^{2} \int \frac{d^3p'}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p'}}}} \left[b_{\vec{p'}}^r u_\beta^r(\vec{p'}) e^{i\vec{p'}\cdot\vec{y}} + c_{\vec{p'}}^{r\dagger} v_\beta^r(\vec{p'}) e^{-i\vec{p'}\cdot\vec{y}} \right] \right\}$$

$$= \sum_{\alpha=1}^{4} \sum_{\beta=1}^{4} \sum_{s=1}^{2} \sum_{r=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \frac{1}{\sqrt{2E_{\vec{p'}}}} \left[\left\{ b_{\vec{p}}^s, b_{\vec{p'}}^r \right\} u_\alpha^s(\vec{p}) u_\beta^r(\vec{p'}) e^{i\vec{p}\cdot\vec{x}+i\vec{p'}\cdot\vec{y}} \right.$$

$$+ \left\{ b_{\vec{p}}^s, c_{\vec{p'}}^{r\dagger} \right\} u_\alpha^s(\vec{p}) v_\beta^r(\vec{p'}) e^{i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}} + \left\{ c_{\vec{p}}^{s\dagger}, b_{\vec{p'}}^r \right\} v_\alpha^s(\vec{p}) u_\beta^r(\vec{p'}) e^{i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}}$$

$$\left. + \left\{ c_{\vec{p}}^s, c_{\vec{p'}}^r \right\} v_\alpha^s(\vec{p}) v_\beta^r(\vec{p'}) e^{i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}} \right]$$

because of eqn(15) every term vanishes.
Therefore

$$\left\{ \psi_\alpha(\vec{x}), \psi_\beta(\vec{y}) \right\} = 0 \tag{16}$$

And

$$\left\{ \psi_\alpha^\dagger(\vec{x}), \psi_\beta^\dagger(\vec{y}) \right\} =$$

$$= \left\{ \sum_{\alpha=1}^{4} \sum_{s=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left[b_{\vec{p}}^{s\dagger} u_\alpha^{s\dagger}(\vec{p}) e^{-i\vec{p}\cdot\vec{x}} + c_{\vec{p}}^s v_\alpha^{s\dagger}(\vec{p}) e^{-i\vec{p}\cdot\vec{x}} \right], \right.$$

$$\left. \sum_{\beta=1}^{4} \sum_{r=1}^{2} \int \frac{d^3p'}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p'}}}} \left[b_{\vec{p'}}^{r\dagger} u_\beta^{r\dagger}(\vec{p'}) e^{-i\vec{p'}\cdot\vec{y}} + c_{\vec{p'}}^r v_\beta^{r\dagger}(\vec{p'}) e^{i\vec{p'}\cdot\vec{y}} \right] \right\}$$

$$= \sum_{\alpha=1}^{4} \sum_{\beta=1}^{4} \sum_{s=1}^{2} \sum_{r=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \frac{1}{\sqrt{2E_{\vec{p'}}}} \left[\left\{ b_{\vec{p}}^{s\dagger}, b_{\vec{p'}}^{r\dagger} \right\} u_\alpha^{s\dagger}(\vec{p}) u_\beta^{r\dagger}(\vec{p'}) e^{-i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}} \right.$$

$$+ \left\{ b_{\vec{p}}^{s\dagger}, c_{\vec{p'}}^r \right\} u_\alpha^{s\dagger}(\vec{p}) v_\beta^{r\dagger}(\vec{p'}) e^{-i\vec{p}\cdot\vec{x}+i\vec{p'}\cdot\vec{y}} + \left\{ c_{\vec{p}}^s, b_{\vec{p'}}^{r\dagger} \right\} v_\alpha^{s\dagger}(\vec{p}) u_\beta^{r\dagger}(\vec{p'}) e^{i\vec{p}\cdot\vec{x}-i\vec{p'}\cdot\vec{y}}$$

$$\left. + \left\{ c_{\vec{p}}^s, c_{\vec{p'}}^r \right\} v_\alpha^{s\dagger}(\vec{p}) v_\beta^{r\dagger}(\vec{p'}) e^{i\vec{p}\cdot\vec{x}+i\vec{p'}\cdot\vec{y}} \right]$$

Also for eqn(15)

$$\left\{ \psi_\alpha^\dagger(\vec{x}), \psi_\beta^\dagger(\vec{y}) \right\} = 0 \tag{17}$$