Solution to Problem Sheet 2

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Solve for problem no. 1

Given

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j^{\mu}A^{\mu} \\
&= -\frac{1}{4}(\partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu})(\partial^{\mu}A^{\nu} - \partial^{\mu}A^{\nu}) - j^{\mu}A^{\mu} \\
&= -\frac{1}{4}(\partial_{\mu}A_{\nu}(\partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}) - \partial_{\nu}A_{\mu}(\partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu})) - j^{\mu}A^{\mu} \\
&= -\frac{1}{2}(\partial_{\mu}A_{\nu}(\partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu})) - j^{\mu}A^{\mu}
\end{aligned}
$$

(a)

$$
\begin{aligned}
\frac{\partial\mathcal{L}}{\partial A_{\nu}} &= -j^{\nu} \\
\frac{\partial\mathcal{L}}{\partial(\partial_{\mu}A_{\nu})} &= -\frac{1}{2}(\partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu})) - \frac{1}{2}\partial^{\mu}A^{\nu} + \frac{1}{2}\partial^{\nu}A^{\mu} \\
&= -(\partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu})) = F^{\mu\nu}
\end{aligned}
$$

Therefore Euler-Langrangian is

$$
\begin{aligned}
\frac{\partial\mathcal{L}}{\partial A_{\nu}} - \frac{\partial\mathcal{L}}{\partial(\partial_{\mu}A_{\nu})} &= -j^{\nu} + \partial_{\mu}F^{\mu}\nu = 0 \\
\partial_{\mu}F^{\mu}\nu &= j^{\nu}
\end{aligned}
$$