

For 3-point connected Green function we get

$$\Gamma_3(G_2)^3 = -G_3 \quad (1)$$

Differentiating with respect to ϕ^c

$$\begin{aligned} & \frac{\delta}{\delta \phi^c} \Gamma_3(G_2)^3 + \Gamma_3 \frac{\delta(G_2)^3}{\delta J} \frac{\delta J}{\delta \phi^c} = \frac{\delta G_3}{\delta J} \frac{\delta J}{\delta \phi^c} \\ \implies & \Gamma_4(G_2)^3 + 3\Gamma_3(G_2)^2 G_3(G_2)^{-1} = -G_4(G_2)^{-1} \\ \implies & \Gamma_4(G_2)^4 + 3\Gamma_3(G_2)^2 G_3 = -G_4 \\ \implies & \Gamma_4(G_2)^4 - 3\Gamma_3(G_2)^2 G_2 \Gamma_3(G_2)^2 = -G_4 \quad [\text{from eqn (1)}] \end{aligned}$$

Digramatically will be

Diagrammatic representation of the differentiation of the 3-point Green function equation. The diagram shows the differentiation of the equation $\Gamma_3(G_2)^3 = -G_3$ with respect to ϕ^c . It uses Feynman diagrams to represent the terms: a 3-point vertex for Γ_3 , a 4-point vertex for Γ_4 , and a 3-point vertex for G_3 . The diagrammatic equation is: $[3\text{-point vertex}]^3 = -[3\text{-point vertex}]$. Differentiating both sides with respect to ϕ^c gives: $3[3\text{-point vertex}]^2 [3\text{-point vertex}] = -[4\text{-point vertex}]$. This is then rearranged to: $[4\text{-point vertex}] = 3[3\text{-point vertex}]^2 [3\text{-point vertex}]^{-1}$. The final diagrammatic equation is: $[4\text{-point vertex}] - 3[3\text{-point vertex}]^2 [3\text{-point vertex}]^{-1} = 0$.