

# 1 Out of Time Order Correlator of $H = xp$ model

The Riemann hypothesis states that non-trivial zeros of the classical zeta function have real part equal to  $1/2$ . The classical zeta function defined by

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} \quad (1)$$

for  $\text{Re } s > 1$ . By the fundamental theorem of arithmetic, which is also equivalent to the Euler product over primes

$$\zeta(s) = \prod_p (1 - p^{-s})^{-1} \quad (2)$$

where  $p$  are all the prime numbers.

Zeros of Riemann zeta function are two different types. Trivial zeros of zeta / Riemann zeta function occurs at all negative integers (for  $s = -2, -4, -6, \dots$ ). For complex  $s (= \sigma + it)$  (with real part between zero and one), zeta function becomes nontrivial ones. And the Riemann hypothesis is for  $s = \frac{1}{2} - it$  zeta function becomes zero  $\zeta(\frac{1}{2} - it) = 0$ . Hilbert-Pólya conjecture suggests that the imaginary parts of the nontrivial zeros are the eigenvalues of a self-adjoint hamiltonian operator  $\hat{H}$ . It is also one of the approach to proving the Riemann hypothesis. Berry-Keating conjectured that the hamiltonian operator of the Hilbert-Pólya conjecture should take the form[1]

$$\hat{H}_{BK} = \frac{1}{2}(\hat{x}\hat{p} + \hat{p}\hat{x}) \quad (3)$$

Here  $x$  and  $p$  are position and momentum operators. This 1d classical Hamiltonian ( $H = xp$ ) related to the Riemann zeros.[?] Berry proposed the Quantum Chaos conjecture, according to which the Riemann zeros are the spectrum of a Hamiltonian obtained by quantization of a classical chaotic hamiltonian, whose periodic orbits are labeled by the prime numbers. Connes took the adelic approach to introduce  $H = xp$  [2]. He showed that using different semiclassical regularization, Riemann zeros appear as missing spectral lines in a continuum.

Now we look into the Berry-Keating and Connes semiclassical approaches to  $H = xp$

## 2 Semiclassical approach

The classical Berry-Keating-Connes (BKC) Hamiltonian is [1, 2]

$$H_0^{cl} = xp \tag{4}$$

which has hyperbolic trajectories

$$x(t) = x_0 e^t \quad p(t) = p_0 e^{-t} \tag{5}$$

So the dynamics is unbounded. There is a continuous spectrum as the quantum level. Berry-Keating and Connes introduced two different types of regularizations and counted the semiclassical states.

## References

- [1] Michael V Berry and Jonathan P Keating.  $H = xp$  and the riemann zeros. *Supersymmetry and trace formulae: chaos and disorder*, pages 355–367, 1999.
- [2] Alain Connes. Trace formula in noncommutative geometry and the zeros of the riemann zeta function. *Selecta Mathematica*, 5(1):29, 1999.