Solution to Problem Sheet 4

Noor E Mustafa Ferdous
email: nooremf@gmail.com

Solve for problem no. 1

(a)

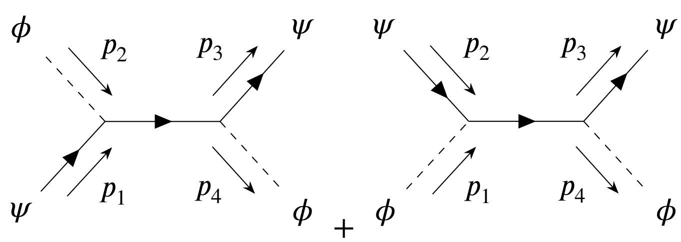

Figure 1: Feynman diagram for $\psi\psi \to \psi\phi$.

Scattering amplitude for this diagram is

$$
\begin{aligned}
i\mathcal{M} &= \int \frac{d^4k}{(2\pi)^4}\delta^4(p_1 - p_2 - k)\delta^4(p_3 - p_4 - k)\frac{i(2\pi)^8}{k^2 - M^2 + i\epsilon} \\
&= (ig)^2 i(2\pi)^4\delta^4(p_3 + p_4 - p_1 - p_2)\frac{1}{(p_1 + p_2)^2 - M^2 + i\epsilon}
\end{aligned}
$$

(b)

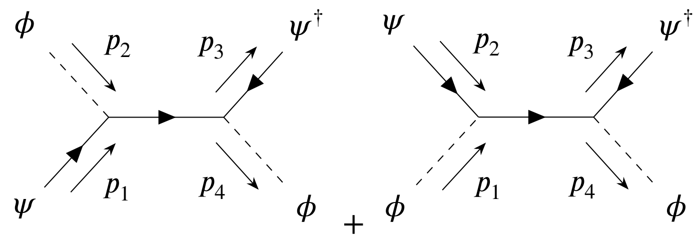

Figure 2: Feynman diagram for $\psi\psi^\dagger \to \phi\phi$.

The scattering amplitude will be

$$
\begin{aligned}
i\mathcal{M} &= (-ig)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i(2\pi)^8}{k^2 - M^2 + i\epsilon} (\delta^4(p_1 + p_2 - k)\delta^4(k - p_3 - p_4)) \\
&= i(-ig)^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \frac{1}{(p_1 + p_2^2 - M^2 + i\epsilon)}
\end{aligned}
$$

Solve for problem no. 3

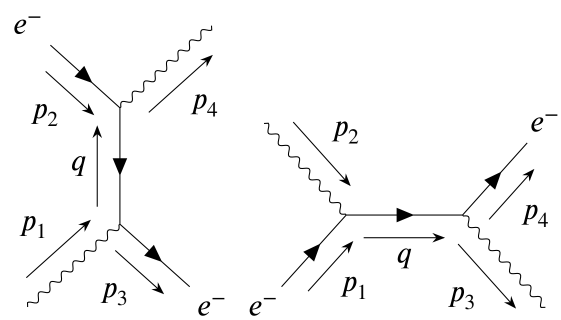

Figure 3: Feynman diagram for $e^-\gamma \to e^-\gamma$

(a) Scattering amplitude will be

$$i\mathcal{M}_1 = (2\pi)^4 \int d^4q \big[\overline{u}(p_4)(-ie\gamma^\mu \epsilon_\mu(p_2))\frac{i(\not{q}+m)}{q^2-m^2+i\epsilon}\epsilon^*_\nu(p_3)u(p_1)\delta^4(p_1-p_3-q)\delta^4(p_2+q-p_4)\big]$$

$$\mathcal{M}_1 = \frac{e^2}{(p_1-p_3)^2-m^2+i\epsilon}\big[\overline{u}(p_4)\gamma^\mu\epsilon_\mu(p_2)\epsilon^*_\mu(p_3)\gamma_\mu u(p_1)\delta^4(p_2+p_3-p_1-p_4))\big]$$

and

$$i\mathcal{M}_2 = (2\pi)^4 \int d^4q \big[\overline{u}(p_4)(-ie\gamma^\mu \epsilon_\mu(p_2))\frac{i(\not{q}+m)}{q^2-m^2+i\epsilon}\epsilon^*_\nu(p_3)u(p_1)\delta^4(p_1+p_2-q)\delta^4(p_4+p_3-q)\big]$$

$$\mathcal{M}_2 = \frac{e^2}{(p_1-p_3)^2-m^2+i\epsilon}\big[\overline{u}(p_4)\gamma^\mu\epsilon_\mu(p_2)\epsilon^*_\mu(p_3)\gamma_\mu u(p_1)\delta^4(p_4+p_3-p_1-p_2)\big]$$

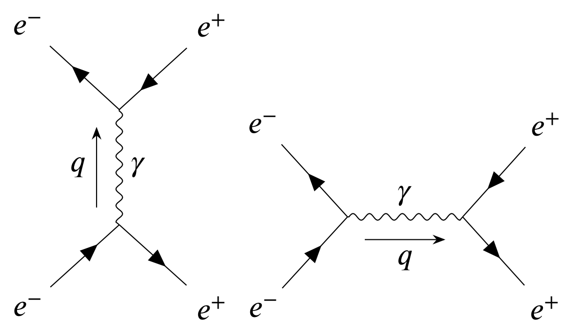

Figure 4: Feynman diagram for $e^-e^+ \to e^-e^+$

(b) Scattering amplitudes will be

$$i\mathcal{M}_1 = \int \overline{u}(p_3)(-ie\gamma^\mu)u(p_1)\frac{i\eta_{\mu\nu}}{q^2+i\epsilon}\overline{v}(p_2)(-ie\gamma_{nu})v(p_4)(2\pi)^8\delta^4(p_1-p_3-q)\delta^4(p_2+q-p_4)\frac{d^4q}{(2\pi)^4}$$

$$= -\frac{ie^2}{(p_1-p_3)^2+i\epsilon}\overline{u}(p_3)\gamma^\mu u(p_1)\overline{v}(p)\gamma_\mu v(p_4)$$

$$-i\mathcal{M}_1 = \frac{ie^2}{(p_1-p_3)^2+i\epsilon}[\overline{u}(p_3)\gamma^\mu u(p_1)]^\dagger[\overline{v}(p_2)\gamma_\mu v(p_4)]^\dagger$$

$$= \frac{ie^2}{(p_1-p_3)^2+i\epsilon}\overline{u}(p_1)\gamma^\mu u(p_3)\overline{v}(p_4)\gamma_\mu v(p_2)$$

$$
\begin{aligned}
i\mathcal{M}_2 &= \int \overline{u}(p_3)(-ie\gamma^{\mu})\overline{v}(p_4)\frac{i\eta_{\mu\nu}}{q^2+i\epsilon}\overline{v}(p_2)(-ie\gamma_{nu})v(p_4)(2\pi)^8\delta^4(p_1+p_2-q)\delta^4(q-p_3-p_4)\frac{d^4q}{(2\pi)^4} \\
&= -\frac{ie^2}{(p_1+p_2+i\epsilon)}\overline{u}(p_3)\gamma^{\mu}\overline{v}(p_4)\overline{v}(p_2)\gamma_{\mu}v(p_4)
\end{aligned}
$$

$$
-i\mathcal{M}_2 = \frac{ie^2}{(p_1+p_2+i\epsilon)}\overline{v}(p_3)\gamma^{\mu}\overline{u}(p_4)\overline{u}(p_2)\gamma_{\mu}v(p_4)
$$

$$
\begin{aligned}
|\mathcal{M}|^2 &= \frac{e^4}{(p_1-p_3)^4}Tr[\overline{u}(p_3)u(p_3\gamma^{\mu}u_{p_1}\overline{u(p_1)})]Tr[v(p_2)\overline{v(p_2)}\gamma^{\mu'}\overline{v(p_4)}v(p_4)] \\
&= \frac{e^4}{(p_1-p_3)^4}[Tr(\not{p}_3\gamma^{\mu'}\not{p}_1\gamma^{\mu})+m^2Tr(\gamma^{\mu'}\gamma^{\mu})][Tr(\not{p}_2\gamma_{\mu'}\not{p}_4\gamma_{\mu})-m^2Tr(\gamma_{\mu'}\gamma_{\mu})] \\
&= \frac{e^4}{(p_1-p_3)^4}[p_3^{\mu'}p_1-p_3p_1\eta^{\mu\mu'}+p_3^{\mu}p_1^{\mu'}+m^2\eta^{\mu\mu}]\times[p_{2\mu'}p_4-p_2p_4\eta_{\mu\mu'}+p_{2\mu}p_{4\mu'}-m^2\eta_{\mu\mu}]
\end{aligned}
$$