MD Samiul Kawser

Ans for (a)

The Partiton fucntion

$$Z[J] = \mathcal{N}_0 e^{V[\frac{\partial}{\partial J_i}]} e^{\frac{1}{2}J_m \Delta_{mn} J_n} \tag{1}$$

Where

$$V[\phi] = \frac{\lambda}{4!}\phi = \frac{\lambda}{4!}\frac{\partial}{\partial J_i} \tag{2}$$

expanding the equation

$$Z[J] = [1 + \frac{\lambda}{4!}(\frac{\partial}{\partial J_i})^4 + \frac{\lambda}{4!}^2(\frac{\partial}{\partial J_i})^4(\frac{\partial}{\partial J_i})^4 + ...]\mathcal{N}_0 e^{\frac{1}{2}J_m \Delta_{mn} J_n} \tag{3}$$

Contribution of the first order in lambda

$$\frac{\lambda}{4!}\frac{\partial^4}{\partial {J_i}^4}[e^{\frac{1}{2}J_m \Delta_{mn} J_n}] \tag{4}$$

Now, taking $e^{\frac{1}{2}J_m \Delta_{mn} J_n} = U$

$$\frac{\partial}{\partial J_i} e^{\frac{1}{2}J_m \Delta_{mn} J_n} = \Delta_{im} J_m U \tag{5}$$

Again

$$\frac{\partial^2}{\partial {J_i}^2} e^{\frac{1}{2}J_m \Delta_{mn} J_n} = \Delta_{ii} U + (J_m \Delta_{im})^2 U \tag{6}$$

Again

$$\frac{\partial^3}{\partial {J_i}^3} e^{\frac{1}{2}J_m \Delta_{mn} J_n} = \Delta_{ii}\Delta_{ii} U + (J_m \Delta_{im})^3 U + 2 J_m \Delta_{im} \Delta_{ii} U \tag{7}$$

Again

$$\frac{\partial^4}{\partial {J_i}^4} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4 U + 6(J_m\Delta_{im})^2\Delta_{ii}U \tag{8}$$

Therefore the partiton function upto first order of lambda is

$$Z[J] = [1 + \frac{\lambda}{4!} 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4 U + 6(J_m\Delta_{im})^2\Delta_{ii}U]\mathcal{N}, e^{\frac{1}{2} J_m \Delta_{mn} J_n} \tag{9}$$

Second order of labmda will be

$$\frac{\lambda^2}{(4!)^2} (\frac{\partial}{\partial J_i})^4 (\frac{\partial}{\partial J_i})^4$$

First term of equation (9) will be

$$\frac{\partial^4}{\partial {J_i}^4} 3\Delta_{ii}\Delta_{ii} = [1 + \frac{\lambda}{4!} 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4 U + 6(J_m\Delta_{im})^2\Delta_{ii}U] \tag{10}$$

Second term

$$\begin{aligned}\frac{\partial^4}{\partial {J_i}^4} 6(J_m\Delta_{im})^2\Delta_{ii}U = {} & 6(\Delta_{ii})(\Delta_{in}J_n)^6 U + 36(\Delta_{ii})^4 U + 36(\Delta_{ii})^3(\Delta_{in}J_n)^2 U \\ & + 30(\Delta_{ii})^2(\Delta_{in}J_n)^2 U + 36(\Delta_{ii})^4 U + 36(\Delta_{ii})(\Delta_{in}J_n)^2 U \\ & + 54(\Delta_{ii})^3(\Delta_{in}J_n)^2 U + 72(\Delta_{ii})^4(\Delta_{in}J_n)^2 U + 24(\Delta_{ii})^3(\Delta_{in}J_n)^4 U \\ & + 30(\Delta_{ii})^2(\Delta_{in}J_n)^4 U\end{aligned}$$

Third term

$$\begin{aligned}\frac{\partial^4}{\partial {J_i}^4} (J_m\Delta_{im})^4 U = {} & 24(\Delta_{ii})^4 U + 12(\Delta_{ii})^2(\Delta_{in}J_n)^4 U + 36(\Delta_{ii})^3(\Delta_{in}J_n)^2 U \\ & + 48(\Delta_{ii})^3(\Delta_{in}J_n)^2 U + 16(\Delta_{ii})^4(\Delta_{in}J_n)^2 U + 20(\Delta_{ii})^4(\Delta_{in}J_n)^2 U \\ & + 4(\Delta_{ii})(\Delta_{in}J_n)^6 U + 60(\Delta_{ii})^3(\Delta_{in}J_n)^2 U + 20(\Delta_{ii})^4(\Delta_{in}J_n)^2 U \\ & + 25(\Delta_{ii})^2(\Delta_{in}J_n)^4 U + 5(\Delta_{ii})(\Delta_{in}J_n)^6 U + 30(\Delta_{ii})^4(\Delta_{in}J_n)^2 U + \\ & + 6(\Delta_{ii})(\Delta_{in}J_n)^6 U + 7(\Delta_{ii})(\Delta_{in}J_n)^6 U + (\Delta_{in}J_n)\end{aligned}$$

So

$$\frac{\lambda^2}{(4!)^2}(\frac{\partial}{\partial J_i})^4(\frac{\partial}{\partial J_i})^4 = \frac{\lambda^2}{(4!)^2}[1+\frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U + 6(J_m\Delta_{im})^2\Delta_{ii}U$$
$$+6(\Delta_{ii})(\Delta_{in}J_n)^6U + 36(\Delta_{ii})^4U + 36(\Delta_{ii})^3(\Delta_{in}J_n)^2U$$
$$+30(\Delta_{ii})^2(\Delta_{in}J_n)^2U + 36(\Delta_{ii})^4U + 36(\Delta_{ii})(\Delta_{in}J_n)^2U$$
$$+54(\Delta_{ii})^3(\Delta_{in}J_n)^2U + 72(\Delta_{ii})^4(\Delta_{in}J_n)^2U + 24(\Delta_{ii})^3(\Delta_{in}J_n)^4U$$
$$+30(\Delta_{ii})^2(\Delta_{in}J_n)^4U + 24(\Delta_{ii})^4U + 12(\Delta_{ii})^2(\Delta_{in}J_n)^4U + 36(\Delta_{ii})^3(\Delta_{in}J_n)^2U$$
$$+48(\Delta_{ii})^3(\Delta_{in}J_n)^2U + 16(\Delta_{ii})^4(\Delta_{in}J_n)^2U + 20(\Delta_{ii})^4(\Delta_{in}J_n)^2U$$
$$+4(\Delta_{ii})(\Delta_{in}J_n)^6U + 60(\Delta_{ii})^3(\Delta_{in}J_n)^2U + 20(\Delta_{ii})^4(\Delta_{in}J_n)^2U$$
$$+25(\Delta_{ii})^2(\Delta_{in}J_n)^4U + 5(\Delta_{ii})(\Delta_{in}J_n)^6U + 30(\Delta_{ii})^4(\Delta_{in}J_n)^2U+$$
$$+6(\Delta_{ii})(\Delta_{in}J_n)^6U + 7(\Delta_{ii})(\Delta_{in}J_n)^6U + (\Delta_{in}J_n)]$$

Ans for (b)

Partition up to first order of lambda

$$Z[J] = [1+\frac{\lambda}{4!}(\frac{\partial}{\partial J_i})^4]\mathcal{N}_0 e^{\frac{1}{2}J_m\Delta_{mn}J_n} \tag{11}$$

The two point function is

$$\langle\phi_x\phi_y\rangle = \frac{1}{Z[0]}\frac{\partial^2 Z[J]}{\partial J_x\partial J_y}\bigg|_{J=0}$$
$$= \frac{1}{Z[0]}\frac{\partial^2}{\partial J_x\partial J_y}[1+\frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U$$
$$+6(J_m\Delta_{im})^2\Delta_{ii}U]\mathcal{N}_0 e^{\frac{1}{2}J_m\Delta_{mn}J_n}\bigg|_{J=0}$$

As we are taking J=0, the $(J_m\Delta_{im})^4U$ term will become zero.

$$\frac{\partial^2 Z[J]}{\partial J_x\partial J_y}[U+\frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^2\Delta_{ii}U]$$
$$= \frac{\partial}{\partial J_y}[\frac{\partial}{\partial J_x} + U\frac{\lambda}{4!}(3\Delta_{ii}\Delta_{ii}\Delta_{xm}J_m + 12\Delta_{xi}J_mU\Delta_{im}\Delta_{ii})]$$
$$= [\Delta_{xy} + \frac{\lambda}{4!}(3\Delta_{ii}\Delta_{ii}\Delta_{xy} + 12\Delta_{xi}\Delta_{ii}\Delta_{iy})]$$

Therefore the two point function

$$\langle \phi_x \phi_y \rangle = \frac{\Delta_{xy} + \frac{\lambda}{4!}(3\Delta_{ii}\Delta_{ii}\Delta_{xy} + 12\Delta_{xi}\Delta_{ii}\Delta_{iy})}{1 + \frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}} \tag{12}$$

Binomial expanding the lower part of the fraction we get

$$\langle \phi_x \phi_y \rangle = [\Delta_{xy} + \frac{\lambda}{4!}(3\Delta_{ii}\Delta_{ii}\Delta_{xy} + 12\Delta_{ix}\Delta_{ii}\Delta_{iy})][1 - \frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}] \tag{13}$$

$$= \Delta_{xy} + \frac{\lambda}{4!}(12\Delta_{ix}\Delta_{ii}\Delta_{iy}) \tag{14}$$

Ans for (c)

So the diagram will be