

4 point function

$$\begin{aligned}
\langle \phi_x \phi_y \phi_z \phi_a \rangle &= \frac{1}{Z[0]} \frac{\partial^4}{\partial J_x \partial J_y \partial J_z \partial J_a} \Big|_{J=0} \\
&= \frac{1}{Z[0]} \frac{\partial^4}{\partial J_x \partial J_y \partial J_z \partial J_a} \left[ 1 + \frac{\lambda}{4!} [3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U \right. \\
&\quad \left. + 6(J_m\Delta_{im})^2\Delta_{ii}U] \right] \mathcal{N}_0 e^{\frac{1}{2}J_m\Delta_{mn}J_n} \Big|_{J=0} \quad (1)
\end{aligned}$$

First term of eqn

$$\begin{aligned}
\frac{\partial^4}{\partial J_x \partial J_y \partial J_z \partial J_a} U &= \frac{\partial^3}{\partial J_y \partial J_z \partial J_a} \Delta_{xm} J_m U \\
&= \frac{\partial^2}{\partial J_z \partial J_a} [\Delta_{xy} U + \Delta_{xm} J_m \Delta_{ym} J_m U] \\
&= \frac{\partial}{\partial J_a} [\Delta_{xz} \Delta_{ym} J_m U + \Delta_{xm} J_m \Delta_{yz} U + \\
&\quad \Delta_{xm} J_m \Delta_{ym} J_m \Delta_{zm} J_m U + \Delta_{xy} \Delta_{zm} J_m U] \\
&= \Delta_{xy} \Delta_{za} + \Delta_{xz} \Delta_{ya} + \Delta_{xa} \Delta_{yz} \quad \text{putting } J=0 \\
&= 3\Delta_{xy} \Delta_{za}
\end{aligned}$$

Second term of equation

$$\begin{aligned}
\frac{\partial^4}{\partial J_x \partial J_y \partial J_z \partial J_a} 3\Delta_{ii}\Delta_{ii}U &= \frac{\partial^3}{\partial J_y \partial J_z \partial J_a} 3\Delta_{ii}\Delta_{ii}\Delta_{xm} J_m U \\
&= \frac{\partial^2}{\partial J_z \partial J_a} 3\Delta_{ii}^2 \Delta_{xy} U + 3\Delta_{ii}^2 \Delta_{xm} J_m \Delta_{ym} J_m U \\
&= \frac{\partial}{\partial J_a} 3\Delta_{ii}^2 \Delta_{xy} \Delta_{zm} J_m U \\
&\quad + 3\Delta_{ii}^2 \Delta_{xz} \Delta_{ym} J_m U + 3\Delta_{ii}^2 \Delta_{xm} J_m \Delta_{yz} U \\
&\quad + 3\Delta_{ii}^2 \Delta_{xm} J_m \Delta_{ym} J_m \Delta_{zm} J_m U \\
&= 9\Delta_{ii}^2 \Delta_{xy} \Delta_{za}
\end{aligned}$$

Third term of equation

$$\begin{aligned}
& \frac{\partial^4}{\partial J_x \partial J_y \partial J_z \partial J_a} (J_m \Delta_{im})^4 U \\
&= \frac{\partial^3}{\partial J_y \partial J_z \partial J_a} [4(\Delta_{im} J_m)^3 \Delta_{ix} U + (J_m \Delta_{im})^4 \Delta_{xm} J_m U] \\
&= 12(J_m \Delta_{im})^2 \Delta_{iy} \Delta_{ix} U + 4(J_m \Delta_{im})^3 \Delta_{ix} \Delta_{ym} J_m U \\
&\quad + 4(J_m \Delta_{im})^3 \Delta_{iy} \Delta_{xm} J_m U + (J_m \Delta_{im})^4 \Delta_{xy} U \\
&\quad + (J_m \Delta_{im})^4 \Delta_{ym} J_m \Delta_{xm} J_m U \\
&= 24 J_m \Delta_{im} \Delta_{zi} \Delta_{iy} \Delta_{xm} J_m U + 12(J_m \Delta_{im})^2 \Delta_{zi} \Delta_{ix} \Delta_{ym} J_m U \\
&\quad + 12(J_m \Delta_{im})^2 \Delta_{zi} \Delta_{ix} \Delta_{ym} J_m U + 12(J_m \Delta_{im})^2 \Delta_{iy} \Delta_{ix} \Delta_{zm} J_m U
\end{aligned}$$