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Ans for (a)

The Partiton fuction

$$Z[J] = \mathcal{N}_0 e^{V[\frac{\partial}{\partial J_i}]} e^{\frac{1}{2} J_m \Delta_{mn} J_n} \quad (1)$$

Where

$$v[\phi] = \frac{\text{lambda}}{4!} \phi = \text{frac} \lambda 4! \frac{\partial}{\partial J_i} \quad (2)$$

expanding the equation

$$Z[J] = [1 + \frac{\lambda}{4!} (\frac{\partial}{\partial J_i})^4 + \frac{\lambda^2}{4!} (\frac{\partial}{\partial J_i})^4 (\frac{\partial}{\partial J_i})^4 + \dots] \mathcal{N}_0 e^{\frac{1}{2} J_m \Delta_{mn} J_n} \quad (3)$$

Contribution of the first order in lambda

$$\frac{\lambda}{4!} \frac{\partial^4}{\partial J_i^4} [e^{\frac{1}{2} J_m \Delta_{mn} J_n}] \quad (4)$$

Now, taking $e^{\frac{1}{2} J_m \Delta_{mn} J_n} = U$

$$\frac{\partial}{\partial J_i} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = \Delta_{im} J_m U \quad (5)$$

Again

$$\frac{\partial^2}{\partial J_i^2} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = \Delta_{ii} U + (J_m \Delta_{im})^2 U \quad (6)$$

Again

$$\frac{\partial^3}{\partial J_i^3} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = \Delta_{ii} \Delta_{ii} U + (J_m \Delta_{im})^3 U + 2 J_m \Delta_{im} \Delta_{ii} U \quad (7)$$

Again

$$\frac{\partial^4}{\partial J_i^4} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U + 6(J_m\Delta_{im})^2\Delta_{ii}U \quad (8)$$

Therefore the partiton function upto first order of lambda is

$$Z[J] = [1 + \frac{\lambda}{4!} 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U + 6(J_m\Delta_{im})^2\Delta_{ii}U] \mathcal{N}_I e^{\frac{1}{2} J_m \Delta_{mn} J_n} \quad (9)$$

Second order of labmda will be

$$\frac{\lambda^2}{(4!)^2} \left(\frac{\partial}{\partial J_i} \right)^4 \left(\frac{\partial}{\partial J_i} \right)^4$$

First term of equation (9) will be

$$\frac{\partial^4}{\partial J_i^4} 3\Delta_{ii}\Delta_{ii} = [1 + \frac{\lambda}{4!} 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U + 6(J_m\Delta_{im})^2\Delta_{ii}U] \quad (10)$$

Second term

$$\begin{aligned} \frac{\partial^4}{\partial J_i^4} 6(J_m\Delta_{im})^2\Delta_{ii}U &= 6(\Delta_{ii})(\Delta_{in}J_n)^6U + 36(\Delta_{ii})^4U + 36(\Delta_{ii})^3(\Delta_{in}J_n)^2U \\ &+ 30(\Delta_{ii})^2(\Delta_{in}J_n)^2U + 36(\Delta_{ii})^4U + 36(\Delta_{ii})(\Delta_{in}J_n)^2U \\ &+ 54(\Delta_{ii})^3(\Delta_{in}J_n)^2U + 72(\Delta_{ii})^4(\Delta_{in}J_n)^2U + 24(\Delta_{ii})^3(\Delta_{in}J_n)^4U \\ &+ 30(\Delta_{ii})^2(\Delta_{in}J_n)^4U \end{aligned}$$

Third term

$$\begin{aligned} \frac{\partial^4}{\partial J_i^4} (J_m\Delta_{im})^4U &= 24(\Delta_{ii})^4U + 12(\Delta_{ii})^2(\Delta_{in}J_n)^4U + 36(\Delta_{ii})^3(\Delta_{in}J_n)^2U \\ &+ 48(\Delta_{ii})^3(\Delta_{in}J_n)^2U + 16(\Delta_{ii})^4(\Delta_{in}J_n)^2U + 20(\Delta_{ii})^4(\Delta_{in}J_n)^2U \\ &+ 4(\Delta_{ii})(\Delta_{in}J_n)^6U + 60(\Delta_{ii})^3(\Delta_{in}J_n)^2U + 20(\Delta_{ii})^4(\Delta_{in}J_n)^2U \\ &+ 25(\Delta_{ii})^2(\Delta_{in}J_n)^4U + 5(\Delta_{ii})(\Delta_{in}J_n)^6U + 30(\Delta_{ii})^4(\Delta_{in}J_n)^2U + \\ &+ 6(\Delta_{ii})(\Delta_{in}J_n)^6U + 7(\Delta_{ii})(\Delta_{in}J_n)^6U + (\Delta_{in}J_n) \end{aligned}$$

So

$$\begin{aligned}
\frac{\lambda^2}{(4!)^2} \left(\frac{\partial}{\partial J_i} \right)^4 \left(\frac{\partial}{\partial J_i} \right)^4 &= \frac{\lambda^2}{(4!)^2} \left[1 + \frac{\lambda}{4!} 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U + 6(J_m\Delta_{im})^2\Delta_{ii}U \right. \\
&\quad + 6(\Delta_{ii})(\Delta_{in}J_n)^6U + 36(\Delta_{ii})^4U + 36(\Delta_{ii})^3(\Delta_{in}J_n)^2U \\
&\quad + 30(\Delta_{ii})^2(\Delta_{in}J_n)^2U + 36(\Delta_{ii})^4U + 36(\Delta_{ii})(\Delta_{in}J_n)^2U \\
&\quad + 54(\Delta_{ii})^3(\Delta_{in}J_n)^2U + 72(\Delta_{ii})^4(\Delta_{in}J_n)^2U + 24(\Delta_{ii})^3(\Delta_{in}J_n)^4U \\
&\quad + 30(\Delta_{ii})^2(\Delta_{in}J_n)^4U + 24(\Delta_{ii})^4U + 12(\Delta_{ii})^2(\Delta_{in}J_n)^4U + 36(\Delta_{ii})^3(\Delta_{in}J_n)^2U \\
&\quad + 48(\Delta_{ii})^3(\Delta_{in}J_n)^2U + 16(\Delta_{ii})^4(\Delta_{in}J_n)^2U + 20(\Delta_{ii})^4(\Delta_{in}J_n)^2U \\
&\quad + 4(\Delta_{ii})(\Delta_{in}J_n)^6U + 60(\Delta_{ii})^3(\Delta_{in}J_n)^2U + 20(\Delta_{ii})^4(\Delta_{in}J_n)^2U \\
&\quad + 25(\Delta_{ii})^2(\Delta_{in}J_n)^4U + 5(\Delta_{ii})(\Delta_{in}J_n)^6U + 30(\Delta_{ii})^4(\Delta_{in}J_n)^2U + \\
&\quad \left. + 6(\Delta_{ii})(\Delta_{in}J_n)^6U + 7(\Delta_{ii})(\Delta_{in}J_n)^6U + (\Delta_{in}J_n) \right]
\end{aligned}$$

Ans for (b)

Partition up to first order of lambda

$$Z[J] = \left[1 + \frac{\lambda}{4!} \left(\frac{\partial}{\partial J_i} \right)^4 \right] \mathcal{N}_0 e^{\frac{1}{2} J_m \Delta_{mn} J_n} \quad (11)$$

The two point function is

$$\begin{aligned}
\langle \phi_x \phi_y \rangle &= \frac{1}{Z[0]} \left. \frac{\partial^2 Z[J]}{\partial J_x \partial J_y} \right|_{J=0} \\
&= \frac{1}{Z[0]} \frac{\partial^2}{\partial J_x \partial J_y} \left[1 + \frac{\lambda}{4!} 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U \right. \\
&\quad \left. + 6(J_m\Delta_{im})^2\Delta_{ii}U \right] \mathcal{N}_0 e^{\frac{1}{2} J_m \Delta_{mn} J_n} \Big|_{J=0}
\end{aligned}$$

As we are taking $J=0$, the $(J_m\Delta_{im})^4U$ term will become zero.

$$\begin{aligned}
&\frac{\partial^2 Z[J]}{\partial J_x \partial J_y} \left[U + \frac{\lambda}{4!} 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^2\Delta_{ii}U \right] \\
&= \frac{\partial}{\partial J_y} \left[\frac{\partial}{\partial J_x} + U \frac{\lambda}{4!} (3\Delta_{ii}\Delta_{ii}\Delta_{xm}J_m + 12\Delta_{xi}J_mU\Delta_{im}\Delta_{ii}) \right] \\
&= [\Delta_{xy} + \frac{\lambda}{4!} (3\Delta_{ii}\Delta_{ii}\Delta_{xy} + 12\Delta_{xi}\Delta_{ii}\Delta_{iy})]
\end{aligned}$$

Therefore the two point function

$$\langle \phi_x \phi_y \rangle = \frac{\Delta_{xy} + \frac{\lambda}{4!}(3\Delta_{ii}\Delta_{ii}\Delta_{xy} + 12\Delta_{xi}\Delta_{ii}\Delta_{iy})}{1 + \frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}} \quad (12)$$

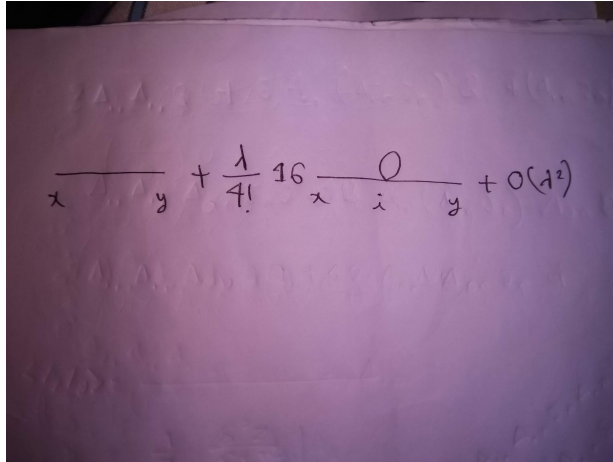
Binomial expanding the lower part of the fraction we get

$$\langle \phi_x \phi_y \rangle = [\Delta_{xy} + \frac{\lambda}{4!}(3\Delta_{ii}\Delta_{ii}\Delta_{xy} + 12\Delta_{ix}\Delta_{ii}\Delta_{iy})][1 - \frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}] \quad (13)$$

$$= \Delta_{xy} + \frac{\lambda}{4!}(12\Delta_{ix}\Delta_{ii}\Delta_{iy}) \quad (14)$$

Ans for (c)

So the diagram will be



A photograph of a piece of white paper with handwritten mathematical notation. The expression is: $\frac{1}{x} \frac{1}{y} + \frac{\lambda}{4!} 16 \frac{0}{x} \frac{0}{i} \frac{0}{y} + O(\lambda^2)$. The handwriting is in black ink and appears to be a student's work.