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Ans for (a)

The Partiton fuction

$$Z[J] = \mathcal{N}_0 e^{V[\frac{\partial}{\partial J_i}]} e^{\frac{1}{2} J_m \Delta_{mn} J_n} \quad (1)$$

Where

$$V[\phi] = \frac{\lambda}{4!} \phi = \frac{\lambda}{4!} \frac{\partial}{\partial J_i} \quad (2)$$

expanding the equation

$$Z[J] = [1 + \frac{\lambda}{4!} (\frac{\partial}{\partial J_i})^4 + \frac{\lambda^2}{4!} (\frac{\partial}{\partial J_i})^4 (\frac{\partial}{\partial J_i})^4 + \dots] \mathcal{N}_0 e^{\frac{1}{2} J_m \Delta_{mn} J_n} \quad (3)$$

Contribution of the first order in lambda

$$\frac{\lambda}{4!} \frac{\partial^4}{\partial J_i^4} [e^{\frac{1}{2} J_m \Delta_{mn} J_n}] \quad (4)$$

Now, taking $e^{\frac{1}{2} J_m \Delta_{mn} J_n} = U$

$$\frac{\partial}{\partial J_i} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = \Delta_{im} J_m U \quad (5)$$

Again

$$\frac{\partial^2}{\partial J_i^2} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = \Delta_{ii} U + (J_m \Delta_{im})^2 U \quad (6)$$

Again

$$\frac{\partial^3}{\partial J_i^3} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = \Delta_{ii} \Delta_{ii} U + (J_m \Delta_{im})^3 U + 2 J_m \Delta_{im} \Delta_{ii} U \quad (7)$$

Again

$$\frac{\partial^4}{\partial J_i^4} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U + 6(J_m\Delta_{im})^2\Delta_{ii}U \quad (8)$$

Therefore the partiton function upto first order of lambda is

$$Z[J] = [1 + \frac{\lambda}{4!} 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U + 6(J_m\Delta_{im})^2\Delta_{ii}U] \mathcal{N}, \quad (9)$$

Second order of lambda will be

$$\frac{\lambda^4}{(4!)^4} (\frac{\partial}{\partial J_i})^4 (\frac{\partial}{\partial J_j})^4 U = \frac{\lambda}{4!} [3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U + 6(J_m\Delta_{im})^2\Delta_{ii}U] \quad (10)$$

First term of equation 10 will be

$$\frac{\lambda^4}{(4!)^4} (\frac{\partial}{\partial J_j})^4 3\Delta_{ii}\Delta_{ii}U = 3\Delta_{ii}\Delta_{ii}[3\Delta_{jj}\Delta_{jj}U + (J_m\Delta_{jm})^4U + 6(J_m\Delta_{jm})^2\Delta_{jj}U] \quad (11)$$

Second term of equation 10 will be

$$\begin{aligned} \frac{\lambda^4}{(4!)^4} (\frac{\partial}{\partial J_j})^4 (J_m\Delta_{im})^4U &= 6\Delta_{ii}[12\Delta_{ij}^2\Delta_{jj}U + 12\Delta_{ij}^2\Delta_{jn}J_n\Delta_{jn}J_nU \\ &\quad + 8\Delta_{in}J_n\Delta_{ij}\Delta_{jj}\Delta_{jn}J_nU + 16\Delta_{in}J_n\Delta_{ij}\Delta_{jj}\Delta_{jn}J_nU \\ &\quad + 8\Delta_{in}J_n\Delta_{ij}(\Delta_{jn}J_n)^3U + (\Delta_{in}J_n)^2\Delta_{jj}(\Delta_{jn}J_n)^2U + 2(\Delta_{in}J_n)^2\Delta_{jj}^2U \\ &\quad + 2(\Delta_{in}J_n)^2\Delta_{jj}(\Delta_{jn}J_n)^2U + 3(\Delta_{in}J_n)^2\Delta_{jj}(\Delta_{jn}J_n)^2U \\ &\quad + (\Delta_{in}J_n)^2(\Delta_{jn}J_n)^4U + (\Delta_{in}J_n)^2\Delta_{jj}^2U] \end{aligned}$$

Third term of the equation 10

$$\begin{aligned} \frac{\lambda^4}{(4!)^4} (\frac{\partial}{\partial J_j})^4 &= 24\Delta_{ij}^4U + 48\Delta_{in}J_n\Delta_{ij}^3\Delta_{jn}J_nU + 60(\Delta_{in}J_n)^2\Delta_{ii}^2(\Delta_{jn}J_n)^2U \\ &\quad + 16(\Delta_{in}J_n)^3\Delta_{ij}\Delta_{jj}(\Delta_{jn}J_n)^2U + 32(\Delta_{in}J_n)^3\Delta_{ij}\Delta_{jj}\Delta_{jn}J_nU \\ &\quad + 16(\Delta_{in}J_n)^3\Delta_{ij}(\Delta_{jn}J_n)^3U + (\Delta_{in}J_n)^4\Delta_{jj}(\Delta_{in}J_n)^2U + 2(\Delta_{in}J_n)^4\Delta_{jj}^2U \\ &\quad + 2(\Delta_{in}J_n)^4\Delta_{jj}^2U + 2(\Delta_{in}J_n)^4\Delta_{jj}(\Delta_{jn}J_n)^2U + 3(\Delta_{in}J_n)^4\Delta_{jj}(\Delta_{jn}J_n)^2U \\ &\quad + (\Delta_{in}J_n)^4\Delta_{jj}^2U + (\Delta_{in}J_n)^4(\Delta_{jn}J_n)^4U \end{aligned}$$

Therefore

$$\begin{aligned}
\frac{\lambda^4}{(4!)^4} \left(\frac{\partial}{\partial J_i} \right)^4 \left(\frac{\partial}{\partial J_j} \right)^4 U &= 3\Delta_{ii}\Delta_{ii}[3\Delta_{jj}\Delta_{jj}U + (J_m\Delta_{jm})^4U + 6(J_m\Delta_{jm})^2\Delta_{jj}U] \\
&+ 6\Delta_{ii}[12\Delta_{ij}^2\Delta_{jj}U + 12\Delta_{ij}^2\Delta_{jn}J_n\Delta_{jn}J_nU \\
&+ 8\Delta_{in}J_n\Delta_{ij}\Delta_{jj}\Delta_{jn}J_nU + 16\Delta_{in}J_n\Delta_{ij}\Delta_{jj}\Delta_{jn}J_nU \\
&+ 8\Delta_{in}J_n\Delta_{ij}(\Delta_{jn}J_n)^3U + (\Delta_{in}J_n)^2\Delta_{jj}(\Delta_{jn}J_n)^2U + 2(\Delta_{in}J_n)^2\Delta_{jj}^2U \\
&+ 2(\Delta_{in}J_n)^2\Delta_{jj}(\Delta_{jn}J_n)^2U + 3(\Delta_{in}J_n)^2\Delta_{jj}(\Delta_{jn}J_n)^2U \\
&+ (\Delta_{in}J_n)^2(\Delta_{jn}J_n)^4U + (\Delta_{in}J_n)^2\Delta_{jj}^2U] \\
&+ 24\Delta_{ij}^4U + 48\Delta_{in}J_n\Delta_{ij}^3\Delta_{jn}J_nU + 60(\Delta_{in}J_n)^2\Delta_{ii}^2(\Delta_{jn}J_n)^2U \\
&+ 16(\Delta_{in}J_n)^3\Delta_{ij}\Delta_{jj}(\Delta_{jn}J_n)^2U + 32(\Delta_{in}J_n)^3\Delta_{ij}\Delta_{jj}\Delta_{jn}J_nU \\
&+ 16(\Delta_{in}J_n)^3\Delta_{ij}(\Delta_{jn}J_n)^3U + (\Delta_{in}J_n)^4\Delta_{jj}(\Delta_{in}J_n)^2U + 2(\Delta_{in}J_n)^4\Delta_{jj}^2U \\
&+ 2(\Delta_{in}J_n)^4\Delta_{jj}^2U + 2(\Delta_{in}J_n)^4\Delta_{jj}(\Delta_{jn}J_n)^2U + 3(\Delta_{in}J_n)^4\Delta_{jj}(\Delta_{jn}J_n)^2U \\
&+ (\Delta_{in}J_n)^4\Delta_{jj}^2U + (\Delta_{in}J_n)^4(\Delta_{jn}J_n)^4U
\end{aligned}$$

Ans for (b)

Partition up to first order of lambda

$$Z[J] = [1 + \frac{\lambda}{4!} \left(\frac{\partial}{\partial J_i} \right)^4] \mathcal{N}_0 e^{\frac{1}{2} J_m \Delta_{mn} J_n} \quad (12)$$

The two point function is

$$\begin{aligned}
\langle \phi_x \phi_y \rangle &= \frac{1}{Z[0]} \left. \frac{\partial^2 Z[J]}{\partial J_x \partial J_y} \right|_{J=0} \\
&= \frac{1}{Z[0]} \frac{\partial^2}{\partial J_x \partial J_y} [1 + \frac{\lambda}{4!} 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U \\
&\quad + 6(J_m\Delta_{im})^2\Delta_{ii}U] \mathcal{N}_0 e^{\frac{1}{2} J_m \Delta_{mn} J_n} \Big|_{J=0}
\end{aligned}$$

As we are taking $J=0$, the $(J_m \Delta_{im})^4 U$ term will become zero.

$$\begin{aligned}
& \frac{\partial^2 Z[J]}{\partial J_x \partial J_y} [U + \frac{\lambda}{4!} 3\Delta_{ii} \Delta_{ii} U + (J_m \Delta_{im})^2 \Delta_{ii} U] \\
&= \frac{\partial}{\partial J_y} [\frac{\partial}{\partial J_x} + U \frac{\lambda}{4!} (3\Delta_{ii} \Delta_{ii} \Delta_{xm} J_m + 12\Delta_{xi} J_m U \Delta_{im} \Delta_{ii})] \\
&= [\Delta_{xy} + \frac{\lambda}{4!} (3\Delta_{ii} \Delta_{ii} \Delta_{xy} + 12\Delta_{xi} \Delta_{ii} \Delta_{iy})]
\end{aligned}$$

Therefore the two point function

$$\langle \phi_x \phi_y \rangle = \frac{\Delta_{xy} + \frac{\lambda}{4!} (3\Delta_{ii} \Delta_{ii} \Delta_{xy} + 12\Delta_{xi} \Delta_{ii} \Delta_{iy})}{1 + \frac{\lambda}{4!} 3\Delta_{ii} \Delta_{ii}} \quad (13)$$

Binomial expanding the lower part of the fraction we get

$$\langle \phi_x \phi_y \rangle = [\Delta_{xy} + \frac{\lambda}{4!} (3\Delta_{ii} \Delta_{ii} \Delta_{xy} + 12\Delta_{ix} \Delta_{ii} \Delta_{iy})] [1 - \frac{\lambda}{4!} 3\Delta_{ii} \Delta_{ii}] \quad (14)$$

$$= \Delta_{xy} + \frac{\lambda}{4!} (12\Delta_{ix} \Delta_{ii} \Delta_{iy}) \quad (15)$$

Ans for (c)

So the diagram will be

$$\frac{x}{y} + \frac{1}{4!} 16 \frac{0}{i} \frac{x}{y} + o(1^2)$$