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Ans for (a)

The Partiton fucntion

$$Z[J] = \mathcal{N}_0 e^{V[\frac{\partial}{\partial J_i}]} e^{\frac{1}{2}J_m \Delta_{mn} J_n} \tag{1}$$

Where

$$v[\phi] = \frac{\lambda}{4!}\phi = \frac{\lambda}{4!}\frac{\partial}{\partial J_i} \tag{2}$$

expanding the equation

$$Z[J] = [1 + \frac{\lambda}{4!}(\frac{\partial}{\partial J_i})^4 + \frac{\lambda}{4!}^2(\frac{\partial}{\partial J_i})^4(\frac{\partial}{\partial J_i})^4 + ...]\mathcal{N}_0 e^{\frac{1}{2}J_m \Delta_{mn} J_n} \tag{3}$$

Contribution of the first order in lambda

$$\frac{\lambda}{4!}\frac{\partial^4}{\partial {J_i}^4}[e^{\frac{1}{2}J_m \Delta_{mn} J_n}] \tag{4}$$

Now, taking $e^{\frac{1}{2}J_m \Delta_m n J_n} = U$

$$\frac{\partial}{\partial J_i} e^{\frac{1}{2}J_m \Delta_{mn} J_n} = \Delta_{im} J_m U \tag{5}$$

Again

$$\frac{\partial^2}{\partial {J_i}^2} e^{\frac{1}{2}J_m \Delta_{mn} J_n} = \Delta_{ii} U + (J_m \Delta_{im})^2 U \tag{6}$$

Again

$$\frac{\partial^3}{\partial {J_i}^3} e^{\frac{1}{2}J_m \Delta_{mn} J_n} = \Delta_{ii}\Delta_{ii} U + (J_m \Delta_{im})^3 U + 2J_m \Delta_{im}\Delta_{ii} U \tag{7}$$

Again

$$\frac{\partial^4}{\partial {J_i}^4} e^{\frac{1}{2}J_m\Delta_{mn}J_n} = 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4 U + 6(J_m\Delta_{im})^2\Delta_{ii}U \tag{8}$$

Therefore the partiton function upto first order of lambda is

$$Z[J] = [1 + \frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4 U + 6(J_m\Delta_{im})^2\Delta_{ii}U]\mathcal{N}_{\prime} e^{\frac{1}{2}J_m\Delta_{mn}J_n} \tag{9}$$

Ans for (b)

Partition up to first order of lambda

$$Z[J] = [1 + \frac{\lambda}{4!}(\frac{\partial}{\partial J_i})^4]\mathcal{N}_0 e^{\frac{1}{2}J_m\Delta_{mn}J_n} \tag{10}$$

The two point function is

$$\begin{aligned}\langle\phi_x\phi_y\rangle &= \frac{1}{Z[0]}\frac{\partial^2 Z[J]}{\partial J_x\partial J_y}\Bigg|_{J=0} \\ &= \frac{1}{Z[0]}\frac{\partial^2}{\partial J_x\partial J_y}[1 + \frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4 U \\ &\quad + 6(J_m\Delta_{im})^2\Delta_{ii}U]\mathcal{N}_0 e^{\frac{1}{2}J_m\Delta_{mn}J_n}\Bigg|_{J=0}\end{aligned}$$

As we are taking J=0, the $(J_m\Delta_{im})^4 U$ term will become zero.

$$\begin{aligned}&\frac{\partial^2 Z[J]}{\partial J_x\partial J_y}[U + \frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^2\Delta_{ii}U] \\ &= \frac{\partial}{\partial J_y}[\frac{\partial}{\partial J_x} + U\frac{\lambda}{4!}(3\Delta_{ii}\Delta_{ii}\Delta_{xm}J_m + 12\Delta_{xi}J_m U\Delta_{im}\Delta_{ii})] \\ &= [\Delta_{xy} + \frac{\lambda}{4!}(3\Delta_{ii}\Delta_{ii}\Delta_{xy} + 12\Delta_{xi}\Delta_{ii}\Delta_{iy})]\end{aligned}$$

Therefore the two point function

$$\langle\phi_x\phi_y\rangle = \frac{\Delta_{xy} + \frac{\lambda}{4!}(3\Delta_{ii}\Delta_{ii}\Delta_{xy} + 12\Delta_{xi}\Delta_{ii}\Delta_{iy})}{1 + \frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}} \tag{11}$$

Binomial expanding the lower part of the fraction we get

$$\langle\phi_x\phi_y\rangle = [\Delta_{xy} + \frac{\lambda}{4!}(3\Delta_{ii}\Delta_{ii}\Delta_{xy} + 12\Delta_{ix}\Delta_{ii}\Delta_{iy})][1 - \frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}] \tag{12}$$

$$= \Delta_{xy} + \frac{\lambda}{4!}(12\Delta_{ix}\Delta_{ii}\Delta_{iy}) \tag{13}$$