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Ans for (a)

The Partiton fuction

$$Z[J] = \mathcal{N}_0 e^{V[\frac{\partial}{\partial J_i}]} e^{\frac{1}{2} J_m \Delta_{mn} J_n} \quad (1)$$

Where

$$V[\phi] = \frac{\lambda}{4!} \phi = \frac{\lambda}{4!} \frac{\partial}{\partial J_i} \quad (2)$$

expanding the equation

$$Z[J] = [1 + \frac{\lambda}{4!} (\frac{\partial}{\partial J_i})^4 + \frac{\lambda^2}{4!} (\frac{\partial}{\partial J_i})^4 (\frac{\partial}{\partial J_i})^4 + \dots] \mathcal{N}_0 e^{\frac{1}{2} J_m \Delta_{mn} J_n} \quad (3)$$

Contribution of the first order in lambda

$$\frac{\lambda}{4!} \frac{\partial^4}{\partial J_i^4} [e^{\frac{1}{2} J_m \Delta_{mn} J_n}] \quad (4)$$

Now, taking $e^{\frac{1}{2} J_m \Delta_{mn} J_n} = U$

$$\frac{\partial}{\partial J_i} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = \Delta_{im} J_m U \quad (5)$$

Again

$$\frac{\partial^2}{\partial J_i^2} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = \Delta_{ii} U + (J_m \Delta_{im})^2 U \quad (6)$$

Again

$$\frac{\partial^3}{\partial J_i^3} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = \Delta_{ii} \Delta_{ii} U + (J_m \Delta_{im})^3 U + 2 J_m \Delta_{im} \Delta_{ii} U \quad (7)$$

Again

$$\frac{\partial^4}{\partial J_i^4} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U + 6(J_m\Delta_{im})^2\Delta_{ii}U \quad (8)$$

Therefore the partiton function upto first order of lambda is

$$Z[J] = [1 + \frac{\lambda}{4!} 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U + 6(J_m\Delta_{im})^2\Delta_{ii}U] \mathcal{N}_0 e^{\frac{1}{2} J_m \Delta_{mn} J_n} \quad (9)$$

Ans for (b)

first order of lambda

$$Z[J] = [1 + \frac{\lambda}{4!} (\frac{\partial}{\partial J_i})^4] \mathcal{N}_0 e^{\frac{1}{2} J_m \Delta_{mn} J_n} \quad (10)$$

The two point function is

$$\begin{aligned} \langle \phi_x \phi_y \rangle &= \frac{1}{Z[0]} \frac{\partial^2 Z[J]}{\partial J_x \partial J_y} \Big|_{J=0} \\ &= \frac{1}{Z[0]} \frac{\partial^2}{\partial J_x \partial J_y} [1 + \frac{\lambda}{4!} 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U \\ &\quad + 6(J_m\Delta_{im})^2\Delta_{ii}U] \mathcal{N}_0 e^{\frac{1}{2} J_m \Delta_{mn} J_n} \Big|_{J=0} \end{aligned}$$

As we are taking J=0, the $(J_m\Delta_{im})^4U$ term will become zero.

$$\begin{aligned} &\frac{\partial^2 Z[J]}{\partial J_x \partial J_y} [U + \frac{\lambda}{4!} 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^2\Delta_{ii}U] \\ &= \frac{\partial}{\partial J_y} [\frac{\partial}{\partial J_x} + U \frac{\lambda}{4!} (3\Delta_{ii}\Delta_{ii}\Delta_{xm}J_m + 12\Delta_{xi}J_mU\Delta_{im}\Delta_{ii})] \\ &= [\Delta_{xy} + \frac{\lambda}{4!} (3\Delta_{ii}\Delta_{ii}\Delta_{xy} + 12\Delta_{xi}\Delta_{ii}\Delta_{iy})] \end{aligned}$$

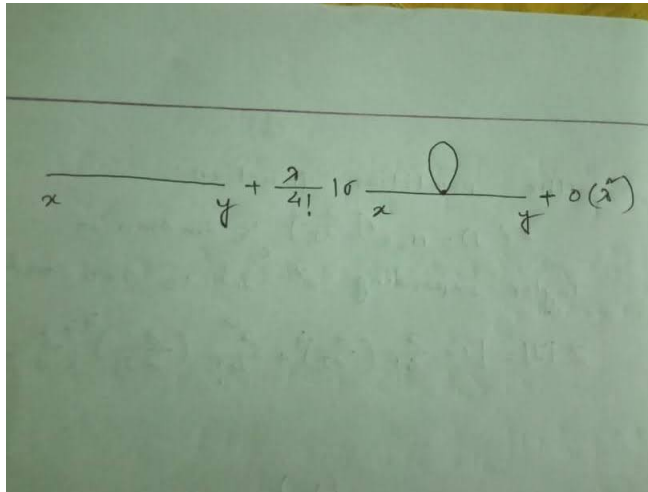
Therefore the two point function

$$\langle \phi_x \phi_y \rangle = \frac{\Delta_{xy} + \frac{\lambda}{4!} (3\Delta_{ii}\Delta_{ii}\Delta_{xy} + 12\Delta_{xi}\Delta_{ii}\Delta_{iy})}{1 + \frac{\lambda}{4!} 3\Delta_{ii}\Delta_{ii}} \quad (11)$$

$$\langle \phi_x \phi_y \rangle = [\Delta_{xy} + \frac{\lambda}{4!} (3\Delta_{ii}\Delta_{ii}\Delta_{xy} + 12\Delta_{ix}\Delta_{ii}\Delta_{iy})] [1 - \frac{\lambda}{4!} 3\Delta_{ii}\Delta_{ii}] \quad (12)$$

$$= \Delta_{xy} + \frac{\lambda}{4!} (12\Delta_{ix}\Delta_{ii}\Delta_{iy}) \quad (13)$$

Ans for (c)



A photograph of a piece of paper with a handwritten mathematical expression. The expression is written in black ink and appears to be a Taylor expansion or a similar mathematical formula. It consists of a horizontal line with several terms. The first term is a horizontal line with 'x' below it. This is followed by a plus sign, then a fraction with 'λ' in the numerator and '4!' in the denominator, followed by a vertical line with 'y' below it. This is followed by a plus sign, then a fraction with 'λ' in the numerator and '4!' in the denominator, followed by a horizontal line with 'x' below it, then a vertical line with a circle around it, and finally a plus sign followed by 'o(x^2)'. The handwriting is somewhat messy and the paper is slightly tilted.

$$\frac{\quad}{x} + \frac{\lambda}{4!} \frac{\quad}{y} + \frac{\lambda}{4!} \frac{\quad}{x} \frac{\quad}{\circ} + o(x^2)$$