

## Ans for (a)

The Partiton fuction

$$Z[J] = \mathcal{N}_0 e^{V[\frac{\partial}{\partial J_i}]} e^{\frac{1}{2} J_m \Delta_{mn} J_n} \quad (1)$$

Where

$$v[\phi] = \frac{\text{lambda}}{4!} \phi = \text{frac} \lambda 4! \frac{\partial}{\partial J_i} \quad (2)$$

expanding the equation

$$Z[J] = [1 + \frac{\lambda}{4!} (\frac{\partial}{\partial J_i})^4 + \frac{\lambda^2}{4!} (\frac{\partial}{\partial J_i})^4 (\frac{\partial}{\partial J_i})^4 + \dots] \mathcal{N}_0 e^{\frac{1}{2} J_m \Delta_{mn} J_n} \quad (3)$$

Contribution of the first order in lambda

$$\frac{\lambda}{4!} \frac{\partial^4}{\partial J_i^4} [e^{\frac{1}{2} J_m \Delta_{mn} J_n}] \quad (4)$$

Now, taking  $e^{\frac{1}{2} J_m \Delta_{mn} J_n}$

$$\frac{\partial}{\partial J_i} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = \Delta_{im} J_m U \quad (5)$$

Again

$$\frac{\partial^2}{\partial J_i^2} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = \Delta_{ii} U + (J_m \Delta_{im})^2 U \quad (6)$$

Again

$$\frac{\partial^3}{\partial J_i^3} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = \Delta_{ii} \Delta_{ii} U + (J_m \Delta_{im})^3 U + 2 J_m \Delta_{im} \Delta_{ii} U \quad (7)$$

Again

$$\frac{\partial^4}{\partial J_i^4} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U + 6(J_m\Delta_{im})^2\Delta_{ii}U \quad (8)$$

Therefore the partiton function upto first order of lambda is

$$Z[J] = [1 + \frac{\lambda}{4!} 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U + 6(J_m\Delta_{im})^2\Delta_{ii}U] \mathcal{N}_0 e^{\frac{1}{2} J_m \Delta_{mn} J_n} \quad (9)$$

## Ans for (b)

Partition up to first order of lambda

$$Z[J] = [1 + \frac{\lambda}{4!} (\frac{\partial}{\partial J_i})^4] \mathcal{N}_0 e^{\frac{1}{2} J_m \Delta_{mn} J_n} \quad (10)$$

The two point function is

$$\langle \phi_i \phi_j \rangle = \frac{1}{Z[0]} \frac{\partial^2 Z[J]}{\partial J_i \partial J_j} \Big|_{J=0} \quad (11)$$

$$= \frac{1}{Z[0]} \frac{\partial^2}{\partial J_i \partial J_j} [1 + \frac{\lambda}{4!} 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U \quad (12)$$

$$+ 6(J_m\Delta_{im})^2\Delta_{ii}U] \mathcal{N}_0 e^{\frac{1}{2} J_m \Delta_{mn} J_n} \Big|_{J=0} \quad (13)$$

As we are taking J=0, the  $(J_m\Delta_{im})^4U$  term will become zero.

$$\frac{\partial^2 Z[J]}{\partial J_i \partial J_j} [U + \frac{\lambda}{4!} 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^2\Delta_{ii}U] \quad (14)$$

$$= \frac{\partial}{\partial J_j} [\frac{\partial}{\partial J_i} + U \frac{\lambda}{4!} (3\Delta_{ii}\Delta_{ii}\Delta_{im}J_m + 12\Delta_{im}J_mU\Delta_{ii}\Delta_{ii})] \quad (15)$$

$$= [\Delta_{ij} + \frac{\lambda}{4!} (3\Delta_{ii}\Delta_{ii}\Delta_{ij} + 12\Delta_{ii}\Delta_{ii}\Delta_{ij})] \quad (16)$$

Therefore the two point function

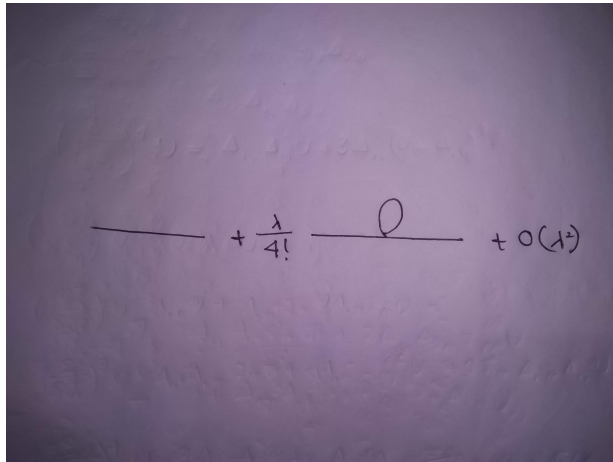
$$\langle \phi_i \phi_j \rangle = \frac{\Delta_{ij} + \frac{\lambda}{4!} (3\Delta_{ii}\Delta_{ii}\Delta_{ij} + 12\Delta_{ii}\Delta_{ii}\Delta_{ij})}{1 + \frac{\lambda}{4!} 3\Delta_{ii}\Delta_{ii}} \quad (17)$$

Binomial expanding the lower part of the fraction we get

$$\langle \phi_i \phi_j \rangle = \Delta_{ij} + \frac{\lambda}{4!} 12\Delta_{ii}\Delta_{ii}\Delta_{ij} \quad (18)$$

**Ans for (c)**

So the diagram will be



A handwritten mathematical expression on a purple background. The expression is:  $\text{---} + \frac{\lambda}{4!} \text{---} 0 \text{---} + o(1^2)$ . The first horizontal line is on the left, followed by a plus sign and the fraction  $\frac{\lambda}{4!}$ . Then another horizontal line, followed by a circled zero, then a third horizontal line, and finally a plus sign and  $o(1^2)$ .