General Relativity and Cosmology

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Very Quick Recap of Relativity

Everywhere we see are events. Spacetime is a collection of events. We can refer events as a stage drama. Coordinates are used to described these events. Coordinates are local.

Manifolds

Manifold is a set endowed with a collection of subsets which But according to physicists, where we can do integration and differentiations on, are called manifolds.

World lines

World lines $\longleftrightarrow$ observables.
squared distance ds will be

$$ds^2 = -dt^2 + d\underline{x} \cdot d\underline{x} \tag{1}$$

Signature of our spacetime will be (-, +, +, +) which is known as East Coast Convention. Where

$A \cdot A > 0$	spacelike
$A \cdot A < 0$	timelike
$A \cdot A = 0$	null or lightlike

Proper Time

Time measured by the observer is called proper time.

Now we define scalar and vectors

Scalar

Quantities that are invariant under coordinate transformation are called scalars.

$$P(x) = P(x^{'})$$

Vector

Collection of tangent lines on a point at a time direct to everywhere are called tangent vector.

T_p is a vector space. If

$$\begin{aligned} U &\in T_p \\ V &\in T_p \end{aligned} \tag{2}$$

$$\alpha U + \beta V \in T_p$$

$$\frac{df}{d\tau} = \frac{\partial f}{\partial x^i}\frac{\partial x^i}{\partial \tau} = (V^i(x)\frac{\partial}{\partial x_i})f \qquad [dx^i = V^i]$$

Functionally,

$$\frac{d}{d\tau} \equiv (V^i(x)\frac{\partial}{\partial x_i}) \tag{3}$$

here $\frac{\partial}{\partial x^i}$ is coordinate basis. However, the coordinates, x^i are observer's contruct. The coordinate basis is independent.

$$\left[\frac{\partial}{\partial x^i}, \frac{\partial}{\partial x^j}\right] = 0 \tag{4}$$

$\frac{d}{d\tau}$ causes a translation along the tangent of the curve. It is independent of the coordinates x's.

Lie Bracket

$$\hat{V} \equiv \frac{d}{d\tau} \equiv (V^i(x)\frac{\partial}{\partial x_i}) \tag{5}$$

$$\begin{aligned}\left[\hat{V}, \hat{W}\right] &= \hat{V}\hat{W} - \hat{W}\hat{V} \\ &= \left(V^j \frac{\partial}{\partial x^j}\right)\left(W^i \frac{\partial}{\partial x^i}\right) - \left(W^j \frac{\partial}{\partial x^j}\right)\left(V^i \frac{\partial}{\partial x^i}\right) \\ &= \left(V^i \frac{\partial W^j}{\partial x^i} - W^i \frac{\partial V^j}{\partial x^i}\right)\frac{\partial}{\partial x^j} \\ &= T^j \frac{\partial}{\partial x^j}\end{aligned} \tag{6}$$

Where

$$T^j = \left[\hat{V}, \hat{W}\right]^j = \left(V^i \frac{\partial W^j}{\partial x^i} - W^i \frac{\partial V^j}{\partial x^i}\right)\frac{\partial}{\partial x^j}$$

$$\hat{V} = \frac{d}{d\tau} = (V^i(x)\frac{\partial}{\partial x_i}) = V'^a(x')\frac{\partial}{\partial x'^a} \tag{7}$$

$$\begin{aligned}x &= x(x') \\ x' &= x'(x) \\ \frac{\partial ?}{\partial x^i} &= \frac{\partial x'^a}{\partial x^i}\frac{\partial ?}{\partial x'^a}\end{aligned}$$

$$\begin{aligned}\hat{V} = V'^a(x')\frac{\partial}{\partial x'^a} &= \left(V^i(x)\frac{\partial x'^a}{\partial x^i}\right)\frac{\partial}{\partial x'^a} \\ \implies V'^a x' &= V^i(x)\frac{\partial x'^a}{\partial x^i}\end{aligned} \tag{8}$$

Diffumorphism maps manifold to itself.