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Ans for (a)

The Partiton fucntion

$$Z[J] = \mathcal{N}_0 e^{V[\frac{\partial}{\partial J_i}]} e^{\frac{1}{2} J_m \Delta_{mn} J_n} \tag{1}$$

Where

$$V[\phi] = \frac{\lambda}{4!}\phi = \frac{\lambda}{4!}\frac{\partial}{\partial J_i} \tag{2}$$

expanding the equation

$$Z[J] = [1 + \frac{\lambda}{4!}(\frac{\partial}{\partial J_i})^4 + \frac{\lambda}{4!}^2(\frac{\partial}{\partial J_i})^4(\frac{\partial}{\partial J_i})^4 + ...]\mathcal{N}_0 e^{\frac{1}{2} J_m \Delta_{mn} J_n} \tag{3}$$

Contribution of the first order in lambda

$$\frac{\lambda}{4!}\frac{\partial^4}{\partial {J_i}^4}[e^{\frac{1}{2} J_m \Delta_{mn} J_n}] \tag{4}$$

Now, taking $e^{\frac{1}{2} J_m \Delta_{mn} J_n} = U$

$$\frac{\partial}{\partial J_i} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = \Delta_{im} J_m U \tag{5}$$

Again

$$\frac{\partial^2}{\partial {J_i}^2} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = \Delta_{ii} U + (J_m \Delta_{im})^2 U \tag{6}$$

Again

$$\frac{\partial^3}{\partial {J_i}^3} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = \Delta_{ii}\Delta_{ii} U + (J_m \Delta_{im})^3 U + 2 J_m \Delta_{im}\Delta_{ii} U \tag{7}$$

Again

$$\frac{\partial^4}{\partial {J_i}^4} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4 U + 6(J_m\Delta_{im})^2\Delta_{ii}U \tag{8}$$

Therefore the partiton function upto first order of lambda is

$$Z[J] = [1 + \frac{\lambda}{4!} 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4 U + 6(J_m\Delta_{im})^2\Delta_{ii}U]\mathcal{N}, \tag{9}$$

Second order of lambda will be

$$\frac{\lambda^4}{(4!)^4}(\frac{\partial}{\partial J_i})^4(\frac{\partial}{\partial J_j})^4 U = \frac{\lambda}{4!}[3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4 U + 6(J_m\Delta_{im})^2\Delta_{ii}U] \tag{10}$$

First term of equation 10 will be

$$\frac{\lambda^4}{(4!)^4}(\frac{\partial}{\partial J_j})^4 3\Delta_{ii}\Delta_{ii}U = 3\Delta_{ii}\Delta ii[3\Delta_{jj}\Delta_{jj}U + (J_m\Delta_{jm})^4 U + 6(J_m\Delta_{jm})^2\Delta_{jj}U] \tag{11}$$

Second term of equation 10 will be

$$\begin{aligned}\frac{\lambda^4}{(4!)^4}(\frac{\partial}{\partial J_j})^4(J_m\Delta_{im})^4 U = 6\Delta_{ii}[&12\Delta_{ij}^2\Delta jjU + 12\Delta_{ij}^2\Delta_{jn}J_n\Delta_{jn}J_nU \\ &+ 8\Delta_{in}J_n\Delta_{ij}\Delta_{jj}\Delta_{jn}J_nU + 16\Delta_{in}J_n\Delta_{ij}\Delta_{jj}\Delta_{jn}J_nU \\ &+ 8\Delta_{in}J_n\Delta_{ij}(\Delta_{jn}J_n)^3U + (\Delta_{in}J_n)^2\Delta_{jj}(\Delta_{jn}J_n)^2U + 2(\Delta_{in}J_n)^2\Delta_{jj}^2U \\ &+ 2(\Delta_{in}J_n)^2\Delta_{jj}(\Delta_{jn}J_n)^2U + 3(\Delta_{in}J_n)^2\Delta_{jj}(\Delta_{jn}J_n)^2U \\ &+ (\Delta_{in}J_n)^2(\Delta_{jn}J_n)^4U + (\Delta_{in}J_n)^2\Delta_{jj}^2U]\end{aligned}$$

Third term of the equation 10

$$\begin{aligned}\frac{\lambda^4}{(4!)^4}(\frac{\partial}{\partial J_j})^4 = {} & 24\Delta_{ij}^4U + 48\Delta_{in}J_n\Delta_{ij}^3\Delta_{jn}J_nU + 60(\Delta_{in}J_n)^2\Delta_{ii}^2(\Delta_{jn}J_n)^2U \\ &+ 16(\Delta_{in}J_n)^3\Delta_{ij}\Delta_{jj}(\Delta_{jn}J_n)^2U + 32(\Delta_{in}J_n)^3\Delta_{ij}\Delta_{jj}\Delta_{jn}J_nU \\ &+ 16(\Delta_{in}J_n)^3\Delta_{ij}(\Delta_{jn}J_n)^3U + (\Delta_{in}J_n)^4\Delta_{jj}(\Delta_{in}J_n)^2U + 2(\Delta_{in}J_n)^4\Delta_{jj}^2U \\ &+ 2(\Delta_{in}J_n)^4\Delta_{jj}^2U + 2(\Delta_{in}J_n)^4\Delta_{jj}(\Delta_{jn}J_n)^2U + 3(\Delta_{in}J_n)^4\Delta_{jj}(\Delta_{jn}J_n)^2U \\ &+ (\Delta_{in}J_n)^4\Delta_{jj}^2U + (\Delta_{in}J_n)^4(\Delta_{jn}J_n)^4U\end{aligned}$$

Therefore

$$\frac{\lambda^4}{(4!)^4}(\frac{\partial}{\partial J_i})^4(\frac{\partial}{\partial J_j})^4U = 3\Delta_{ii}\Delta ii[3\Delta_{jj}\Delta_{jj}U + (J_m\Delta_{jm})^4U + 6(J_m\Delta_{jm})^2\Delta_{jj}U]$$
$$+ 6\Delta_{ii}[12\Delta_{ij}^2\Delta jjU + 12\Delta_{ij}^2\Delta_{jn}J_n\Delta_{jn}J_nU$$
$$+ 8\Delta_{in}J_n\Delta_{ij}\Delta_{jj}\Delta_{jn}J_nU + 16\Delta_{in}J_n\Delta_{ij}\Delta_{jj}\Delta_{jn}J_nU$$
$$+ 8\Delta_{in}J_n\Delta_{ij}(\Delta_{jn}J_n)^3U + (\Delta_{in}J_n)^2\Delta_{jj}(\Delta_{jn}J_n)^2U + 2(\Delta_{in}J_n)^2\Delta_{jj}^2U$$
$$+ 2(\Delta_{in}J_n)^2\Delta_{jj}(\Delta_{jn}J_n)^2U + 3(\Delta_{in}J_n)^2\Delta_{jj}(\Delta_{jn}J_n)^2U$$
$$+ (\Delta_{in}J_n)^2(\Delta_{jn}J_n)^4U + (\Delta_{in}J_n)^2\Delta_{jj}^2U]$$
$$+ 24\Delta_{ij}^4U + 48\Delta_{in}J_n\Delta_{ij}^3\Delta_{jn}J_nU + 60(\Delta_{in}J_n)^2\Delta_{ii}^2(\Delta_{jn}J_n)^2U$$
$$+ 16(\Delta_{in}J_n)^3\Delta_{ij}\Delta_{jj}(\Delta_{jn}J_n)^2U + 32(\Delta_{in}J_n)^3\Delta_{ij}\Delta_{jj}\Delta_{jn}J_nU$$
$$+ 16(\Delta_{in}J_n)^3\Delta_{ij}(\Delta_{jn}J_n)^3U + (\Delta_{in}J_n)^4\Delta_{jj}(\Delta_{in}J_n)^2U + 2(\Delta_{in}J_n)^4\Delta_{jj}^2U$$
$$+ 2(\Delta_{in}J_n)^4\Delta_{jj}^2U + 2(\Delta_{in}J_n)^4\Delta_{jj}(\Delta_{jn}J_n)^2U + 3(\Delta_{in}J_n)^4\Delta_{jj}(\Delta_{jn}J_n)^2U$$
$$+ (\Delta_{in}J_n)^4\Delta_{jj}^2U + (\Delta_{in}J_n)^4(\Delta_{jn}J_n)^4U$$

Ans for (b)

Partition up to first order of lambda

$$Z[J] = [1 + \frac{\lambda}{4!}(\frac{\partial}{\partial J_i})^4]\mathcal{N}_0 e^{\frac{1}{2}J_m\Delta_{mn}J_n} \tag{12}$$

The two point function is

$$\langle\phi_x\phi_y\rangle = \frac{1}{Z[0]}\frac{\partial^2 Z[J]}{\partial J_x\partial J_y}\bigg|_{J=0}$$
$$= \frac{1}{Z[0]}\frac{\partial^2}{\partial J_x\partial J_y}[1 + \frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U$$
$$+ 6(J_m\Delta_{im})^2\Delta_{ii}U]\mathcal{N}_0 e^{\frac{1}{2}J_m\Delta_{mn}J_n}\bigg|_{J=0}$$

As we are taking J=0, the $(J_m\Delta_{im})^4U$ term will become zero.

$$\frac{\partial^2 Z[J]}{\partial J_x \partial J_y}[U+\frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}U+(J_m\Delta_{im})^2\Delta_{ii}U]$$
$$=\frac{\partial}{\partial J_y}[\frac{\partial}{\partial J_x}+U\frac{\lambda}{4!}(3\Delta_{ii}\Delta_{ii}\Delta_{xm}J_m+12\Delta_{xi}J_mU\Delta_{im}\Delta_{ii})]$$
$$=[\Delta_{xy}+\frac{\lambda}{4!}(3\Delta_{ii}\Delta_{ii}\Delta_{xy}+12\Delta_{xi}\Delta_{ii}\Delta_{iy})]$$

Therefore the two point function

$$\langle\phi_x\phi_y\rangle=\frac{\Delta_{xy}+\frac{\lambda}{4!}(3\Delta_{ii}\Delta_{ii}\Delta_{xy}+12\Delta_{xi}\Delta_{ii}\Delta_{iy})}{1+\frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}} \tag{13}$$

Binomial expanding the lower part of the fraction we get

$$\langle\phi_x\phi_y\rangle=[\Delta_{xy}+\frac{\lambda}{4!}(3\Delta_{ii}\Delta_{ii}\Delta_{xy}+12\Delta_{ix}\Delta_{ii}\Delta_{iy})][1-\frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}] \tag{14}$$
$$=\Delta_{xy}+\frac{\lambda}{4!}(12\Delta_{ix}\Delta_{ii}\Delta_{iy}) \tag{15}$$

Ans for (c)

So the diagram will be