Ans for (a)

The Partiton fucntion

$$Z[J] = \mathcal{N}_0 e^{V[\frac{\partial}{\partial J_i}]} e^{\frac{1}{2} J_m \Delta_{mn} J_n} \tag{1}$$

Where

$$v[\phi] = \frac{lambda}{4!} \phi = frac \lambda 4! \frac{\partial}{\partial J_i} \tag{2}$$

expanding the equation

$$Z[J] = [1 + \frac{\lambda}{4!} (\frac{\partial}{\partial J_i})^4 + \frac{\lambda}{4!}^2 (\frac{\partial}{\partial J_i})^4 (\frac{\partial}{\partial J_i})^4 + ...] \mathcal{N}_0 e^{\frac{1}{2} J_m \Delta_{mn} J_n} \tag{3}$$

Contribution of the first order in lambda

$$\frac{\lambda}{4!} \frac{\partial^4}{\partial {J_i}^4} [e^{\frac{1}{2} J_m \Delta_{mn} J_n}] \tag{4}$$

Now, taking $e^{\frac{1}{2} J_m \Delta_m n J_n}$

$$\frac{\partial}{\partial J_i} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = \Delta_{im} J_m U \tag{5}$$

Again

$$\frac{\partial^2}{\partial {J_i}^2} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = \Delta_{ii} U + (J_m \Delta_{im})^2 U \tag{6}$$

Again

$$\frac{\partial^3}{\partial {J_i}^3} e^{\frac{1}{2} J_m \Delta_{mn} J_n} = \Delta_{ii} \Delta_{ii} U + (J_m \Delta_{im})^3 U + 2 J_m \Delta_{im} \Delta_{ii} U \tag{7}$$

Again

$$\frac{\partial^4}{\partial {J_i}^4} e^{\frac{1}{2}J_m\Delta_{mn}J_n} = 3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U + 6(J_m\Delta_{im})^2\Delta_{ii}U \tag{8}$$

Therefore the partiton function upto first order of lambda is

$$Z[J] = [1 + \frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U + 6(J_m\Delta_{im})^2\Delta_{ii}U]\mathcal{N}_{\prime}e^{\frac{1}{2}J_m\Delta_{mn}J_n} \tag{9}$$

Ans for (b)

Partition up to first order of lambda

$$Z[J] = [1 + \frac{\lambda}{4!}(\frac{\partial}{\partial J_i})^4]\mathcal{N}_0 e^{\frac{1}{2}J_m\Delta_{mn}J_n} \tag{10}$$

The two point function is

$$\langle\phi_i\phi_j\rangle = \frac{1}{Z[0]}\frac{\partial^2 Z[J]}{\partial J_i \partial J_j}\bigg|_{J=0} \tag{11}$$

$$= \frac{1}{Z[0]}\frac{\partial^2}{\partial J_i \partial J_j}[1 + \frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^4U \tag{12}$$

$$+ 6(J_m\Delta_{im})^2\Delta_{ii}U]\mathcal{N}_0 e^{\frac{1}{2}J_m\Delta_{mn}J_n}\bigg|_{J=0} \tag{13}$$

As we are taking J=0, the $(J_m\Delta_{im})^4U$ term will become zero.

$$\frac{\partial^2 Z[J]}{\partial J_i \partial J_j}[U + \frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}U + (J_m\Delta_{im})^2\Delta_{ii}U] \tag{14}$$

$$= \frac{\partial}{\partial J_j}[\frac{\partial}{\partial J_i} + U\frac{\lambda}{4!}(3\Delta_{ii}\Delta_{ii}\Delta_{im}J_m + 12\Delta_{im}J_mU\Delta_{ii}\Delta_{ii})] \tag{15}$$

$$= [\Delta_{ij} + \frac{\lambda}{4!}(3\Delta_{ii}\Delta_{ii}\Delta_{ij} + 12\Delta_{ii}\Delta ii\Delta_{ij})] \tag{16}$$

Therefore the two point function

$$\langle\phi_i\phi_j\rangle = \frac{\Delta_{ij} + \frac{\lambda}{4!}(3\Delta_{ii}\Delta_{ii}\Delta_{ij} + 12\Delta_{ii}\Delta ii\Delta_{ij})}{1 + \frac{\lambda}{4!}3\Delta_{ii}\Delta_{ii}} \tag{17}$$

Binomial expanding the lower part of the fraction we get

$$\langle\phi_i\phi_j\rangle = \Delta_{ij} + \frac{\lambda}{4!}12\Delta_{ii}\Delta_{ii}\Delta_{ij} \tag{18}$$

Ans for (c)

So the diagram will be