$$\frac{dv}{dt} - g + av = 0$$

for a retarding force proportional to the velocity.

(a) Define a function **void velocity (double t0, double v0)** that will integrate the above ODE by using **RK4** method and write the values of t and v in a data file. Take $a = 0.2\ s^{-1}$. 3

(b) Plot $v(t)$ vs. t upto a large value of t ($t \sim 100$) and save the plot as **velocity.png**. Take $v(0) = 0$. What is the velocity of the body after 10 sec (write down the answer in your answer script)? 3

(c) What is the approximate ratio of $v(15)$ to the terminal velocity? Write the value in answer script. 1

3. The Lane-Emden equation of astrophysics is given by

$$\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} + y^s = 0$$

(a) Write a function **double lane_emden(double x, double s)** that will return the values of y by using Euler-Cromer method. Take the initial conditions as $y(0) = 1$ & $\dot{y}(0) = 0$. 4

(b) Plot $y(x)$ vs. x for $x \epsilon (0,20]$ with an increment of 0.01 for $s = 0, 1, 2, 3,$ and 4 in a single plot and save the plot as **emden.png**. From the graph locate the first zero of $y(x)$ for all the six cases of s and write the results in your answer script as $x_0, x_1, \ldots..$ for $s = 0, 1, \ldots.$ respectively. [Hints: you may use a break statement when y<0.0 within the loop that is used to find y since your target is to get the first zero of y]

3. The differential equations of motion of a charged particle in crossed electric and magnetic fields ($\boldsymbol{E} = (E, 0, 0)$ and $\boldsymbol{B} = (0, 0, B)$) are given by

$$\frac{dv_x}{dt} = +\frac{qB}{m}v_y - \frac{\gamma}{m}v_x + \frac{qE}{m}$$

$$\text{and } \frac{dv_y}{dt} = -\frac{qB}{m}v_x - \frac{\gamma}{m}v_y$$

where q is the charge of the particle, m is the mass and γ is a damping factor. The velocity is in the xy-plane with components v_x and v_y.

(a) Write a C++ program to solve the above coupled differential equations using any suitable numerical methods. Take $v_x(0) = 4$ & $v_y(0) = 0$ and $q = m = E = 1, B = 2, \gamma = 0.1$ (MKS units). 4

(b) Write the values of v_x and v_y as functions of time t in a file and plot (i) v_x and v_y vs time and (ii) v_x vs v_y using gnuplot. Save the plots. 4

3. The differential equation for the population of a radioactive daughter element is

$$\frac{dN_2(t)}{dt} = \lambda_1 \exp(-\lambda_1 t) - \lambda_2 N_2$$

where $\lambda_1 \exp(-\lambda_1 t)$ is the rate of production resulting from the decay of the parent element, $\lambda_1 = 0.1\ s^{-1}$, $\lambda_2 = 0.08\ s^{-1}$.

(a) Define a function **void population(double t0,double N20)** that will use **RK4 to** integrate this ODE from $t =$ ... initial condition $N_2(0) = 0$. 4

(b) Tabulate and plot $N_2(t)$ vs. t. Save the plot as **population.png**.

3. The differential equation of a Van der Pol oscillator is given by

$$\frac{d^2x}{dt^2} - c(1 - x^2)\frac{dx}{dt} + kx = 0$$

Consider $c = k = 1$ here and write a code in C++ using suitable method to solve the Van der Pol equation for different set of initial conditions (i) $x(0) = 1, \dot{x}(0) = 1$ (ii)$x(0) = 2, \dot{x}(0) = 1$ and (iii) $x(0) = 10, \dot{x}(0) = 3$. Plot $x(t)$ vs t in each case and save the plots as **.png** files. 7

3. Lorentz proposed the following system of differential equations as a simple model of atmospheric convection:

$$\frac{dx}{dt} = 10(y - x)$$

$$\frac{dy}{dt} = x(27 - z) - y$$

$$\frac{dz}{dt} = xy - \frac{8}{3}z$$

The variables x, y, z represent physical quantities such as temperatures and flow velocities, while the numbers 10, 27, and 8/3 represent properties of the atmospheric system.

(a) Solve the above system of differential equations by any suitable numerical technique and write the values of x, y, z in a data file for $t\epsilon[0: 100]$ with an increment of 0.01. 5

(b) Plot (i) x Vs y, (ii) x Vs. z and (iii) y Vs z. Save the plots as **.png** files. 4