MONROE COMMUNITY COLLEGE

MTH 211 – SLN

Unit 4 Written Assignment

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*See Blackboard for the deadline for submitting your assignment.

Directions:

- Be sure to follow the submission instructions given in Blackboard when submitting your completed assignment. Do NOT email me your completed assignment.
- Only methods covered in this course up to the current unit may be used on this assignment.
- In all problems you must show sufficient work to support your final answers. All such work must be done in this assignment document. If additional space is needed, you may add pages, but your work must be submitted in order.
- Be sure to include this page as a cover page for your assignment when you submit it and please make sure your name is written in the designated spot above.
- **The work you submit must be your own. While I cannot prevent students from discussing problems, suspicion of duplicated work will be investigated and penalties may result. <u>In addition, solutions taken from online calculators or the equivalent will be not be accepted and will be considered cheating.</u>**
- Be sure to include this page as a cover page for your assignment when you submit it and please make sure your name is written in the designated spot above. If you do not have access to a printer, you may write your solutions on regular paper, but each page must consist of solutions to only those problems on the corresponding page of the original exam.

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$a = 8$

[A] Find the 3rd degree Taylor Polynomial for $f(x) = \sqrt[3]{x}$ centered at $x = 8$. Clearly show all derivatives involved as well as the values obtained from those derivatives as was done in Example 2 from Section 11.1 of the book and Examples 1 and 2 in the Unit 4.1 Summary Notes. Simplify the coefficients in your final answer (no factorials in the final answer; do not use decimals).

$$T_3(X) = \sum_{K=0}^{3} \frac{f^{(K)}(a)}{K!} \cdot (X-a)^K \qquad a=8$$

$f(a) = a^{1/3}$

$\frac{f(a)}{0!} = 2$ $\qquad \frac{f''(a)}{2!} = \frac{-1/144}{2} = -\frac{1}{288}$

$f'(a) = \frac{1}{3}a^{-2/3}$

$= \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$

$\frac{f'(a)}{1!} = 1/12$

$f''(a) = -\frac{2}{9} \cdot a^{-5/3}$

$\frac{f'''(a)}{3!} = \frac{\frac{5}{3456}}{6} \to \frac{5}{20736}$

$= -2/9 \cdot \frac{1}{\sqrt[3]{a^5}}$

$f'''(a) = +\frac{10}{27} \cdot a^{-8/3}$

$= -2/9 \cdot \frac{1}{32}$

$= \frac{10}{27} \cdot \frac{1}{\sqrt[3]{a^8}} \to \frac{5}{3456}$

(continue) on seperate page

$= -\frac{2}{288} = -\frac{1}{144}$

[B] Find a bound for $|R_3(x)|$ on the interval $[6, 12]$, where $R_3(x)$ is the remainder associated with the 3rd degree Taylor Polynomial that you obtained for $f(x) = \sqrt[3]{x}$ in problem [A]. See Example 8a in Section 11.1 of the book and Example 3 in the Unit 4.1 Summary Notes for an example.

$$|R_3(x)| \leq \frac{|x-8|^4}{4!} \cdot \max\{|f^{4}(x)|\}$$

$\approx .0014$

$$|R_3(X)| \leq \frac{|x-8|^4}{4!} \cdot .0014$$

$x^{1/3}$

$\frac{1}{3}X^{-2/3}$

$-\frac{2}{9}X^{-5/3}$

$\frac{10}{27}X^{-8/3}$

$-\frac{80}{81}X^{-11/3} = -\frac{80}{81} \cdot \frac{1}{\sqrt[3]{X^{11}}}$

$[6, 12]$

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A.

$$T_3(x) = \frac{f(a)}{0!} + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$$

$$= \boxed{2 + \frac{1}{12}(x-8) - \frac{1}{288}(x-8)^2 + \frac{5}{20736}(x-8)^3}$$

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[C] Determine the center-, radius-, and interval-of convergence for the following power series.

(C. 1) $\sum_{k=0}^{\infty} \frac{1}{2^k}(x+5)^k$

a_k

center

$x+5=0$

$x=-5$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}(x)}{a_n(x)}\right| < 1$$

$$\lim \left|\frac{(x+5)^{n+1}}{2^{n+1}} \cdot \frac{2^n}{(x+5)^n}\right| < 1$$

$$\lim_{n\to\infty} \left|\frac{(x+5)}{2}\right| \to \left|\frac{x+5}{2}\right| = \frac{|x+5|}{2} < 1$$

$$|x+5| < 2$$

cont. on seperate page

Answers. Center: -5 Radius: 2 Interval: $(-7, -3)$

(C. 2) $\sum_{k=2}^{\infty} \frac{x^k}{\ln k} = f(x)$

$$f(-1) = \sum_{k=2}^{\infty} \frac{(-1)^k}{\ln k}$$

1. $a_k \geq a_{k+1}$ ✓
2. $\lim_{k\to\infty} a_k = 0$ ✓

$a_k = \frac{1}{\ln k}$

$\ln(k+1) > \ln k$,

$\frac{1}{\ln(k+1)} < \frac{1}{\ln(k)}$

$\lim_{k\to\infty} \frac{1}{\ln(k)} = 0$

$$\lim_{k\to\infty} \left|\frac{a_{k+1}(x)}{a_k(x)}\right| < 1$$

$$\lim \left|\frac{x^{k+1}}{\ln(k+1)} \cdot \frac{\ln(k)}{x^k}\right| < 1$$

$$\lim \left|x \cdot \frac{\ln k}{\ln(k+1)}\right| < 1$$

$$|x| \cdot \boxed{\lim_{k\to\infty} \left|\frac{\ln k}{\ln(k+1)}\right|} < 1$$

1

$$|x| < 1 \qquad x \in (-1, 1)$$

Answers. Center: 0 Radius: 1 Interval: $[-1, 1)$

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(1)

$-2 < x+5 < 2$ $\quad x \in (-7, -3)$

$-7 < x < -3$

$f(-7) = \sum_{k=0}^{\infty} \frac{-2^k}{2^k} = (-1)^k$

$\sum_{k=0}^{\infty} (-1)^k$

$r = -1$

$|r| < 1$

$-1 \leq r < 1$

$f(-3) = \sum_{k=0}^{\infty} \frac{2^k}{2^k} = \sum_{k=0}^{\infty} (1)^k$

$\boxed{C_2}$ noor mustafa

$$f(1) = \sum_{k=2}^{\infty} \frac{1}{\ln k} > \sum \frac{1}{k} = \infty \rightarrow \text{diverge}$$

$$k > \ln k$$

$$\sum \frac{1}{k} < \sum \frac{1}{\ln k}$$

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[D] (D.1) Give the Taylor Series for $f(x) = \sin x$ centered at $x = 0$ using summation notation (i.e. using Σ). Also indicate the interval of convergence. You do not have to show any work for this part, just provide the requested information which you should know from the notes/book.

Series: $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n+1}}{(2n+1)!}$

Interval of Convergence: $(-\infty, \infty)$

(D.2) Use your answer in (D.1) to obtain the Taylor Series centered at $x = 0$ for $g(x) = x^2 \sin(x^2)$. Write your answer in the form $\sum_{k=0}^{\infty} a_k x^{p(k)}$.

$$\sin(x^2) = \sum_{k=0}^{\infty} \frac{(-1)^k (x^2)^{2k+1}}{(2k+1)!} = \sum \frac{(-1)^k x^{4k+2}}{(2k+1)!} \cdot x^2$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k x^{4(k+1)}}{(2k+1)!}$$

$+1$

$x^{4(k+1)}$

Series: $x^2 \sin(x^2) = \sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^{4(k+1)}}{(2k+1)!}$

(D.3) Use the first four non-zero terms of the series determined in (D.2) to approximate the following integral rounding your final answer to the nearest ten-thousandth.

$\int_0^1 x^2 \sin(x^2)\, dx$

K	0	1	2	3
value	x^4	$\frac{-x^8}{6}$	$\frac{x^{12}}{5!}$	$\frac{-x^{16}}{7!}$
$\int$	$\frac{x^5}{5}$	$\frac{-x^9}{9\cdot 3!}$	$\frac{x^{13}}{13\cdot 5!}$	$\frac{-x^{17}}{17\cdot 7!}$

exponent rule to integrate

$$\frac{1}{5} - \frac{1}{9\cdot 3!} + \frac{1}{13\cdot 5!} - \frac{1}{17\cdot 7!}$$

$$\frac{1}{5} - \frac{1}{54} + \frac{1}{1560} - \frac{1}{85680} = 0.18211$$