MONROE COMMUNITY COLLEGE

MTH 211 – SLN

Unit 3 Written Assignment

Printed Name: Noor Mustafa

*See Blackboard for the deadline for submitting your assignment.

Directions:

- Be sure to follow the submission instructions given in Blackboard when submitting your completed assignment. Do NOT email me your completed assignment.
- Only methods covered in this course up to the current unit may be used on this assignment.
- In all problems you must show sufficient work to support your final answers. All such work must be done in this assignment document. If additional space is needed, you may add pages, but your work must be submitted in order.
- **The work you submit must be your own. While I cannot prevent students from discussing problems, suspicion of duplicated work will be investigated and penalties may result. <u>In addition, solutions taken from online calculators or the equivalent will be not be accepted and will be considered cheating.</u>**
- Be sure to include this page as a cover page for your assignment when you submit it and please make sure your name is written in the designated spot above. If you do not have access to a printer, you may write your solutions on regular paper, but each page must consist of solutions to only those problems on the corresponding page of the original exam.

[A] Determine whether each of the following series Converges or Diverges. Indicate your answer by circling the appropriate word. For each, you must justify your answer by indicating which series test you are using and how it determines your answer. Make sure that your justifications are complete (all requirements of the series test must be addressed) and clearly communicated.

(A.1) $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{1+k^2}}$ CONVERGES DIVERGES (Circle One)

① $A_{K+1} \leq A_K$

② $\lim_{K\to\infty} a_K = 0$

Justification/Work:

$a_K = \frac{1}{\sqrt{1+K^2}}$

$\frac{1}{\sqrt{1+K^2}} > \frac{1}{\sqrt{1+(1+K)^2}}$

Bigger fraction

$\lim_{K\to\infty} \frac{1}{\sqrt{1+K^2}} \to \frac{1}{\infty} \to 0$

converges because it passes the alternating series test

(A.2) $\sum_{k=1}^{\infty} \frac{\sqrt[k]{4}}{7}$ CONVERGES DIVERGES (Circle One)

Justification/Work:

$\sum \frac{\sqrt[K]{4}}{7} = \frac{}{7} + \frac{}{7} + \frac{}{7}$

$\frac{1}{7} \sum_{K=1}^{\infty} 4^{1/K}$

$\boxed{\text{nth term}}$

$\lim_{K\to\infty} a_K = \lim_{K\to\infty} 4^{1/K} = 4^0 = 1$

$1 \neq 0$

Diverges by using the n^{th} term test

(A.3) $\sum_{k=1}^{\infty} \frac{6}{\sqrt[3]{k^2}}$ CONVERGES DIVERGES (Circle One)

Justification/Work:

$6 \sum_{k=1}^{\infty} \frac{1}{k^{2/3}} \rightarrow$ P-series

$p = 2/3 \leq 1$

* when $p \leq 1$ it diverges
* when $p > 1$ it converges

(A.4) $\sum_{k=1}^{\infty} \frac{(k+1)!}{k \cdot 2^k}$ CONVERGES DIVERGES (Circle One)

Justification/Work: $\boxed{\text{The ratio test}}$

$\lim_{k \to \infty} \left| \frac{a_{k+1}}{a_k} \right| = \lim_{k \to \infty} \left| \frac{(k+2)!}{(k+1)2^{k+1}} \cdot \frac{k \cdot 2^k}{(k+1)!} \right|$

$a_k = \frac{(k+1)!}{k \cdot 2^k}$

$= \lim_{k \to \infty} \frac{1}{2} \left| \frac{(k+2) \cdot k}{k+1} \right|$

$a_{k+1} = \frac{(k+2)!}{(k+1) 2^{k+1}}$

$= \frac{1}{2} \lim_{k \to \infty} \frac{k(k+2)}{k+1}$

$= \frac{1}{2} \lim_{k \to \infty} \frac{k^2 + 2k + 1 - 1}{k+1}$

$= \frac{1}{2} \lim_{k \to \infty} \left(\frac{(k+1)^{\cancel{2}}}{\cancel{k+1}} - \cancel{\frac{1}{k+1}} \right)$

$= \infty$

$\infty > 1$ so it diverges

the integral test

(A.5) $\sum_{k=1}^{\infty} \frac{k}{e^{3k^2}}$ CONVERGES DIVERGES (Circle One)

Justification/Work:

$f(k) = a_k = \frac{k}{e^{3k^2}}$

$f(x) = x \cdot e^{-3x^2}$

The sum would also be finite

. Both would converge

$\int_1^{\infty} x \cdot e^{-3x^2} \, dx = \frac{1}{6}\int_3^{\infty} e^{-u} \, du$

$u(x) = 3x^2$

$\frac{du}{6} = 6x \, dx$

$u(1) = 3$

$u(\infty) = \infty$

$= -\frac{1}{6} e^{-u} \Big|_3^{\infty}$

$= \frac{1}{6} e^{-u} \Big|_{\infty}^{3}$

$\frac{1}{6} \cdot \left(e^{-3} - \cancel{e^{-\infty}}\right)$

$\cancel{\frac{1}{e^{\infty}}}$

. it converges b/c its a finite integral

$\rightarrow = \frac{1}{6} \cdot e^{-3}$

(A.6) $\sum_{k=1}^{\infty} \frac{1}{k^3 + \ln k}$ CONVERGES DIVERGES (Circle One)

limit comparison test

Justification/Work:

$\frac{1}{k^3 + \ln k} \leq \frac{1}{k^3}$

$\frac{1}{1} \leq \frac{1}{1}$

$\sum_{k=1}^{\infty} \frac{1}{k^3 + \ln k} \leq \sum_{k=1}^{\infty} \frac{1}{k^3}$

$\leq L$ conv

P-series

$p = 3 > 1$

it has to be smaller than a finite number.

The sum of $\frac{1}{k^3}$ is smaller or equ to a finite number. which means it must converge.

[B] Consider the following infinite series: $\frac{2}{5} - \frac{4}{15} + \frac{8}{45} - \frac{16}{135} + \frac{32}{405} - \cdots$

Common ratio
2/3

(B.1) Express the sum of above using sigma/summation notation (i.e. using Σ).

$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{5}\left(\frac{2}{3}\right)^{n}$

$\frac{(2/3)}{(4/15)} = 2/3$

$\frac{2}{5} \cdot \frac{2}{3} = \frac{4}{15}$

$\frac{(4/15)}{(8/45)} = 2/3$

(B.2) Explain how we know the above series converges. Provide a complete justification.

① $\sum |a_n|$ converges

② $r = 2/3 < 1$

③ $\sum a_n$ converges

(B.3) Determine the exact value that the series converges to. Express your final answer as a fraction in lowest terms.

$\sum_{n=0}^{\infty} a r^n = \frac{e_0}{1-r}$ $\frac{e_1\ ar}{1-r}$

Seq $(-1)^{n+1} \times (3/5)(2/3)^n, x, 1, 100$

$= \frac{1}{5}$

[C] For any value of x, the series below will be geometric. For which values of x will the series DIVERGE?

$$\sum_{k=0}^{\infty} \left(\frac{x}{4}\right)^{2k}$$

$x \geq 4$

[D] Consider the following Telescoping Series. (See Example 3b in Section 10.3 of the text for a similar problem)

$$\sum_{k=1}^{\infty} \frac{3}{9k^2+3k-2} \rightarrow \frac{3}{3k(3k-1)+2(3k-1)}$$

(D.1) Obtain the partial fraction decomposition for $\frac{3}{9k^2+3k-2}$. Show all work!!!

$$\frac{3}{(3k-1)(3k+2)} = \frac{A}{(3k-1)} + \frac{B}{(3k+2)}$$

① $k=1$

② $k=2$

$$3 = A(3k+2) + B(3k-1)$$

$$3 = 5A + 2B$$

$$3 = 8A + 5B$$

A cont. on seperate page

(D.2) Obtain an explicit formula for S_n (the n^{th} partial sum). The formula should be simplified so that it does not contain the $\cdots$ in it.

$$S_n = \sum_{k=1}^{n} \left(\frac{1}{(3k-1)} - \frac{1}{(3k+2)}\right) = \left(\frac{1}{2}-\frac{1}{5}\right) + \left(\frac{1}{5}-\frac{1}{8}\right) + \left(\frac{1}{8}-\frac{1}{11}\right) + \cdots + \frac{1}{3(n-1)-1} - \frac{1}{3(n-1)+2}$$

A cont. on seperate page

(D.3) Determine if the series converges or diverges by calculating the appropriate limit of S_n. If the series converges, indicate its sum. Your solution must clearly show the limit being calculated using limit notation. Be sure to clearly indicate whether or not the series converges or diverges and clearly indicate the sum of the series.

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\frac{1}{2} - \frac{1}{3n+2}\right)$$

$$= 1/2$$

so A converges

Noor

D_1

$$\left[\begin{array}{cc|c} 5 & 2 & 3 \\ 8 & 5 & 3 \end{array}\right] \quad \frac{1}{5} \cdot R_1 \to r_2$$

$$\left[\begin{array}{cc|c} 1 & 2/5 & 3/5 \\ 8 & 5 & 3 \end{array}\right] \quad -8 \cdot R_1 + R_2 \to R_2$$

$$\left[\begin{array}{cc|c} 1 & 2/5 & 3/5 \\ 0 & 9/5 & -9/5 \end{array}\right]$$

$$A = 3/5 + 2/5 \cancel{B} \Rightarrow A = 1$$

$$9/5 \, B = -9/5 \Rightarrow B = -1$$

$$\frac{3}{(3k-1)(3k+2)} = \frac{1}{3k-1} - \frac{1}{3k+2}$$

$$\sum_{k=1}^{\infty} \left(\frac{1}{(3k-1)} - \frac{1}{(3k+2)} \right)$$

$\boxed{D_2}$ $\boxed{\text{Proof}}$

$$\cancel{\frac{1}{2} + \frac{1}{5} + \frac{1}{8} + \cdots + \frac{1}{3(n-1)-1} + \frac{1}{3n-1}}$$

$$\cancel{\frac{1}{3} \sum \frac{1}{k} \quad 3k+5 > 3k+2 \quad \frac{1}{3n-4} + \frac{1}{3n-1}}$$

$$= \left(\frac{1}{2} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{8}}\right) + \left(\cancel{\frac{1}{8}} - \frac{1}{11}\right)$$

$$+ \left(\cancel{\frac{1}{3(n-1)-1}} - \frac{1}{3(n-1)+2}\right) \qquad \frac{1}{3n \cancel{-5 \cdot 2 - 1}} \; -1$$

$$+ \left(\cancel{\frac{1}{3n-1}} - \frac{1}{3n+2}\right)$$

$$= \frac{1}{2} - \frac{1}{3n+2}$$

[E] Consider the following series.

lim a - B = A - B

$$\sum_{k=1}^{\infty} \frac{(-1)^k \cdot 3^k}{4^k + 1}$$

(E.1) Show that $\lim_{k\to\infty} \frac{3^k}{4^k+1} = 0$. Your answer must follow from a determinate form.

$$\lim_{k\to\infty} \frac{3^k}{4^k(1+\frac{1}{4^k})} = \lim_{k\to\infty} \left(\frac{3}{4}\right)^k \cdot \frac{1}{(1+\frac{1}{4^k})}$$

$$= \lim_{k\to\infty} \left(\frac{3}{4}\right)^k \cdot \lim_{k\to\infty} \frac{1}{1+\frac{1}{4^k}}$$

0   1

$$0 \cdot 1 = 0$$

$$\lim_{k\to\infty} \frac{3^k}{4^k+1} = 0 \checkmark$$

(E.2) Use Theorem 10.18 in section 10.6 of the textbook to determine the **smallest** n guaranteeing that $|R_n| < .001$. You may use trial-and-error to come up with the answer.

(1) $\left(\frac{3}{4}\right)^k \cdot \frac{1}{(1+\frac{1}{4^k})} \geq \left(\frac{3}{4}\right)^{k+1} \cdot \frac{1}{(1+\frac{1}{4^{k+1}})}$

(2) converges

$|R_n \leq a_{n+1} = .001$ cont. on seperate page

(E.3) Calculate S_n for the value of n you obtained in (E.2). Round your answer to the nearest thousandth. You may use the *sum* and *seq* features of a TI Graphing Calculator, Excel, or any other program for computing partial sums (try a web search) for this part.

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} \frac{(-1)^k \cdot 3^k}{4^k+1}$$

$$= -.306$$

(E.4) Is the approximation found in (E.3) an overestimate or underestimate? Explain why.

n is 23   The N is odd so its an underestimate.

$\boxed{E_2}$

$$\frac{1}{1000} = \left(\frac{3^{\cancel{n+1}}}{4^{\cancel{n+1}}}\right)^{n+1} \cdot \frac{1}{1+\frac{1}{4}^{n+1}}$$

$$\ln(.001) = (n+1) \cdot \ln \tfrac{3}{4} - \ln\left(1 + \tfrac{1}{4}^{n+1}\right)$$

$$f(24) = \frac{3^{(23+1)}}{4^{(23+1)} + 1}$$

$$= .00100$$

$$n = 23$$

[F] The following series satisfies the conditions of the *Integral Test* and can be shown to converge.

$$s = \sum_{k=1}^{\infty} e^{-k/2}$$

(F.1) Approximate the sum of this series using S_3. Round to the nearest ten-thousandth.

$S_3 = \sum_{K=1}^{3} e^{-K/2} = e^{-1/2} + e^{-1} + e^{-3/2} \approx \boxed{1.1927}$

(F.2) Recall the inequality below that provides upper and lower bounds for R_n.

$$\int_{n+1}^{\infty} f(x)\,dx < R_n < \int_{n}^{\infty} f(x)\,dx$$

Find the values of the integrals above (to the nearest ten-thousandth) to first obtain the specific inequality satisfied by R_3. Then use your result from (F.1) to obtain an inequality satisfied by S (the infinite sum). Show sufficient work to support your answer. You must handle the improper integrals as we did in Unit 2 (with limits).

**See Example 8 in the Unit 3.2 section summary for more guidance.

$\int_4^{\infty} e^{-x/2}\,dx < R_3 < \int_3^{\infty} e^{-x/2}\,dx$

$\lim_{t\to\infty} \int_4^{t} e^{-x/2} < R_3 < \lim_{t\to\infty} \int_3^{t} e^{-x/2}\,dx$

$\lim_{t\to\infty} \left(-2e^{-x/2} \Big|_4^t \right)$

Inequality for R_3: $.2707 < R_3 < .4463$ Inequality for S:

cont on sec page

$1.4682 < S < 1.6438$

(F.3) Convince yourself that that given series is Geometric and then use this fact to determine the EXACT (no decimals) value of the sum. Then approximate the sum by rounding to the nearest ten-thousandth. The result should agree with your inequality in (F.2). Show work to support your answer.

$S = \sum_{K=1}^{\infty} e^{-K/2} = \sum_{K=1}^{\infty} \left(e^{-1/2}\right)^K = \frac{e^{-1/2}}{1-e^{-1/2}}$

$= 1.5415$

$\boxed{F_2}$

$$\lim_{t\to\infty}\left(2\cdot e^{-2} - \cancel{2\cdot e^{-t/2}}\right)$$

$$2\cdot e^{-2} \approx .2707$$

$$.2707 < R_3 < \int_3^{\infty} e^{-x/2}\,dx$$

$$\lim_{t\to\infty}\left(2e^{-x/2}\Big]_t^3\right)$$

$$2\cdot e^{-3/2} \approx .4463$$

$$.2707 < R_3 < .4463$$

$$R_3 = S - S_3$$

$$+S_3 \qquad +S_3$$

$$R_3 + S_3 = S$$

$.2707 + 1.1975 = 1.4682$

$.4463 + 1.1975 = 1.6438$

$$1.4682 < S < 1.6438$$