# MONROE COMMUNITY COLLEGE

# MTH 211 – SLN

## Unit 2 Written Assignment

Printed Name: Noor mustafa

*See Blackboard for the deadline for submitting your assignment.

Directions:

- Be sure to follow the submission instructions given in Blackboard when submitting your completed assignment. Do NOT email me your completed assignment.
- Only methods covered in this course up to the current unit may be used on this assignment.
- In all problems you must show sufficient work to support your final answers. All such work must be done in this assignment document. If additional space is needed, you may add pages, but your work must be submitted in order.
- **The work you submit must be your own. While I cannot prevent students from discussing problems, suspicion of duplicated work will be investigated and penalties may result. <u>In addition, solutions taken from online calculators or the equivalent will be not be accepted and will be considered cheating.</u>**
- Be sure to include this page as a cover page for your assignment when you submit it and please make sure your name is written in the designated spot above. If you do not have access to a printer, you may write your solutions on regular paper, but each page must consist of solutions to only those problems on the corresponding page of the original exam.

Noor

[A] Give the **FORM** of the *partial fraction decomposition* of the rational function below. Do not find any of the unknown constants.

$$\frac{x^4+6}{(x^2+2x+20)(x+4)^3} =$$

$$\frac{x^4+6}{(x^2+2x+20)(x+4)^3} = \frac{A}{x+4} + \frac{B}{(x+4)^2} + \frac{C}{(x+4)^3} + \boxed{\frac{Dx+E}{(x^2+2x+20)}}$$

[B] Use the method of **Partial Fractions** to determine the following integrals.

(B. 1) $\int \frac{x}{x^2-a^2}\,dx$ (here $a$ represents a positive real number)

$$\int \frac{x}{x^2-a^2}\,dx$$

$$\int \frac{x}{(x-a)(x+a)}\,dx$$

$$\frac{x}{(x-a)(x+a)} = \frac{A}{x-a} + \frac{B}{x+a}$$

pick x's $\quad x = A(x+a) + B(x-a)$

Let $x=a$ $\quad a = A(a+a) + \cancel{B(a-a)}$ 0

$$a = 2a \cdot A \iff 2A = 1,\ A = \tfrac{1}{2}$$

N001

(B.2) $\int \frac{16x^2 - 17x + 8}{(5x-4)(x^2+4)}\,dx$

$$\frac{16x^2-17x+8}{(5x-4)(x^2+4)} = \frac{A}{(5x-4)} + \frac{Bx+C}{(x^2+4)}$$

$$16x^2-17x+8 = A(x^2+4) + (Bx+C)(5x-4)$$

$x=0$

$$\frac{8}{4} = \frac{A4 - 4C}{4}$$

$$\boxed{2 = A - C} \quad 2 = \frac{796}{116} - C$$

ex $= 4/5$

$$16\left(\tfrac{4}{5}\right)^2 - 17\left(\tfrac{4}{5}\right) + 8 = A\left(\left(\tfrac{4}{5}\right)^2 + 4\right)$$

$$16 \cdot \frac{16}{25} - \frac{17 \cdot 4}{5} + 8 = A\left(\frac{16}{25} + 4\right)$$

$$\frac{16^2 + 20 \cdot 17 + 8 \cdot 25}{25} = A \cdot \frac{116}{25}$$

$$\frac{796}{\cancel{25}} = \frac{A \cdot 116}{\cancel{25}}$$

$$\frac{A \cdot 116}{116} = \frac{796}{116}$$

$$A = \frac{796}{116}$$

cont. on sep. sheets

B.2

Next

$$C = \frac{796}{116} - \frac{116 \cdot 2}{116}$$

$$C = \frac{796 - 232}{116} = \frac{564}{116}$$

$$x = 1, \quad 16 - 17 + 8 = A(\underbrace{1^2 + 4}_{5}) + (B + C)\underbrace{(5 - 4)}_{1}$$

$$7 = 5A + B + C$$

$$B = 7 - 5A - C$$

$$B = 7 - \frac{5 \cdot 796}{116} - \frac{564}{116}$$

$$B = 7 \cdot 116 - \frac{5 \cdot 796}{116} - \frac{564}{116}$$

$$B = 812 - \frac{3980}{116} - \frac{564}{116}$$

$$B = \frac{-3732}{116}$$

3.2

Noor

$$= \int \frac{A}{5x-4} + \frac{Bx}{x^2+4} + \boxed{\frac{C}{x^2+4}} \rightarrow \tan\theta$$

$x(\theta) = 2 \cdot \tan\theta$ $\quad 2^2 \cdot \tan^2\theta + 2^2$

$dx = 2\sec^2\theta \, d\theta$ $\quad 2^2(\tan^2\theta - 1)$

$$= \int \frac{C}{\cancel{2}(\cancel{\tan^2\theta + 1})} (\cancel{2}\cancel{\sec^2\theta}\, d\theta)$$

$$= \frac{C}{2} \int d\theta$$

$$= \frac{C}{2}\theta + K$$

$\tan^{-1}\left(\frac{x}{2}\right) = \cancel{2} \cdot \tan\theta$

$\theta = \tan^{-1}\left(\frac{x}{2}\right)$

$$= \frac{C}{2} \cdot \tan^{-1}\left(\frac{x}{2}\right) + K$$

$\int \frac{A}{5x-4} dx$ $\quad \frac{A}{5} \int \frac{1}{u} du = \frac{A}{5} \ln(5x-4) + K$

$u = 5x-4$

$\frac{du}{5} = \cancel{5} dx$ $\quad \int \frac{Bx}{x^2+4} dx$

$u = x^2+4$ $\quad \frac{du}{2} = \cancel{2}x \, dx$

$= \frac{B}{2} \int \frac{1}{u}$ $\quad \frac{B}{2} \ln(x^2+4) + K$

Noor

[C] For each of the following integrals, indicate the precise trigonometric substitution that you would use to obtain the antiderivatives. Do not do anything more than that (i.e. don't actually find the antiderivatives or apply the substitution). Write each answer after the word "Let".

A sample format of an answer would be: Let $9x = 13 \cot \theta$

(C. 1) $\int \sqrt{10 + 9x^2}\, dx$

$\sqrt{9x^2} = \sqrt{10 \tan^2 \theta}$

$\frac{3x}{3} = \frac{\sqrt{10} \tan \theta}{3}$

Let $x = \frac{\sqrt{10}}{3} \tan \theta$

(C. 2) $\int \frac{x^6}{\sqrt{4x^2 - 1}}\, dx$

Let $2x = \sec \theta$

cont. on seperate page

(C. 3) $\int x^6 \sqrt{3 - x^2}\, dx$

continue on seperate page

Let $x = \sqrt{3} \cdot \sin \theta$

Noor

$\boxed{C_2}$

$1 + \tan^2\theta = \sec^2\theta - 1$

$u = 2b$

$$\int \frac{(u/2)^6}{\sqrt{u^2-1}}\,du$$

$$= \frac{1}{2^7}\int \frac{u^6}{\sqrt{u^2-1}}\,du$$

$u(\theta) = \sec\theta$

$du = \sec\theta \cdot \tan\theta \, d\theta$

$$\frac{1}{2^7}\int \frac{\sec^6\theta}{\cancel{\tan\theta}}\,(\sec\theta\,\cancel{\tan\theta})$$

$$\boxed{\frac{1}{2^7}\int \sec^7\theta \, d\theta}$$

Noor

$C_3$

$\sqrt{a^2 - x^2}$

Let $x = \sqrt{3} \cdot \sin\theta$

$= \sqrt{a} \cdot \sin\theta$

$\int (\sqrt{3} \cdot \sin\theta)^6 \sqrt{3 - 3\sin^2\theta}$

$(\sqrt{3} \cdot \cos\theta \, d\theta)$

$3\sqrt{3} \int \sin^6\theta \sqrt{1 - \sin^2\theta}$

$\sin^6\theta \cos^2\theta \, d\theta$

$x = \sqrt{3} \cdot \sin\theta$

Noor

[D] Use the method of **Trigonometric Substitution** to determine the following integrals. You must express your final answers in terms of $x$ and do not express the final answers using a composition of trig. and inverse trig. functions.

(D.1) $\int \frac{3}{4x\sqrt{1-x^2}}\,dx = \frac{3}{4}\int \frac{1}{x\sqrt{1-x^2}}\,dx$

$\frac{3}{4}\int \frac{1}{\sin\theta\sqrt{1-\sin^2\theta}}(\cos\theta\,d\theta)$

$u(\theta)=\sin\theta$

$dx=\cos\theta\,d\theta$

$=\frac{3}{4}\int \frac{1}{\sin\theta\,\cancel{\cos\theta}}(\cancel{\cos\theta}\,d\theta)$

$=\frac{3}{4}\int \csc\theta\,d\theta$

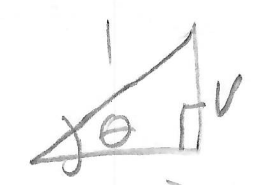
$\frac{3}{4}(-\ln|\csc\theta+\cot\theta|+C)$

$\int \frac{1}{\sin\theta}\,d\theta$

$v=\sin\theta$

$dv=\cos\theta\,d\theta$

$\frac{dv}{\cos\theta}=d\theta$ $\quad \int \frac{1}{v}\frac{dv}{\cos\theta}$




$y^2+v^2=1$

$y=\sqrt{1-v^2}$

$\int \frac{1}{v}\frac{1}{\sqrt{1-v^2}}\,dv$

$D_1$

Nour

$$x(\theta) = \sec\theta$$

$$\frac{3}{4}\int \frac{1}{x\sqrt{1-x^2}}\,dx$$

$$dx = \sec\theta\cdot\tan\theta\,d\theta$$

$$= \frac{3}{4}\int \frac{1}{\cancel{\sec\theta}\,\cancel{\underbrace{\sqrt{1-\sec^2\theta}}_{\tan\theta}}}\left(\cancel{\sec\theta}\cdot\cancel{\tan\theta}\,d\theta\right)$$

$$\frac{3}{4}\int d\theta$$

$$= \frac{3}{4}\theta + C$$

$$\theta = \sec^{-1}(x)$$

$$= \frac{3}{4}\sec^{-1}(x) + C$$

NOUR

(D.2) $\int \frac{1}{(1+x^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{1+x^2}^{\,3}}\,dx$

$x(\theta) = \tan\theta$

$dx = \sec^2\theta\,d\theta$

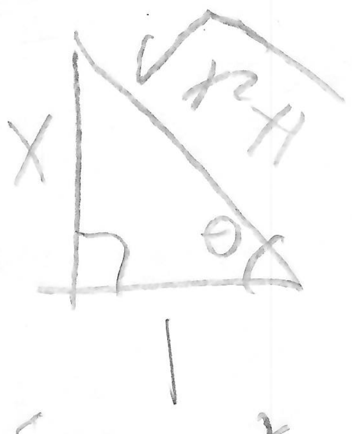
$$\int \frac{1}{\sqrt{\underbrace{(1+\tan^2\theta)}_{\sec^2\theta}^{\,3}}}\left(\sec^2\theta\,d\theta\right)$$

$$= \int \frac{1}{\cancel{\sec^3\theta}}\,\cancel{\sec^2\theta}\,d\theta$$

$$= \int \frac{1}{\sec\theta}\,d\theta$$

$$= \int \cos\theta\,d\theta$$

$$= \sin\theta + C$$




$\sin\theta = \frac{x}{\sqrt{x^2+1}}$

$$\sin\theta = \frac{x}{\sqrt{x^2+1}} + C$$

$$= \frac{x}{\sqrt{x^2+1}} + C$$

Nour

[E] Use ***L'Hopital's Rule*** to determine the exact value of the following limits. If a given limit doesn't exist, but can be assigned a "value" of $\pm\infty$, do so. Otherwise, if a limit doesn't exist, indicate DNE.

(E. 1) $\lim_{x\to a}\left[\dfrac{x-a}{a\ln x - a\ln a}\right]$ where $a$ is a nonzero real number.

$$\lim_{x\to a} \frac{x-a}{a\ln x - a\ln a} \to \frac{1}{a/x}$$

plug in a

$$\frac{1}{a/a} = \underline{1}$$

(E. 2) $\lim_{t\to\infty} e^{-t}\ln\left(\dfrac{1}{t}\right)$

$$= \lim_{t\to\infty} e^{-t}\ln\frac{1}{t}$$

$$= \lim_{t\to\infty}\frac{\ln \frac{1}{t}}{e^{t}}$$

$$= \lim_{t\to\infty}\frac{-\frac{t}{t^2}}{e^{t}}$$

$$= \lim_{t\to\infty}\frac{-1}{te^{t}}$$

$$= 0$$

Moor

[F] Let $f(x)$ be a function whose first second and third derivatives are continuous on an interval containing zero, except possibly at zero and suppose that:

$$\lim_{x\to 0} f(x) = 0, \quad \lim_{x\to 0} f'(x) = 0, \quad \lim_{x\to 0} f''(x) = 3, \quad \text{and} \quad \lim_{x\to 0} f'''(x) = 5.$$

Use ***L'Hopital's Rule*** to evaluate the following limits.

(F.1) $\lim_{x\to 0}\left[\frac{x^3}{f(x)}\right]$

$$= \lim_{x\to 0} \frac{x^3}{f(x)}$$

$$= \lim_{x\to 0} \frac{3x^2}{f'(x)}$$

$$= \lim_{x\to 0} \frac{6x}{f''(x)}$$

$$= 0$$

(F.2) $\lim_{x\to 0}\left(1+x^2\right)^{1/f(x)}$

$$\lim_{x\to 0} \frac{\ln(1+x^2)}{f(x)} = \lim_{x\to 0} \frac{\frac{2x}{1+x^2}}{f'(x)}$$

$$= \lim_{x\to 0} \frac{2x}{f'(x)(1+x^2)}$$

$$= \lim_{x\to 0} \frac{2}{f''(x)(1+x^2)+2xf'(x)} = 2/3$$

$$= \lim_{x\to 0} (1+x^2)^{1/f(x)} = e^{2/3}$$

Noar

[G] For each of the following improper integrals:

1. Rewrite the integral in terms of their defining limits.
2. Determine the value of the improper integral or indicate that it diverges.

(G. 1) $\int_0^{\infty} \frac{1}{x^p}\, dx$ where $p$ is a real number with $p < 1$

* cont. on seperate page

$$\int_0^1 \frac{1}{x^p}\, dx + \int_1^{\infty} \frac{1}{x^p}\, dx$$

$$= \lim_{a \to 0} \int_a^1 \frac{1}{x^p}\, dx + \lim_{b \to \infty} \int_1^b \frac{1}{x^p}\, dx$$

$$= \lim_{a \to 0} \left[\frac{1}{1-p} x^{1-p}\right]_a^1 + \lim_{b \to \infty} \left[\frac{1}{1-p} x^{1-p}\right]_1^b$$

(G. 2) $\int_0^{\pi} \frac{x\cos x - \sin x}{x^2}\, dx$ $\left[\text{Hint: } \int \frac{x\cos x - \sin x}{x^2}\, dx = \frac{\sin x}{x} + C\right]$

$$\lim_{a \to 0^+} \int_a^{\pi} \frac{x\cos x - \sin x}{x^2}\, dx$$

$$\lim_{a \to 0^+} \left(\frac{\sin x}{x}\right)_a^{\pi}$$

$$= -1$$

(it converges to $-1$)

$\boxed{G.1}$ NOOV

$$= \frac{1}{1-p}\left(\underbrace{\lim_{a\to 0}(1-a^{1-p})}_{1} + \underbrace{\lim_{b\to\infty}(b^{1-p}-1)}_{\infty}\right)$$

diverges becuase it goes to infinity.