

$$1) \frac{dx}{dt} = 3x - y - z$$

$$\frac{dy}{dt} = x + y - z + t$$

$$\frac{dz}{dt} = x - y + z + ze^t$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{bmatrix} 3 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 0 \\ t \\ ze^t \end{pmatrix}$$

$$x' = Ax + t$$

$$A = \begin{bmatrix} 3 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$A - \lambda I = 0 \rightarrow \begin{bmatrix} 3-\lambda & -1 & -1 \\ 1 & 1-\lambda & -1 \\ 1 & -1 & 1-\lambda \end{bmatrix} = 0$$

$$(3-\lambda)((1-\lambda)^2 - 1) + 1(1-\lambda+1) - 1(-1-1+\lambda) = 0$$

$$\lambda = 1 \quad \lambda = 2 \quad \lambda = 2$$

①  $\lambda_1 = 1$  :

$$\begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{aligned} x - y &= 0 \rightarrow x = y \\ x - z &= 0 \rightarrow x = z \end{aligned}$$

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda_2 = 2$$

$$\begin{pmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$z=0 \rightarrow x - y - z = 0$$

$$v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$y=0$$

$$v_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$X(t) = c_1 e^t v_1 + c_2 e^{2t} v_2 + c_3 t e^{2t} v_3$$

$$X(t) = c_1 e^t \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_3 t e^{2t} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

general solution of homogeneous form

$$\Phi = \begin{bmatrix} e^t & e^{2t} & te^{2t} \\ e^t & e^{2t} & 0 \\ e^t & 0 & te^{2t} \end{bmatrix} \quad F = \begin{bmatrix} 0 \\ t \\ ze^t \end{bmatrix}$$

$$\Phi^{-1} \cdot F = 0$$

$$\Phi^{-1} = \begin{bmatrix} -\frac{1}{e^t} & \frac{1}{e^t} & \frac{1}{e^t} \\ \frac{1}{e^{2t}} & 0 & -\frac{1}{e^{2t}} \\ \frac{1}{te^{2t}} & -\frac{1}{te^{2t}} & 0 \end{bmatrix}$$

$$\Phi^{-1} \cdot F =$$

$$\begin{pmatrix} -\frac{1}{e^t} & \frac{1}{e^t} & \frac{1}{e^t} \end{pmatrix} \begin{pmatrix} 0 \\ t \\ ze^t \end{pmatrix} = \left(-\frac{1}{e^t}\right) \cdot 0 + \frac{1}{e^t} t + \frac{1}{e^t} \cdot ze^t$$

$$\begin{pmatrix} \frac{1}{ze^t} & 0 & -\frac{1}{e^{2t}} \end{pmatrix} \begin{pmatrix} 0 \\ t \\ ze^t \end{pmatrix} = \frac{1}{e^{2t}} \cdot 0 + 0 \cdot t + -\frac{1}{e^{2t}} \cdot ze^t$$

$$\begin{pmatrix} \frac{1}{te^{2t}} & -\frac{1}{te^{2t}} & 0 \end{pmatrix} \begin{pmatrix} 0 \\ t \\ ze^t \end{pmatrix} = \frac{1}{te^{2t}} \cdot 0 + \left(-\frac{1}{te^{2t}}\right) t + 0 \cdot ze^t$$

$$= \begin{pmatrix} -te^{-t} + e^t + 2t \\ -ze^{-t} \\ -e^{-2t} \end{pmatrix} \leftarrow \Phi^{-1} \cdot F$$

$$\Phi \Phi^{-1} \cdot F$$

$$= \begin{pmatrix} -te^{-t} + e^t + 2t \\ ze^{-t} \\ \frac{1}{2}e^{2t} \end{pmatrix}$$

$$\Phi \cdot \Phi^{-1} \cdot F$$

$$= \begin{bmatrix} e^t & e^{2t} & te^{2t} \\ e^t & e^{2t} & 0 \\ e^t & 0 & te^{2t} \end{bmatrix} \begin{bmatrix} -te^{-t} + e^t + 2t \\ ze^{-t} \\ \frac{1}{2}e^{2t} \end{bmatrix}$$

$$= \begin{pmatrix} e^t(-te^{-t} + e^t + 2t) + e^{2t} \cdot ze^{-t} + te^{2t} \cdot \frac{1}{2}e^{2t} \\ e^t(-te^{-t} + e^t + 2t) + e^{2t} \cdot ze^{-t} + 0 \cdot \frac{1}{2}e^{2t} \\ e^t(-te^{-t} + e^t + 2t) + 0 \cdot ze^{-t} + te^{2t} \cdot \frac{1}{2}e^{2t} \end{pmatrix}$$

$$= \begin{bmatrix} -t + e^{2t} + 2te^t + ze^{3t} + \frac{1}{2}te^{4t} \\ -t + e^{2t} + 2te^t + ze^{3t} \\ -t + e^{3t} + 2te^t + \frac{1}{2}te^{4t} \end{bmatrix}$$

Quiz #14) Noor Mustafa (Math 225)

$$\Phi_p = t \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + e^{2t} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + te^t \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} + e^{3t} \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} + te^{4t} \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix}$$

$$X(t) = c_1 e^t \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_3 te^{2t} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} + e^{2t} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + te^t \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} + e^{3t} \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + te^{4t} \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix}$$



(2)  $\frac{dA}{dt} = \tan t A$   $\frac{dB}{dt} = \tan t B$   $\frac{dC}{dt} = \tan t C$

$$\frac{dA}{dt} = 3(150) + 10 - \frac{B}{L_B} - 40 \frac{A}{L_A} - 20 \frac{A}{L_A}$$

$$\frac{dB}{dt} = 40 \frac{A}{L_A} + 10 \frac{C}{L_C} - 10 \frac{B}{L_B} - 30 \frac{B}{L_B} - 10 \frac{B}{L_B}$$

$$\frac{dC}{dt} = 30 \frac{B}{L_C} - 10 \frac{C}{L_C} - 20 \frac{C}{L_C}$$

$$L_A = 200 + (50 + 10 - 40 - 20)t \quad t = 200$$

$$L_B = 100 + (40 + 10 - 10 - 30 - 10)t \quad t = 100$$

$$L_C = 100 + (30 - 10 - 20)t \quad t = 100$$

$$\frac{dA}{dt} = 150 + \frac{B}{10} - \frac{6A}{20}$$

$$\frac{dB}{dt} = \frac{A}{5} + \frac{C}{10} - \frac{B}{2}$$

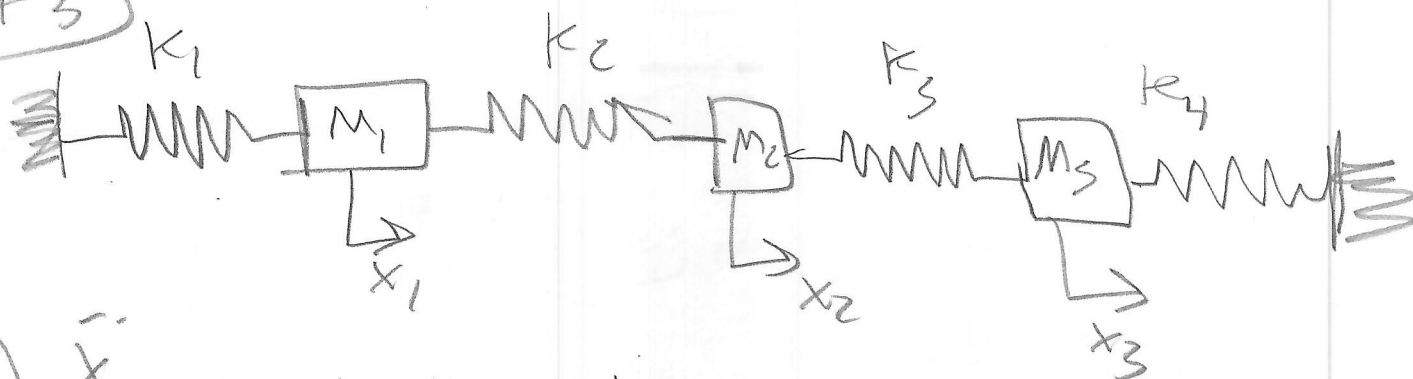
$$\frac{dC}{dt} = \frac{3B}{10} - \frac{3C}{10}$$

$$A(0) = 20$$

$$B(0) = 20$$

$$C(0) = 0$$

#3



$$m_1 \ddot{x}_1 = -k_1 x_1 + k_2 (x_2 - x_1)$$

$$m_2 \ddot{x}_2 = -k_2 (x_2 - x_1) + k_3 (x_3 - x_2)$$

$$m_3 \ddot{x}_3 = -k_3 (x_3 - x_2) + k_4 (x_3)$$

$$\ddot{x}_1 = \frac{-k_1 - k_2}{m_1} x_1 + \frac{k_2}{m_1} x_2 = \ddot{x}_4$$

$$\ddot{x}_2 = \frac{k_2}{m_2} x_1 + \frac{-k_2 - k_3}{m_2} x_2 + \frac{k_3}{m_2} x_3 = \ddot{x}_5$$

$$\ddot{x}_3 = \frac{k_3}{m_3} x_2 + \frac{-k_3 + k_4}{m_3} x_3 = \ddot{x}_6$$

$$\dot{x}_1 = \dot{x}_4$$

$$\dot{x}_2 = \dot{x}_5$$

$$\dot{x}_3 = \dot{x}_6$$

Quiz # 14 (Noor Mustafa) Math 225

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-k_1 - k_2}{m_1} & k_2 & 0 & 0 & 0 & 0 \\ \frac{k_2}{m_2} & \frac{-k_2 - k_1}{m_2} & \frac{k_3}{m_2} & 0 & 0 & 0 \\ 0 & \frac{k_3}{m_3} & \frac{-k_3 + k_4}{m_3} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

$\left. \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} \right\} \text{pos}$   
 $\left. \begin{matrix} x_4 \\ x_5 \\ x_6 \end{matrix} \right\} \text{velocity}$

$$X(0) = 0$$

↑  
(column vector of  $X$ )