

Q47 #6 MTH 225

$$1. y''' - y'' - 4y' + 4y = 5 - e^x + e^{-x}$$

$$y''' - y'' - 4y' + 4y = 0$$

$$y = C_1 e^x + C_2 e^{-2x} + C_3 e^{2x}$$

$$(e^{4x})''' - (e^{4x})'' - 4(e^{4x})' + 4e^{4x} = 0$$

$$e^{4x}(y^3 - y^2 - 4y + 4) = 0$$

$$(e^{4x})''' = y^3 e^{4x} \quad (e^{4x})' = y^2 e^{4x}$$

$$(e^{4x})' = e^{4x} y$$

$$y^3 e^{4x} - y^2 e^{4x} - 4e^{4x} y + 4e^{4x} = 0$$

$$e^{4x}(y^3 - y^2 - 4y + 4) = 0$$

$$(y-1)(y+2)(y-2) = 0$$

$$y=1 \quad y=-2 \quad y=2$$

$$y = C_1 e^x + C_2 e^{-2x} + C_3 e^{2x}$$

$$y''' - y'' - 4y' + 4y = 5 - e^x + e^{-x}$$

$$y = \frac{5}{4} + \frac{e^x x}{3} + \frac{1}{6} e^{-x}$$

$$y=1, y=-2, y=2$$

1.

The general solution is $y_h + y_p$

$$y = c_1 e^x + c_2 e^{-2x} + c_3 e^{2x} + \frac{5}{4} + \frac{e^x x}{3} + \frac{1}{6} e^{-x}$$

$$2. y'' + 2y' + y = x^2 e^{-x}$$

$$y'' + 2y' + y = 0 \quad y = e^{4x}$$

$$(e^{4x})'' + 2(e^{4x})' + e^{4x} = 0$$

$$(e^{4x})'' = 16e^{4x} \quad (e^{4x})' = 4e^{4x}$$

$$e^{4x}(16 + 8 + 1) = 0$$

$$e^{4x}(y^2 + 2y + 1) = 0 \quad y = -1 \text{ with multiplicity of 2}$$

$$y = c_1 e^{-x} + c_2 x e^{-x}$$

$$y'' + 2y' + y = x^2 e^{-x}$$

$$y = a_0 x^4 e^{-x} + a_1 x^3 e^{-x} + a_2 x^2 e^{-x} = N$$

$$(N)'' + 2(N)' + N = 0$$

$$2 \cdot 12a_0 e^{-x} x^2 + 6a_1 e^{-x} x + 2a_2 e^{-x} = x^2 e^{-x}$$

$$a_0 = \frac{1}{12}, a_2 = 0, a_1 = 0$$

$$y = \frac{1}{12} x^4 e^{-x} + 0 x^3 e^{-x} + 0 x^2 e^{-x}$$

$$y = \frac{e^{-x} x^4}{12}$$

$$y = \frac{e^{-x} x^4}{12}$$

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2 - general solution is

$$y = c_1 e^{-x} + c_2 x e^{-x} + \frac{e^{-x} + 4}{12}$$

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$$3. \quad y'' - 4y' + 8y = x^3, \quad y(0) = 2 \quad y'(0) = 4$$

$$y'' - 4y' + 8y = 0$$

$$y = e^{rx}$$

$$(e^{rx})'' = r^2 e^{rx}$$

$$(e^{rx})'' - 4(e^{rx})' + 8e^{rx} = 0$$

$$(e^{rx})' = e^{rx} r$$

$$r^2 e^{rx} - 4r e^{rx} + 8e^{rx} = 0$$

$$e^{rx}(r^2 - 4r + 8) = 0$$

$$(r^2 - 4r + 8) = 0$$

$$r = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(8)}}{2} = 2 \pm 2i$$

$$r = \frac{-(-4) - \sqrt{(-4)^2 - 4(1)(8)}}{2} = 2 - 2i$$

$$y = e^{2x} (C_1 \cos(2x) + C_2 \sin(2x))$$

$$y(x) = x^3$$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$y'' - 4y' + 8y = x^3$$

$$6a_0 x + 2a_1 (3a_0 x^2 + 2a_1 x + a_2) + 8(a_0 x^3 + a_1 x^2 + a_2 x + a_3) = x^3$$

$$a_0 = \frac{1}{8} \quad a_2 = \frac{3}{32} \quad a_1 = \frac{3}{16} \quad a_3 = 0$$

3. (Quiz #6) Noel Mustafey MTH 224

$$y = \frac{1}{8}x^3 + \frac{3}{16}x^2 + \frac{3}{32}x + 0$$

$$y = \frac{x^3}{8} + \frac{3x^2}{16} + \frac{3x}{32}$$

general solution

$$y = e^{2x} \left(\cos(2x) - \frac{3}{84} \sin(2x) \right) + \frac{x^3}{8} + \frac{3x^2}{16} + \frac{3x}{32}$$