Quiz #9 | MTH 225 | NOUR

(1)

$$\frac{m\partial^2 x}{\partial t^2} = -kx - \beta\frac{\partial x}{\partial t}$$

$$\frac{\partial^2 x}{\partial t^2} + \frac{k}{m}x + \frac{\beta}{m}\frac{\partial x}{\partial t} = 0$$

$$\frac{\partial^2 x}{\partial t^2} + \frac{\beta}{m}\frac{\partial x}{\partial t} + \frac{k}{m}x = 0$$

spring constant $= \frac{\text{weight attached}}{\text{difference in length}} = \frac{W}{S}$

$= \frac{8\,lb}{8\,ft - 4\,ft}$

$= 2\ lb/ft$

$k = 2\ lb/ft$

$m = \frac{W}{g} = \frac{8\,lb}{32\,ft/s^2} = \frac{1}{4}\ slugs$

$m = \frac{1}{4}\ slugs$

$\beta = \sqrt{2}$

$$\frac{\partial^2 x}{\partial t^2} + 4\sqrt{2}\frac{\partial x}{\partial t} + 8x = 0$$

$$r^2 + 4\sqrt{2}r + 8 = 0$$

$$r^2 + 2\sqrt{2}r + 2\sqrt{2}r + 8 = 0$$

$$r(r + 2\sqrt{2}) + 2\sqrt{2}(r + 2\sqrt{2}) = 0$$

$$(r + 2\sqrt{2})^2 = 0$$

$$r_1 = r_2 = -2\sqrt{2}$$

① Noor | Mth 225 | Quiz #9

$$x(t) = c_1 e^{-2\sqrt{2}t} + c_2 e^{-2\sqrt{2}t}$$

$$= e^{-2\sqrt{2}t}(c_1 + c_2 t)$$

$x(0) = 0$ ft, $x'(0) = 4$ ft/s

$$x(0) \to e^{-2\sqrt{2}(0)}(c_1 + c_2(0)) = 0$$

$$1 \cdot (c_1) = 0$$

$$c_1 = 0$$

$$x(t) = e^{-2\sqrt{2}t}(c_2 t)$$

$$x'(t) = (-2\sqrt{2})e^{-2\sqrt{2}t}(c_2 t) + c_2 e^{-2\sqrt{2}t}$$

$$= c_2 e^{-2\sqrt{2}t}(-2\sqrt{2}t + 1)$$

$$x'(0) = 4 \to c_2 \cdot e^{-2\sqrt{2}(0)}(-2\sqrt{2}(0) + 1) = 0 + 4$$

$$= c_2 \cdot 1 \quad (0 + 1) = 4$$

$$\boxed{c_2 = 4}$$

equation of motion is:

$$x(t) = 4te^{-2\sqrt{2}t}$$

(2) (M+H 225) Quiz #9 Noori

$m\frac{d^2x}{dt^2} + kx = f(t)$

$m = 2$

$k = 32$

$F = 68e^{-2t}\cos 4t$

$x(0) = 0$

$x'(0) = 0$

$2\frac{d^2x}{dt^2} + 32x = 68e^{-2t}\cos 4t$

$x'' + 16x = 34e^{-2t}\cos(4t)$

$r^2 + 16 = 0$

$r = \pm 4$

$x_h = c_1\cos 4t + c_2 \sin 4t$

$x_p = Ae^{-2t}\cos 4t + Be^{-2t}\sin 4t$

$x_p' = -2Ae^{-2t}\cos 4t - 4Ae^{-2t}\sin 4t - 2Be^{-2t}\sin 4t + 4Be^{-2t}\cos 4t$

$x_p'' = 4Ae^{-2t}\cos 4t + 8Ae^{-2t}\sin 4t + 8Ae^{-2t}\sin 4t - 16Ae^{-2t}\cos 4t + 4Be^{-2t}\sin 4t - 8Be^{-2t}\cos 4t - 8Be^{-2t}\cos 4t - 16Be^{-2t}\sin 4t$

$4Ae^{-2t}\cos 4t + 8Ae^{-2t}\sin 4t + 8Ae^{-2t}\sin 4t - 16Ae^{-2t}\cos 4t + 4Be^{-2t}\sin 4t - 8Be^{-2t}\cos 4t - 8Be^{-2t}\cos 4t - 16Be^{-2t}\sin 4t + 16(Ae^{-2t}\cos 4t + Be^{-2t}\sin 4t) = 34e^{-2t}\cos(4t)$

(2) Quiz #9 | Noor | MTH 225

$-4A - 16B = 34$

$16A + 4B = 0$

$A = \frac{1}{2}, \quad B = -2$

$X_p = \frac{1}{2}e^{-2t}\cos 4t - 2e^{-2t}\sin 4t$

$X(t) = X_h + X_p$

$X(t) = C_1\cos 4t + C_2 \sin 4t + \frac{1}{2}e^{-2t}\cos 4t - 2e^{-2t}\sin 4t$

$X(0) = 0$

$0 = C_1 + \frac{1}{2}$

$C_1 = -\frac{1}{2}$

$X'(0) = 0$

$X'(t) = -4\left(\frac{1}{2}\right)\sin 4t + 4C_2\cos 4t - e^{-2t}\cos 4t - 2e^{-2t}\sin 4t + 4e^{-2t}\sin 4t - 8e^{-2t}\cos 4t$

$0 = 4C_2 - 1 - 8$

$C_2 = \frac{9}{4}$

general solution

$X(t) = -\frac{1}{2}\cos 4t + \frac{9}{4}\sin 4t + \frac{1}{2}e^{-2t}\cos 4t - 2e^{-2t}\sin 4t$

3 | NOOR | MTH 225 | QUIZ #9 | (A)

$L=1$

$w_0 = 48EI$

$$EI\frac{d^4y}{dx^4} = w(x)$$

$$EI\frac{d^4y}{dx^4} = 48EIx$$

$$\frac{d^4y}{dx^4} = 48x$$

$$\frac{d^3y}{dx^3} = \frac{48x^2}{2} + C_1 = 24x^3 + C_1$$

$$\frac{d^2y}{dx^2} = \frac{24x^3}{3} + C_1x + C_2 = 8x^3 + C_1x + C_2$$

$y''(0) = 0$

$0 = 0 + 0 + C_2 \rightarrow C_2 = 0$

$$\frac{d^2y}{dx^2} = 8x^3 + C_1x$$

$$\frac{dy}{dx} = \frac{8x^4}{4} + \frac{C_1x^2}{2} + C_3 = 2x^4 + \frac{C_1x^2}{2} + C_3$$

$$y = \frac{2}{5}x^5 + \frac{C_1x^3}{6} + C_3x$$

$y(1) = 0$

$$0 = \frac{2}{5} + \frac{C_1}{6} + C_3 \rightarrow 12 + 5C_2 + 30C_3 = 0$$

$y''(1) = 0$

$0 = 8 + C_1 \rightarrow C_1 = -8$

3 | Noor | MTH 225 | Quiz #9

$12+5(-8)+30c_3=0$

$12-40+30c_3=0 \Rightarrow 30c_3=28$

$c_3=\frac{28}{30}=\frac{14}{15}$

$y=\frac{2}{5}x^5-\frac{8x^3}{6}+\frac{14}{15}x$

$$y=\frac{2}{5}x^5-\frac{4}{3}x^3+\frac{14}{15}x$$

3B $y(0)=y'(0)=0$

$y(0)=0,\ y'(0)=0,\ y''(1)=0,\ y'''(1)=0$

$EI\frac{d^4y}{dx^4}=48EIx$

$\frac{d^4y}{dx^4}=48x$

$\frac{d^3y}{dx^3}=\frac{48x^2}{2}+C_1=24x^2+C_1$

$\frac{d^2y}{dx^2}=\frac{24x^3}{3}+C_1x+C_2=8x^3+C_1x+C_2$

$\frac{dy}{dx}=\frac{8x^4}{4}+\frac{C_1x^2}{2}+C_2x+C_3=2x^4+\frac{C_1x^2}{2}+C_2x+C_3$

$y'(0)=0$

$0=0+0+0+C_3 \rightarrow C_3=0$

$\frac{dy}{du}=2u^4+\frac{C_1u^2}{2}+C_2u$

$y=\frac{2}{5}u^5+\frac{C_1u^3}{6}+\frac{C_2u^2}{2}+C_4$

$y(0)=0$

$0=0+0+0+C_4 \rightarrow C_4=0$

$y=\frac{2}{5}u^5+\frac{C_1u^3}{6}+\frac{C_2u^2}{2}$

$y''(1)=0$

$0=8+C_1+C_2$

$y'''(1)=0$

$0=24+C_1 \rightarrow C_1=-24$

$0=8+(-24)+C_2 \rightarrow C_2=16$

$y=\frac{2}{5}u^5-\frac{24}{6}u^3+\frac{16}{2}u^2$

$y=\frac{2}{5}u^5-4u^3+8u^2$