Noor Mustafa | Quiz 1 Math 425

(1a)

$y=\sum_{n=0}^{\infty} c_n x^n \quad y'=\sum_{n=1}^{\infty} n c_n x^{n-1} \quad y''=\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2}$

$(x-1)y''+y'=0$

$(x-1)\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2}+\sum_{n=1}^{\infty} n c_n x^{n-1}=0$

$x\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2}-\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2}+\sum_{n=1}^{\infty} n c_n x^{n-1}=0$

$\sum_{n=2}^{\infty} n(n-1)c_n x^{n-1}-\sum_{n=3}^{\infty} n(n-1)c_n x^{n-2}-n(n-1)c_n x^{n-2}\Big|_{n=2}$

$+\sum_{n=2}^{\infty} n c_n x^{n-1}+\left[n c_n x^{n-1}\right]_{n=1}=0$

$k=n-1 \quad k=n-2 \quad k=n-1$

$n=k+1 \quad n=k+2 \quad n=k+1$

$\sum_{k=1}^{\infty}(k+1)(k)c_{k+1}x^k-\sum_{k=1}^{\infty}(k+2)(k+1)c_{k+2}x^k$

$+\sum_{k=1}^{\infty}(k+1)c_{k+1}x^k-2c_2x^0+c_1x^0=0$

$\sum_{k=1}^{\infty}\left(x^k(k+1)\left[kc_{k+1}-(k+2)c_{k+2}+c_{k+1}\right]\right)$

$-2c_2+c_1=0$

$(k+1)\left[kc_{k+1}-(k+2)c_{k+2}+c_{k+1}\right]=0$

$kc_{k+1}-(k+2)c_{k+2}+c_{k+1}=0$

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1a

$$c_{k+2} = \frac{(k+1)(c_{k+1})}{(k+2)}$$

$-2c_2 + c_1 = 0$

$c_2 = \frac{1}{2} c_1$

$c_3 = \frac{2}{3} c_2 = \frac{1}{3} c_1$

$c_4 = \frac{3}{4} c_3 = \frac{1}{4} c_1$

$c_5 = \frac{4}{5} c_4 = \frac{1}{5} c_1$

$c_6 = \frac{5}{6} c_5 = \frac{1}{6} c_1$

$$y = \sum_{n=1}^{\infty} c_n x^n$$

$$\boxed{\begin{aligned} &\ast\ y_1 = 1 \cdot x + \tfrac{1}{2} + 2 + \tfrac{1}{3} x^3 + \tfrac{1}{4} x^4 \\ &\quad = \sum_{k=1}^{\infty} \frac{x^k}{k} \\ &\ast\ y_2 = c_0 = 1 \end{aligned}}$$

$$\boxed{\begin{aligned} y_1 &\rightarrow c_1 = 1 \\ &\quad\ c_0 = 0 \\ y_2 &\rightarrow c_1 = 0 \\ &\quad\ c_0 = 1 \end{aligned}}$$

$$\boxed{\begin{aligned} y &= c_1 y_1 + c_2 y_2 \\ y &= c_1 y_1 + c_2 \end{aligned}}$$

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1b) $(x-1)y''-xy'+y=0$

$y=\sum_{n=0}^{\infty} c_n x^n \quad y'=\sum_{n=0}^{\infty} n c_n x^{n-1} \quad y''=\sum_{n=0}^{\infty} n(n-1)c_n x^{n-2}$

$(x-1)\sum_{n=0}^{\infty} c_n (n-1)(n) x^{n-2} - x\sum_{n=0}^{\infty} n c_n x^{n-1}$

$+\sum_{n=0}^{\infty} c_n x^n = 0$

$\sum_{n=2}^{\infty} c_n(n-1)n x^{n-1} - \sum_{n=2}^{\infty} c_n(n-1)n x^{n-2} - \sum_{n=1}^{\infty} n c_n x^n$

$+\sum_{n=0}^{\infty} c_n x^n = 0$

$\sum_{n=2}^{\infty} c_n(n-1)n x^{n-1} - \sum_{n=3}^{\infty} c_n(n-1)n x^{n-2}$

$- c_n(n-1)n x^{n-2}\Big|_{n=2} - \sum_{n=1}^{\infty} c_n n x^n + \sum_{n=1}^{\infty} c_n x^n$

$+ c_n x^n\Big|_{n=0} = 0$

$k=n-1 \quad k=n-2 \quad k=n$

$n=k+1 \quad n=k+2 \quad n=k$

$\sum_{k=1}^{\infty} c_{k+1} k(k+1)x^k - \sum_{k=1}^{\infty} c_{k+2}(k+1)(k+2)x^k$

$-\sum_{k=1}^{\infty} c_k k x^k + \sum_{k=1}^{\infty} c_k x^k - c_2(2)x^2 + c_0 x^0 = 0$

$\sum_{k=1}^{\infty} x^k \big(c_{k+1} k(k+1) - c_{k+2}(k+1)(k+2) - c_k k + c_k$

$+ c_0 - 2c_2 = 0$

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$$c_{k+1}k(k+1) - c_{k+2}(k+1)(k+2) + c_k(1-k) = 0$$

$$c_{k+1}k(k+1) + c_k(1-k) = c_{k+2}(k+1)(k+2)$$

$$c_{k+2} = \frac{k}{k+2}c_{k+1} + \frac{1-k}{(k+1)(k+2)}c_k$$

$c_2 = \frac{1}{2}c_0$

$c_0 = ?$

$c_1 = ?$

$c_2 = \frac{1}{2}c_0$

$c_3 = \frac{1}{3}c_2 + 0 = \frac{1}{6}c_0$

$c_4 = \frac{1}{2}c_3 + \frac{-1}{12}c_2 = \frac{1}{12}c_0 - \frac{1}{24}c_0 = \frac{1}{24}c_0$

$y = c_1 y_1 + c_2 y_2$

$y_1 \to c_0 = 1$, $c_1 = 0$ $\qquad y_1 = 1 + 0x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \dots$

$y_2 \to c_0 = 0$, $c_1 = 1$ $\qquad y_2 = 0 + x + 0x^2 \dots \qquad y_2 = x$

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2a) $9x^2y'' + 9x^2y' + 2y = 0$

$$y = \sum_{n=0}^{\infty} c_n (\cancel{x - x_0})^{n+r} \Rightarrow \sum_{n=0}^{\infty} c_n x^{n+r}$$

$$y' = \sum_{n=0}^{\infty} c_n (n+r) x^{n+r-1}$$

$$y'' = \sum_{n=0}^{\infty} c_n (n+r)(n+r-1) x^{n+r-2}$$

$$9x^2 \sum_{n=0}^{\infty} c_n (n+r)(n+r-1) x^{n+r-2} + 9x^2 \sum_{n=0}^{\infty} c_n (n+r) x^{n+r-1}$$
$$+ 2 \sum_{n=0}^{\infty} c_n x^{n+r} = 0$$

$$\sum_{n=0}^{\infty} 9 c_n (n+r)(n+r-1) x^{n+r} + \sum_{n=0}^{\infty} c_n (n+r) x^{n+r+1}$$
$$+ \sum_{n=0}^{\infty} 2 c_n x^{n+r} = 0$$

$$\sum_{n=1}^{\infty} \underbrace{9 c_n (n+1)(n+r-1)}_{n=k+1} x^{n+r} + 9 c_0 r(r-1) x^r$$
$$+ \underbrace{\sum_{n=0}^{\infty} 9 c_n (n+r) x}_{n=k}{}^{n+r+1} + \underbrace{\sum_{n=1}^{\infty} 2 c_n x^{n+r}}_{n=k+1} + 2c_0 x^r = 0$$

$$\sum_{k=0}^{\infty} 9 c_{k+1} (k+1+r)(k+r) x^{k+r+1} + \sum_{k=0}^{\infty} 9 c_k (k+r) x^{k+r+1}$$
$$+ \sum_{k=0}^{\infty} 2 c_{k+1} x^{k+1+r} + 9 c_0 r(r-1) x^r + 2 c_0 x^r = 0$$

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$$\sum_{k=0}^{\infty} x^r x^{k+1} [9c_{k+1}(k+1+r)(k+r) + 9c_k(k+r) + 2c_{k+1}] + x^r\, 9c_0 r(r-1) + 2c_0 = 0$$

$$9c_{k+1}(k+1+r)(k+r) + 9c_k(k+r) + 2c_{k+1} = 0$$

$$9c_0 r(r-1) + 2c_0 = 0$$

$$9r(r-1)+2=0$$

$$9r^2-9r+2=0$$

$$r = \frac{9 \pm \sqrt{9^2-4(9)(2)}}{18}$$

$$r = \frac{9\pm 3}{18} \qquad r_1 = 2/3 \quad r_2 = 1/3$$

$$9c_{k+1}(k+1+2/3)(k+2/3) + 9c_k(k+2/3) + 2c_{k+1} = 0$$

$$c_{k+1}[9(k+1+2/3)(k+2/3)+2] = -9(k+2/3)c_k$$

$$c_{k+1} = \frac{-9(k+2/3)}{9(k+1+2/3)(k+2/3)+2} c_k$$

$$c_1 = \frac{-9(2/3)}{9(5/3)(2/3)+2} c_0$$

$$c_{k+2} = \frac{-9(k+1/3)}{9(k+1\cancel{2}+1/3)(k+1/3)+2} c_k$$

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$y = c_0 x^{0+r} + c_1 x^{1+r} + c_2 x^{2+r} + c_3 x^{3+r} + \dots$

$y_1 = 1x^{2/3} - \frac{1}{2} x^{5/3} + \frac{5}{28} x^{8/3} +$

$C_2 = \dfrac{-9(1+2/3)}{9(1+1+2/3)(1+2/3)+2}$ $\quad C_1 = \dfrac{-15}{42} - \frac{1}{2}$

$C_3 = \dfrac{-9(2+2/3)}{9(3+2/3)+(2+2/3)+2} C_2$

$C_1 = \dfrac{-9(1/3)}{9(4/3)(1/3)+2}$

$C_2 = \dfrac{-9(1+1/3)}{9(2+1/3)(1+1/3)+2} \quad -\frac{1}{2}$

$C_3 = \dfrac{-9(2+1/3)}{9(3+1/3)(2+1/3)+2} \quad -\frac{1}{5}$

$y(x) = c_1 y_1 + c_2 y_2$

$r = 2/3 \quad y_1 = 1x^{2/3} - \frac{1}{2} x^{5/3} + \frac{5}{28} x^{8/3} - \frac{1}{21} x^{1/3} - \dots$
$+ \sum_{n=4}^{\infty} c_n x^{n+2/3}$

$r = 1/3 \quad y_2 = 1x^{1/3} - \frac{1}{2} x^{4/3} + \frac{1}{5} x^{7/3} - \frac{7}{120} x^{10/3} + \dots$
$\sum_{n=4}^{\infty} c_n x^{n+1/3}$

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$$xy'' - xy' + y = 0$$

$$y = \sum_{n=0}^{\infty} c_n x^{n+r} \quad y' = \sum_{n=0}^{\infty} c_n (n+r) x^{n+r-1}$$

$$y'' = \sum_{n=0}^{\infty} c_n (n+r)(n+r-1) x^{n+r-2}$$

$$x \sum_{n=0}^{\infty} c_n (n+r)(n+r-1) x^{n+r-2} - x \sum_{n=0}^{\infty} c_n (n+r) x^{n+r-1}$$
$$+ \sum_{n=0}^{\infty} c_n x^{n+r} = 0$$

$$\sum_{n=0}^{\infty} c_n (n+r)(n+r-1) x^{n+r-1} - \sum_{n=0}^{\infty} c_n (n+r) x^{n+r}$$
$$+ \sum_{n=0}^{\infty} c_n x^{n+r} = 0$$

$$\sum_{n=1}^{\infty} c_n (n+r)(n+r-1) x^{n+r-1} + [c_0 r(r-1) x^{r-1}]$$
$$- \sum_{n=0}^{\infty} c_n (n+r) x^{n+r} + \sum_{n=0}^{\infty} c_n x^{n+r} = 0$$

$n = k+1 \quad n = k \quad n = k$

$$\sum_{k=0}^{\infty} c_{k+1} (k+r+1)(k+r) x^{k+r} - \sum_{k=0}^{\infty} c_k (k+1) x^{k+r}$$
$$+ \sum_{k=0}^{\infty} c_k x^{k+r} + [c_0 r(r-1) x^{r-1}] = 0$$

$$\sum_{k=0}^{\infty} x^r x^k [c_{k+1}(k+r+1)(k+r) - c_k (k+r) + c_k]$$
$$+ x^r x^{-1} c_0 r(r-1) = 0$$

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$$c_{k+1}(k+r+1)(k+r) - c_k(k+r) + c_k = 0$$

$$c_0 \, r(r-1) = 0$$

$$r_1 = 1 \quad r_2 = 0$$

$$c_{k+1}(k+r+1)(k+r) - c_k(k+r) - c_k$$

$$c_{k+1} = \frac{(k+r-1)}{(k+r+1)(k+r)} c_k$$

$\underline{r=0}$

$$c_{k+1} = \frac{k-1}{(k+1)k} c_k$$

$$c_1 = \frac{-1}{0} c_0$$

$$y_1(x) = y_2(x) \int \frac{e^{-\int P(x)\,dx}}{y_2^2(x)} dx$$

$$y_1(x) = x \int \frac{e^{-\int -1\,dx}}{x^2} dx$$

$$y_1(x) = x \int \frac{e^{x+2}}{x^2} dx$$

$\underline{r=1}$

$$c_{k+1} = \frac{k}{(k+2)(k+1)} c_k$$

$$c_1 = \frac{0}{2} c_0$$

$$c_2 = \frac{1}{6} - 0$$

$$\underline{y_2 = x}$$

$$y(x) = c_1 y_1 + c_2 y_2$$