

$$1) y'' + y = \cos^2 x$$

$$y = y_h + y_p$$

$$c_1 y_1 + c_2 y_2$$

$$y'' + y = 0$$

$$(e^{yx})'' + e^{yx} = 0$$

$$(y^2 e^{yx}) + e^{yx} = 0$$

$$y^2 + 1 = 0$$

$$y = \pm i$$

$$y_1 = e^{ix}$$

$$y_2 = e^{-ix}$$

$$y_1 = \cos(x)$$

$$y_2 = \sin(x)$$

$$e^{\pm ix} = (\cos(x) \pm i \sin(x))$$

$$y_G = c_1 (\cos(x)) + c_2 \sin(x)$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$u_1' = \frac{-y_2 g(x)}{W}$$

$$u_2' = \frac{y_1 g(x)}{W}$$

$$u_1 = \int \frac{-y_2 g(x)}{W} dx$$

$$u_2 = \int \frac{y_1 g(x)}{W} dx$$

$$W = \begin{vmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{vmatrix} = 1$$

$$u_1 = \int \frac{-\sin(x) \cos^2(x)}{1} dx$$

$$u_2 = \int \frac{\cos(x) \cos^2(x)}{1} dx$$

$$u_1 = \frac{1}{3} \cos^3(x) + C$$

$$u_2 = \sin(x) - \frac{1}{3} \sin^3(x)$$

$$y_p = u_1 y_1 + u_2 y_2$$

$$y_p = \frac{1}{3} \cos^3(x) \cos(x) + \left( \sin(x) - \frac{1}{3} \sin^3(x) \right) \sin(x)$$

$$y = y_g + y_p$$

$$y = C_1 \cos(x) + C_2 \sin(x) + \frac{1}{3} \cos^4(x) + \sin^2(x) - \frac{1}{3} \sin^4(x)$$

QUIZ #7 MATH 225 NOOR MUSTAFA

$$2) y'' + 2y' + y = e^{-x} \ln x$$

$$y = y_h + y_p$$

$$y'' + 2y' + y = 0 \quad y = e^{\gamma x}$$

$$(e^{\gamma x})'' + 2(e^{\gamma x})' + e^{\gamma x}$$

$$e^{\gamma x} (\gamma^2 + 2\gamma + 1) = 0 \quad e^{\gamma x} \neq 0$$

$$\gamma^2 + 2\gamma + 1 = 0$$

$$\gamma = \frac{-2 \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(-1)}$$

$\gamma = -1$  with multiplicity of 2

$$y_h = c_1 e^{-x} + c_2 x e^{-x}$$

$$y_p = u_1 y_1 + u_2 y_2 \quad g(x) = e^{-x} \ln(x)$$

$$w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$u_2' = \frac{y_1 g(x)}{w}$$

$$u_2 = \int \frac{y_1 g(x)}{w} dx$$

$$u_1' = \frac{-y_2 g(x)}{w}$$

$$u_1 = \int \frac{-y_2 g(x)}{w} dx$$

$$y_1 = e^{-x}$$

$$y_2 = e^{-x} X$$

$$w = y_1 y_2' - y_1' y_2$$

$$2) \quad w = e^{-x}(-e^x x + e^{-x}) - (-e^{-x})e^{-x} x$$

$$w = e^{-2x}$$

$$u_1 = \int \frac{e^{-x} x e^{-x} \ln(x)}{e^{-2x}} dx$$

$$= -\int x \ln(x) dx$$

← integrate by parts

$$= -\left(\frac{1}{2} x^2 \ln(x) - \int \frac{x}{2} dx\right)$$

$$\int \frac{x}{2} dx = \frac{x^2}{4}$$

$$= -\left(\frac{1}{2} x^2 \ln(x) - \frac{x^2}{4}\right)$$

$$= -\frac{1}{2} x^2 \ln(x) + \frac{x^2}{4} + C$$

$$u_1 = -\frac{1}{2} x^2 \ln(x) + \frac{x^2}{4} + C$$

$$u_2 = \int \frac{e^{-x} e^{-x} \ln(x)}{e^{-2x}} dx$$

$$u_2 = x \ln(x) - x$$

$$y_p = u_1 y_1 + u_2 y_2$$

$$y_p = \left(-\frac{1}{2} x^2 \ln(x) + \frac{x^2}{4}\right) e^{-x} + (x \ln(x) - x) e^{-x} x$$

(Note: modified due to ANTH 225)

2)

$$y_p = \frac{2e^{-x}x^2 \ln(x) - 3e^{-x}x^2}{4}$$

$$y = y_g + y_p$$

$$y = c_1 e^{-x} + c_2 x e^{-x} + \frac{2e^{-x}x^2 \ln(x) - 3e^{-x}x^2}{4}$$

general  
solution

$$y_1 = x^{-1/2} \cos(x) \quad y_2 = x^{-1/2} \sin(x)$$

③  $x^2 y'' + x y' + (x^2 - \frac{1}{4}) y = \frac{3}{2}$

$$y = y_G + y_P$$

$$y_G = C_1 x^{-1/2} \cos(x) + C_2 x^{-1/2} \sin(x)$$

$$y_P = u_1 y_1 + u_2 y_2$$

$$y_1' = -x^{-1/2} \sin(x) - \cos(x) \frac{1}{2} x^{-3/2}$$

$$w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$y_2' = x^{-1/2} \cos(x) - \frac{\sin(x)}{2x^{3/2}}$$

$$u_1' = \frac{-y_2 y(x)}{w} \quad u_2' = \frac{y_1 y(x)}{w}$$

$$w = y_1 y_2' - y_2 y_1'$$

$$w = (x^{-1/2} \cos(x)) (x^{-1/2} \cos(x) - \frac{\sin(x)}{2x^{3/2}}) - (x^{-1/2} \sin(x)) (-x^{-1/2} \sin(x) - \frac{\cos(x)}{2x^{3/2}})$$

$$w = \frac{\cos^2(x)}{x} + \frac{\sin^2(x)}{x}$$

$$w = \frac{1}{x}$$



Ques 78) Method

$$u_1' = \frac{-x^{-1/2} \cos x \cdot x^{-1/2}}{1/x} = (-x^{-1/2} \cos x (x^{-1/2}) \cdot x)$$

$$= -\cos x$$

$$u_1 = \int -\cos x$$

$$u_1 = -\sin x$$

$$u_2' = \frac{x^{-1/2} \sin x \cdot x^{-1/2}}{1/x} \cdot \frac{x}{1} = \sin(x)$$

$$u_2 = \int \sin(x)$$

$$u_2 = -\cos x$$

$$y = C_1 x^{-1/2} \cos(x) + C_2 x^{-1/2} \sin x - \sin(x) [x^{-1/2} \cos(x)] - \cos(x) [x^{-1/2} \sin(x)]$$

$$y = C_1 x^{-1/2} \cos(x) + C_2 x^{-1/2} \sin x - 2 \sin(x) \cos(x) x^{-1/2}$$