Noor mustafa | math 225

1. $(x+y)^2\,dx + (2xy + x^2 - 1)\,dy = 0 \qquad y(1) = 1$

$M = x^2 + 2xy + y^2 \qquad N = 2xy + x^2 - 1$

$\frac{\partial M}{\partial y} = 2x + 2y \qquad \frac{\partial N}{\partial x} = 2y + 2x$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial f}{\partial x} = x^2 + 2xy + y^2 \qquad \frac{\partial f}{\partial y} = 2xy + x^2 - 1$

$f(x,y) = \int (x^2 + 2xy + y^2)\,dx$

$f(x,y) = \frac{x^3}{3} + x^2y + xy^2 + h(y)$

$\frac{\partial f}{\partial y} = x^2 + 2xy + h'(y)$

$\cancel{x^2 + 2xy} + h'(y) = \cancel{2xy + x^2} - 1$

$h'(y) = -1 \qquad h(y) = -y + C$

$C = \frac{x^3}{3} + x^2y + xy^2 - y + C$

$\frac{x^3}{3} + x^2y + xy^2 - y = C \qquad y(1) = 1$

$\frac{1}{3} + 1 + 1 - 1 = C$

$C = 4/3 \qquad \boxed{\frac{x^3}{3} + x^2y + xy^2 - y = 4/3}$

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$\cdot \; x\frac{dy}{dx} - (1-x)y = xy^2$

$$\frac{x}{xy^2}\frac{dy}{dx} - \frac{(1-x)y}{xy^2} = 1$$

$$\frac{1}{y^2}\frac{dy}{dx} - \left(\frac{1}{x}-1\right)\frac{1}{y} = 1$$

let $z = \frac{1}{y}$ then $\frac{dz}{dx} = -\frac{1}{y^2}\frac{dy}{dx}$

$$-\frac{dz}{dx} - \left(\frac{1}{x}-1\right)z = 1$$

$$\frac{dz}{dx} + \left(\frac{1}{x}-1\right)z = -1$$

$$e^{\int\left(\frac{1}{x}-1\right)dx}$$

$$= e^{(\ln x - x)+c}$$

$$= xe^{-x+c}$$

$$d(zxe^{-x}) = \int(-1)(xe^{-x})dx$$

$$zxe^{-x+c} = -\int xe^{-x+c}dx$$

$$= -\left[\frac{e^{-x+c}}{-1}x - \int 1\cdot\frac{e^{-x+c}}{-1}dx\right]$$

$$= xe^{-x+c} - \int e^{-x+c}dx$$

$$= xe^{-x+c} - \frac{e^{-x+c}}{-1} + K \rightarrow = xe^{-x+c} + e^{-x+c} + K$$

$\star$ c is an arbitrary constant

door multara) math 225

2.
$$zx = x+1+ke^{x+c}$$

$$z = \frac{1+x+ke^{x+c}}{x}$$

$$\frac{1}{y} = \frac{1+x+ke^{x+c}}{x}$$

$$y = \frac{x}{1+x+ke^{x+c}}$$

$$z = \frac{1}{y}$$

C is an arbitrary constant

$$y(x) = \frac{x}{1+x+ke^{x+c}}$$

$$= \frac{x}{1+x+ke^{x}e^{c}}$$

$$ke^{c} = m$$

$$= \frac{x}{1+xme^{x}}$$

$$y(x) = \frac{x}{1+xme^{x}}$$

Noor mustufa | Math 225 | 1   euler $y = x^r$

3. $x^3y'''' + 5x^2y''' - 3xy'' = 0$

$x^3(x^r)'''' + 5x^2(x^r)''' - 3x(x^r)'' = 0$

$(x^r)'''' = rx^{r-4}(r-1)(r-2)(r-3)$

$(x^r)''' = rx^{r-3}(r-1)(r-2)$

$(x^r)'' = rx^{r-2}(r-1)$

$x^3rx^{r-4}(r-1)(r-2)(r-3) + 5x^2rx^{r-3}(r-1)(r-2)$
$-3xrx^{r-2}(r-1) = 0$

$x^3rx^{r-4}(r-1)(r-2)(r-3) = 0$

$x^3x^{r-4} \to x^{3+r-4}$

$rx^{3+r-4}(r-1)(r-2)(r-3) = 0$ applying exponent rule

$= rx^{r-1}(r-1)(r-2)(r-3)$

$5x^2rx^{r-3}(r-1)(r-2) = 5rx^{r-1}(r-1)(r-2)$

$3xrx^{r-2}(r-1) = 3rx^{r-1}(r-1)$

$0 = rx^{r-1}(r-1)(r-2)(r-3) + 5rx^{r-1}(r-1)(r-2) - 3rx^{r-1}(r-1)$

$0 = rx^{r-1}(r^2-3r+2)(r-3) + 5rx^{r-1}(r^2-3r+2) - 3rx^{r-1}(r-$

$0 = rx^{r-1}(r^3-6r^2+11r-6) + 5rx^{r-1}(r^2-3r+2) - 3rx^{r-1}(r$

Nour Mustafa) math 225 2

3c) $rx^{r-1}(r^3-6r^2+11r-6)$

$0 = r^3 r x^{r-1} - 6r^2 r x^{r-1} + 11 r r x^{r-1} - 6 r x^{r-1}$

$0 = r^4 x^{r-1} - 6r^3 x^{r-1} + 11r^2 x^{r-1} - 6 r x^{r-1}$

$0 = r^4 x^{r-1} - 6r^3 x^{r-1} + 11r^2 x^{r-1} - 6r x^{r-1}$

$+5rx^{r-1}(r^2-3r+2) - 3rx^{r-1}(r-1)$

$5rx^{r-1}(r^2-3r+2) \Rightarrow 5r^3x^{r-1} - 15r^2x^{r-1} + 10rx^{r-1}$

$-3rx^{r-1}(r-1) \Rightarrow -3r^2x^{r-1} + 3rx^{r-1}$

$0 = r^4 x^{r-1} - 6r^3 x^{r-1} + 11r^2 x^{r-1} - 6rx^{r-1} + 5r^3 x^{r-1}$
$\quad -15r^2 x^{r-1} + 10rx^{r-1} - 3r^2 x^{r-1} + 3rx^{r-1}$

$0 = r^4 x^{r-1} - r^3 x^{r-1} - 7r^2 x^{r-1} + 7rx^{r-1}$ $\boxed{\text{*combine like terms}}$

$r^4x^{r-1} - r^3x^{r-1} - 7r^2x^{r-1} + 7rx^{r-1} = 0$

$x^{r-1}(r^4 - r^3 - 7r^2 + 7r) = 0$

$x^{r-1} \neq 0$

$r^4 - r^3 - 7r^2 + 7r = 0$

$= r(r^3 - r^2 - 7r + 7) = 0$

Noor mustafa) math 225 3

$0 = r(r^3 - r^2 - 7r + 7)$

$0 = r(r^2(r-1) - 7(r-1))$

$= r\,(r-1)(r^2-7)$

$= r(r-1)\,(r+\sqrt{7})(r-\sqrt{7})$

$r=0 \quad r=1 \quad r=-\sqrt{7} \quad r=\sqrt{7}$

$y = c_1 x^0 + c_2 x^1 + c_3 x^{-\sqrt{7}} + c_4 x^{\sqrt{7}}$

$y = c_1 + c_2 x + \frac{c_3}{x^{\sqrt{7}}} + c_4 x^{\sqrt{7}}$

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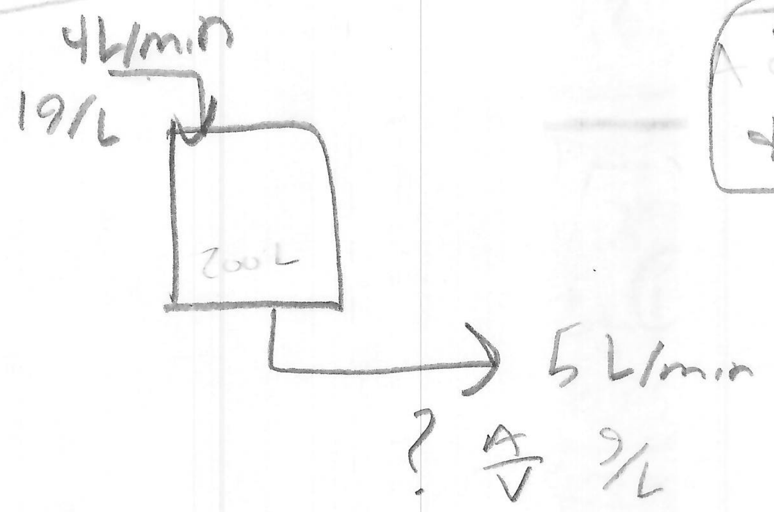
1.



200 → starting volume

40,200 → A(0)

initial condition

$$\frac{dA}{dt} = R_{in} - R_{out} = 4 \cdot 1 - 5\frac{A}{V}$$

$$\frac{g}{min} = 4 - \frac{5A}{V} \qquad \frac{4-5A}{200-t}$$

$$V(t) = 200 + t\,[r_{in} - r_{out})$$

$$= 200 + t\,[4-5] = 200 - t$$

$$\frac{dA}{dt} + \frac{5}{200-t}A = 4 \qquad \frac{dA}{dt} + P(t)A = Q(t)$$

$$e^{\int \left(\frac{5}{200-t}\right) dt}$$

$$e^{-5\ln(200-t) + K}$$

$$K = \ln C$$

$$e^{\ln[(200-t)^{-5}] + \ln C}$$

Noor mydrea) math 225 (2)

4.) $= C(200-t)^{-5}_{-5}$

$$\frac{dA}{dt}\left[C(200-t)^{-5}\right. = 4C(200-t)^{-5}$$

$$A\cancel{C}(200-t)^{-5} = \int 4\cancel{C}(200-t)^{-5}\,dt$$

$u = 200-t$

$$= 4C\left(-\int u^{-5}\,du\right)$$

$$= -4C\left(-\frac{1}{4}u^{-4} + m\right.$$

$$= \frac{1}{u^4} + M$$

$$= \frac{1}{(200-t)^4} + M$$

$$A(200-t)^{-5} = (200-t)^{-4} + m$$

$$A = (200-t) + m(200-t)^5$$

$$40{,}200 = (200 + m(200)^5$$

$$M = 1.25 \times 10^{-7}$$

$$\boxed{A = (200-t) + 1.25\times10^{-7}(200-t)^5}$$

5) $x' = 4x - y$ $\quad x(0) = 4$

$y' = x + 2y$ $\quad y(0) = 3$

$$\begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\det[A - \lambda I] = \begin{vmatrix} 4-\lambda & -1 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$(4-\lambda)(2-\lambda) + 1 = 0$

$\lambda^2 - 6\lambda + 8 + 1 = 0$

$\lambda^2 - 6\lambda + 9 = 0$

$(\lambda - 3)^2$

$\lambda = 3, 3$

for $\lambda = 3$:

$$\begin{bmatrix} 4-3 & -1 \\ 1 & 2-3 \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}\begin{pmatrix} K_1 \\ K_2 \end{pmatrix} = 0$$

$K_1 = K_2 = 1$ $\quad K_{\lambda=3} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Noor mustafa) math 225

repeated eigenvalues

$$\begin{bmatrix} x \\ y \end{bmatrix} = c_1 e^{\lambda t} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} + c_2 \left(e^{\lambda t} + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} + t e^{\lambda t} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \right)$$

$$\begin{bmatrix} 4-3 & -1 \\ 1 & 2-3 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$p_1 - p_2 = 1$ $\qquad$ $p_1 = p_2 + 1$

$$P = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$ $p_1 = 1$, $p_2 = 0$

$$\begin{bmatrix} x \\ y \end{bmatrix} = c_1 e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \left(e^{3t} t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{3t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$$

$$\begin{bmatrix} 4 \\ 3 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$$

$$\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_1 \end{bmatrix} + \begin{bmatrix} c_2 \\ 0 \end{bmatrix}$$

$4 = c_1 + c_2$ $\qquad$ $c_1 = 3$, $c_2 = 1$

$3 = c_1$

Noor Mustafa Math 225 (3) #5

$$\begin{bmatrix} x \\ y \end{bmatrix} = 3e^{3t} + e^{3t} t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{3t} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$x = 4e^{3t} + e^{3t} t$$

$$y = 3e^{3t} + e^{3t} t$$

Nour Mustafa | Math 226 (1)

$y_c = c_1 y_1 + c_2 y_2$

1) $y'''' - 4y'' = 5x^2 - e^{2x}$

$y = e^{mx}$

$y'' = m^2 e^{mx}$

$y^{(4)} = m^4 e^{mx}$

$m^4 e^{mx} - 4(m^2 e^{mx}) = 0$

$m^4 e^{mx} - 4m^2 e^{mx} = 0$

$\cancel{e^{mx}}(m^4 - 4m^2) = 0$

$m^2(m^2 - 4) = 0 \quad m \pm 2, 0, 0$

$y_c = c_1 e^{2x} + c_2 e^{-2x} + c_3 e^0 + c_4 x e^0$

$y_c = c_1 e^{2x} + c_2 e^{-2x} + c_3 + c_4 x$

$y_p = (Ax^4 + Bx^3 + Cx^2 + De^{2x} + Exe^{2x}$

$y_p' = 4Ax^3 + 3Bx^2 + 2Cx + 2De^{2x} + Ee^{2x} + 2Exe^{2x}$

$y_p'' = 12Ax^2 + 6Bx + 2C + 4De^{2x} + 4Ee^{2x} + 4Exe^{2x}$

$y_p''' = 24Ax + 6B + 8De^{2x} + 12Ee^{2x} + 8Exe^{2x}$

$y_p'''' = 24A + 16De^{2x} + 32Ee^{2x} + 16Exe^{2x}$

$24A + 16De^{2x} + 32Ee^{2x} + 16Exe^{2x}$

$- 4(12Ax^2 + 6Bx + 2C + 4De^{2x} + 4Ee^{2x} + 4Exe^{2x})$

$= 5x^2 - e^{2x}$

$24A + 16De^{2x} + 32Ee^{2x} + 16Exe^{2x} - 48Ax^2 - 24Bx - 8C$
$- 16De^{2x} - 16Ee^{2x} - 16Exe^{2x} = 5x^2 - e^{2x}$

$16Ee^{2x} - 48Ax^2 - 24xB + 24A - 8C = 5x^2 - e^{2x}$

$e^{2x}: 16E = -1 \quad E = \frac{-1}{16}$

$x^2 = -48A = 5 \quad A = -\frac{5}{48} \qquad D = 0$

$x = -24B = 0 \qquad B = 0$

constant $= 24A - 8C = 0 \quad C = -\frac{5}{16}$

$y = c_1 e^{2x} + c_2 e^{-2x} + c_3 + c_4 x - \frac{5}{48}x^4 - \frac{5}{16}x^2$
$- \frac{1}{16}xe^{2x}$

Nour Mubarak Math 225 (1)

7) $y'' + 4y = f(t)$ $y(0) = 1$ $y'(0) = 2$

$$f(t) = \begin{cases} 2, & 0 \le t < \pi/2 \\ 0, & \pi/2 \le t < \pi \\ 4, & t \ge \pi \end{cases}$$

$f(t) = 2u(t) + (0-2)u(t - \pi/2) + (4-0)u(t-\pi)$

$L\{y''\} = s^2 Y - sy(0) - y'(0) \Rightarrow s^2 Y - s - 2$

$L\{4y\} = 4Y$

$L\{2u(t)\} = \frac{2}{s}$ $u(t-c) \to \frac{e^{-cs}}{s}$

$L\{-2u(t - \pi/2)\} = \frac{-2e^{-\pi/2 s}}{s}$

$L\{4u(t-\pi)\} = \frac{4e^{-\pi s}}{s}$

$s^2 Y - s - 2 + 4Y = \frac{2}{s} - \frac{2e^{-\pi/2 s}}{s} + \frac{4e^{-\pi s}}{s}$

$Y(s^2+4) = \frac{2}{s} - \frac{2e^{-\pi/2 s}}{s} + \frac{4e^{-\pi s}}{s} + s + 2$

$Y = \frac{2}{s(s^2+4)} - \frac{2e^{-\pi/2 s}}{s(s^2+4)} + \frac{4e^{-\pi s}}{s(s^2+4)} + \frac{s}{(s^2+4)} + \frac{2}{(s^2+4)}$

voor mutatie Math 2.25 (2)

$$\frac{2}{s(s^2+4)} = \frac{A}{s} + \frac{Bs+C}{s^2+4} = \frac{1}{2}\frac{1}{s} - \frac{1}{2}\frac{s}{s^2+4}$$

$$2 = A(s^2+4) + Bs^2 + Cs$$

$0 = A + B$

$0 = C$

$2 = A \cdot 4$

$A = 1/2$

$C = 0$

$B = -1/2$

$$\frac{-2}{s(s^2+4)} = \frac{D}{s} + \frac{Es+F}{s^2+4}$$

$$-2 = D(s^2+4) + Es^2 + Fs$$

$0 = D + E$

$0 = F$

$-2 = 4D$

$D = \frac{1}{2}$

$E = -1/2$

$F = 0$

$$= -\frac{1}{2}\frac{1}{s} + \frac{1}{2}\frac{s}{s^2+4}$$

$$\frac{4}{s(s^2+4)} = \frac{H}{s} + \frac{Is+J}{s^2+4} = \frac{1}{s} - \frac{s}{s^2+4}$$

$$4 = H(s^2+4) + Is^2 + Js$$

$0 = H + I$

$0 = J$

$4 = 4H$

$H = 1$

$I = -1$

$J = 0$

user mustererymath 225 (5)

$$Y = \frac{1}{2}\frac{1}{s} - \frac{1}{2}\frac{s}{s^2+4} + \frac{1}{2}\frac{e^{-\frac{\pi}{2}s}}{s} - \frac{1}{2}\frac{se^{-\frac{\pi}{2}s}}{s^2+4}$$

$$+ \frac{e^{-\pi s}}{s} - \frac{se^{-\pi s}}{s^2+4} + \frac{s}{s^2+4} + \frac{2}{s^2+4}$$

$$L^{-1}\left\{\frac{1}{2}\frac{1}{s}\right\} = \frac{1}{2}$$

$$L^{-1}\left\{-\frac{1}{2}\frac{s}{s^2+4}\right\} = -\frac{1}{2}\cos(2t)$$

$$L^{-1}\left\{\frac{s}{s^2+4}\right\} = \cos(2t)$$

$$L^{-1}\left\{\frac{2}{s^2+4}\right\} = \sin(2t)$$

$$L^{-1}\left\{\frac{1}{2}\frac{e^{-\frac{\pi}{2}s}}{s}\right\} = \frac{1}{2}u\left(t-\frac{\pi}{2}\right)$$

$$L^{-1}\left\{\frac{e^{-\pi s}}{s}\right\} = u(t-\pi)$$

$$L^{-1}\left\{-\frac{1}{2}\frac{se^{-\frac{\pi}{2}s}}{s^2+4}\right\} = -\frac{1}{2}u(t-\pi/2)\cos(2(t-\pi/2)$$

$$= -\frac{1}{2}u(t-\pi/2)\cos(2t-\pi)$$

$$L^{-1}\left\{\frac{-se^{-\pi s}}{s^2+4}\right\} = -u(t-\pi)\cos(2(t-\pi))$$

$$= -u(t-\pi)\cos(2t-2\pi)$$

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2)

$$y(t) = \frac{1}{2} + \frac{1}{2}\cos 2t + \sin 2t + \frac{1}{2}u(t-\pi/2)\left[1-\cos(2t-\pi)\right] + u(t-\pi)\left[1-\cos(2t-2\pi)\right]$$

voor Mustafa) Math 223) 1

) $\det(A-\lambda I) = \begin{vmatrix} 1-\lambda & 1 & 2 \\ 3 & -1-\lambda & 6 \\ -2 & 0 & -4-\lambda \end{vmatrix} = 0$

$0 = (1-\lambda)\left[(-1-\lambda)(-4-\lambda) - 0(6)\right] - 1\left[3(-4-\lambda) - (-2)(6)\right]$

$- 1\left[3(-4-\lambda - (-2)(6)\right] + 2\left[3(0) - (-2)(-1-\lambda)\right]$

$(1-\lambda)\left[\lambda^2 + 5\lambda + 4\right] - \left[-3\lambda\right] + 2\left[-2-2\lambda\right] =$

$\cancel{4} \cancel{+\lambda} - 4\lambda^2 - \lambda^3 \cancel{+3\lambda} \cancel{-4} \cancel{-4\lambda} = 0$

$\lambda^3 + 4\lambda^2 = 0$

$\lambda^2(\lambda + 4)$ $\quad \lambda = -4, 0, 0$

$\lambda^2 = 0, 0 \quad \lambda = -4$

$\lambda = -4$

$\begin{bmatrix} 5 & 1 & 2 \\ 3 & 3 & 6 \\ -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$5k_1 + k_2 + 2k_3 = 0$ $\qquad k_1 = 0$

$3k_1 + 3k_2 + 6k_3 = 0$

$-2k_1 = 0$

Moor mujtaba | math 225 ②

$K_2 + 2K_3 = 0$ $\qquad K_{\lambda=-4} = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$

$3K_2 + 6K_3 = 0$

$K_3 = 1 \quad K_2 = -2$

$\boxed{\text{for } \lambda = 0}$

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & -1 & 6 \\ -2 & 0 & -4 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 = R_2 + R_3$

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 2 \\ -2 & 0 & -4 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 = R_1 - R_2$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 0 \\ -2 & 0 & -4 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$K_1 + K_2 + 2K_3 = 0$

$2K_2 = 0 \qquad K_2 = 0$

$-2K_1 - 4K_3 = 0 \qquad K_1 = -2K_3$

$K_{\lambda=0} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

Noor Mustafa | Math 225 | (3)

8)

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & -1 & 6 \\ -2 & 0 & -4 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 3 & -1 & 6 & 0 \\ -2 & 0 & -4 & 1 \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 3 & -1 & 6 & 0 \\ 1 & 1 & 2 & -2 \\ -2 & 0 & -4 & 1 \end{array}\right]$$

$R_2 = R_1 - 3R_2$

$$\left[\begin{array}{ccc|c} 3 & -1 & 6 & 0 \\ 0 & -4 & 0 & 6 \\ -2 & 0 & -4 & 1 \end{array}\right]$$

$3P_1 - P_2 + 6P_3 = 0$

$-4P_2 = 6$

$-2P_1 - 4P_3 = 1$

$P_2 = -3/2$

$P_1 = -\frac{1}{2} - 2P_3$

$P_3 = 0$

$P_1 = -1/2$

$P_2 = -3/2$

Nacer [illegible]) math 225 (8)

$p = \begin{bmatrix} -1/2 \\ -3/2 \\ 0 \end{bmatrix}$

$$X = c_1 e^{\lambda t} K\lambda_1 + c_2 e^{\lambda_2 t} K\lambda_2 + c_3 (t e^{\lambda_2 t} K_{\lambda_2} + e^{\lambda_2 t} p)$$

$$\boxed{X = c_1 e^{-4t} \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + c_3 t \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} -1/2 \\ -3/2 \\ 0 \end{bmatrix}}$$

$$\boxed{X = c_1 e^{-4t} \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + c_3 (t \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -1/2 \\ -3/2 \\ 0 \end{bmatrix}}$$

your mistakes | Math 225 | ①

9.) $x=0$

$y'' - 2xy' + 5y = 0$

$y = \sum_{n=0}^{\infty} c_n x^n$ $\quad y'' = \sum_{n=2}^{\infty} n(n-1)c_n x^{n-2}$

$y' = \sum_{n=1}^{\infty} c_n n x^{n-1}$

$\sum_{n=2}^{\infty} c_n n(n-1) x^{n-2} - 2x \sum_{n=1}^{\infty} c_n n x^{n-1} + 5\sum_{n=0}^{\infty} c_n x^n$

$= 0$

$\sum_{n=2}^{\infty} c_n n(n-1) x^{n-2} - \sum_{n=1}^{\infty} 2c_n n x^n + 5\sum_{n=0}^{\infty} c_n x^n$

$\sum_{n=3}^{\infty} c_n n(n-1) x^{n-2} - \sum_{n=1}^{\infty} 2c_n n x^n + 5\sum_{n=1}^{\infty} c_n x^n$

$+ c_n n(n-1)x^{n-2}\big|_{n=2} + 5c_n x^n\big|_{n=0}$

$n = k+2 \quad n=k \quad n=k$

$\sum_{k=1}^{\infty} c_{k+2}(k+2)(k+1)x^k - \sum_{k=1}^{\infty} 2c_k k x^k + \sum_{k=1}^{\infty} 5c_k x^k$

$+2c_2 + 5c_0 = 0$

Noor Mustafa) math 225) (2)

9)

$$\sum_{k=1}^{\infty} x^k \left[C_{k+2}(k+2)(k+1) - 2kC_k + 5C_k \right]$$

$$+2C_2 + 5C_0 = 0$$

$$C_{k+2}(k+2)(k+1) - 2kC_k + 5C_k = 0$$

$$C_{k+2} = \frac{2k-5}{(k+2)(k+1)} C_k$$

$$2C_2 + 5C_0 = 0$$

$$C_2 = \frac{-5}{2} C_0$$

$y_1 = 1 + \frac{-5}{2}x^2 + \frac{5}{24}x^4 + \frac{1}{48}x^6 + \dots$

$y_1 \rightarrow C_0 = 1$
$C_1 = 0$

$$C_2 = \frac{-5}{2}$$

$$C_3 = 0$$

$$C_4 = \frac{-1}{12} \cdot \left(\frac{-5}{2}\right) = \frac{5}{24}$$

$$C_5 = 0$$

$$C_6 = \frac{3}{30} \cdot \frac{5}{24} = \frac{1}{48}$$

$y_2 \rightarrow C_0 = 0$
$C_1 = 1$

$$C_2 = 0$$

$$C_3 = -\frac{1}{2}$$

$$C_4 = 0$$

$$C_5 = \frac{1}{20} \cdot \frac{-1}{2} = \frac{-1}{40}$$

$$C_6 = 0$$

$$C_7 = \frac{5}{42} \cdot \left(\frac{-1}{40}\right) = \frac{-1}{336}$$

$y_2 = x + \frac{1}{2}x^3 - \frac{1}{40}x^5 - \frac{1}{336} \dots$

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$$y = c_1 y_1 + c_2 y_2$$

$$y_1 = 1 - \frac{5}{2}x^2 + \frac{5}{24}x^4 + \frac{1}{48}x^6 + \dots$$

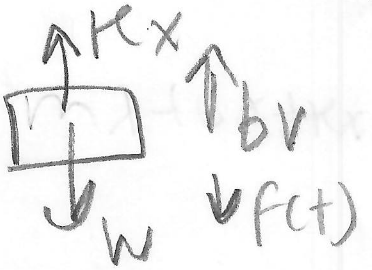
$$y_2 = x - \frac{1}{2}x^3 - \frac{1}{40}x^5 - \frac{1}{336}x^7 + \dots$$

$y_1 \to c_0 = 1$
$c_1 = 0$

$y_2 \Rightarrow c_0 = 0$
$c_1 = 1$

10. Nour mustafa Math 225 (1)

$k = \frac{4k_1k_2}{k_1+k_2}$ $\Sigma F = ma$



$w + f(t) - kx - b\underset{\dot{x}}{v} = m\underset{\ddot{x}}{a}$

$mg + 3\sin(2t) = m\ddot{x} + 6\dot{x} + kx$

$m\ddot{x} + 6\dot{x} + kx = mg + 3\sin(2t)$

$\frac{24}{32}\ddot{x} + \frac{3}{2}\dot{x} + 45x = 24 + 3\sin 2t$

hookes law
$F = kx$

$15 = k_1 \frac{6}{12}$ $k = \frac{4(30)(18)}{(30+18)}$

$k_1 = 30$ lb/ft $k = 45$ lb/ft

$15 = k_2 \frac{10}{12}$

$k_2 = 18$ lb/ft

voor mystery) mum z=5) 2

1) $x(0)=0$ ← equilibrium

$\dot{x}(0)=0$ ← velocity is zero

$$\frac{24}{32}x'' + \frac{3}{2}x' + 45x = 24 + 3\sin(2t)$$

$$L\{\frac{24}{32}\ddot{x}\} = \frac{3}{4}s^2 L\{x\} \cancel{-s x(0)} - \cancel{\dot{x}(0)}$$

$$L\{\frac{3}{2}\dot{x}\} = \frac{3}{2} s L\{x\} - \cancel{x(0)}$$

$$L\{45x\} = 45\, L\{x\}$$

$$L\{24\} = \frac{24}{s}$$

$$L\{3\sin(2t)\} = \frac{6}{s^2+4}$$

$$L\{x\}\left[\frac{3}{4}s^2 + \frac{3}{2}s + 45\right] = \frac{24}{s} + \frac{6}{s^2+4}$$

$$\cancel{3}L\{x\}\left[\frac{1}{4}s^2 + \frac{1}{2}s + 15\right] = \cancel{3}\left[\frac{8}{s} + \frac{2}{s^2+4}\right]$$

$$L\{x\}\left[s^2 + 2s + 60\right] = \frac{32}{s} + \frac{8}{s^2+4}$$

$$L\{x\} = \frac{32}{s(s^2+2s+60)} + \frac{8}{(s^2+4)[s^2+2s+60]}$$

$s^2+2s+60$

$s^2+2s+1-1+60$

$(s+1)^2+59$ → use this later

$$\frac{32}{s[s^2+2s+60]} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+60}$$

$32 = A(s^2+2s+60) + Bs^2 + Cs$

$0 = A + B$

$0 = 2A + C$

$32 = 60A$

$A = 8/15$

$B = -8/15$

$C = -\frac{16}{15}$

$$= \frac{8}{15s} - \frac{8}{15}\frac{s}{s^2+2s+60} - \frac{16}{15}\frac{1}{s^2+2s+60}$$

$$\frac{8}{(s^2+4)(s^2+2s+60)} = \frac{Ds+E}{s^2+4} + \frac{Fs+G}{s^2+2s+60}$$

$$8 = (Ds+E)(s^2+2s+60) + (Fs+G)(s^2+4)$$

s^3 : $0 = D + F$

s^2 : $0 = 2D + E + G$

s : $0 = 60D + 2E + 4F$

constant $8 = 60E + 4G$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 60 & 2 & 4 & 0 \\ 0 & 60 & 0 & 4 \end{bmatrix} \begin{bmatrix} D \\ E \\ F \\ G \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 8 \end{bmatrix}$$ ← Plugged it into a calculator

$D = -\frac{1}{197}$

$E = \frac{28}{197}$

$F = \frac{1}{197}$

$G = -\frac{26}{197}$

10)

$$= -\frac{1}{197}\frac{s}{s^2+4} + \frac{28}{197}\frac{1}{s^2+4} + \frac{1}{197}\frac{s}{s^2+2s+60}$$
$$- \frac{26}{197}\frac{1}{s^2+2s+60}$$

$$L\{X3\} = \frac{8}{15}\frac{1}{s} - \frac{8}{15}\frac{s}{(s-1)^2+59} - \frac{16}{15}\frac{1}{(s-1)^2+59}$$
$$-\frac{1}{197}\frac{s}{s^2+4} + \frac{28}{197}\frac{1}{s^2+4} + \frac{1}{197}\frac{s}{(s+1)^2+59}$$
$$-\frac{26}{197}\frac{1}{(s+1)^2+59}$$

$$-\frac{8}{15}\frac{s}{(s+1)^2+59} = -\frac{8}{15}\frac{(s+1)-1}{(s+1)^2+59}$$

$$= -\frac{8}{15}\frac{s+1}{(s+1)^2+59} + \frac{8}{15}\frac{1}{(s+1)^2+59}$$

$$L\{x\} = \frac{8}{15}\frac{1}{s} - \frac{8}{15}\frac{s+1}{(s+1)^2+59} - \frac{8}{15}\frac{1}{(s+1)^2+59}$$

$$- \frac{1}{197}\frac{s}{s^2+4} + \frac{28}{197}\frac{1}{s^2+4} + \frac{1}{197}\frac{(s+1)}{(s+1)^2+59}$$

$$- \frac{27}{197}\frac{1}{(s+1)^2+59}$$

$$L^{-1}\left\{-\frac{8}{15}\frac{s+1}{(s+1)^2+59}\right\} = \frac{-8}{15}e^{-t}\cos(\sqrt{59}\,t)$$

$$L^{-1}\left\{-\frac{8}{15}\frac{1}{(s+1)^2+59}\right\} = -\frac{8}{15\sqrt{59}}e^{-t}\sin(\sqrt{59}\,t)$$

$$L^{-1}\left\{-\frac{1}{197}\frac{s}{s^2+4}\right\} = -\frac{1}{197}\cos(2t)$$

$$L^{-1}\left\{\frac{28}{197}\frac{1}{s^2+4}\right\} = \frac{14}{197}\sin(2t)$$

$$L^{-1}\left\{\frac{1}{197}\frac{(s+1)}{(s+1)^2+59}\right\} = \frac{1}{197}e^{-t}\cos(\sqrt{59}\,t)$$

$$L^{-1}\left\{-\frac{27}{197}\frac{1}{(s+1)^2+59}\right\} = \frac{-27}{197\sqrt{59}}e^{-t}\sin(\sqrt{59}\,t)$$

$$L^{-1}\left\{\frac{8}{15}\frac{1}{s}\right\} = \frac{8}{15}$$

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$$X(t) = \frac{8}{15} - \frac{8}{15}e^{-t}\cos(\sqrt{59}t) - \frac{8}{15\sqrt{59}}e^{-t}\sin(\sqrt{59}t)$$
$$-\frac{1}{197}\cos(2t) + \frac{14}{197}\sin(2t) + \frac{1}{197}e^{-t}\cos(\sqrt{59}t)$$
$$-\frac{27}{197\sqrt{59}}e^{-t}\sin(\sqrt{59}t)$$

final answer after further simplification

$$x(t) = \frac{8}{15} - \frac{1561}{2955}e^{-t}\cos(\sqrt{59}t) - \frac{1}{197}\cos(2t)$$
$$+\frac{14}{197}\sin(2t) - \frac{1981}{2955\sqrt{59}}e^{-t}\sin(\sqrt{59}t)$$