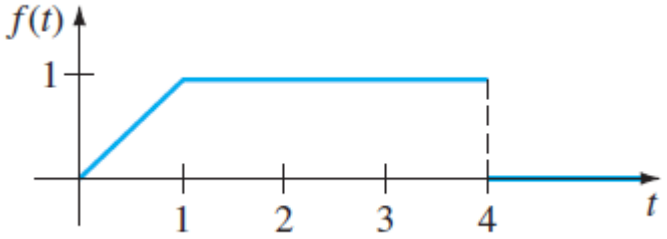
Math 225 – Exam #3

Clearly and neatly show all work with proper notation for each problem. Solutions with no work will receive no credit. Simplify all answers completely.

1. (8) Find $\mathcal{L}\{te^{3t}\sin 5t\}$.

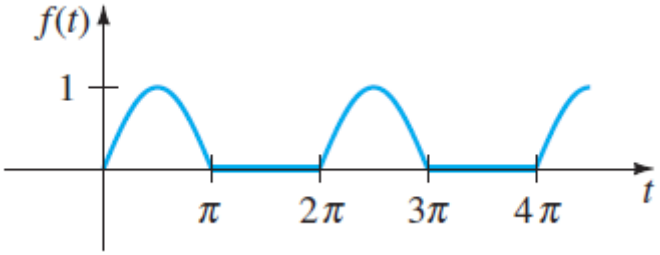
2. (8) Express f in terms of unit step functions, then find $\mathcal{L}\{f(t)\}$.



3. (10) Obtain the Laplace transform of the solution of the following initial value problem. In other words, solve for $Y(s)$. Do **NOT** apply the inverse transform in order to find $y(t)$.

$$y'' - 4y = f(t), \qquad y(0) = 1, \qquad y'(0) = -2,$$

where $f(t)$ is the periodic half-wave rectification of sine given by



4. (10) Evaluate the following inverse Laplace transform.

$$\mathcal{L}^{-1}\left\{\frac{s^2+5s}{s^3-2s^2+3s-6}\right\}$$

5. (10) Use the Laplace transform to solve the following initial value problem.

$$y'' + 6y' + 9y = e^{-3t}, \qquad y(0) = 0, \qquad y'(0) = 5$$

6. (10) Find a general solution to the following system.

$$\frac{dx}{dt} = -6x + 2y \qquad x(0) = 5$$
$$\frac{dy}{dt} = -3x + 1y \qquad y(0) = 0$$

7. (12) Find a general solution of the system. Your general solution should be in terms of only real-valued functions.

$$\mathbf{X}' = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \mathbf{X}$$

8. (12) Solve the following 2nd-order differential equation.

$$y'' + 4y' + 13y = \delta(t - \pi), \qquad y(0) = 1, \qquad y'(0) = 0$$