Quiz 3 | Substitution | Noor Mujtaba

① $x\frac{dy}{dx} - y = \sqrt{x^2+y^2} \quad x>0$

$y = ux$

$x\frac{dux}{dx} - ux = \sqrt{x^2+u^2x^2}$

$x\frac{dux}{dx} - ux = \sqrt{x^2(1+u^2)}$

$x\frac{dux}{dx} - ux = x\sqrt{1+u^2}$

$\frac{d(ux)}{dx} - u = \sqrt{1+u^2}$

$\frac{d(ux)}{dx} = x\frac{du}{dx} + u\frac{dx}{dx} = u + x\frac{du}{dx}$

$x\frac{du}{dx} = \sqrt{1+u^2}$

$\int \frac{du}{\sqrt{1+u^2}} = \int \frac{dx}{x}$

$\ln|\sqrt{1+u^2}+u| = \ln|x| + \ln|C|$

$\ln|\sqrt{1+u^2}+u| = \ln|Cx|$

$\sqrt{1+u^2}+u = Cx$

$\sqrt{1+\frac{y^2}{x^2}} + \frac{y}{x} = Cx$

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(2) $x^2 \frac{dy}{dx} + y^2 = xy \quad x > 0$

$y = ux$

$x^2 \frac{dy}{dx} = u^2x^2 = ux^2$

$\frac{d ux}{dx} = u - u^2$

$u + \frac{x\,du}{dx} = u - u^2$

$\frac{x\,du}{dx} = -u^2$

$-\frac{du}{u^2} = \frac{dx}{x}$

$\frac{1}{u} = \ln|x| + c$

$u = \frac{1}{\ln|x| + c}$

$y = ux$

$y = \frac{x}{\ln|x| + c}$

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3. $\sqrt{y}\frac{dy}{dx} + y^{3/2} = 1 \quad y(0) = 4$

$y = u^2 \qquad \frac{du^2}{dx} = 2u\frac{du}{dx}$

$u\left[2u\frac{du}{dx}\right] + (u^2)^{3/2} = 1$

$2u^2\frac{du}{dx} + u^3 = 1$

$\frac{du}{dx} = \frac{1-u^3}{2u^2}$

$\int \frac{2u^2}{1-u^3}\,du = \int dx$

$-\frac{2}{3}\ln(|1-u^3|) = x + C$

$\ln|(1-u^3)^{-2/3}| = x + C$

$(1-u^3)^{-2/3} = e^{x+C} = Ke^x$

$y = y^{1/2}$

$(1-y^{3/2})^{-2/3} = Ke^x$

$K = (1-4^{3/2})^{-2/3} \rightarrow \frac{1}{49^{1/3}}$

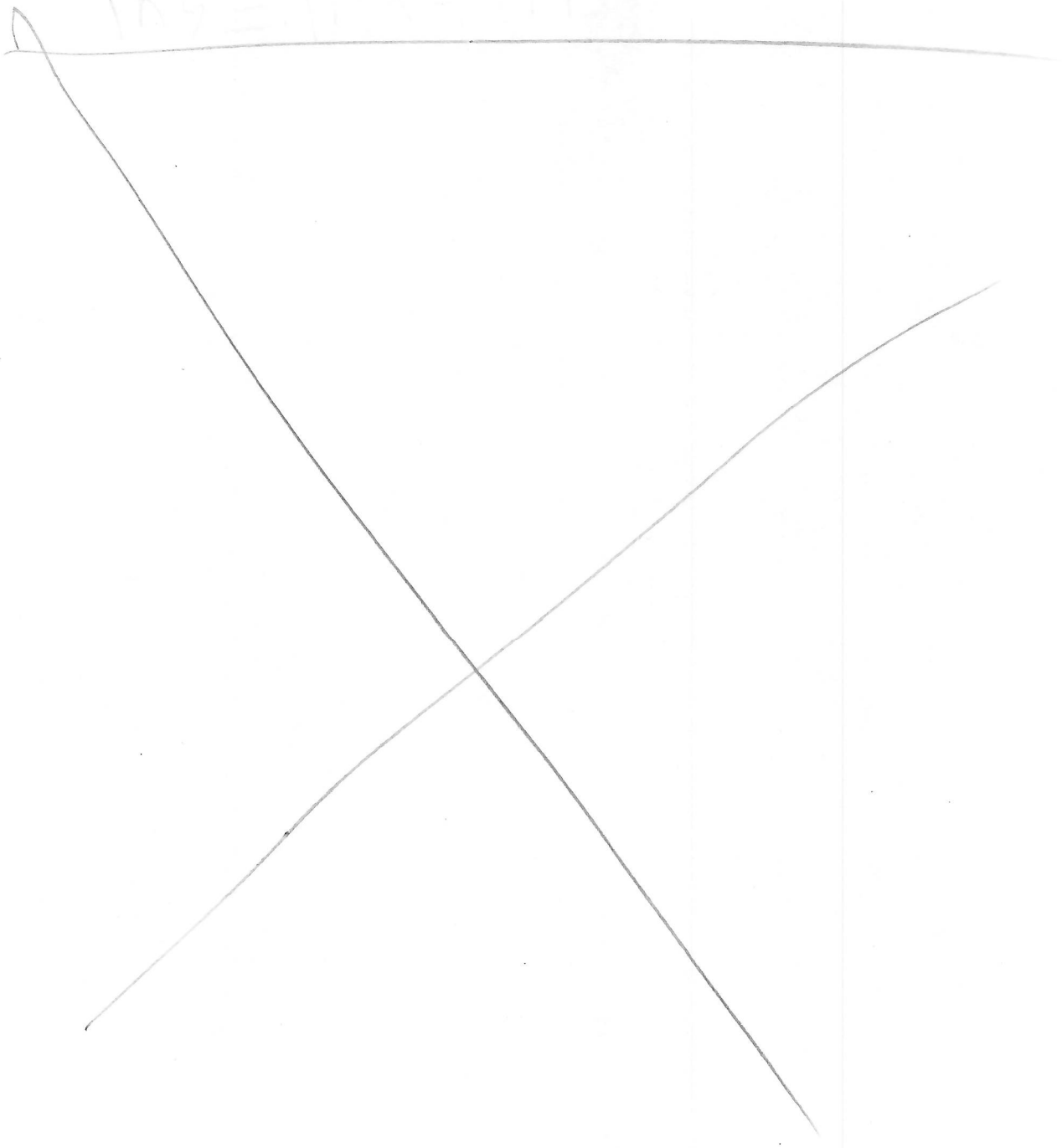
$K = \frac{1}{49^{1/3}}$

Quiz 3 Model Solutions 1st Page

(4) $y\,dx + x(\ln x - \ln y - 1)\,dy = 0$

$x \ln x - x \ln y - x = N$

$M = y$



Quiz 3 substitutions hour Mustafa continued 2nd page

(4) $y\,dx + x(\ln x - \ln y - 1)\,dy = 0$ $\quad y(1) = e$

$dy = d(ux) \qquad y = ux$

$$x\,du - (x\ln u + x)\frac{d(ux)}{dx} = 0$$

$$x\,du - (x\ln u + x)\left[u + x\frac{du}{dx}\right] = 0$$

$$x\,du - xu[\ln u + 1] - x^2[\ln u + 1]\frac{du}{dx} = 0$$

$$-xu\ln u - x^2(\ln u + 1)\frac{du}{dx} = 0$$

$$\int \frac{-dx}{x} = \int \frac{\ln(u)+1}{u\ln(u)}\,du$$

$$\ln(x) + \ln C_1 = \left(\int \frac{1}{u}\,du + \int \frac{1}{u\ln u}\right)du$$

$$\ln(x) + C_1 = \ln u + \ln(|\ln(u)|) + C_2$$

$$-\ln|x| = \ln u + \ln(|\ln(u)|) + C$$

$$-\ln(x) + C = \ln(u\ln(u))$$

$$-\ln(x) + C = \ln\left(\frac{y}{x}\ln\left(\frac{y}{x}\right)\right)$$

continued →

4a(3) Not multiplied 3rd page continued

(4)

$-\ln(x) + c = \ln(y) + \ln(\ln(\frac{y}{x}))$ $-\ln(x)$

$\ln(y) + \ln(\ln(\frac{y}{x})) + c = 0$

at $x=1$

$c = 1 + \ln(1)$ $y(1) = e$

$c = 1$

$1 = \ln(y) + \ln(\ln(\frac{y}{x}))$

$e = y \ln(\frac{y}{x})$

$e^{e} = e^{y} \frac{y}{x}$