

Quiz 1 Nour Mustafa

1-
(a)

$$e^x, \frac{\partial y}{\partial x} = e^{-y} + e^{-2x-y}$$

$$\begin{aligned} M(x,y) + N(x,y) \frac{\partial y}{\partial x} &= 0 \\ M(x,y) &= -e^{-y} - e^{-2x-y} \\ N(x,y) &= e^x, \\ F &= (F_y = N, F_x = M) \\ M_y &= e^{-y} + e^{-2x-y} = 2x - y \\ N_x &= e^x \end{aligned}$$

wrong
verre

$$e^x, \frac{\partial y}{\partial x} = e^{-y} [1 + e^{-2x}]$$

$$e^y, \frac{\partial y}{\partial x} = e^{-x} + e^{-3x}$$

$$\int e^y y \, dy = \int e^{-x} + e^{-3x} \, dx$$

$$ye^y - e^y + C = -e^{-x} - \frac{1}{3}e^{-3x} + C$$

$$ye^y - e^y = -e^{-x} - \frac{1}{3}e^{-3x} + C$$

Q.2 1) Near Mutation -

(P.B) $y \ln x \frac{dy}{dx} = \frac{y+1}{x}$

$$\frac{y}{y+1} \frac{dy}{dx} = \frac{1}{x \ln x}$$

$$\int \frac{y}{y+1} dy = \int \frac{1}{x \ln x} dx \quad u = \ln x$$

$$\int 1 - \frac{1}{y+1} dy \quad \frac{d}{dx} (\ln x) \rightarrow \frac{1}{x}$$

$$y + C + \int -\frac{1}{y+1} dy \quad \int \frac{1}{u} du$$

$$u = y+1$$

$$y + C - \int \frac{1}{u} du$$

$$y + C - (\ln(u) + C)$$

$$y - \ln(u) + C$$

$$y - \ln(y+1) + C = \ln(\ln x) + C$$

$$y - \ln(y+1) = \ln(\ln x) + C$$

2u' 2 1) Now mtrifa

$$(2) (a) \quad xy' + (3x+1)y = e^{-3x}, \quad x > 0$$

$$\frac{dy}{dx} + P(x)y = Q(x) \quad P(x) = \frac{3x+1}{x}$$

$$y = u \cdot v$$

$$Q(x) = \frac{e^{-3x}}{x}$$

$$u \frac{dv}{dx} + v \frac{du}{dx} + P(x)uv = Q(x)$$

$$u \frac{dv}{dx} + v \left[\frac{du}{dx} + P(x)u \right] = Q(x)$$

$$u \frac{dv}{dx} = Q(x)$$

particular solution

$$\frac{du}{dx} + P(x)u = 0$$

general solution

$$\frac{1}{u} du = -P(x) dx$$

$$\ln(u) = -\int P(x) dx$$

$$y' + \frac{3x+1}{x}y = \frac{e^{-3x}}{x}$$

$$\ln(u) = -[3x + \ln x] + C$$

$$u = e^{-3x} \cdot e^{-\ln x} \cdot e^C$$

$$u = \frac{C e^{-3x}}{x}$$

continued
on next
page

(Add multipliers) ex 2.1
2 a) continue

$$u \frac{dv}{dx} = Q(x)$$

$$\frac{(e^{-3x})}{x} \frac{dv}{dx} = \frac{e^{-3x}}{x}$$

$$dv = \frac{1}{x} dx$$

$$v = \frac{1}{x} + \frac{k}{x}$$

$$y = u \cdot v$$

$$y = \frac{(e^{-3x})}{x} \cdot \frac{1}{x} + \frac{k}{x}$$

$$y = e^{-3x} + \frac{k e^{-3x}}{x}$$

Quiz 1) Nach mustern

2B) $xy' + y = e^x$, $y(1) = 2$

$$P(x) =$$

$$Q(x) =$$

$$\frac{dy}{dx} + P(x)y = Q(x) \quad P(x) = \frac{1}{x}$$

$$u \quad \frac{du}{dx} = Q(x)$$

$$Q(x) = \frac{e^x}{x}$$

$$\frac{du}{dx} + P(x)u = 0$$

$$\ln(u) = -\int P(x) dx$$

$$\ln(u) = -\ln(x) + \ln(C)$$

$$u = e^{-\ln(x)} \cdot C$$

$$u = \frac{C}{x}$$

$$\frac{C}{x} \frac{dv}{dx} = \frac{e^x}{x}$$

$$dv = \frac{e^x}{C} dx$$

$$v = \frac{e^x}{C} + \frac{k}{C}$$

$$y = u \cdot v$$

$$\frac{C}{x} \cdot \frac{e^x}{C} + \frac{k}{C}$$

$$y = \frac{e^x}{x} + \frac{k}{x}$$

$$2 = \frac{e}{1} + \frac{k}{1}$$

$$(k = 2 - e) \quad k \approx -0.718$$