Math 225 – Exam #1

Clearly and neatly show all work with proper notation for each problem. Solutions with no work will receive no credit. Simplify all answers completely.

1. (10) Solve the following initial-value problem. Give an implicit solution.

$$\left(x + ye^{y/x}\right)dx - xe^{y/x}dy = 0, \qquad y(1) = 0$$

2. (10) Solve the following differential equation. Give an explicit solution for y.

$$\cos x \frac{dy}{dx} + (\sin x)y = 1$$

3. (10) Solve the following initial-value problem. Give an explicit solution for y.

$$(x + 1)\frac{dy}{dx} + y = \ln x\,, \qquad y(1) = 10$$

4. (10) Solve the following differential equation. Give an implicit solution.

$$(3x^2y + e^y)dx + (x^3 + xe^y - 2y)dy = 0$$

5. (10) Solve the following differential equation. Give an explicit solution for y.

$$\frac{dy}{dx} = y(xy^3 - 1)$$

For questions 6 and 7, be sure to set up am equation and solve. You may round all k values to 2 decimal places. Round final answers to whole numbers.

6. (18) A large tank is partially filled with 75 liters of lemon water in which 100 hectograms of sugar is dissolved. Lemon water containing sugar flows into the tank at a rate of 4 L/min, and the well-mixed solution then flows out of the tank at a rate of 2 L/min. The concentration $\gamma(t)$ of sugar in the incoming lemon water (in hg/L) varies with time according to the expression

$$\gamma(t) = 1 - \cos t$$

Set up and solve a differential equation to find the amount of sugar in the tank at a given time t. If the tank can hold 95 liters of lemon water, how much sugar will there be in the tank when it begins to overflow?

7. (12) While you are letting the filter remove more of the chlorine from the water, you are begin to acclimate your fish to the temperature of the water in the fish tank. In order to not shock the fish, you place the bag that the fish is in into the tank to let the water in the bag slowly reach the temperature of the water in the tank. The water in the bag is currently 27^{o}F, while the tank water is 24^{o}C. After 2 minutes the water temperature in the bag has dropped by one degree. If the rate of change of temperature of the water in the bag is proportional to the difference between the temperature of the water in the bag and the water in the tank (Newton's Law of Heating/Cooling), how long until the temperature in the bag is within four tenths of a degree of the temperature of the water? **Note: other conditions than temperature should also be considered when acclimating fish to a new tank.*