1 a) $A = (2,3,1)$

$B = (0,3,1)$

$\vec{AB} = \langle -2,0,0 \rangle$

1B) $\langle 1,-1,4 \rangle$

$$\vec{u} = \frac{\langle 1,-1,4 \rangle}{\sqrt{18}}$$

$$\left\langle \frac{1}{\sqrt{18}}, \frac{-1}{\sqrt{18}}, \frac{4}{\sqrt{18}} \right\rangle$$

1(C) 

$|v| = 4$

$\vec{v} = \langle |v| \cos\theta, |v| \sin\theta \rangle$

$\vec{v} = \langle -4\cos\frac{3\pi}{4}, 4\sin\frac{3\pi}{4} \rangle$

$\vec{u} = -2\sqrt{2}\,\hat{i} + 2\sqrt{2}\,\hat{j}$

4D) $|r - 6| \leq 1$

tells us ~~that~~ in 2D its a filled in circle with a radius ~~up~~ to 1. In 3D its a filled in sphere with a radius of up to 1.

Question #1

2 $u = \langle 1, 0, 1 \rangle$

$v = \langle 2, 1, 2 \rangle$ $w = \langle -1, 1, 1 \rangle$

2a) $v \cdot w = -2 + 1 + 2 = 1$

2b) $\vec{v} \times \vec{w} = \begin{vmatrix} i & j & k \\ 2 & 1 & 2 \\ -1 & 1 & 1 \end{vmatrix} = \langle -1, -4, 3 \rangle$

2c) $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$-1(x+1) + -4(y+1) + 3(z-2) = 0$

2d) $r(t) = \langle 0, 0, 0 \rangle + t\langle 2, 1, 2 \rangle$

$r(t) = \langle 2t, t, 2t \rangle$

3) $a(t) = \langle 0, 0, 5 \rangle$

$v(0) = \langle 3, -1, -1 \rangle$ and $r(0) = \langle 2, -5, 1 \rangle$

$v(t) = \int 5k \, dt$

$= 5tk + 3i - j + k$

$v(t) = 3i - j + (5t - 1)k$

$r(t) = \int 3i - j + (5t - 1)k \, dt$

$= 3t\hat{i} - t\hat{j} + \left(\frac{5t^2}{2} - t\right)k$
$+ 2i - 5\hat{j} + k$

$r(t) = (3t + 2)\hat{i} + (-t - 5)\hat{j} + \left(\frac{5t^2}{2} - t + 1\right)\hat{k}$

4) $r(t) = \langle \cos(2t), \sin(2t), 2t \rangle$

4a) $r'(t) = \langle -2\sin(2t), 2\cos(2t), 2 \rangle$

$r'(\frac{\pi}{2}) = \langle -2\sin(\pi), 2\cos(\pi), 2 \rangle$

$= \langle 0, -2, 2 \rangle$

4B) $\vec{r_0} + tv$

$r(\frac{\pi}{2}) = (\cos\pi, \sin\pi, \pi)$

$\langle -1, 0, \pi \rangle$

$\langle -1, 0, \pi \rangle + T\langle 0, -2, 2 \rangle$

$= \langle -1, -2t, \pi + 2t \rangle$

4C) $x - y + z = 1$

$-1 + 2t + (\pi + 2t) = 1$

$4t = 2 - \pi$

$t = \frac{2-\pi}{4}$

$\left\langle -1, -2\left(\frac{2\pi}{4}\right), \pi + 2\left(\frac{2\pi}{4}\right) \right\rangle$

1D

$$L = \int_0^{2\pi} \sqrt{4\sin^2 2t + 4\cos^2 2t + 4}\, \cancel{dt}$$

$$= \int_0^{2\pi} \sqrt{4+4}$$

$$= \int_0^{2\pi} \sqrt{8}$$

$$2\pi\sqrt{8}$$

$$L = 17.772$$

$r(t) = 2\cos t, 2\sin t, e^t \quad 0 \le t \le \pi$

$x + \sqrt{3}y = 1$

plane

$r'(t) = \langle -2\sin t, 2\cos t, e^t \rangle \quad \vec{n} = \langle 1, \sqrt{3}, 0 \rangle$

for $r'(t)$ to be parallel to plane

$r'(t) \cdot \vec{n} = 0$

$$\frac{-2\sin t + 2\sqrt{3}\cos t}{2} = \frac{0}{2}$$

$$-\sin t + \sqrt{3}\cos t = 0$$

$$\sqrt{3}\cos t = \sin t$$

$$\sqrt{3} = \tan t$$

$$t = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

$$r\left(\frac{\pi}{3}\right) = \langle 2\cos\left(\frac{\pi}{3}\right), 2\sin\left(\frac{\pi}{3}\right), e^{\pi/3} \rangle$$

$$= \langle 1, \sqrt{3}, e^{\pi/3} \rangle$$

Question 5

5) $y - x - 3z = -7$

$4x - 4y = 4$

$r(t) = \vec{r_0} + t(\vec{v})$

point of line

direction of line

$$\begin{cases} y - x - 3z = -7 \\ 4x - 4y = 4 \end{cases}$$

$y = 0$

$-x - 3z = -7$

$4x = 4$

$\vec{r_0} = \langle 1, 0, 2 \rangle$

$x = 1, y = 0, z = 2$

$\vec{v} = \vec{n_1} \times \vec{n_2}$

$\vec{N_1} = \langle -1, 1, -3 \rangle$

$\vec{n_2} = \langle 4, -4, 0 \rangle$

$\vec{v} = \langle -12, -12, 0 \rangle$

$r(t) = \langle 1 - 12t, -12t, 2 \rangle$

7

a) $r(t) = \langle \cos t, 3t, 2\sin(2t) \rangle$

$T(t) = \frac{r'(t)}{\|r'(t)\|}$ $= \langle -\sin t, 3, 4\cos(2t) \rangle$

$t=0$

$= \frac{\langle 0, 3, 4 \rangle}{\sqrt{25}} = \langle 0, \frac{3}{5}, \frac{4}{5} \rangle$

13) $\int_0^1 (t\,i - t^3 j)\,dt$

$\frac{t^2}{2} i - \frac{t^4}{4} j \Big|_0^1$

$\frac{1}{2} i - \frac{1}{4} j$

8

a) $$\lim_{(x,y)\to(3,-3)} \frac{xy}{\cos(\sin(x+y))}$$

$$= \frac{-9}{1}$$

$$= -9$$

B) $$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

$$\lim_{x\to 0} \frac{0}{x^2} = \frac{0}{2x} = \frac{0}{2} = 0$$

$$\lim_{y\to 0} \frac{0}{y^2} = \frac{0}{2y} = \frac{0}{2} = 0$$

$y = x$

$$\hookrightarrow \lim_{x\to 0} \frac{x^2}{2x^2} = \frac{1}{2}$$

So limit does not exist

DNE

question 4

1) a) $f(x,y) = \dfrac{2x+7y}{7x-9y}$

$f_x = \dfrac{(7x-9y)\cdot 2 + (2x+7y)\cdot 7}{(7x-9y)^2}$

B) $F(x,y,z) = e^{xyz}$

$f_x = e^{xyz}(yz)$

$\to f_{xz} = e^{xyz}\cdot(y) + e^{xyz}\cdot(xy)(yz)$

C) $f(x,y) = x^8y^4 + 9x^8y$

$F_y = 4x^8y^3 + 9x^8$

D) $h(s,t) = e^{-4t}\tan(\pi s)$

$D_1 h = \nabla h = \langle e^{-4t}\cdot\sec^2(\pi s)\cdot\pi,$

$e^{-4t}\cdot(-4)\cdot\tan(\pi s)\rangle$

question 10

10 a) $F(x,y) = \frac{x}{y}$

$f_x = \frac{1}{y}$

$f_y = -\frac{x}{y^2}$

10 B) $F_x(7,7) = \frac{1}{7}$

$f_y(7,7) = -\frac{7}{49} \to -\frac{1}{7}$

10 C) $(7,7,1)$

$\nabla F(7,7)$

$z - 1 = F_x^{(7,7)}(x-7) + f_y^{(7,7)}(y-7)$

$z - 1 = \frac{1}{7}(x-7) - \frac{1}{7}(y-7)$

10 d) $L(x,y)$ of f at $(7,7)$

$L(x,y) - 1 = \frac{1}{7}(x-7) - \frac{1}{7}(y-7)$

$+1$

(11) $f(x,y) = e^x \sin^2(y)$

a) $\langle e^x \cdot \sin^2 y, e^x \cdot 2\sin y\cos y \rangle$

B) $\langle 1, 0 \rangle$ ← plugged into equation from A $(0, \pi/2)$

C) $D_A f = \nabla F(0, \frac{\pi}{2}) \cdot \vec{u}$

$= \langle 1, 0 \rangle \cdot \frac{\langle -6, 8 \rangle}{10} = \frac{-6}{10}$

D) $\max_u \left\{ D_u f \big|_{(0, \pi/2)} \right\}$

$\hat{u} = \frac{\nabla F}{|\nabla F|} = \langle 1, 0 \rangle$

- the max $|\nabla F| = 1$

12) $f(x,y)\ xy + \frac{1}{x} + \frac{1}{y}$

a) $\begin{cases} f_x = y - \frac{1}{x^2} = 0 \\ f_y = x - \frac{1}{y^2} = 0 \end{cases}$ $\nabla f = 0$

$\hookrightarrow x = \frac{1}{y^2}$

$\hookrightarrow y = \frac{1}{(\frac{1}{y^2})^2} = y - y^4 = 0$

crit points $(1,1)$

$y = 1,\ 1$

$x = 1$

B) $f_{xx} = \frac{2}{x^3}$

$f_{yy} = \frac{2}{y^3}$

$f_{xy} = 1$

$f_{yx} = 1$

c) $D(x,y) = \left(\frac{2}{x^3} \cdot \frac{2}{y^3}\right) - 1$

$= \left(\frac{4}{x^3y^3}\right) - 1$

saddle points and max does not exist

D) $\frac{2}{1^3} = 2$

e) $(1,1)$

$D(1,1) = 3 > 0$

$f_{xx}(1,1) > 0$

$(1,1)$ is a local min

3 is the local min

(13) x^2+2y^2 $\quad x^2+y^2=1$

$2x = \lambda 2x \rightarrow 2x(1-\lambda)=0$ $\quad x=0$ or $\lambda=1$

$x=0 \rightarrow 2y(2-\lambda)=0$ $\quad y=0$ or $\lambda=2$

$4y = \lambda 2y$

$y = \frac{0}{4-2\lambda} = 0$

$x=0 \rightarrow y \pm 1$

$(0,1), (0,-1)$

or

$y=0$ $\quad x \pm 1$

$(0,1), (0,-1)$

$(1,0), (-1,0)$

$\Rightarrow$ max $=2$

min $=1$

14) $f(x,y) = x^2 - 2xy + 2y$

$F_x = 2x - 2y = 0$ $y=1$

$F_y = -2x + 2 = 0$

$\hookrightarrow x = 1$ $(1,1)$

(x, y)	$x^2 - 2xy + 2y$
$(1,1)$	1
$(0,0)$	0
$(2,2)$	0
$(0,2)$	4
$(2,0)$	4

max = 4
min = 0