Unit 1.5 Supplemented Practice Problems

Write and solve a differential equation to answer each of the following questions.

1) The rate of change of R is inversely proportional to the product of R and t. When $t = 1, R = 4$ and when $t = 2, R = 6$. Determine R explicitly as a function of t.

2) The rate of change of P is directly proportional to r and inversely proportional to the square of P. When $r = 0, P = 3$ and when $r = 1, P = 6$. Determine P explicitly as a function of r.

3) The rate of change W is inversely proportional to $1 + x^2$. Given that $W(0) = 0$ and $W(1) = \pi$, determine $\lim_{x \to \infty} W(x)$.

4) The rate of change of y is proportional to y. When $x = 0, y = 4$ and when $x = 3, y = 10$. What is the value of y when $x = 6$?

5) The rate of change of V is proportional to V. When $t = 2, V = 16{,}000$ and when $t = 4, V = 12{,}500$. What is the value of V when $t = 0$?

6) The isotope ^{226}Ra has a half-life of approximately 1599 years. If 10 grams are currently present, how much will be present in 1000 years? How long will it take until 1 gram remains?

7) When object is removed from a furnace and placed in an environment with a constant temperature of 80°F, its core temperature is 1500°F. One hour after it is removed, the core temperature is 1120°F. Use Newton's Law of Cooling to find the core temperature 5 hours after the object is removed from the furnace.

   Recall: Newton's Law of Cooling says the object will cool at a rate that is proportional to the difference between the temperature of the object and the temperature of the surrounding medium (the ambient temperature).