9.1: Parametric Curves

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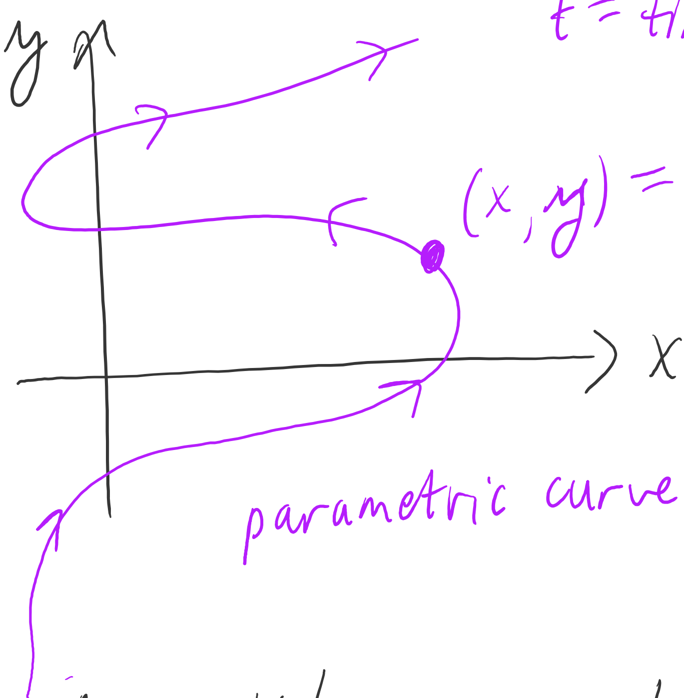
t = time parameter



A particle moves along C, but can't describe location with $y=f(x)$ b/c fails VLT. But x, y func. of time t.

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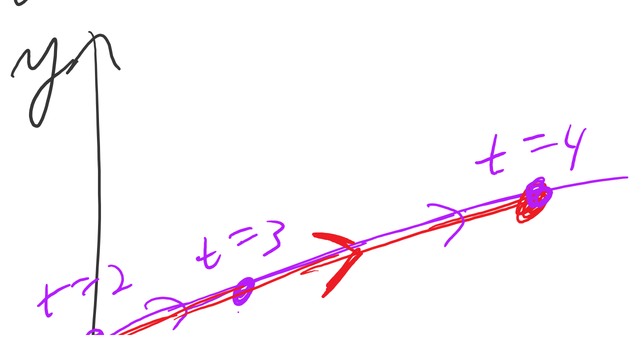
ex. Sketch & identify curve given by

$x = t^2 - 2t$, $y = t+1$

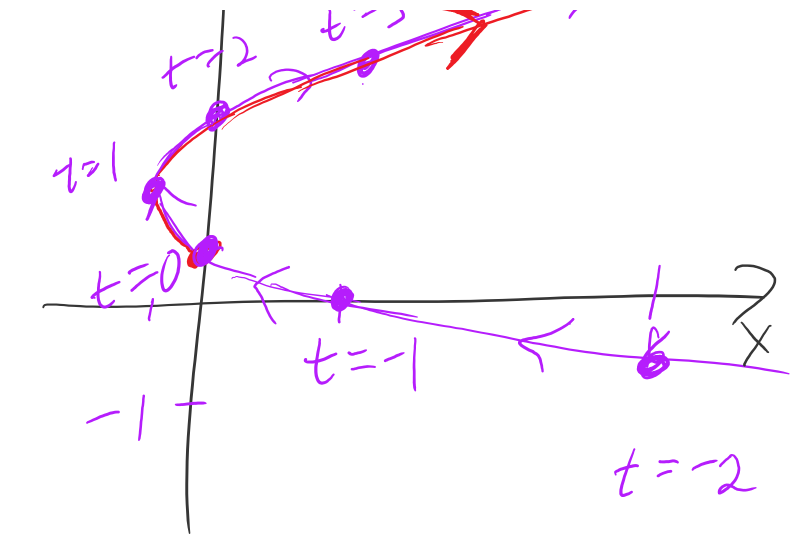
$$y - 1 = t$$

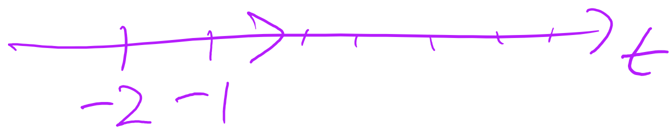
Soln:

t	x	y
-2	8	-1 ✓



-2	8	-1
-1	3	0
0	0	1
1	-1	2
2	0	3
3	3	4
4	8	5





Note

1) particle moves in direction of inc. t

2) Could find curve algebraically, but no direction given.

$$x = t^2 - 2t = (y-1)^2 - 2(y-1)$$

$$x = y^2 - 4y + 3$$

3) Sometimes domain of t is restricted.

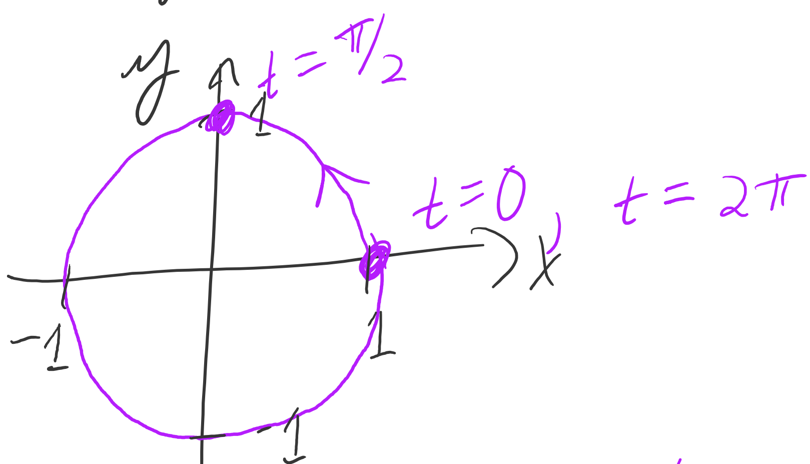
ex. $0 \leq t \leq 4$ ⇝ red curve above

Def. For $x = f(t)$, $y = f(t)$, $a \leq t \leq b$, the points $(f(a), g(a))$ and $(f(b), g(b))$ are the <u>initial</u> and <u>terminal pts</u> respectively.

<u>ex:</u> $x = \cos(t)$, $y = \sin(t)$, $0 \le t \le 2\pi$.

$$x^2 + y^2 = \cos^2(t) + \sin^2(t) = 1$$

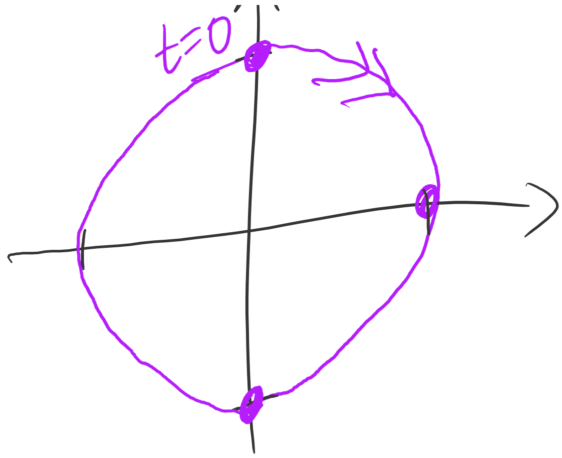
$x^2 + y^2 = 1$ ⇝ unit circle



traverses unit circle once.

<u>ex.</u> $x = \sin(2t)$, $y = \cos(2t)$, $0 \le t \le 2\pi$.

<u>$t=0$:</u> $x = 0$
$y = 1$



$y = 1$

$t = \frac{\pi}{2}$ $\quad x = \sin(\pi) = 0$

$y = \cos(\pi) = -1$

$t = \frac{\pi}{4}$ $\quad x = \sin\left(\frac{\pi}{2}\right) = 1$

$y = \cos\left(\frac{\pi}{2}\right) = 0$

Start at $(0,1)$, traverse the circle clockwise, twice.

* Different param. curve from last ex.

Fact circle w/center (h,k) & radius r has param. eq'ns (traverse once cc-wise) to

$$x = h + r\cos(t), \quad y = k + r\sin(t), \quad 0 \le t \le 2\pi.$$

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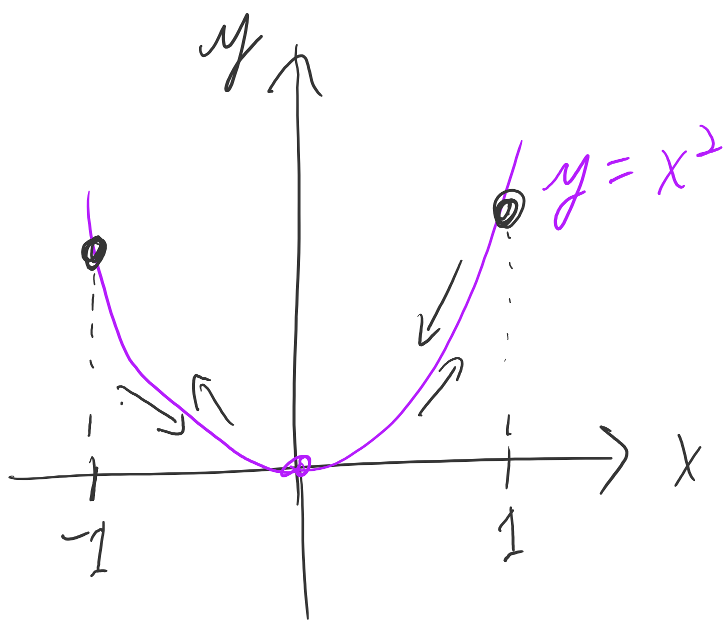
ex. $x = \sin(t)$, $y = \sin^2(t)$, t?

$y = \sin(t)^2 = x^2 \Rightarrow$ on parab. $y = x^2$.

Note $-1 \leq \sin(t) \leq 1$

$\Rightarrow -1 \leq x \leq 1$



traverse back & forth forever

b/c no restriction on t.

9.3: Polar Coordinates

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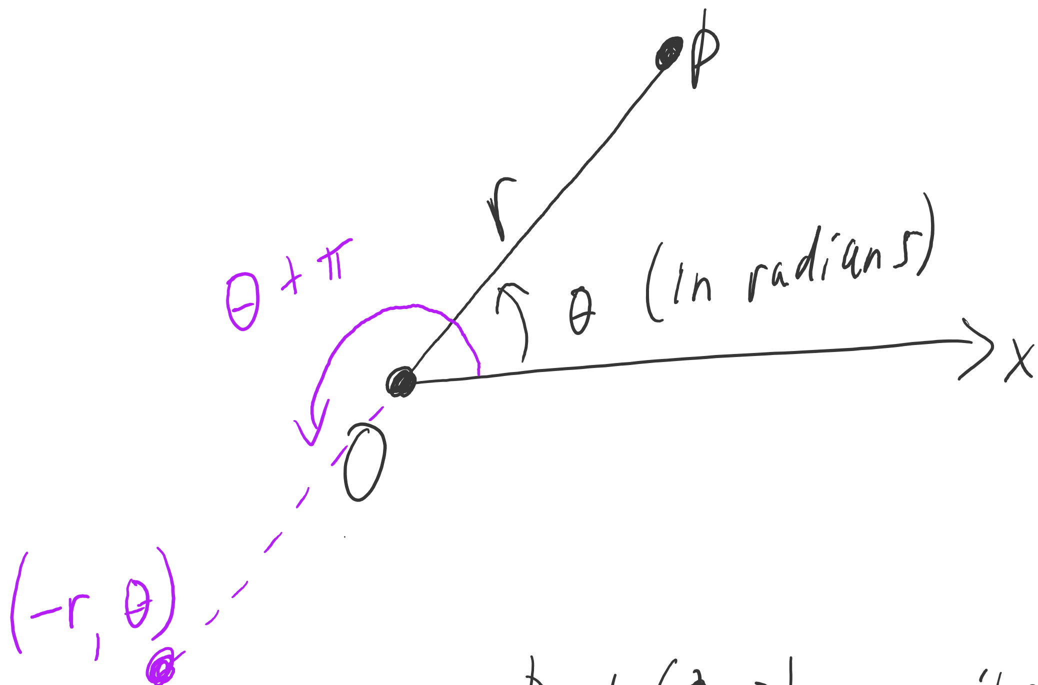
(Alternate labeling of xy-plane)



For any point $P \neq (0,0)$, write $P = (r, \theta)$

where

- r = directed distance from O to P
- θ = angle between x-axis and segment from O to P.

Note $(-r, \theta)$ and $(r, \theta + \pi)$ rep. same point on plane.

Fact 1) If P has Cartesian coord. (x,y) and polar coord. (r,θ), then

$$x = r\cos(\theta) \quad \& \quad y = r\sin(\theta).$$

2) $r^2 = x^2 + y^2$

3) $\tan(\theta) = \frac{y}{x}$

ex. Convert $\left(2, \frac{\pi}{3}\right)$ from polar to Cartesian.

Sol'n: $r = 2$, $\theta = \frac{\pi}{3}$

$$x = r\cos(\theta) = 2\cos\left(\frac{\pi}{3}\right) = 1.$$

$$y = r\sin(\theta) = 2\sin\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$$\boxed{(1, \sqrt{3})}$$

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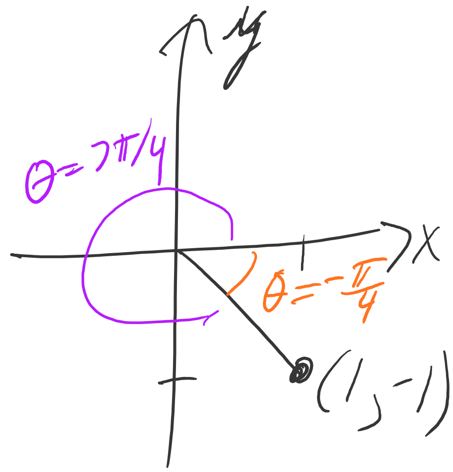
ex. Convert $(1,-1)$ from Cart. to polar.

Let $r > 0$.

$$r = \sqrt{x^2+y^2} = \sqrt{1^2+(-1)^2} = \sqrt{2}$$

$$\tan(\theta) = \frac{y}{x} = \frac{-1}{1} = -1$$



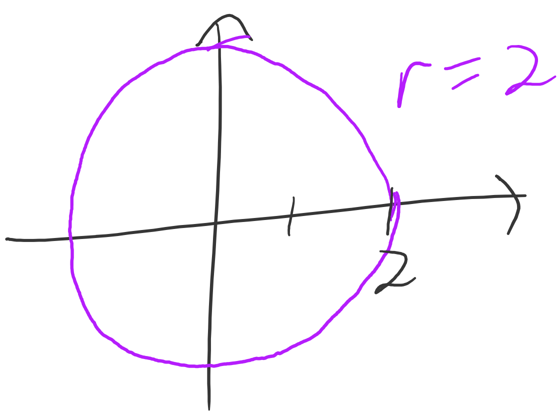
Choose $\theta = -\pi/4$

$\rightsquigarrow \boxed{\left(\sqrt{2}, -\pi/4\right)}$ or $\left(\sqrt{2}, \frac{7\pi}{4}\right)$.

$\rightsquigarrow \boxed{\left(\sqrt{2}, -\pi/4\right)}$ or $\left(\sqrt{2}, 7\pi/4\right)$.

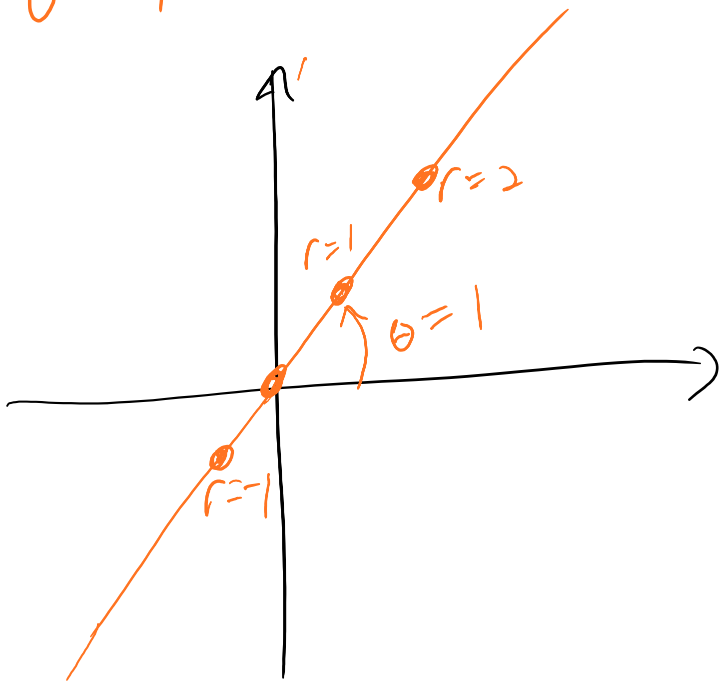
<u>Polar Curves</u> The graph of a polar eq'n $F(r, \theta) = 0$ is the set of all points w/ at least one polar rep. satisfying the eq'n.

<u>ex.</u> $r = 2$ (i.e. $F(r,\theta) = r - 2 = 0$)



$\theta = 1$

$\theta = 1$



ex. $r = 2\cos(\theta)$

Convert to Cart. to see graph

$$x = r\cos(\theta) \Rightarrow \cos(\theta) = \frac{x}{r}$$

$$r = 2\cos(\theta) = \frac{2x}{r}$$

$$\Rightarrow r^2 = 2x = x^2 + y^2$$

$$\Rightarrow x^2 + y^2 - 2x = 0$$

$$(x-1)^2 + y^2 = 1$$

$r = 2\cos(\theta)$