MONROE COMMUNITY COLLEGE

MTH 211 – SLN

Unit 5 Written Assignment

Printed Name: Noor Mustafa

*See Blackboard for the deadline for submitting your assignment.

Directions:

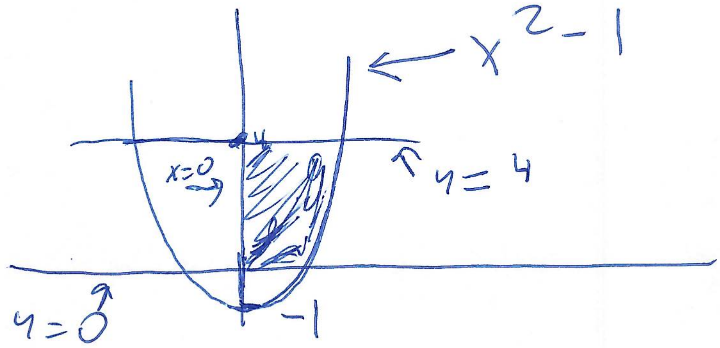
- Be sure to follow the submission instructions given in Blackboard when submitting your completed assignment. Do NOT email me your completed assignment.
- Only methods covered in this course up to the current unit may be used on this assignment.
- In all problems you must show sufficient work to support your final answers. All such work must be done in this assignment document. If additional space is needed, you may add pages, but your work must be submitted in order.
- **The work you submit must be your own. While I cannot prevent students from discussing problems, suspicion of duplicated work will be investigated and penalties may result. In addition, solutions taken from online calculators or the equivalent will be not be accepted and will be considered cheating.**
- Be sure to include this page as a cover page for your assignment when you submit it and please make sure your name is written in the designated spot above. If you do not have access to a printer, you may write your solutions on regular paper, but each page must consist of solutions to only those problems on the corresponding page of the original exam.

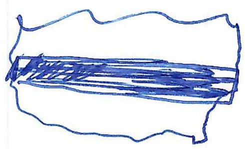
In all problems where you are asked to evaluate (without approximation) a definite integral, you must clearly demonstrate your use of the Fundamental Theorem of Calculus.

Noor Mustafa

[A] Consider the region in the first quadrant that is bounded by $y = x^2 - 1$, $y = 0$, $y = 4$, and $x = 0$ which is to be revolved about the y-axis.

(A.1) Sketch the region bounded by the functions above and include a single **horizontal** representative rectangle.





(A.2) If the region is revolved about the y-axis, will the representative rectangle generate a disk, washer, or shell?

Disk

(A.3) Write an expression for ΔV_i, the volume of the representative element in (A.2). Be sure to use appropriate notation.

$y = x^2 - 1$

$\Delta V_i = \pi (y+1) \Delta y$

$x = \pm\sqrt{y+1}$

$A = \pi r^2$

$r = \sqrt{y+1}$

$A = \pi(y+1)$

(A.4) Write the corresponding Riemann Sum that approximates the volume of the solid of revolution generated by revolving the region about the y-axis.

$\Delta y = \frac{b-a}{n}$

$\Delta y = \frac{4-0}{n}$

$\lim_{n\to\infty} \sum_{i=1}^{n} \pi (y_i + 1) \frac{4}{n}$, $y_i = 0 + \frac{4i}{n}$

$y_i = \frac{4i}{n}$

(A.5) Write and **evaluate** a definite integral to obtain the exact volume of the solid.

$\int_0^4 \pi (y+1)\,dy = \pi \int_0^4 (y+1)\,dy \rightarrow \pi \left(\frac{y^2}{2} + y\right)\Big|_0^4$

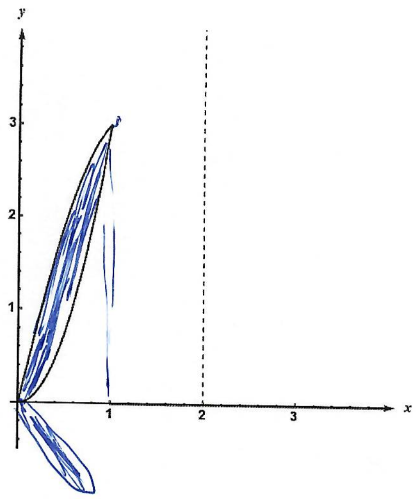
$\rightarrow \pi(12)$

$= 12\pi$

$y = 1+2$ $\quad$ $y = 3(1)^2$

[B] Consider the region bounded by $y = x + 2\sin\left(\frac{\pi}{2}x\right)$ and $y = 3x^2$ (see the graph below).

$3 = 3$



$\cancel{x + 2\sin\left(\frac{\pi}{2}x\right) = 3x^2}$

$\cancel{2\sin\left(\frac{\pi}{2}x\right) = 3x^2 - x}$

$\cancel{\sin\left(\frac{\pi}{2}x\right) = \frac{3x^2 - x}{2}}$

♣ For each part of this problem you are <u>encouraged</u> to show work to make it clear how you obtained your definite integrals.

(B.1) **<u>Write and evaluate</u>** a definite integral whose value gives the <u>area</u> of the bounded region. Simplify your answer, but don't approximate it using decimals.

$$\int_0^1 x + 2\sin\left(\frac{\pi}{2}x\right) - 3x^2 \, dx$$

$f(x) = x + 2\sin\left(\frac{\pi}{2}x\right)$

$g(x) = 3x^2$

$a = 0$

$b = 1$

$$= \frac{x^2}{2} - \frac{4\cos\left(\frac{\pi}{2}x\right)}{\pi} - x^3 \Big]_0^1$$

$$= \left(\frac{1}{2} - 0 - 1\right) - \left(-\frac{4}{\pi}\right)$$

$$= \frac{4}{\pi} - \frac{1}{2}$$

(B.2) Write a definite integral whose value gives the <u>volume</u> of the solid generated by revolving the region about the x-axis. **DO NOT ATTEMPT TO EVALUATE THE INTEGRAL; JUST SET IT UP.**

* washer method

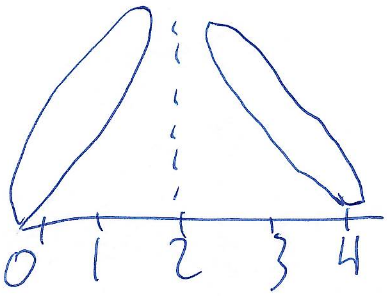
$$\pi\int_0^1 \left(x+2\sin\left(\tfrac{\pi}{2}x\right)\right)^2 - 9x^4\,dx$$

$$\pi\int f(x)^2 - 9(x)^2$$

(B.3) Write a definite integral whose value gives the <u>volume</u> of the solid generated by revolving the region about the line $x = 2$ (this line is shown in the figure). **DO NOT ATTEMPT TO EVALUATE THE INTEGRAL; JUST SET IT UP.**

* Shell method

$$2\pi\int_0^1 (x-2)\left(\left(x+2\sin\left(\tfrac{\pi}{2}x\right)\right)-(3x^2)\right)dx$$



[C] Use the table below to complete (C.1) and (C.2). Show your substitutions into the appropriate formula for each. Round answers to the nearest hundredth when appropriate.

x	3	6	9	12	15
$g(x)$	2.5	5	3.5	4	6

~~Δx~~ $\Delta x = 6$

(C.1) Use Mid(2) to approximate $\int_3^{15} g(x)dx$.

$$\sum_{i=1}^{2} f(x_i)\Delta x = \begin{matrix} g(6)\cdot 6 + g(12)\cdot 6 \\ 30 + 24 = 54 \end{matrix}$$

Mid(2) = 54

(C.2) Use Simp(4) to approximate $\int_3^{15} g(x)dx$. $\Delta x = 3$ $\quad \frac{12}{4} = 3$

$$\frac{1}{3}\left(g(3) + 4g(6) + 2g(9) + 4g(12) + g(15)\right)$$

$$\frac{1}{3}(2.5 + 20 + 7 + 16 + 6) \approx 17.1666667$$

Simp(4) = 17.167

[D] Write a definite integral giving the arc length of the curve given by $y = \ln\sqrt{x}$ on the interval from 1 to 10. Then approximate this arc length by using Trap(3) to estimate the integral. You must show all appropriate calculations to demonstrate your use of this rule (see the supplemented practice problems from the Arc Length section for an example). Round your final answer to the nearest hundredth if appropriate.

$$\int_a^b \sqrt{1+f'(x)^2}\,dx$$

$y = \frac{1}{2}\ln(x)$

$y' = \frac{1}{2x}$

cont. on seperate page

$$\int_1^{10} \sqrt{1+\frac{1}{4x^2}}\,dx$$

trapezoid rule

$n=3$ $\quad \frac{10-1}{3} = 3$ $\quad \Delta x = 3$

$$\frac{1}{2}(3)\left(f(1)+2f(4)+2f(7)+f(10)\right)$$

$$\frac{1}{2}(3)\left(\frac{\sqrt{5}}{2}+2\frac{\sqrt{65}}{8}+2\frac{\sqrt{197}}{14}+\frac{\sqrt{401}}{20}\right)$$

$\boxed{\text{D.}}$

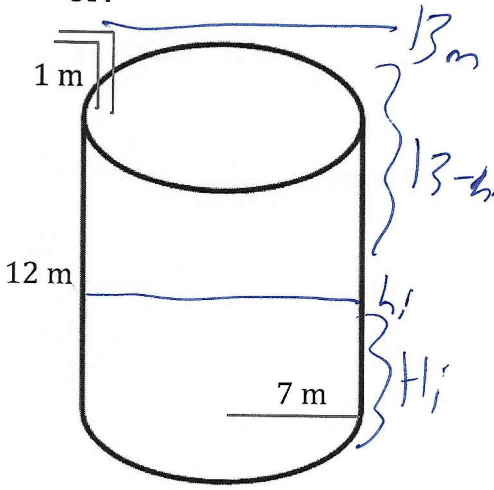
$$\frac{1}{2}(3)\left(\frac{\sqrt{5}}{2}\cancel{3} + 2\frac{\sqrt{65}}{8} + 2\frac{\sqrt{197}}{14} + \frac{\sqrt{401}}{20}\right)$$

$$\approx 9{,}209914793$$

$$= 9{,}210$$

*In the last three problems, be sure to show sufficient work to indicate how you are arriving at your integral. In particular, your work must clearly indicate what your variable represents.

[E] A cylindrical tank with radius 7 meters and a height of 12 meters is currently filled with water to a height of 8 meters. The water is to be pumped out to a height 1 meter above the top of the tank. Write an integral that gives the amount of work that is required to pump out enough water to leave 2 meters of water in the tank. (Note: the density of water is 1000 kg/m^3 and $g = 9.8$ m/sec^2). **DO NOT ATTEMPT TO EVALUATE THE INTEGRAL; JUST SET IT UP.**



density = 1000 kg/m^3

~~(1000)(49π)~~

$\Delta F_i = (1000)(49\pi)\Delta h$

$13 - h_i = D_i$

$D w_i = (49{,}000\,\pi\,\Delta H_i)(13 - h_i)$

$\int_0^8 (49{,}000\,\pi)^{(9.8)}(13-h)\,dh$

$= \int_0^8 (49{,}000\,\pi)(9.8)(13-h)\,dh$

$= \int_0^8 (49\pi)(9.8)(1000)(13-h)\,dh$

Work = $\int_0^8 (49\pi)(9.8)(1000)(13-h)\,dh$ (integral only) Units: m

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[F] An empty tank has the shape of a right circular cone. The height of the tank is 16 feet and the radius at the bottom is 4 feet. Write an integral giving the amount of work required to fill the tank with fuel through a hole in the base (at ground level) if the fuel source is at ground level. The weight-density of the fuel to be pumped is 60 lb/ft³. **DO NOT ATTEMPT TO EVALUATE THE INTEGRAL; JUST SET IT UP.**

$\frac{\sqrt{272} \cdot a}{4}$

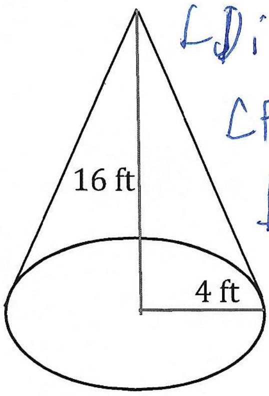
density = 60 lb/ft

$LD_i = 16 - h_i$

$CF_i = 60\pi\left(4 - \frac{h}{\sqrt{67}}\right)^2 \Delta h$

$\Delta V_i = \pi\left(4 - \frac{h}{\sqrt{67}}\right)^2 \Delta h$

$dH \uparrow$



h, h', 4, a

16, $\sqrt{272}$, 4

$\frac{a}{4}$

$a^2 + h^2 = \frac{272 \cdot a^2}{4}$

$h^2 = a^2\left(\frac{272}{4} - 1\right)$

$a = \sqrt{\frac{h^2}{67}}$

$a = \frac{h}{\sqrt{67}}$

$r = 4 - a$

$r = 4 - \frac{h}{\sqrt{67}}$

$\Delta f_i = \text{density} \cdot \Delta V_i$

$60 \cdot \pi\left(4 - \frac{h}{\sqrt{67}}\right)^2 \Delta h$

$\Delta f_i = 60\pi\left(4 - \frac{h}{\sqrt{67}}\right)^2 \Delta h$

$\int_0^{16} (60\pi)\left(4 - \frac{h}{\sqrt{67}}\right)^2 (16-h)\,dh$

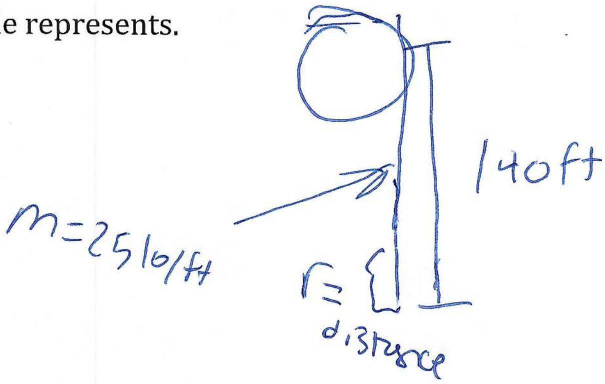
Work = $\int_0^{16} (60\pi)\left(4 - \frac{h}{\sqrt{67}}\right)^2 (16-h)\,dh$ (integral only) Units: Ft

[G] A 140 ft length of steel chain weighing 25 lb/ft is dangling from a pulley. How much work is required to wind the entire chain on the pulley? In this problem you are to **EVALUATE** the integral required to answer the question.

Include a diagram that clearly illustrates what your variable represents.

$$W = \int_a^b F \cdot dr$$

$F(r)$



$$= \int_0^{140} 25\ \text{lb/ft}\ dr$$

$a = 0$

$b = 140$

$r =$ distance

$f(r) = 25 \cdot r$

$F(r) = 25 \cdot r$

$25\ \text{lb/ft} \cdot r\ \text{ft}$

$$W_i = \int_a^b F(r)\,dr$$

$$W_i = \int_0^{140} 25r\,dr$$

$$= \frac{25}{2} r^2 \Big|_0^{140}$$

$$= \frac{25}{2} \cdot 140^2 \ (\text{lb} \cdot \text{ft})$$

$$= 2.45 \times 10^5\ \text{lb} \cdot \text{ft}$$

Work = 2.45×10^5 lb-ft