Unit 5.6: Work

The final application that we look at in this chapter is that of ***work***. In its simplest form, *work* is the product of *force* and *displacement*. Work is done by a force when the force causes an object to move. We will only consider movement along a straight line and in the positive direction, thus we can also think of displacement as the distance that the object was moved by the force. In examples where the forces and/or displacements involved are constant, we can simply use the formula $W = FD$. However, we will be encountering examples where these variables aren't necessarily constant and thus we will turn to definite integrals to carry out the calculations. Before turning to definite integrals, let's look at two examples that illustrate the basic concept of work.

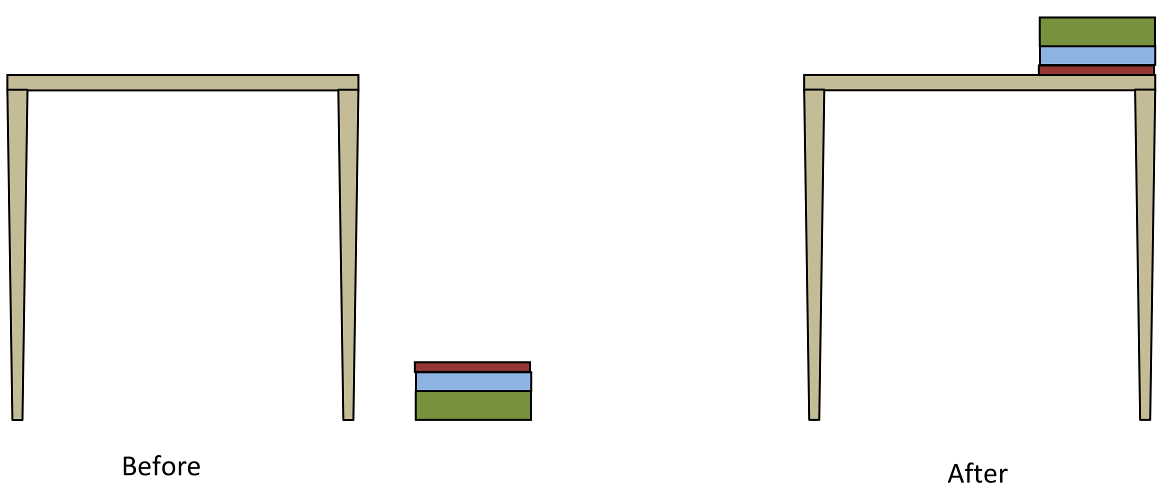
Example 1 Determine how much work is done in lifting a 2 lb book, from the ground, up 4 feet, and onto a table.

Solution: As in most of our examples, the force required to move an object will be equal to the weight of the object. Thus by the formula $W = FD$, we obtain

$$Work = (2\text{ lbs})(4\text{ ft}) = \boxed{8\text{ ft}\cdot\text{lbs}}$$

Take note of the final units in our answer. We simply multiply our units of force and displacement/distance. One might also encounter units such as inch-pounds, foot-tons, newton-meters (also known as joules), etc…

Example 2 Three books are stacked on the floor, one on top of the other. The top book is 1 inch thick and weighs 2 pounds, the middle book is 2 inches thick and weighs 1 pound, and the bottom book is 3 inches thick and weighs 3 pounds. Determine how much work is done in lifting the top book, then middle, then bottom up and onto a tabletop that is 3 feet from the ground, stacking the books in the reverse order. See the figure below.



Solution: If we simply wanted to simultaneously lift all three books and set them on the table in the same order, we would be moving 6 pounds a total of 3 feet and will therefore have done 18 foot-pounds of work. However, we have to be a little more careful here. Because we are reversing the order, each book which has a different weight (force), will travel a different distance. Thus we will separately calculate the work required for each book and then take the sum. The bottom of the top book begins 5 inches off of the ground and must be moved to the surface of the table which is 36 inches from the ground. Therefore, the top book will move a distance of 31 inches, thus requiring (2 lb.)(31 in.) = 62 in-lbs of work. The bottom of the middle book begins 3 inches off of the ground and must be moved to the top of the 1-inch book which is 37 inches from the ground. Therefore the middle book must be moved 34 inches, thus requiring (1 lbs.)(34 in.) = 34 in-lbs of work. Finally the bottom of the bottom book begins on the ground and must be moved to the top of the middle book which will be 39 inches from the ground. Thus the bottom book must be moved 39 inches, thus requiring (3 lbs.)(39 in.) = 117 in-lbs of work.

This gives us a grand total of $\boxed{\text{213 in-lbs or 17 ¾ ft-lbs.}}$

The main purpose of the second example was to motivate the approach that we will have to take when we get to examples where our variables are changing continuously. In that example, different parts (each book) of the whole (the entire stack) had different weights/forces and traveled different distances. Thus we broke the stack into its parts and did the work calculation for each, followed by the sum. In the narrated examples that go along with the Work handout, I will be focusing on problems where fluid will be pumped into or out of a tank as well as problems where chains will be lifted by a winch. Be sure to read through the additional examples in the book involving Hooke's Law, force and pressure.

Notation. Again, in this section I can't stress enough, the importance of you getting to learn to use the notation of calculus and the derivation of a definite integral to do a calculation. The goal of this section is not to learn to find work—it's to use a simple concept like work, to learn how to construct an integral. Thus in each of our examples, we will be looking at representative elements, forming increment formulas and then writing the definite integrals. Let's look at some of the notation that we will be using. In most of our examples we will be taking an object and breaking it up into "slices" in order to approximate the work required in moving each slice. In these examples, we will use the following notation.

ΔW_i = increment of work (the work done in moving a representative slice)

ΔF_i = increment of force (typically the weight of a representative slice)

ΔV_i = increment of volume (typically the volume of a representative slice)

D_i = distance/displacement (distance traveled by representative slice; this is not an increment)

Note: $\Delta W_i = \Delta F_i \cdot D_i$ and weight $= \Delta F_i = \Delta V_i \cdot \text{density}$