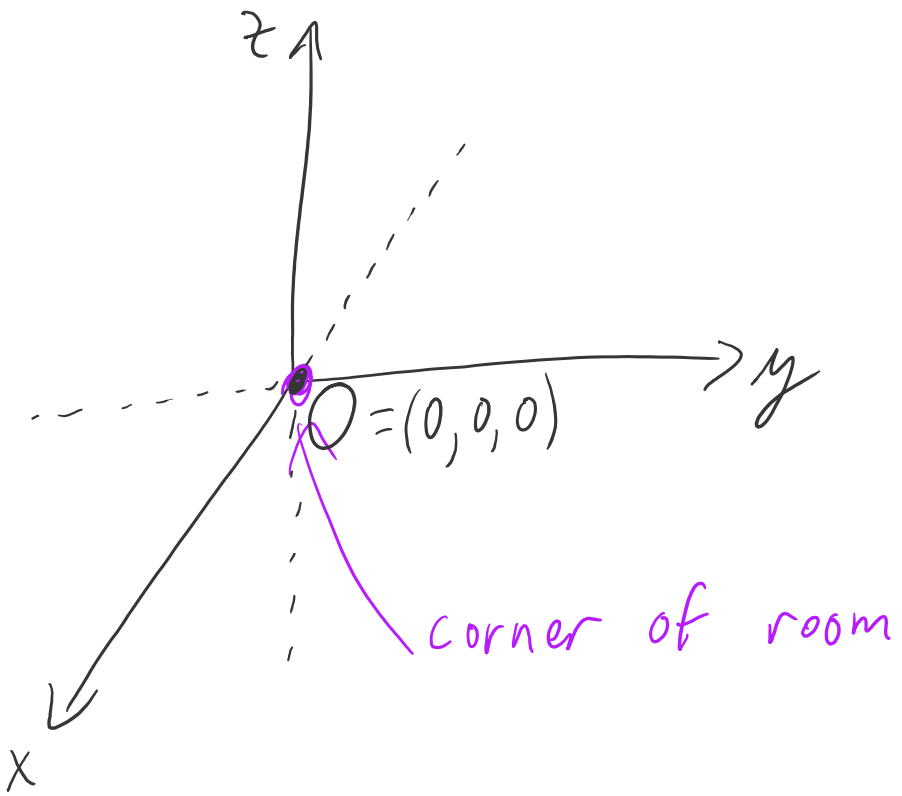
# 10.1: Three-Dimensional Coordinate Systems

Thursday, August 13, 2020 9:00 PM

## 3-dimensional coordinate systems

- $\mathbb{R}^2 = \{(x, y) : x, y \in \mathbb{R}\}$ is 2-dim'l.
- $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$
  $= \{(x, y, z) : x, y, z \in \mathbb{R}\}$ is 3-D.
  = points in space.



Def. 1) x, y, z axes are the coordinate axes.

2) The ~~coordt~~ three coordinate planes are

(i) left wall : xz-plane ($y = 0$)

(ii) back/right wall : yz-plane ($x = 0$)

(iii) floor : xy-plane ($z = 0$)

3) Coord. planes divide space into 8 octants.

first: foreground det. by pos. axes (standing here looking at

corner of room)

second: behind back/right wall, on your level,

⋮

etc.

4) Point in space $P = (a, b, c)$

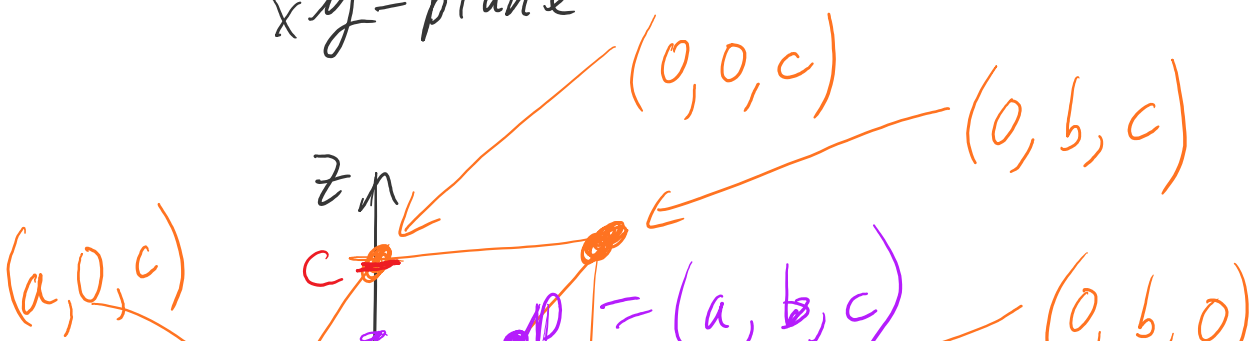
a = directed dist. from point P to yz-plane

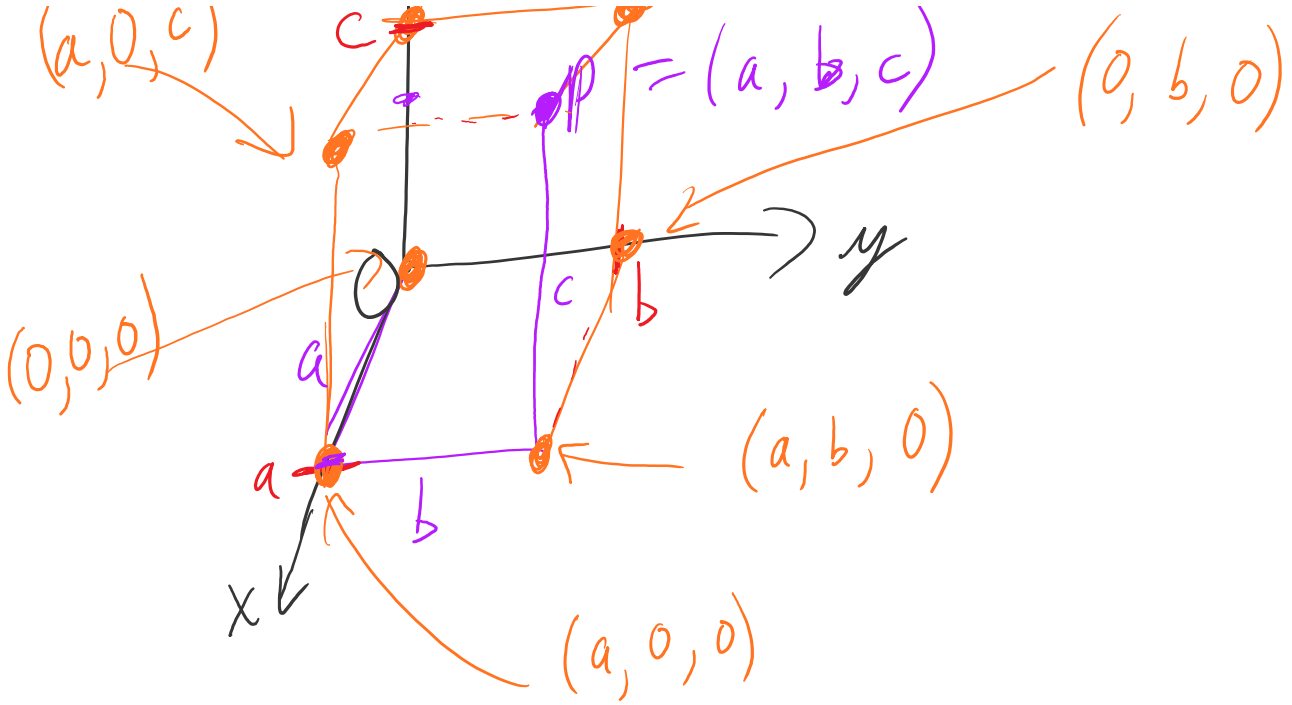
b = " " " " " "

xz-plane

c = " " " " " "

xy-plane






$P = (a,b,c)$ determines a box with corners $P$, $(a,0,0)$, $(a,b,0)$, $(0,b,0)$, $(a,0,c)$, $(0,0,c)$, $(0,b,c)$, $O = (0,0,0)$.

**Def.** • $(a,b,0)$ is the projection of $(a,b,c)$ onto the $xy$-plane.

• $(a,0,c)$ " " " " " " " "

" " $xz$-plane.

• $(0,b,c)$ " " " "

" $yz$-plane.

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Note 1) An eq'n in $x$ & $y$ ⇝ curve in $\mathbb{R}^2$.

2) " " " $x,y,z$ ⇝ surface in $\mathbb{R}^3$

3) Unless otherwise stated, assume in $\mathbb{R}^3$.

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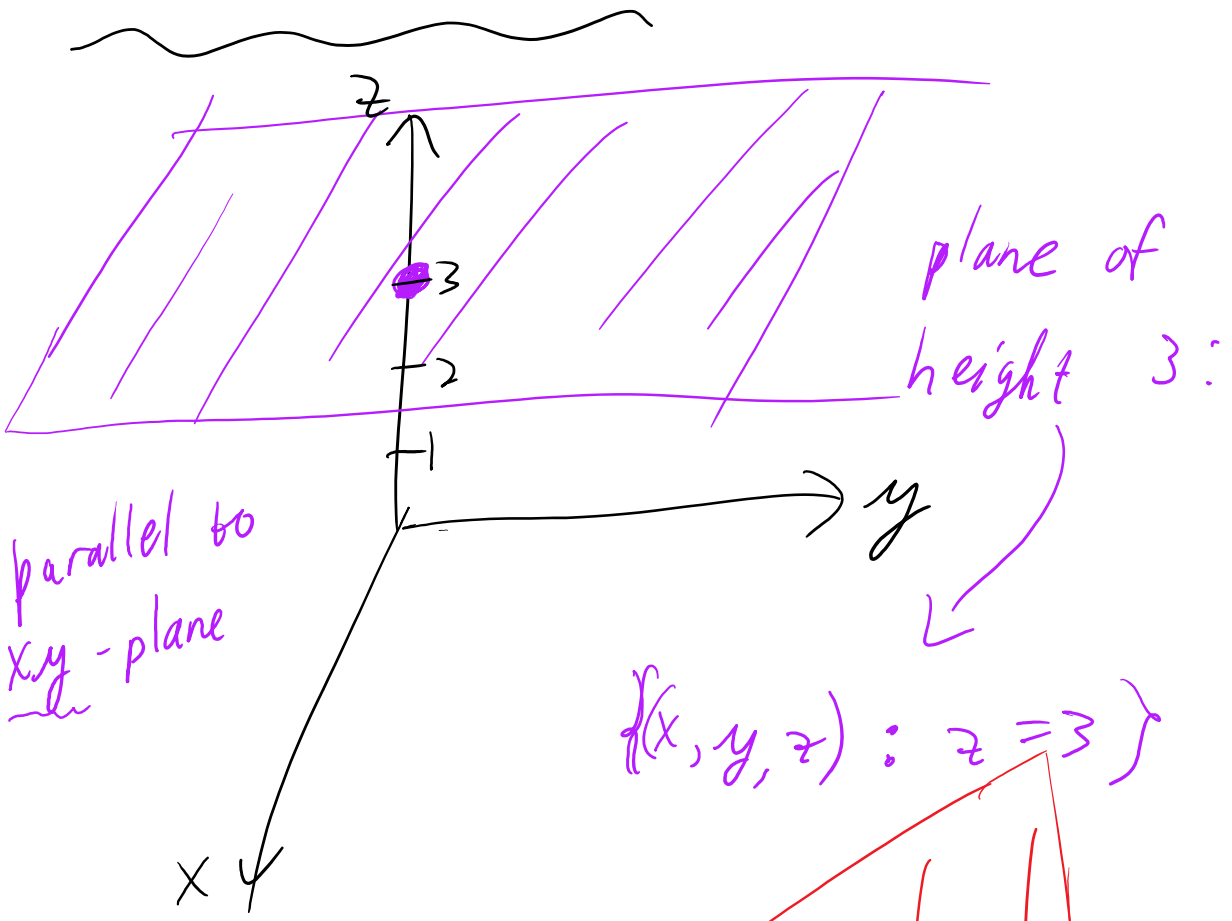
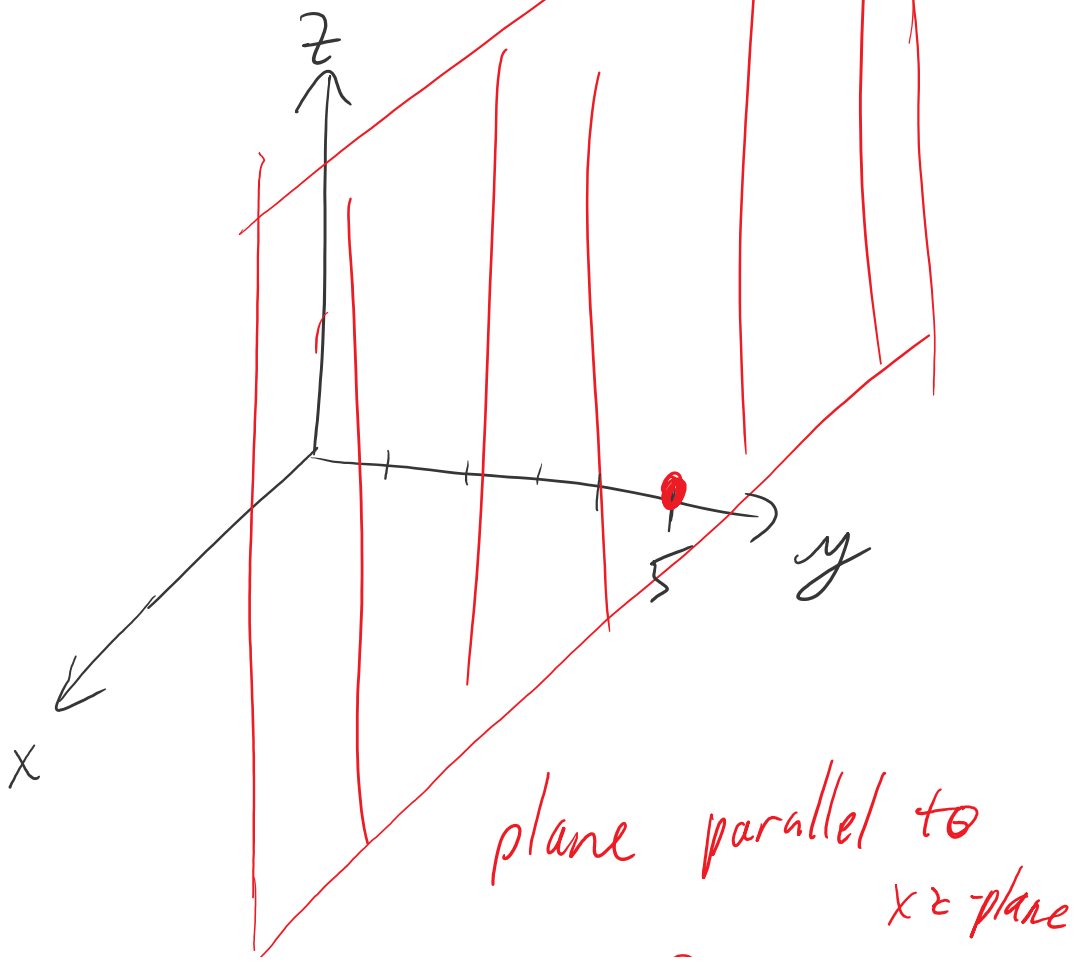
Ex. 1 What surface in $\mathbb{R}^3$ is given by

(a) $z=3$ ? (b) $y=5$


z
3
2
1
y
x
plane of height 3:
parallel to xy-plane
{(x, y, z) : z = 3}

(b)


z
y
x
plane parallel to xz-plane

$xz$-plane

$\{(x,y,z) : y = 5\}$

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**Fact** $x = k \rightsquigarrow$ plane in $\mathbb{R}^3$ parallel to $yz$-plane

(graph of $x = 0$ is the $yz$-plane)

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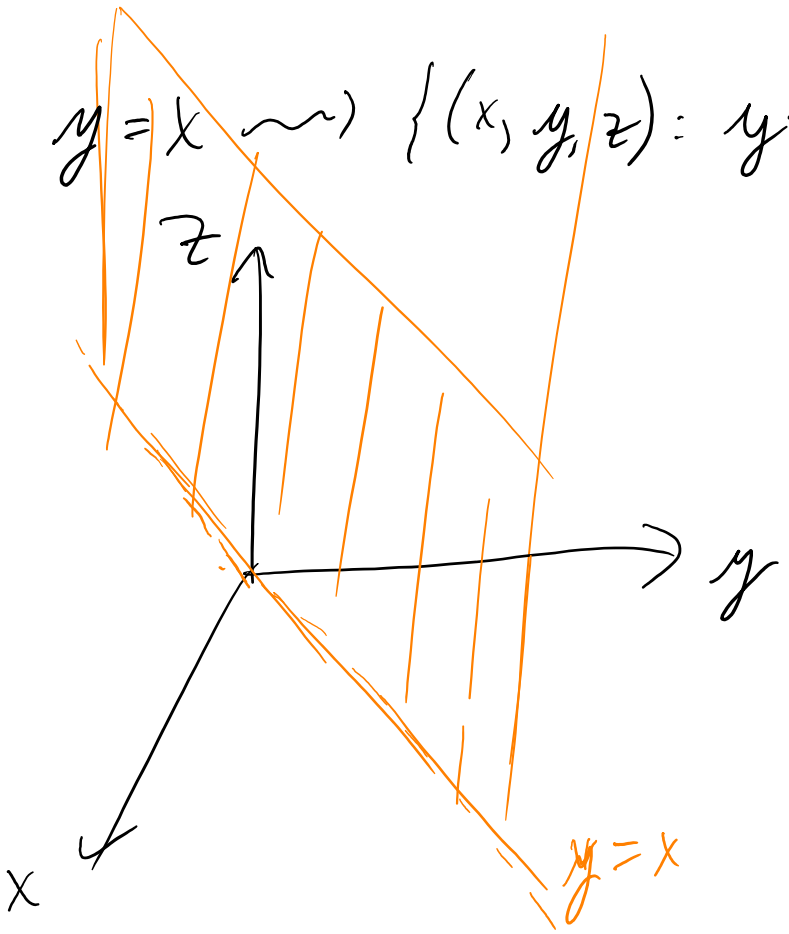
ex. $y = x \rightsquigarrow \{(x,y,z) : y = x\}$




not parallel to any coord. axes.

Distance formula

$P_1 = (x_1, y_1, z_1)$ , $P_2 = (x_2, y_2, z_2)$

$$|P_1P_2| = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$$

(same idea in 2-D)

Ex. 4 Find eq'n of sphere w/radius r and cent. $(h, k, \ell)$.

Soln (see text)

$$(x-h)^2 + (y-k)^2 + (z-\ell)^2 = r^2$$

Ex. 5 Show $x^2+y^2+z^2+4x-6y+2z+6=0$ is the eqn of a sphere and find center & radius.

Soln: $(x^2+4x+4)+(y^2-6y+9)+(z^2+2z+1)=-6+4+9+1$

$$(x+2)^2+(y-3)^2+(z+1)^2=8$$

$$r=\sqrt{8}=\boxed{2\sqrt{2}=r}$$

$$\boxed{C=(-2,3,-1)}$$

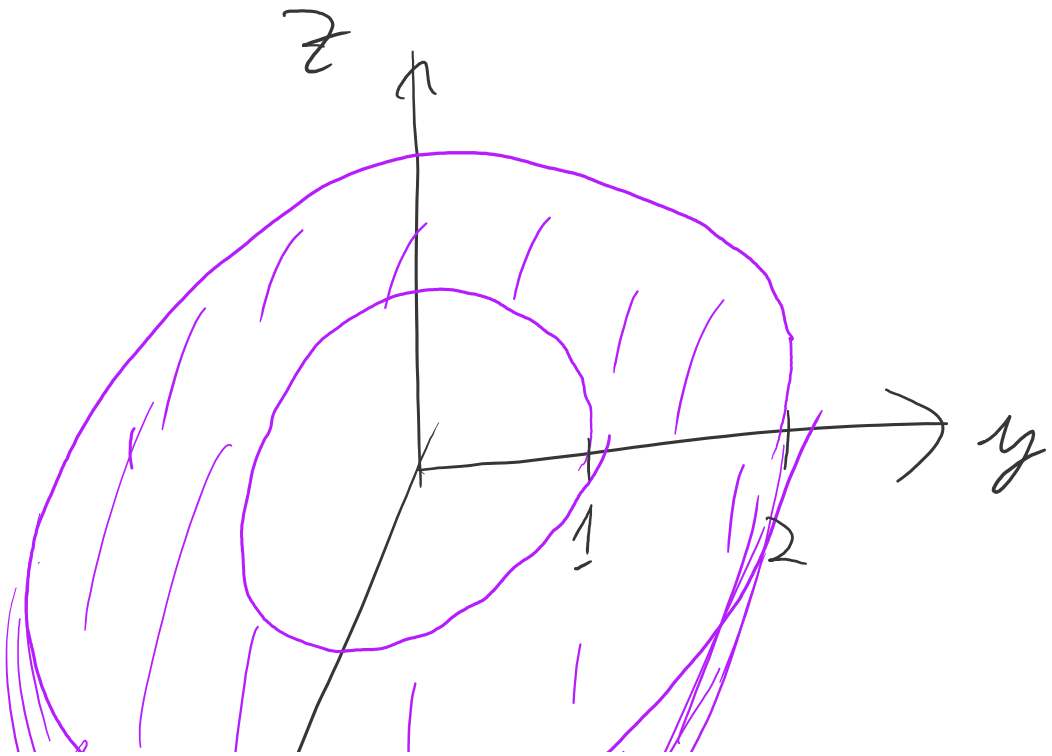
Ex. 6 What region in $\mathbb{R}^3$ is rep. by

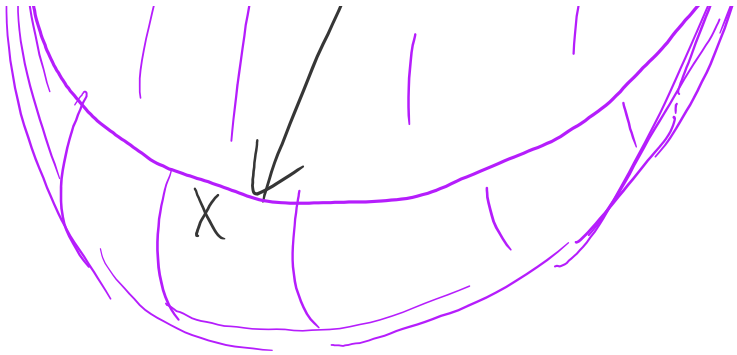
$$1 \le x^2+y^2+z^2 \le 4, \quad z \le 0 \ ?$$

$$1 \le \sqrt{x^2+y^2+z^2} \le 2, \quad z \le 0$$

dist. from $(x,y,z)$ to $(0,0,0)$

lower half space





region between spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$, below (and at) xy-plane.