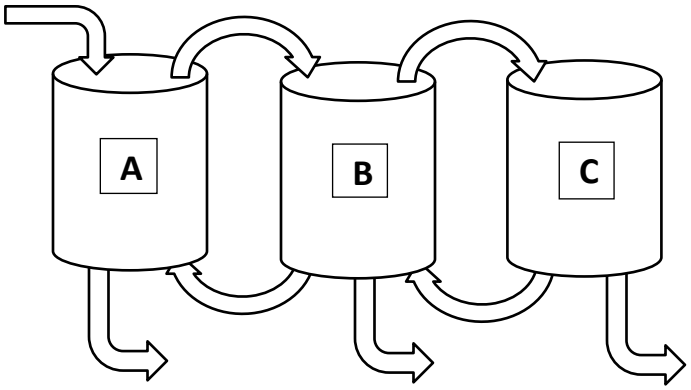
Math 225 – Quiz #14: Systems of Equations III

Clearly and neatly show all work for each problem. Solutions with no work will receive no credit.

1. Solve the following system of equations.

$$\frac{dx}{dt} = 3x - y - z$$
$$\frac{dy}{dt} = x + y - z + t$$
$$\frac{dz}{dt} = x - y + z + 2e^t$$

2. Three tanks are filled with a dye solution. Tanks A and B initially contains 20 hg of dye and tank C is initially pure water. A solution that contains 3 hg/L of dye is pumped into tank A at a rate of 50 L/hr. The solution in tank A flows out to tank B at a rate of 40 L/hr and out an exhaust spout at a rate of 20 L/hr. The solution in tank B flows into tank A at a rate of 10 L/hr, into tank C at a rate of 30 L/hr, and out an exhaust spout at a rate of 10 L/hr. The solution in tank C flows into tank B at a rate of 10 L/hr and out an exhaust spout at a rate of 20 L/hr. If tank A holds 200 L, tank B holds 100 L, and tank C holds 100 L, set up a system to determine the amount of dye in each tank at a given time t.



3. Consider the following three mass-four spring system below. The displacement of the three masses from their equilibrium positions are given by $x_1, x_2, \text{and } x_3$. (Note: right is considered to be the positive direction on the diagram). Set up a **first-order system** of differential equations (in matrix form) that represents the spring-mass system. You do NOT need to solve the system of differential equations.