MONROE COMMUNITY COLLEGE

MTH 211 – SLN

Unit 1 Written Assignment

Printed Name: Noor Mustafa

*See Blackboard for the deadline for submitting your assignment.

Directions:

- Be sure to follow the submission instructions given in Blackboard when submitting your completed assignment. Do NOT email me your completed assignment.
- Only methods covered in this course up to the current unit may be used on this assignment.
- In all problems you must show sufficient work to support your final answers. All such work must be done in this assignment document. If additional space is needed, you may add pages, but your work must be submitted in order.
- **The work you submit must be your own. While I cannot prevent students from discussing problems, suspicion of duplicated work will be investigated and penalties may result. <u>In addition, solutions taken from online calculators or the equivalent will be not be accepted and will be considered cheating.</u>**
- Be sure to include this page as a cover page for your assignment when you submit it and please make sure your name is written in the designated spot above. If you do not have access to a printer, you may write your solutions on regular paper, but each page must consist of solutions to only those problems on the corresponding page of the original exam.

Noor mustafa

[A] Use appropriate substitutions to determine the following integrals. You must show all work leading to your final answer and must use correct notation throughout.

(A.1) $\int \frac{x}{\sqrt{x+9}}\, dx$

$u(x) = x+9$

$du = dx$

$x(u) = u-9$

$\int \frac{x}{\sqrt{u}}\, du$

$\int \frac{u-9}{\sqrt{u}}\, du$

$\int \frac{u}{\sqrt{u}}\, du - \int \frac{9}{\sqrt{u}}\, du$

$\int u^{1/2}\, du - \int 9u^{-1/2}\, du$

$\frac{u^{3/2}}{3/2} - 9\frac{u^{1/2}}{1/2}$

$\frac{2}{3}u^{3/2} - 9 \cdot 2u^{1/2} + C \Rightarrow \frac{2}{3}u^{3/2} - 18 \cdot u^{1/2} + C$

$\frac{2}{3}(x+9)^{3/2} - 18(x+9)^{1/2} + C$

(A.2) $\int \frac{x}{\sqrt{12-4x^2-x^4}}\, dx$ [hint: complete the square]

$-x^4 - 4x^2 + 12$

$-(x^4 + 4x^2 - 12)$

$-(x^4 + 4x^2 + 4 - 4 - 12)$

$-((x^2+2)^2 - 16)$

$= -(x^2+2)^2 + 4^2$

$\int \frac{x}{\sqrt{4^2 - (x^2+2)^2}}\, dx$

$u(x) = x^2+2$

$\frac{du}{2} = \frac{2x\, dx}{2}$

$\frac{du}{2} = x\, dx$

$\int \frac{1}{\sqrt{4^2 - u^2}}\, du$

$u(\theta) = 4\sin\theta$

$\frac{du}{d\theta} = 4\cos\theta\, d\theta$

* continued on separate sheet

1 of 2

Noor Mustafa

A. 2

$$\int \frac{1}{\sqrt{4^2 - 4^2\sin^2\theta}} (4 \cdot \cos\theta \, d\theta)$$

$$\frac{1}{\cancel{4}} \int \frac{\cancel{4} \cdot \cos\theta}{\sqrt{1-\sin^2\theta}} \, d\theta \qquad \cos^2\theta = 1 - \sin^2\theta$$

$$= \int d\theta = \theta = \sin^{-1}\left(\frac{u}{4}\right) + C$$

$$\sin^{-1}\left(\frac{u}{4}\right) = \theta$$

$$\sin^{-1}\left(\frac{x^2+2}{4}\right) + C$$

1. complete square
2. u - subsitution
3. trig substitution
4. simplified

Moar mstafa

(A.3) $\int \frac{7e^x}{9+4e^{2x}}\,dx$ [hint: $e^{2x} = (e^x)^2$]

$\tan^2\theta + 1 = \sec^2\theta$

$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$\int \frac{7e^x}{9+(2e^x)^2}\,dx$

$u(x) = 2e^x$

$\frac{du}{2} = e^x\,dx$

$= \frac{7}{2}\int \frac{1}{9+u^2}\,du$

$= \frac{7}{2}\int \frac{1}{3^2+u^2}\,du$

$u(\theta) = 3\cdot\tan\theta$

$du = 3\cdot\sec^2\theta\,d\theta$

$= \frac{7}{2}\int \frac{1}{3^2+3^2\cdot\tan^2\theta}(3\cdot\sec^2\theta\,d\theta)$

$= \frac{7}{\cancel{2}}\int \frac{1}{3^{\cancel{2}}(1+\tan^2\theta)}(\cancel{3}\cdot\sec^2\theta\,d\theta)$

$= \frac{7}{6}\int \frac{1}{\cancel{\sec^2\theta}}\cancel{\sec^2\theta}\,d\theta$

$= \frac{7}{6}\theta + C = \frac{7}{6}\tan^{-1}\left(\frac{u}{3}\right) + C$

$= \frac{7}{6}\tan^{-1}\left(\frac{2e^x}{3}\right) + C$

(A.4) $\int \sec^n(x)\tan^3(x)\,dx$, where n is a positive integer.

$1+\tan^2 x = \sec^2 x$

$\int \sec^{n-1}(x)\cdot\tan^2 x\,(\sec x\cdot\tan x)\,dx$

$\int \sec^{n-1}(x)\cdot(\sec^2 x - 1)\cdot(\sec x\cdot\tan x\,dx)$

$u(x) = \sec x$

$du = \sec x\cdot\tan x\,dx$

$= \int u^{n-1}(u^2-1)\,du$

$= \int (u^{n+1} - u^{n-1})\,du$

$= \frac{u^{n+2}}{n+2} - \frac{u^n}{n} + C$

$= \frac{(\sec x)^{n+2}}{n+2} - \frac{(\sec x)^n}{n} + C$

Nour mustafa

[B] For the integral $\int_0^{\sqrt{\pi}} x\cos^2(x^2)\ dx$, carry out the following to determine its value:

Apply a basic u-substitution followed by an appropriate trig. identity to first rewrite the definite integral below in terms of u. Include the appropriate limits of integration that follow from your substitution. Then evaluate the integral using the Fundamental Theorem of Calculus. Please note: when grading this problem, I will be focusing on your correct use of natation and the procedures that you use, so do not be "sloppy" or incomplete. **Simplify, but do not approximate your answer (no decimals).**

$u(x)=x^2$

$\frac{du}{2}=\cancel{2}x\,dx$

$u(0)=0^2=0$

$u(\sqrt{\pi})=\sqrt{\pi}^2=\pi$

$\frac{1}{2}\int_{u=0}^{u=\pi}\cos^2(u)\,du$

$=\frac{1}{4}\int_0^{\pi}(u^2+\cos(2u))\,du$

$=\frac{1}{4}\cdot\left[u+\frac{\sin 2u}{2}\right]_0^{\pi}$

$=\frac{1}{4}\cdot\left[\pi+\frac{\sin 2\pi}{2}\right)-\left(0+\frac{\sin(0)}{2}\right)$

$=\boxed{\frac{\pi}{4}}$

[C] Use Integration by Parts to determine the following integrals.

(C.1) $\int x^2e^{x/2}\,dx$ $\quad\int u\,dv=uv-\int v\,du$

$u(x)=x^2$

$du=2x\,dx$

$\int dv=\int e^{x/2}\,dx$

$v(x)=\frac{e^{x/2}}{1/2}=2e^{x/2}$

$=x^2\cdot 2e^{x/2}-\int(2e^{x/2})\cdot(2x)\,dx$

$=2x^2e^{x/2}-4\int x\cdot e^{x/2}\,dx$

$u(x)=x$

$du=dx$

$v(x)=2e^{x/2}$

$=2x^2e^{x/2}-4[2xe^{x/2}$

1 of 2

$\boxed{C.1}$ $\boxed{\text{Noor Mustafa}}$

$$= 2x^2 e^{x/2} - 4\left[2x e^{x/2} - \int (2e^{x/2})\,dx\right]$$

$$2\int e^{x/2}\,dx = 2\frac{e^{x/2}}{1/2}$$

$$= 4e^{x/2}$$

$$= 2x^2 e^{x/2} - 8xe^{x/2} + 16e^{x/2} + C$$

$\boxed{C.2}$

$$= \frac{\ln x \cdot x^{n+1}}{n+1} - \int \frac{x^{n+1}}{n+1} \cdot \frac{1}{\cancel{x}}\,dx$$

$$= \frac{\ln x \cdot x^{n+1}}{n+1} - \frac{1}{n+1} \cdot \int x^{n}\,dx$$

$$= \frac{\ln x \cdot x^{n+1}}{n+1} - \frac{x^{n+1}}{(n+1)^2} + C$$

Nour mustafa

(C.2) $\int x^n \ln(x)\, dx$ where n is a real number with $n \neq -1$.

integration by parts

$u(x) = \ln x$

$du = \frac{1}{x} dx$

$x\, du = dx$

$\int x^n \cdot u(x\, du)$

$\int x^{n+1} \cdot u\, du$

$u = \ln x$

$e \quad e$

$x = e^u$

$\int (e^u)^{n+1} \cdot u\, du$

doesnt work

$\int u\, dv = uv - \int v\, du$

$u(x) = \ln x$

$dv = x^n dx$

$du = \frac{1}{x} dx$

$v(x) = \frac{x^{n+1}}{n+1}$

continues on seperate page

1 of 2

***Problems [D] and [E] are conceptual problems intended to test your understanding of the associate process. They intentionally differ from the more routine problems you've seen.**

[D] For each of the following integrals, find a function, $f(x)$, for which the integral could be determined using a basic u-substitution. Then using the function that you found, determine the integral.

(D.1) $\int f(x) \sin(\ln x)\, dx$

$f(x) = \frac{1}{x}$

$\int \frac{\sin(\ln x)}{x} dx$

$u(x) = \ln x$

$du = \frac{1}{x} dx$

$= \int \sin(u)\, du$

$= -\cos(u) + C$

$= -\cos(\ln x) + C$

$\boxed{C_2}$ $\boxed{\text{Nour}}$

$$= \frac{\ln x - x^{n+1}}{n+1} - \int \frac{x^{n+1}}{n+1} - \frac{1}{x} dx$$

$$= \frac{\ln x - x^{n+1}}{n+1} - \frac{1}{n+1} - \int x^n dx$$

$$= \frac{\ln x - x^{n+1}}{n+1} - \frac{x^{n+1}}{(n+1)^2} + C$$

Noor mustafa

(D. 2) $\int \frac{f(x)}{(\cos x + e^x)^2} dx$

$f(x) = -\sin x + e^x$

$\int \frac{(-\sin x + e^x)}{((\cos x + e^x)^2} dx$

$u(x) = \cos x + e^x$

$du = (-\sin x + e^x)\, dx$

$\int \frac{(-\sin x + e^x)}{((\cos x + e^x)^2} dx$

$= \frac{1}{u^2} du$

$= \int u^{-2} du$

$= \frac{u^{-1}}{-1} + C$

$= -\frac{1}{u} + C$

$= -\frac{1}{\cos x + e^x} + C$

$\frac{1}{\cos x + e^x} + C$

[E] Let $g(x)$ be a function with derivative and antiderivatives given below (note: these are made up functions).

email him

$$g'(x) = der(x) \quad \text{and} \quad \int g(x)\, dx = ant(x) + C$$

Use integration by parts to determine the following integral. You will have to express your final answer in terms of the function(s): $g(x)$, $der(x)$, and/or $ant(x)$.

$\int x \cdot der(2x)\, dx$ $\quad \int u\, dv = u \cdot v - \int v\, du$

$u(x) = x$

$du = dx$

$dv = der(2x)\, dx$

$v(x) = \int der(2x)\, dx$

$= 2\int g'(x)\, dx$

$= x \cdot (2 \int \underbrace{g'(x)}_{der(x)} dx) - \int (2) \underbrace{g'(x)}_{der(x)} dx)\, dx$

$= 2x \int der\, x\, dx - 2\int (\int der\, x\, dx)\, dx$

Noor mustafa

[F] Translate the following statement into a differential equation (don't solve it).

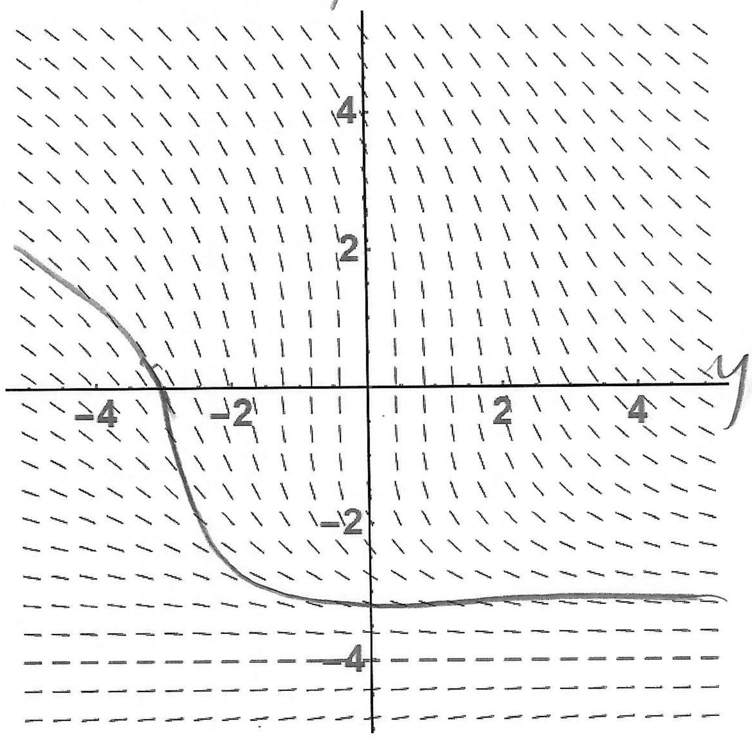
"N changes at a rate (with respect to t) that is directly proportional to the square of N and inversely proportional to t."

$$\frac{dn}{dt} \propto \frac{n^2}{t}$$

$$\frac{dn}{dt} = C \cdot \frac{n^2}{t}$$

[G] The slope field for the differential equation $\frac{dy}{dx} = -\frac{16+4y}{x^2+y^2}$ is given below. Use the slope field to sketch the solution satisfying $y(-3) = 0$. Then use the slope field to determine $\lim_{x \to \infty} y(x)$ for the function that you sketched.

Phase field



$$y' = -\frac{16+4y}{x^2+y^2}$$

$$(x^2+y^2)\frac{dy}{dx} = -16+4y$$

$$x^2y' + y^2y' = -16+4y$$

$$y^2y' - 4y = -x^2y' - 16$$

$$(y, y')$$

$\lim_{x \to \infty} y(x) =$ 0

Nour Mustafa

[H] Consider the differential equation $\dfrac{dy}{dx} = \dfrac{xe^{4y}}{x^2+1}$

(H.1) Find the general solution of the above differential equation. You may leave your answer in implicit form for this part.

$u(x) = x^2+1$

$\frac{du}{2} = x\,dx$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln|x^2+1| + C$

$e^{-4y}(dx)\frac{dy}{\cancel{dx}} = \frac{x e^{4y}}{x^2+1}(dx)$

$\int e^{-4y}\,dy = \int \frac{x}{x^2+1}\,dx$

$\frac{e^{-4y}}{-4} = \frac{1}{2}\ln|x^2+1| + C_1$

$e^{-4y} = -2\ln|x^2+1| + C_2$

(H.2) Find the particular solution of the above differential equation satisfying $y(0) = 0$. Write your answer in EXPLICIT form [i.e. $y = f(x)$] simplifying where possible.

$e^{-4y(x)} = -2\ln|x^2+1| + C$

$e^{\cancel{-4y(0)}} = \cancel{-2\ln|0^2+1|} + C$

$1 = C_2$

$e^{-4y} = -2\ln|x^2+1| + 1$

$\ln(e^{-4y}) = \ln(-2\cdot\ln|x^2+1| + 1)$

$\frac{-4y(x)}{-4} = \frac{\ln(-2\cdot\ln|x^2+1|+1)}{-4}$

X on separate page

Lof c

$\boxed{H_2}$ Noor Mustafa

$$y(x) = -\frac{1}{4} \ln\left[-2 \cdot \ln|x^2+1| + 1\right]$$