Quiz #10 | Moor | MATH 225

1. $\mathcal{L}\{\sin 2t\} = \frac{2}{s^2+4}$

2. a. $\mathcal{L}\{y'\} = sf(s) - f(0)$

$\mathcal{L}\{6y\} = 6f(s)$

$\mathcal{L}\{e^{4t}\} = \frac{1}{s-4}$

$sf(s) - f(0) + 6f(s) = \frac{1}{s-4}$

$F(s)[s+6] - 2 = \frac{1}{s-4}$

$F(s) = \frac{\frac{1}{s-4} + 2}{s+6} = \frac{1+2(s-4)}{\frac{s-4}{s+6}}$

$F(s) = \frac{2s-7}{(s-4)(s+6)} = \frac{A}{s-4} + \frac{B}{s+6}$

$s = 2 = A+B$

$= -7 = 6A - 4B$

$8 = 4A + 4B$

$-7 = 6A - 4B$

$1 = 10A$

$A = \frac{1}{10}$

$B = \frac{19}{10}$

2. A. Quiz 10) Nour | MTH 225

$$F(s) = \frac{1}{10}\,\frac{1}{s-4} + \frac{19}{10}\,\frac{1}{s+6}$$

$$s = \frac{1}{10}e^{4t} + \frac{19}{10}e^{-6t}$$

2.

B. $y'' - 4y' = 6e^{3t} - 3e^{-t}$, $y(0) = 1$, $y'(0) = -1$

$\mathcal{L}\{y'' - 4y'\} = \mathcal{L}\{6e^{3t} - 3e^{-t}\}$

$\mathcal{L}\{y'' - 4y'\}$

$= \mathcal{L}\{y''\} - 4\mathcal{L}\{y'\}$

$\mathcal{L}\{y''\} = s^2\mathcal{L}\{y\} - sy(0) - y'(0)$

$\mathcal{L}\{y'\} = s\mathcal{L}\{y\} - y(0)$

$= s^2\mathcal{L}\{y\} - sy(0) - y'(0) - 4(s\mathcal{L}\{y\} - y(0))$

$\mathcal{L}\{6e^{3t} - 3e^{-t}\}$

$\mathcal{L}\{e^{3t}\} = \frac{1}{s-3}$

$\mathcal{L}\{e^{-t}\} = \frac{1}{s+1}$

$= 6\frac{1}{s-3} - 3\frac{1}{s+1}$

$= \frac{6}{s-3} - \frac{3}{s+1}$

$s^2\mathcal{L}\{y\} - s(1) + 1 - 4(s\mathcal{L}\{y\} - 1) = \frac{6}{s-3} - \frac{3}{s+1}$

Noor Mostafa MATH 225 QUIZ 10 (2)

(2-B.)

$$s^2L\{y\} - 4sL\{y\} - s + 5 = \frac{3s+15}{(s-3)(s+1)}$$

$$s^2L\{y\} - 4sL\{y\} = \frac{3s+15}{(s-3)(s+1)} + s - 5$$

$$sL\{y\}(s-4) = \frac{3s+15}{s^2-2s-3} + s - 5$$

$$\frac{sL\{y\}(s-4)}{s(s-4)} = \frac{3s+15}{s^2-2s-3} + \frac{s}{s(s-4)} - \frac{5}{s(s-4)}$$

$$L\{y\} = \frac{s^3 - 7s^2 + 10s + 30}{s(s^2-2s-3)(-4+s)}$$

$$y = L^{-1}\left\{\frac{s^3 - 7s^2 + 10s + 30}{s(s^2-2s-3)(-4+s)}\right\}$$

$$= L^{-1}\left\{\frac{5}{2s}\right\} - L^{-1}\left\{\frac{3}{5(s+1)}\right\} - L^{-1}\left\{\frac{2}{s-3}\right\} + L^{-1}\left\{\frac{11}{10(s-4)}\right\}$$

$$L^{-1}\left\{\frac{5}{2s}\right\} = \frac{5}{2}$$

$$L^{-1}\left\{\frac{3}{5(s+1)}\right\} = \frac{3}{5}e^{-t}$$

$$L^{-1}\left\{\frac{2}{s-3}\right\} = 2e^{3t}$$

$$L^{-1}\left\{\frac{11}{10(s-4)}\right\} = \frac{11}{10}e^{4t}$$

$$y = \frac{5}{2} - \frac{3}{5}e^{-t} - 2e^{3t} + \frac{11}{10}e^{4t}$$

Quiz 10) Noor Murtaza / MTH 225

2. $y''' + 2y'' - y' - 2y = \sin 3t$ $\quad y(0)=0$, $y'(0)=0$, $y''(0)=1$

(c) $L\{y''' + 2y'' - y' - 2y\} = L\{\sin 3t\}$

$L\{y'''\} + 2L\{y''\} - L\{y'\} - 2L\{y\}$

$L\{y'''\} = s^3 L\{y\} - s^2 y(0) - s y'(0) - y''(0)$

$L\{y''\} = s^2 L\{y\} - s y(0) - y'(0)$

$L\{y'\} = s L\{y\} - y(0)$

$= s^3 L\{y\} - s^2 y(0) - s y'(0) - y''(0) + 2(s^2 L\{y\} - s y(0) - y'(0)) - (s L\{y\} - y(0)) - 2L\{y\}$

$L\{\sin(3t)\} = \dfrac{3}{s^2+9}$

$s^3 L\{y\} - s^2 y(0) - s y'(0) - y''(0) + 2(s^2 L\{y\} - s y(0) - y'(0)) - (s L\{y\} - y(0)) - 2L\{y\} = \dfrac{3}{s^2+9}$

$y(0) = 0$, $y'(0) = 0$, $y''(0) = 1$

$s^3 L\{y\} - s^2 \cdot 0 - s \cdot 0 - 1 + 2(s^2 L\{y\} - s \cdot 0 - 0) - (s L\{y\} - 0) - 2L\{y\} = \dfrac{3}{s^2+9}$

$s^3 L\{y\} + 2s^2 L\{y\} - s L\{y\} - 2L\{y\} - 1 = \dfrac{3}{s^2+9}$

Quiz 10 Noor mustafa MTH 225 (7)

2. (c) $s^3\mathcal{L}\{y\} + 2s^2\mathcal{L}\{y\} - 5\mathcal{L}\{y\} - 2\mathcal{L}\{y\}$

$= \frac{3}{s^2+9} + 1$

$s^3\mathcal{L}\{y\} + 2s^2\mathcal{L}\{y\} - 5\mathcal{L}\{y\} - 2\mathcal{L}\{y\}$

$= \mathcal{L}\{y\}(s^3 + 2s^2 - s - 2)$

$\mathcal{L}\{y\}(s^2-1)(s+2) = \frac{3}{s^2+9} + 1$

$\mathcal{L}\{y\} = \frac{3}{(s^2+9)(s^2-1)(s+2)} + \frac{1}{(s+2)(s^2-1)}$

$\mathcal{L}\{y\} = \frac{s^2+12}{(s^2+9)(s^2-1)(2+s)}$

$y = \mathcal{L}^{-1}\left\{\frac{s^2+12}{(s^2+9)(s^2-1)(2+s)}\right\}$

$\frac{s^2+12}{(s+1)(s-1)(s+2)(s^2+9)} = \frac{a_1 s + a_0}{s^2+9} + \frac{a_2}{s+1} + \frac{a_3}{s-1} + \frac{a_4}{s+2}$ ← partial fraction

$s^2+12 = (a_1 s + a_0)(s+1)(s-1)(s+2) + a_2(s^2+9)(s-1)(s+2) + a_3(s^2+9)(s+1)(s+2) + a_4(s^2+9)(s+1)(s-1)$

2-6. $a_2 = -\frac{13}{20}$

(7)

Nour Matrizes
Quiz 10

$a_3 = \frac{13}{60}$

$a_4 = \frac{16}{39}$

$s^2 + 12 = -2a_0 + \frac{774}{65} + a_1 s^4 - \frac{3s^4}{130}$
$+ a_0 s^3 + 2a_1 s^3 + 2a_0 s^2 + \frac{29s^2}{26} - a_1 s^2$
$- a_0 s - 2a_1 s$

$s^2 + 12 = -2a_0 + \frac{774}{65} + a_1 s^4 - \frac{3}{130} s^4 + a_0 s^3$
$+ 2a_1 s^3 + 2a_0 s^2 + \frac{29}{26} s^2 - a_1 s^2 - a_0 s - 2a_1 s$

$-2a_0 + \frac{774}{65} = 12$

$-a_0 - 2a_1 = 0$

$2a_0 + \frac{29}{26} - a_1 = 1$

$a_0 + 2a_1 = 0$

$a_1 - \frac{3}{130} = 0$

$a_0 = -\frac{3}{65}$

$a_1 = \frac{3}{130}$

$$\frac{\frac{3}{130} s + \left(-\frac{3}{65}\right)}{s^2 + 9} + \frac{-\frac{13}{20}}{s+1} + \frac{\frac{13}{60}}{s-1} + \frac{\frac{16}{39}}{s+2}$$

2. C) $\left\{\frac{3s-6}{130(s^2+9)} - \frac{13}{20(s+1)} + \frac{13}{60(s-1)} + \frac{16}{39(s+2)}\right\}$

$= L^{-1}\{$ ↙ $\}$

$\frac{3s-6}{130(s^2+9)} = \frac{3s}{130(s^2+9)} - \frac{6}{130(s^2+9)}$

$= L^{-1}\left\{\frac{3s}{130(s^2+9)} + \frac{13}{60(s+1)} + \frac{16}{39(s+2)} - \frac{6}{130(s^2+9)} - \frac{13}{20(s+1)}\right\}$

$L^{-1}\left\{\frac{s}{s^2+9}\right\} = \cos 3t$

$L^{-1}\left\{\frac{1}{s^2+9}\right\} = \frac{1}{3}\sin 3t$

$L^{-1}\left\{\frac{1}{s+1}\right\} = e^{-t}$

$L^{-1}\left\{\frac{1}{s-1}\right\} = e^{t}$

$L^{-1}\left\{\frac{1}{s+2}\right\} = e^{-2t}$

$= \frac{3}{130}\cos(3t) - \frac{6}{130}\cdot\frac{1}{3}\sin 3t - \frac{13}{20}e^{-t} + \frac{13}{60}e^{t} + \frac{16}{39}e^{-2t}$

$$\frac{6}{130} - \frac{1}{5}\sin(3t) = \frac{\sin(3t)}{65}$$

$$y = \frac{3}{130}\cos(3t) - \frac{\sin(3t)}{65} - \frac{13}{20}e^{-t} + \frac{13}{60}e^{t} + \frac{16}{39}e^{-2t}$$