QUIZ #8 Noor Mustafa | MTH 225

$a_n x^n y^{(n)} + \dots + a_1 x y' + a_0 y = 0$

① $x^3 y''' - 6y = 0$

$y = x^r$

$x^3 (x^r)''' - 6x^r = 0$

$(x^r)''' = r x^{r-3} (r-1)(r-2)$

$x^3 r x^{r-3} (r-1)(r-2) - 6x^r = 0$

$x^3 r x^{r-3} (r-1)(r-2) = r x^r (r-1)(r-2)$

$= r x^r (r-1)(r-2) - 6x^r$

$r x^r (r-1)(r-2) - 6x^r = 0$

$x^r (r(r-1)(r-2) - 6) = 0$

$x^r \neq 0$

$r((r-1)(r-2) - 6) = 0$

$r^3 - 3r^2 + 2r - 6 = 0$

$(r-3)(r^2+2) = 0$

$r = 3 \qquad r = \pm\sqrt{2}\, i$

$\downarrow$

$y = c_1 x^3 \qquad r_1 \neq r_2$

$r_1 = a + iB \quad r_2 = a - iB$

(Quiz #8) MTH 1225) Noor Mustafa)

(1)

$$x^0 \left(c_2 \cos\left(\sqrt{2}\ \ln(x)\right) + c_3 \sin\left(\sqrt{2}\ \ln(x)\right)\right)$$

$$c_2 \cos\left(\sqrt{2}\ \ln(x)\right) + c_3 \sin\left(\sqrt{2}\ \ln(x)\right)$$

$$y = c_1 x^3 + c_2 \cos\left(\sqrt{2}\ \ln(x)\right) + c_3 \sin\left(\sqrt{2}\ \ln(x)\right)$$

general solution

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② $xy'' - 4y' = x^4$

$y = y_g + y_p$

$y = x^r$ $\quad xy'' - 4y' = 0$

$x(x^r)'' - 4(x^r)' = 0$

$(x^r)'' = rx^{r-2}(r-1)$

$(x^r)' = rx^{r-1}$

$xrx^{r-2}(r-1) - 4rx^{r-1} = 0$

$r^2x^{r-1} - 5rx^{r-1} = 0$

$x^{r-1}(r^2 - 5r) = 0$

$x^{r-1} \neq 0$

$r^2 - 5r = 0$

$$r_{1,2} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(0)}}{2}$$

$r_1 = 5 \quad r_2 = 0$

$y = c_1x^{r_1} + c_2x^{r_2}$

$y = c_1x^5 + c_2x^0$

$y = c_1x^5 + c_2$

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(2)

$$y_p = \frac{xy'' - 4y'}{x} = \frac{x^4}{x}$$

$$y'' - \frac{4y'}{x} = x^3$$

$$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0 = g(x)$$

$$u_1 = \int \frac{-y_2\, g(x)}{w}\, dx$$

$$u_2 = \int \frac{y_1\, g(x)}{w}\, dx$$

$y_1 = x^5$

$y_2 = 1$

$y_1' = 5x^4$

$y_2' = 0$

$w(y_1, y_2) = x^5 \cdot 0 - 5x^4 \cdot 1$

$w = -5x^4$

$$u_1 = \int \frac{-1(x^3)}{-5x^4}\, dx$$

$$= \frac{-1}{-5} \int \frac{x^3}{x^4}\, dx$$

$$= \frac{1}{5} \int \frac{1}{x}\, dx$$

$$= \frac{1}{5} \ln(x) + C$$

$$u_1 = \frac{1}{5} \ln(x)$$

$$u_2 = \int \frac{x^5 x^3}{-5x^4}\, dx$$

$$= \frac{1}{-5} \int \frac{x^5 x^3}{x^4}\, dx$$

$$= \left(\frac{1}{-5}\right) \int x^4\, dx$$

power rule →

$$= \left(-\frac{1}{5}\right) \frac{x^{4+1}}{4+1}$$

$$= -\frac{1}{5} \cdot \frac{x^5}{5}$$

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$$= -\frac{x^5}{25} + C$$

$$u_2 = -\frac{x^5}{25}$$

$$y_p = u_1 y_1 + u_2 y_2$$

$$y_p = \frac{1}{5}\ln(x)\,x^5 + \left(-\frac{x^5}{25}\right)\cdot 1$$

$$y_p = \frac{1}{5}x^5\ln(x) - \frac{x^5}{25}$$

$y = y_g + y_p \leftarrow$ formula for general solution

$$\boxed{y = c_1 x^5 + C_2 + \frac{1}{5}x^5\ln(x) - \frac{x^5}{25}}$$

$\uparrow$
general
solution

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(3) $x^2y'' - 5xy' + 8y = 0$, $y(2) = 32$, $y'(2) = 0$

$ax^2y'' + bxy' + cy = 0$ $\quad y = x^r$

$$x^2(x^r)'' - 5x(x^r)' + 8x^r = 0$$

$$(x^r)'' = rx^{r-2}(r-1)$$

$$(x^r)' = rx^{r-1}$$

$$x^2rx^{r-2}(r-1) - 5xrx^{r-1} + 8x^r = 0$$

$$r^2x^r - 6rx^r + 8x^r = 0$$

$$x^r(r^2 - 6r + 8) = 0$$

$$x^r \neq 0$$

$a = 1$
$b = -6$
$c = 8$

$$r_{1,2} = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(8)}}{2(1)}$$

$$r_1 = 4 \quad r_2 = 2$$

$$y = c_1x^4 + c_2x^2$$

$$32 = c_1 \cdot 2^4 + c_2 \cdot 2^2$$

$$c_1 = \frac{8 - c_2}{4}$$

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(3)

$$y = \frac{8-c_2}{4} x^4 + c_2 x^2$$

$$y = \frac{x^4(8-c_2)}{4} + c_2 X^2$$

$$0 = 2^3(8-c_2) + 2c_2 \cdot 2$$

$$2^3(-c_2+8) + 4c_2 = 0$$

$$8(8-c_2) + 4c_2 = 0$$

$$64 - 8c_2 + 4c_2 = 0$$

$$64 - 4c_2 = 0$$

$$\frac{-4c_2}{-4} = \frac{-64}{-4}$$

$$c_2 = 16$$

for $y = \frac{x^4(8-c_2)}{4} + c_2 x^2$

$$y = \frac{x^4(8-16)}{4} + 16x^2$$

$$y = -2x^4 + 16x^2$$