Quiz #2 exact equations | Noor Mustafa

① $(2xy^2-3)dx+(2x^2y+4)dy=0$

$M(x,y)+N(x,y)\frac{dy}{dx}=0$

$M_y = 4xy$

$N_x = 4xy$

$F_y = N$

$\int M\,dx = \int 2xy^2-3\,dx$

$F = x^2y^2-3x+g(y)$

$F_y = 2x^2y+g'(y)$

$2x^2y+4 = 2x^2y+g'(y)$

$4 = g'(y)$

$g(y) = 4y+C$

$F = x^2y^2-3x+4y+C = 0$

$x^2y^2-3x+4y+C=0$

Quiz 2) non-exact

(2) $M(x,y) + N(x,y)\frac{dy}{dx} = 0$

$M = 1 + \ln x - \frac{y}{x}$ $\quad M_y = -\frac{1}{x}$

$N = 1 - \ln x$ $\quad N_x = -\frac{1}{x}$

$F = \int M\,dx = \int 1 + \ln x - \frac{y}{x}$

$F = x - y\ln x + C + \int \ln(x)\,dx$

$F = x + x\ln x - x - y\ln x + g(y)$

$F_y = -\ln(x) + g'(y) \Rightarrow N$

$F_y = N$

$1 - \ln(x) = -\ln(x) + g'(y)$

$g'(y) = 1$

$g(y) = y + C$ ← plug back to F

$$x\ln(x) - y\ln(x) + y + C = 0$$

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(3) $M(x,y) + N(x,y)\frac{dy}{dx} = 0$

$M = [y^2 - 4][(x+1)^{-3}]$ $\quad M_y = 2y$

$N = [-y - xy][(x+1)^{-3}]$ $\quad N_x = -y$

$(y^2 - 4)dx = (y + xy)dy$

$\frac{N_x - M_y}{m} = \frac{-3y}{y^2-4}$

$\frac{M_y - N_x}{N} = \frac{y}{-y(1+x)} = \frac{-3}{x+1}$

$e^{\int \frac{-3}{x+1}dx} = e^{-3\ln(|x+1|)}$

$= (x+1)^{-3}$

$M = [y^2 - 4][(x+1)^{-3}]$ $\quad M_y = \frac{2y}{(x+1)^{-3}}$

$N = [-y - xy][(x+1)^{-3}]$ $\quad N_x = \frac{2y}{(x+1)^3}$

$= \frac{-y(1+x)}{(x+1)^2} = \frac{-y}{(x+1)^{-2}}$

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(3) $F = \int m\,dx = \int \frac{y^2-4}{(x+1)^3} = \frac{y^2-4}{-2(x+1)^2}$

$= \frac{y^2-4}{-2(x+1)^2} + g(y)$

$F_y = \frac{2y}{-2(x+1)^2} = \frac{-y}{(x+1)^2} + g'(y)$

$F_y = N$

$\frac{-y}{(x+1)^2} = \frac{-y}{(x+1)^2} + g'(y)$

$g'(y) = 0$

$g(y) = C$

$0 = \frac{y^2-4}{-2(x+1)^2} + C$

$0 = \frac{1^2-4}{-2(1)^2} + C$

$0 = -\frac{3}{2} + C$

$-\frac{3}{2} = C$

$\frac{y^2-4}{-2(x+1)^2} + \frac{-3}{2} = 0$

$y^2 - 4 + 3(x+1)^2 = 0$