Quiz #7 | MTH 265 | Noor Mustafa

1) $y'' + y = \cos^2 x$

$y = y_G + y_P$

$c_1 y_1 + c_2 y_2$

$y'' + y = 0$

$(e^{yx})'' + e^{yx} = 0$

$(y^2 e^{yx}) + e^{yx} = 0$

$y^2 + 1 = 0$

$e^{\pm ix} = \cos(x) \pm i\sin(x)$ $\quad y = \pm i$

$y_1 = e^{ix}$

$y_2 = e^{-ix}$

$\boxed{y_G = c_1\cos(x) + c_2\sin(x)}$

$y_1 = \cos(x)$

$y_2 = \sin(x)$

$$w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$u_1' = \dfrac{-y_2\, g(x)}{w}$ $\qquad u_2' = \dfrac{y_1\, g(x)}{w}$

$u_1 = \int \dfrac{-y_2\, g(x)}{w}\, dx$ $\qquad u_2 = \int \dfrac{y_1\, g(x)}{w}\, dx$

$$w = \begin{vmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{vmatrix} = 1$$

$u_1 = \int \dfrac{-\sin(x)\cos^2(x)}{1}\, dx$ $\qquad u_2 = \int \dfrac{\cos(x)\cos^2(x)}{1}\, dx$

$u_1 = \frac{1}{3}\cos^3(x) + C$ $\qquad u_2 = \sin(x) - \frac{1}{3}\sin^3(x)$

QUIZ #7 | MTH 225 | Mooy Musterfo

$y_p = u_1 y_1 + u_2 y_2$

$y_p = \frac{1}{3}\cos^3(x)\cos(x) + (\sin(x) - \frac{1}{3}\sin^3 x)\sin(x)$

$y = y_g + y_p$

$y = C_1 \cos(x) + C_2 \sin(x) + \frac{1}{3}\cos^4(x) + \sin^2(x) - \frac{1}{3}\sin^4(x)$

Quiz #7 MTH 225 Noor Mustafa

2) $y'' + 2y' + y = e^{-x} \ln x$ $y = y_g + y_p$

$y'' + 2y' + y = 0$ $y = e^{yx}$

$(e^{yx})'' + 2(e^{yx})' + e^{yx}$

$e^{yx}(y^2 + 2y + 1) = 0$ $e^{yx} \neq 0$

$y^2 + 2y + 1 = 0$

$$y = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(1)}}{2 \cdot 1}$$

$y = -1$ with multiplicity of 2

$y_g = c_1 e^{-x} + c_2 x e^{-x}$

$y_p = u_1 \cdot y_1 + u_2 y_2$ $g(x) = e^{-x} \ln(x)$

$$w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$u_2' = \frac{y_1 g(x)}{w}$$

$$u_2 = \int \frac{y_1 g(x)}{w} dx$$

$$u_1' = \frac{-y_2 (g(x))}{w}$$

$$u_1 = \int \frac{-y_2 g(x)}{w} dx$$

$y_1 = e^{-x}$

$y_2 = e^{-x} x$

$w = y_1 y_2' - y_1' y_2$

Quiz #7 Nour Mustafa MTH 225

2)

$$W = e^{-x}(-e^{x}x + e^{-x}) - (-e^{-x})e^{-x}x$$

$$W = e^{-2x}$$

$$u_1 = \int -\frac{e^{-x}\, x e^{-x} \ln(x)}{e^{-2x}}\, dx$$

$$= -\int x\ln(x)\, dx$$ ← integrate by parts

$$= -\left(\frac{1}{2}x^2\ln(x) - \int \frac{x}{2}\, dx\right)$$

$$\int \frac{x}{2}\, dx = \frac{x^2}{4}$$

$$= -\left(\frac{1}{2}x^2\ln(x) - \frac{x^2}{4}\right)$$

$$= -\frac{1}{2}x^2\ln(x) + \frac{x^2}{4} + C$$

$$u_1 = -\frac{1}{2}x^2\ln(x) + \frac{x^2}{4} + C$$

$$u_2 = \int \frac{e^{-x} e^{-x} \ln(x)}{e^{-2x}}\, dx$$

$$u_2 = x\ln(x) - x$$

$$y_p = u_1 y_1 + u_2 y_2$$

$$y_p = \left(-\frac{1}{2}x^2\ln(x) + \frac{x^2}{4}\right)e^{-x} + (x\ln(x) - x)e^{-x}x$$

Noor Mustafa) Quiz #7) MTH 225

2)

$$y_p = \frac{2e^{-x}x^2\ln(x) - 3e^{-x}x^2}{4}$$

$y = y_g + y_p$

$$y = c_1e^{-x} + c_2xe^{-x} + \frac{2e^{-x}x^2\ln(x) - 3e^{-x}x^2}{4}$$

general solution