Math 225

Quiz 12

$x' = -6x + 5y$

$y' = -5x + 4y$

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#1.

$y' = -5x + 4y$

$y'' = -5x' + 4y'$

$y'' = -5(-6x + 5y) + 4y'$

$y'' = 30x - 25y + 4y'$

$y'' = 30 \cdot \frac{1}{5}(4y - y') - 25y + 4y'$

$y'' = 24y - 6y' - 25y + 4y'$

$y'' + 2y' + y = 0$

$m^2 + 2m + 1 = 0$

$m = -1, -1$

$y(t) = (c_1 + c_2 t)e^{-t}$ ← general solution

$c_1, c_2 =$ arbitrary

$y'(t) = c_2 e^{-t} - (c_1 + c_2 t)e^{-t}$

$c_2 e^{-t} - (c_1 + c_2 t)e^{-t} = -5x + 4(c_1 + c_2 t)e^{-t}$

$$\boxed{x(t) = (c_1 + c_2 t)e^{-t} - \frac{1}{5}e^{-t}}$$

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#2 $x' = 2x - 7y$

$y' = 5x + 10y + 4z$

$z' = 5y + 2z$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 2 & -7 & 0 \\ 5 & 10 & 4 \\ 0 & 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 2-\lambda & -7 & 0 \\ 5 & 10-\lambda & 4 \\ 0 & 5 & 2-\lambda \end{bmatrix}$$

$$= 2-\lambda \begin{bmatrix} 10-\lambda & 4 \\ 5 & 2-\lambda \end{bmatrix} - 7 \begin{bmatrix} 5 & 4 \\ 0 & 2-\lambda \end{bmatrix}$$

$$= (2-\lambda)(\lambda^2 - 12\lambda) - 5(\lambda - 2)$$

$$0 = (\lambda - 2)(\lambda - 5)(-\lambda + 7)$$

$\lambda = 2$ $\quad \lambda = 5$ $\quad \lambda = 7$

for $\lambda = 2$:

$$A - 2I = \begin{bmatrix} 0 & -7 & 0 \\ 5 & 8 & 4 \\ 0 & 5 & 0 \end{bmatrix}$$

$R_3 \to R_3/5$

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#2

$$\begin{bmatrix} 0 & -7 & 0 \\ 5 & 8 & 4 \\ 0 & 1 & 0 \end{bmatrix}$$

$R_1 \to R_1 + 7R_3$ $R_2 \to R_2 - 8R_3$

$$\begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 4 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$y = 0$

$5x + 4z = 0$

$x = -\frac{4}{5}z$

$$v_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{4}{5}z \\ 0 \\ z \end{bmatrix} = \begin{bmatrix} -\frac{4}{5} \\ 0 \\ 1 \end{bmatrix} z$$

$$v_1 = \begin{bmatrix} -\frac{4}{5} \\ 0 \\ 1 \end{bmatrix}$$

For $\lambda = 7$

$$A - 7I = \begin{bmatrix} -5 & -7 & 0 \\ 5 & 3 & 4 \\ 0 & 5 & -5 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

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#2 $R_2 \to R_2 + R_1$

$$= \begin{bmatrix} -5 & -7 & 0 \\ 0 & -4 & 4 \\ 0 & 5 & -5 \end{bmatrix}$$

$R_2 \to R_2/-4$ $\quad$ $R_3 \to R_3/5$

$$= \begin{bmatrix} -5 & -7 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} -5 & -7 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1 - 7R_2$

$$\begin{bmatrix} -5 & 0 & -7 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x = -7/5 \qquad y = 3$

$$v_2 = \begin{bmatrix} \cancel{x} \\ y \\ 3 \end{bmatrix} = \begin{bmatrix} -7/5\,3 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} -7/5 \\ 1 \\ 1 \end{bmatrix} 3$$

$$V_2 = \begin{bmatrix} -7/5 \\ 1 \\ 1 \end{bmatrix}$$

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#2

For $\lambda = 5$

$$A - 5I = \begin{bmatrix} -3 & -7 & 0 \\ 5 & 5 & 4 \\ 0 & 5 & -3 \end{bmatrix}$$

$R_1 \to R_2$

$$= \begin{bmatrix} 5 & 5 & 9 \\ -3 & -2 & 0 \\ 0 & 5 & -3 \end{bmatrix}$$

$R_2 \to R_2 + \frac{3}{5} R_1$

$$= \begin{bmatrix} 5 & 5 & 9 \\ 0 & -4 & \frac{12}{5} \\ 0 & 5 & -3 \end{bmatrix}$$

$R_2 \to R_3$

$$= \begin{bmatrix} 5 & 5 & 9 \\ 0 & 5 & -3 \\ 0 & -4 & \frac{12}{5} \end{bmatrix}$$

$R_3 \to R_3 + \frac{4}{5} R_2$

$$\begin{bmatrix} 5 & 5 & 9 \\ 0 & 5 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1 - R_2$

$$= \begin{bmatrix} 5 & 0 & 7 \\ 0 & 5 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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#2

$$x = -\tfrac{7}{5}\,3$$

$$y = \tfrac{3}{5}\,3$$

$$v_3 = \begin{bmatrix} x \\ y \\ 3 \end{bmatrix} = \begin{bmatrix} -\tfrac{7}{5}3 \\ \tfrac{3}{5}3 \\ 3 \end{bmatrix} = \begin{bmatrix} -\tfrac{7}{5} \\ \tfrac{3}{5} \\ 1 \end{bmatrix} 3$$

$$v_3 = \begin{bmatrix} -\tfrac{7}{5} \\ \tfrac{3}{5} \\ 1 \end{bmatrix}$$

$$x(t) = c_1 e^{\lambda_1 t}[v_1] + c_2 e^{\lambda_2 t}[v_2] + c_3 e^{\lambda_3 t}[v_3]$$

$$\boxed{x(t) = c_1 e^{2t}\begin{bmatrix} -\tfrac{4}{5} \\ 0 \\ 1 \end{bmatrix} + c_2 e^{7t}\begin{bmatrix} -\tfrac{2}{5} \\ 1 \\ 1 \end{bmatrix} + c_3 e^{5t}\begin{bmatrix} -\tfrac{7}{5} \\ \tfrac{3}{5} \\ 1 \end{bmatrix}}$$

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#3 $x' = \begin{pmatrix} 1 & 3 \\ -2 & 6 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$$\begin{pmatrix} 1 & 3 \\ -2 & 6 \end{pmatrix} = \begin{vmatrix} 1-\lambda & 3 \\ -2 & 6-\lambda \end{vmatrix} = \lambda^2 - 7\lambda + 12$$

$$= (\lambda - 3)(\lambda - 4)$$

$$\lambda_1 = 3 \quad \lambda_2 = 4$$

for $\lambda_1 = 3$

$$\begin{pmatrix} -2 & 3 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\rightarrow -2\lambda_1 + 3\lambda_2 = 0$$

$$\vec{X} = \begin{pmatrix} \frac{3}{2} X_2 \\ X_2 \end{pmatrix} \Rightarrow \vec{X}^{(1)} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \; X_2 = 2$$

$\lambda_2 = 4$

$$\begin{pmatrix} -3 & 3 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow -x_1 + x_2 = 0$$

$$\vec{X} = \begin{pmatrix} x_2 \\ x_2 \end{pmatrix} \Rightarrow \vec{X}^{(2)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \; X_2 = 1$$

$$\vec{x}(t) = c_1 e^{3t} \begin{pmatrix} 3 \\ 2 \end{pmatrix} + c_2 e^{4t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

* c_1 and c_2 are arbitrary constants

$$X(0) = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

#3

$x(0) = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 4 \end{pmatrix} = c_1 \begin{pmatrix} 3 \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3c_1 + c_2 \\ 2c_1 + c_2 \end{pmatrix}$

$\rightarrow 3c_1 + c_2 = 1$

$2c_1 + c_2 = 4$

$c_1 = -3, \; c_2 = 10$

$x(t) = -3e^{3t} \begin{pmatrix} 3 \\ 2 \end{pmatrix} + 10e^{4t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

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#4

$x' = \begin{bmatrix} 2 & 4 \\ -1 & 6 \end{bmatrix} x, \quad x(0) = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$

$\begin{bmatrix} 2-\lambda & 4 \\ -1 & 6-\lambda \end{bmatrix} = 0$

$(2-\lambda)(6-\lambda) + 4 = 0$

$\lambda^2 - 8\lambda + 16 = 0$

$(\lambda - 4)^2 = 0$

$\lambda = 4, 4$

$\begin{bmatrix} 2-4 & 4 \\ -1 & 6-4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$\begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$\rightarrow -x + 2y = 0 \rightarrow x = 2y$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

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4

$$-2\alpha_1 + 4\alpha_2 = 2$$

$$-\alpha_1 + 2\alpha_2 = 1$$

$$\alpha_1 = 2\alpha_2 - 1$$

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$X(t) = c_1 e^{4t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 \left[t e^{4t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + e^{4t} \begin{bmatrix} -1 \\ 0 \end{bmatrix} \right]$$

$$X(0) = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

$$c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 2c_1 - c_2 \\ c_1 \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

$$c_1 = 6$$

$$c_2 = 13$$

$$\boxed{X(t) = 6 e^{4t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 13 \left[t e^{4t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + e^{4t} \begin{bmatrix} -1 \\ 0 \end{bmatrix} \right]}$$