Quiz #6 MTH 225

1. $y''' - y'' - 4y' + 4y = 5 - e^x + e^{-x}$

$y''' - y'' - 4y' + 4y = 0$

$y = C_1 e^x + C_2 e^{-2x} + C_3 e^{2x}$

$(e^{yx})''' - (e^{yx})'' - 4(e^{yx})' + 4e^{yx} = 0$

$e^{yx}(y^3 - y^2 - 4y + 4) = 0$

$(e^{yx})''' = y^3 e^{yx} \quad (e^{yx})'' = y^2 e^{yx}$

$(e^{yx})' = e^{yx} y$

$y^3 e^{yx} - y^2 e^{yx} - 4e^{yx} y + 4e^{yx} = 0$

$e^{yx}(y^3 - y^2 - 4y + 4) = 0$

$y = 1, y = -2, y = 2$

$(y-1)(y+2)(y-2) = 0$

$y = 1 \quad y = -2 \quad y = 2$

$y = C_1 e^x + C_2 e^{-2x} + C_3 e^{2x}$

$y''' - y'' - 4y' + 4y = 5 - e^x + e^{-x}$

Quiz #6 | MTH 225

1. y_{p_1}

$g(x) = 5 \quad y = a_0$

$a_0''' - a_0'' - 4a_0' + 4a_0 = 5$

$4a_0 = 5$

$a_0 = \frac{5}{4}$ $\quad y = \frac{5}{4}$

y_{p_2}

$y''' - y'' - 4y' + 4y = -e^x$

$(a_0 x e^x)''' - (a_0 x e^x)'' - 4(a_0 x e^x)' + 4a_0 x e^x = -e^x$

$(a_0 x e^x)''' = a_0(e^x x + 3e^x)$

$a_0(e^x x + 3e^x) - (a_0 x e^x) - 4(a_0 x e^x) + 4a_0 x e$

$= -e^x$

$(a_0 x e^x)'' = a_0(e^x x + 2e^x)$

$(a_0 x e^x)' = a_0(e^x + e^x x)$

$a_0(e^x x + 3e^x - a_0(e^x x + 2e^x) - 4a_0(e^x + e^x x)$

$+ 4a_0 x e^x = -e^x$

$-3a_0 e^x = -e^x$

diff equations | quiz #8

1.

$y = \frac{1}{3} x e^x$ particular solution $\rightsquigarrow$

$y = \frac{e^x x}{3}$ $\leftarrow y_{p2}$

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y_{p3}) $g(x) = e^{-x}$ $\quad y = a_0 e^{-x}$

$(a_0 e^{-x})''' = -a_0 e^{-x}$

$(a_0 e^{-x})'' = a_0 e^{-x}$

$(a_0 e^{-x})' = -a_0 e^{-x}$

$-a_0 e^{-x} - a_0 e^{-x} - 4(-a_0 e^{-x}) + 4 a_0 e^{-x} = e^{-x}$

$6 a_0 e^{-x} = e^{-x}$

$y = \frac{1}{6} e^{-x}$

Quiz #6 (MTH 225)

1. $y = \frac{5}{4} + \frac{e^x x}{3} + \frac{1}{6}e^{-x}$

$y = c_1 e^x + c_2 e^{-2x} + c_3 e^{2x}$

The general solution is

$$y = c_1 e^x + c_2 e^{-2x} + c_3 e^{2x} + \frac{5}{4} + \frac{e^x x}{3} + \frac{1}{6}e^{-x}$$

Quiz #6 MTH 225

2. $y'' + 2y' + y = x^2 e^{-x}$

$y = e^{gx}$

$y'' + 2y' + y = 0$

$(e^{gx})'' + 2(e^{gx})' + e^{gx} = 0$

$(e^{gx})'' = g^2 e^{gx}$ $(e^{gx})' = e^{gx} g$

$e^{gx}(g^2 + 2g + 1) = 0$

$e^{gx}(g^2 + 2g + 1) = 0$ $g = -1$ with multiplicity of 2

$y = c_1 e^{-x} + c_2 x e^{-x}$

$y'' + 2y' + y = x^2 e^{-x}$

$y = a_0 x^4 e^{-x} + a_1 x^3 e^{-x} + a_2 x^2 e^{-x} = N$

$(N)'' + 2(N)' + N = 0$

$12 a_0 e^{-x} x^2 + 6 a_1 e^{-x} x + 2 a_2 e^{-x} = x^2 e^{-x}$

$a_0 = \frac{1}{12}, a_2 = 0, a_1 = 0$

$y = \frac{1}{12} x^4 e^{-x} + 0\, x^3 e^{-x} + 0 \cdot x^2 e^{-x}$

$y = \frac{e^{-x} x^4}{12}$

$y = \frac{e^{-x} x^4}{12}$

$N'' = g^2 e^{gx}$

$N' = e^{gx} g$

Quiz # 6

2- general solution is

$$y = c_1 e^{-x} + c_2 x e^{-x} + \frac{e^{-x} x^4}{12}$$

Quiz #6 / MTH 225

3. $y'' - 4y' + 8y = x^3$, $y(0) = 2$, $y'(0) = 4$

$y'' - 4y' + 8y = 0$ $\quad y = e^{rx}$ $\quad (e^{rx})'' = r^2 e^{rx}$

$(e^{rx})'' - 4(e^{rx})' + 8e^{rx} = 0$ $\quad (e^{rx})' = e^{rx} r$

$r^2 e^{rx} - 4e^{rx} r + 8e^{rx} = 0$

$e^{rx}(r^2 - 4r + 8) = 0$

$(r^2 - 4r + 8) = (r + 2$

$r = \frac{-(-4) + \sqrt{(-4)^2 - 4(1)(8)}}{2} = 2 + 2i$

$r = \frac{-(-4) - \sqrt{(-4)^2 - 4(1)(8)}}{2} = 2 - 2i$

$y = e^{2x}(c_1 \cos(2x) + c_2 \sin(2x))$

$g(x) = x^3$ $\quad y = a_0 x^3 + a_1 x^2 + a_2 x + a_3$

$y'' - 4y' + 8y = x^3$

$8a_0 x + 2a_1 - 4(3a_0 x^2 + 2a_1 x + a_2) + 8(a_0 x^3 + a_1 x^2 + a_2 x + a_3) = x^3$

$a_1 x^2 (a_2 x + a_3) = x^3$

$a_0 = \frac{1}{8}$ $\quad a_2 = \frac{3}{32}$ $\quad a_1 = 0$ $\quad a_3 = 0$

Quiz #6 | MTH 225

3.

$$S = a_0 t^3 + a_1 t^2 + a_2 t + a_3$$

$$(a_0 t^3 + a_1 t^2 + a_2 t + a_3)'' = 6a_0 t + 2a_1$$

$$(a_0 t^3 + a_1 t^2 + a_2 t + a_3)' = 3a_0 t^2 + 2a_1 t + a_2$$

$$6a_0 x^3 + 2a_1 - 12a_0 t^2 - 8a_1 t - 4a_2 +$$

$$8a_0 x^3 + 8a_1 t^2 + 8a_2 x + 8a_3 = t^3$$

$$4a_0 t^3 + (-12a_0 + 8a_1) t^2 + (6a_0 - 8a_1 + 8a_2) t +$$

$$+ (2a_1 - 4a_2 + 8a_3) = 1 - t^3$$

$$\begin{cases} 0 = 2a_1 - 4a_2 + 8a_3 \\ 0 = 6a_0 - 8a_1 + 8a_2 \\ 0 = -12a_0 + 8a_1 \\ 1 = 8a_0 \end{cases}$$

$$a_0 = \frac{1}{8}$$

$$\begin{cases} 0 = -12\left(\frac{1}{8}\right) + 8a_1 \\ 0 = 2a_1 - 4a_2 + 8a_3 \\ 0 = 6\left(\frac{1}{8}\right) - 8a_1 + 8a_2 \end{cases}$$

20.2 #4 (MTH 225)

3.

$a_1 = -\frac{3}{2} + 8a_1$

$a_1 = \frac{3}{16}$

$\left[\begin{array}{l} 0 = \frac{3}{4} - 8\left(\frac{3}{16}\right) + 8a_2 \\ 0 = 2\left(\frac{3}{16}\right) - 4a_2 + 8a_3 \end{array}\right]$

$0 = 8a_2 - \frac{3}{4}$

$a_2 = \frac{3}{32}$

$\left[0 = \frac{3}{8} - 4\left(\frac{3}{32}\right) + 8a_3\right]$

$0 = 8a_3$

$a_3 = 0$

$a_0 = \frac{1}{8}, a_1 = \frac{3}{16}, a_2 = \frac{3}{32}$

$a_3 = 0$

$y = \frac{1}{8}x^3 + \frac{3}{16}x^2 + \frac{3}{32}x + 0$

$y = \frac{x^3}{8} + \frac{3x^2}{16} + \frac{3x}{32}$

3. (Quiz #6) Noor Mustafa MTH 226

$$y = \frac{1}{8}x^3 + \frac{3}{16}x^2 + \frac{3}{32}x + 0$$

$$y = \frac{x^3}{8} + \frac{3x^2}{16} + \frac{3x}{32}$$

$y = y_h + y_p$

$y = e^{2x}(c_1\cos(2x) + c_2\sin(2x))$

general solution

$$y = e^{2x}\left(\cos(2x) - \frac{3}{64}\sin(2x)\right) + \frac{x^3}{8} + \frac{3x^2}{16} + \frac{3x}{32}$$