Quiz #6 | MTH 225

1. $y''' - y'' - 4y' + 4y = 5 - e^x + e^{-x}$

$y''' - y'' - 4y' + 4y = 0$

$y = c_1 e^x + c_2 e^{-2x} + c_3 e^{2x}$

$(e^{yx})''' - (e^{yx})'' - 4(e^{yx})' + 4e^{yx} = 0$

$e^{yx}(y^3 - y^2 - 4y + 4) = 0$

$(e^{yx})''' = y^3 e^{yx} \quad (e^{yx}) = y^2 e^{yx}$

$(e^{yx})' = e^{yx} y$

$y^3 e^{yx} - y^2 e^{yx} - 4e^{yx} y + 4e^{yx} = 0$

$e^{yx}(y^3 - y^2 - 4y + 4) = 0$

$y = 1, y = -2, y = 2$

$(y-1)(y+2)(y-2) = 0$

$y = 1 \quad y = -2 \quad y = 2$

$y = c_1 e^x + c_2 e^{-2x} + c_3 e^{2x}$

$y''' - y'' - 4y' + 4y = 5 - e^x + e^{-x}$

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1. y_{p_1}

$g(x) = 5 \quad y = a_0$

$a_0''' - a_0'' - 4a_0' + 4a_0 = 5$

$4a_0 = 5$

$a_0 = \frac{5}{4} \qquad y = \frac{5}{4}$

y_{p_2}

$y''' - y'' - 4y' + 4y = -e^x$

$(a_0 x e^x)''' - (a_0 x e^x)'' - 4(a_0 x e^x)' + 4a_0 x e^x = -e^x$

$(a_0 x e^x)''' = a_0(e^x x + 3e^x) = y'''$

$a_0(e^x x + 3e^x) - (a_0 x e^x) - 4(a_0 x e^x) + 4a_0 x e^x = -e^x$

$(a_0 x e^x)'' = a_0(e^x x + 2e^x) = y''$

$(a_0 x e^x)' = a_0(e^x + e^x x) = y'$

$a_0(e^x x + 3e^x - a_0(e^x x + 2e^x) - 4a_0(e^x + e^x x) + 4a_0 x e^x = -e^x$

$-3a_0 e^x = -e^x$

diff equation quiz ~~#3~~

1.

$y = \frac{1}{3} x e^{x}$ particular solution to

$y = \frac{e^{x} x}{3}$ y_{p_2}

y_{p_3} $g(x) = e^{-x}$ $y = a_0 e^{-x}$

$(a_0 e^{-x})''' = -a_0 e^{-x}$

$(a_0 e^{-x})'' = a_0 e^{-x}$

$(a_0 e^{-x})' = -a_0 e^{-x}$

$-a_0 e^{-x} - a_0 e^{-x} - 4(-a_0 e^{-x}) + 4a_0 e^{-x} = e^{-x}$

$6 a_0 e^{-x} = e^{-x}$

$y = \frac{1}{6} e^{-x}$

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$y = c_1 e^{x} + c_2 e^{-2x} + c_3 e^{2x}$

1. $y = \frac{5}{4} + \frac{e^{x}x}{3} + \frac{1}{6}e^{-x}$

The general solution is

$y = c_1 e^{x} + c_2 e^{-2x} + c_3 e^{2x} + \frac{5}{4} + \frac{e^{x}x}{3} + \frac{1}{6}e^{-x}$

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2. $y'' + 2y' + y = x^2 e^{-x}$

$y = e^{yx}$

$y'' + 2y' + y = 0$

$(e^{yx})'' + 2(e^{yx})' + e^{yx} = 0$

$(e^{yx})'' = y^2 e^{yx}$ $(e^{yx})' = e^{yx} y$

$e^{yx}(y^2 + 2y + 1) = 0$

$e^{yx}(y^2 + 2y + 1) = 0$ $y = -1$ with multiplicity of 2

$y = c_1 e^{-x} + c_2 x e^{-x}$

$y'' + 2y' + y = x^2 e^{-x}$

$y = a_0 x^4 e^{-x} + a_1 x^3 e^{-x} + a_2 x^2 e^{-x} = N$

$(N)'' + 2(N)' + N = 0$

$12 a_0 e^{-x} x^2 + 6 a_1 e^{-x} x + 2 a_2 e^{-x} = x^2 e^{-x}$

$a_0 = \frac{1}{12}, a_2 = 0, a_1 = 0$

$y = \frac{1}{12} x^4 e^{-x} + 0 \, x^3 e^{-x} + 0 \cdot x^2 e^{-x}$

$y = \frac{e^{-x} x^4}{12}$

$y = \frac{e^{-x} x^4}{12}$

$N'' = y^2 e^{yx}$

$N' = e^{yx} y$

Quiz # 6

2. general solution is

$$y = c_1 e^{-x} + c_2 x e^{-x} + \frac{e^{-x} x^4}{12}$$

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3. $y'' - 4y' + 8y = x^3$, $y(0) = 2$ $y'(0) = 4$

$y'' - 4y' + 8y = 0$ $y = e^{yx}$ $(e^{yx})'' = y^2 e^{yx}$

$(e^{yx})'' - 4(e^{yx})' + 8e^{yx} = 0$ $(e^{yx})' = e^{yx} y$

$y^2 e^{yx} - 4e^{yx} y + 8e^{yx} = 0$

$e^{yx}(y^2 - 4y + 8) = 0$

$(y^2 - 4y + 8) = (y + 2$

$$y = \frac{-(-4) + \sqrt{(-4)^2 - 4(1)(8)}}{2} = 2 + 2i$$

$$y = \frac{-(-4) - \sqrt{(-4)^2 - 4(1)(8)}}{2} = 2 - 2i$$

$y = e^{2x}(c_1 \cos(2x) + c_2 \sin(2x))$

$g(x) = x^3$ $y = a_0 x^3 + a_1 x^2 + a_2 x + a_3$

$y'' - 4y' + 8y = x^3$

$6a_0 x + 2a_1 ( ) a_0 x^2$ [illegible] $+ 8(a_0 x^3 +$

$a_1 x^2 + a_2 x + a_3) = x^3$

$a_0 = \frac{1}{8}$ $a_1 = \frac{3}{32}$ $a_2 = \frac{}{16}$ $a_3 = 0$

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7.

$y = a_0 t^3 + a_1 t^2 + a_2 t + a_3$

$(a_0 t^3 + a_1 t^2 + a_2 x + a_3)'' = 6a_0 t + 2a_1$

$(a_0 t^3 + a_1 x^2 + a_2 t + a_3)' = 3a_0 t^2 + 2a_1 t + a_2$

$6a_0 x + 2a_1 - 12a_0 t^2 - 8a_1 x - 4a_2 +$

$8a_0 x^3 + 8a_1 t^2 + 8a_2 x + 8a_3 = x^3$

$8a_0 t^3 + (-12a_0 + 8a_1)x^2 + (6a_0 - 8a_1 + 8a_2)x$
$+ (2a_1 - 4a_2 + 8a_3) = 1 \cdot t^3$

$$\left[\begin{aligned} 0 &= 2a_1 - 4a_2 + 8a_3 \\ 0 &= 6a_0 - 8a_1 + 8a_2 \\ 0 &= -12a_0 + 8a_1 \\ 1 &= 8a_0 \end{aligned}\right]$$

$a_0 = \frac{1}{8}$

$$\left[\begin{aligned} 0 &= -12\left(\tfrac{1}{8}\right) + 8a_1 \\ 0 &= 2a_1 - 4a_2 + 8a_3 \\ 0 &= 6\left(\tfrac{1}{8}\right) - 8a_1 + 8a_2 \end{aligned}\right]$$

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3.

$a_1 = \frac{-3}{2} + 8a_1$

$a_1 = \frac{3}{16}$

$\left[0 = \frac{3}{4} - 8\left(\frac{3}{16}\right) + 8a_2 \right.$

$\left. 0 = 2\left(\frac{3}{16}\right) - 4a_2 + 8a_3 \right]$

$0 = 8a_2 - \frac{3}{4}$

$a_2 = \frac{3}{32}$

$\left[0 = \frac{3}{8} - 4\left(\frac{3}{32}\right) + 8a_3 \right]$

$0 = 8a_3$

$a_3 = 0$

$a_0 = \frac{1}{8}, a_1 = \frac{3}{16}, a_2 = \frac{3}{32}, a_3 = 0$

$y = \frac{1}{8}x^3 + \frac{3}{16}x^2 + \frac{3}{32}x + 0$

$y = \frac{x^3}{8} + \frac{3x^2}{16} + \frac{3x}{32}$

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3.

$$y = \frac{1}{8}x^3 + \frac{3}{16}x^2 + \frac{3}{32}x + 0$$

$$y = y_h + y_p$$

$$y = \frac{x^3}{8} + \frac{3x^2}{16} + \frac{3x}{32}$$

$$y = e^{2x}(c_1\cos(2x) + c_2\sin(2x))$$

general solution

$$y = e^{2x}\left(1\cos(2x) - \frac{3}{64}\sin(2x)\right) + \frac{x^3}{8} + \frac{3x^2}{16} + \frac{3x}{32}$$