

(diff equation) (non linear)

1. $(x + y e^{y/x}) dx - x e^{y/x} dy = 0, \quad y(1) = 0$

$$F_x = M = x + y e^{y/x} \quad F_y = N = -x e^{y/x}$$

$$M_y = e^{y/x} + y e^{y/x} \left(\frac{1}{x}\right) \quad N_x = -e^{y/x} - x e^{y/x} \left(-\frac{y}{x^2}\right)$$

$$N_x = -e^{y/x} - x e^{y/x} \left(-\frac{y}{x^2}\right)$$

$$M_y = e^{y/x} \left[1 + \frac{y}{x}\right]$$

$$N_x = e^{y/x} \left[-1 + \frac{y}{x}\right]$$

$$\left[\frac{M_y - N_x}{N} \right] \left[\frac{N_x - M_y}{M} \right]$$

$$\frac{e^{y/x} \left[1 + \frac{y}{x}\right] - e^{y/x} \left[-1 + \frac{y}{x}\right]}{-x e^{y/x}}$$

$$= \frac{\left[1 + \frac{y}{x}\right] + \left[1 - \frac{y}{x}\right]}{-x}$$

$$= -\frac{2}{x}$$

$$u(x) = e^{\int -\frac{2}{x} dx} = e^{-2 \ln(x)}$$

(continues)

$$1. e^{\ln(\frac{1}{x^2})}$$

$$F_1 = m = \frac{x}{x^2} + \frac{ye^{y/x}}{x^2}$$

$$N = \frac{-x e^{y/x}}{x^2} \rightarrow -\frac{e^{y/x}}{x}$$

$$m_y = \frac{1}{x^2} \left[e^{y/x} + y e^{y/x} \left(\frac{1}{x} \right) \right] = \frac{e^{y/x}}{x^2} \left[1 + \frac{y}{x} \right]$$

$$N_x = \frac{1}{x^2} e^{y/x} + \left(-\frac{1}{x} \right) e^{y/x} \left(\frac{-y}{x^2} \right)$$

$$= \frac{1}{x^2} e^{y/x} + \frac{y}{x^3} e^{y/x} =$$

$$\frac{e^{y/x}}{x^2} \left[1 + \frac{y}{x} \right]$$

$$F = \int N dy = \int \frac{-e^{y/x}}{x} dy = -\frac{1}{x} \int e^{y/x} dy$$

$$= -\frac{1}{x} (x e^{y/x}) + g(x)$$

$$F = -e^{y/x} + g(x)$$

$$F_x = \frac{-y}{x^2} (-e^{y/x}) + g'(x) = m$$

$$= \frac{ye^{y/x}}{x^2} + g'(x) = \frac{1}{x} + \frac{ye^{y/x}}{x^2}$$

continues

$$g'(x) = \frac{1}{x} \quad g(x) = \ln(x) + C$$

(diff eqs) (our method)

$$1. F = -e^{y/x} + \ln(x) + C = 0$$

$$y(1)=0 \Rightarrow e^{y/x} = \ln(x) + C$$

$$\rightarrow C = 1$$

$$e^{y/x} = \ln(x) + 1$$

Mod
mustafa / dit
erwary

$$2. \cos x \frac{dy}{dx} + (\sin x) y = 1$$

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$\frac{dy}{dx} + \tan(x)y = \frac{1}{\cos(x)}$$

$$\frac{dy}{dx} + \underbrace{(\tan x)}_p y = \underbrace{\sec(x)}_q$$

$$e^{\int \tan(x) dx} = e^{\ln |\sec(x)|}$$
$$= \sec(x)$$

$$\int \frac{d}{dx} [\sec(x)y] = \int \sec^2(x) +$$

$$\sec(x)y = \tan(x) + c$$

$$y = \frac{\tan(x)}{\sec(x)} + \frac{c}{\sec(x)}$$

3. $(x+1) \frac{dy}{dx} + y = \ln x \quad y(1) = 10$

$$\frac{dy}{dx} + \left(\frac{1}{x+1} \right) y = \frac{\ln x}{x+1}$$

P

$$\frac{d}{dx} [e^{\int P} y] = \frac{\ln x}{x+1} e^{\int P}$$

$$\int \frac{d}{dx} [(x+1)y] = \int \ln |x+1|$$

$e^{\int \frac{1}{x+1}} = e^{\ln(x+1)} = x+1$

↑ integrate by parts

$$\frac{(x+1)y}{x+1} = \frac{x \ln(x) - x + e}{x+1}$$

$y(1) = 10 \quad y = \frac{x \ln(x)}{x+1} - \frac{x}{x+1} + \frac{C}{x+1}$

$$10 = \frac{0}{2} - \frac{1}{2} + \frac{C}{2}$$

$$10 = -\frac{1}{2} + \frac{C}{2}$$

$$C = 21$$

3. diff equation
Nouf mustafa

$$y = \frac{x \ln(x)}{x+1} - \frac{x}{x+1} + \frac{2}{x+1}$$

4. (not ez work
now must be)

$$F_x = M = 2x^2y + e^y \quad F_y = N = x^3 + \cancel{x}e^{2-2y}$$

$$M_y = 2x^2 + e^y - 1 \quad N_x = 3x^2 + e^y$$

$$F = \int M dx = \int 2x^2y + e^y dx \\ = x^3y + xe^y + g(y)$$

$$F_y = x^3 + xe^y + g'(y) = N$$

$$x^3 + xe^y - 2y = x^3 + xe^y + g'(y)$$

$$g'(y) = -2y \quad g(y) = -y^2 + C$$

$$F = x^3y + \cancel{xe^y} - y^2 + C = 0$$

$$x^3y + xe^y - y^2 + C = 0$$

5. (diff equation
now mustafa)

⊗ bemerkung

$$\frac{\partial y}{\partial x} = 9(x^3 - 1)$$

$$\frac{\partial y}{\partial x} + y = x^4$$

$$P(X) = 1$$

$$f(x) = x$$

$$n = 4$$

$$y = y^{-3}$$

$$\frac{\partial y}{\partial y} = \frac{1}{y^3} = \frac{-3}{y^4}$$

$$\cancel{\frac{\partial u}{\partial u} = \frac{-3}{y^4}}$$

$$\partial y = \frac{-y^4}{3} \partial u$$

$$y^3 = \frac{1}{u}$$

$$y = \frac{1}{u^{1/3}}$$

$$\frac{-y^4}{3} \frac{\partial u}{\partial x} + y = x^4$$

$$\frac{\partial u}{\partial x} - \frac{3y}{y^4} = \frac{-3 \times y^4}{y^4} = -3x$$

$$\frac{\partial u}{\partial x} - \frac{3}{y^3} = -3x$$

$$\frac{\partial u}{\partial x} - \frac{3}{1/u} = -3x$$

$$\frac{\partial u}{\partial x} - 3u = -3x$$

⊗ continues

5. non constant diff equation

$$\frac{\partial u}{\partial x} - 3u = -3x$$

$$u = a \cdot b$$

$$a \frac{\partial b}{\partial x} + b \frac{\partial a}{\partial x} - 3ab = -3x$$

$$a \frac{\partial b}{\partial x} + b \left(\frac{\partial a}{\partial x} - 3a \right) = -3x$$

$$\frac{\partial a}{\partial x} - 3a = 0$$

$$\frac{\partial a}{\partial x} = 3a$$

$$\ln(a) = 3x + C$$

$$a = Ke^{3x}$$

$$a \frac{\partial b}{\partial x} = -3x$$

$$\frac{\partial b}{\partial x} = \frac{-3x}{Ke^{3x}}$$

$$\partial b = \frac{-3}{K} x e^{-3x} dx$$

$$b = \frac{-3}{K} \left[\int x e^{-3x} dx \right]$$

$$b = \frac{-3}{K} \left[\frac{x e^{-3x}}{-3} - \frac{e^{-3x}}{9} \right]$$

$$b = \frac{e^{-3x}}{K} \left[x + \frac{1}{3} \right] + \frac{C}{K}$$

$$u = Ke^{3x} \left[\frac{e^{-3x}}{K} \left(x + \frac{1}{3} \right) + \frac{C}{K} \right]$$

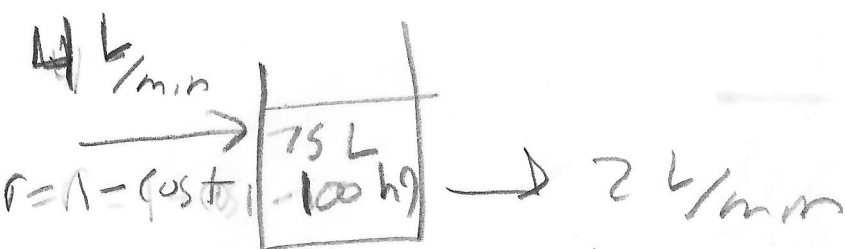
$$u = Ce^{3x} + x + \frac{1}{3}$$

$$y = \frac{1}{(Ce^{3x} + x + \frac{1}{3})^{1/3}}$$

6.

differential equation

Your master



$$\frac{dA}{dt} = R_{in} - R_{out}$$

$$A(t) = ? \frac{\text{kg}}{\text{L}}$$

$$A(0) = \frac{100}{75}$$

$$\frac{dA}{dt} = 4(1 - \cos t) - 2A$$

$$\frac{dA}{dt} = 4 - 4 \cos t - 2A$$

$$\frac{dA}{dt} + 2A = 4 - 4 \cos t$$

use
linear
equation

$$A = U \cdot V \quad V \frac{dU}{dt} + U \frac{dV}{dt} + 2UV = 4 - 4 \cos t$$

$$V \frac{dU}{dt} + U \left[\frac{dV}{dt} + 2V \right] = 4 - 4 \cos t$$

$$\frac{dV}{dt} + 2V = 0$$

$$\int \frac{dV}{V} = \int -2 dt$$

$$\ln V = -2t + C$$

$$V = e^{-2t} + C$$

$$V = Ke^{-2t}$$

$$V \frac{dU}{dt} = 4 - 4 \cos t$$

$$Ke^{-2t} \frac{dU}{dt} = 4 - 4 \cos t$$

$$\frac{dU}{dt} = \frac{4e^{2t}}{K} - \frac{4e^{2t} \cos t}{K}$$

$$\frac{dU}{dt} = \frac{4}{K} [e^{2t} - e^{2t} \cos t]$$

6. differentiation
equations normal method

$$\int du = \frac{4}{K} \int e^{2t} - e^{2t} \cos t \, dt$$

$$u = \frac{4}{K} \left[\frac{1}{2} e^{2t} \int -e^{2t} \cos t \, dt \right]$$

$$u = e^{2t} \quad \int u \, dv = u \, v - \int v \, du$$

$$\int dv = \int \cos t \, dt \quad \int e^{2t} \cos t \, dt = e^{2t} \sin t - \int 2e^{2t} \sin t \, dt$$

$$v = \sin t \quad u = e^{2t} \\ dv = \cos t$$

$$\int 2e^{2t} \sin t \, dt = -2e^{2t} \cos t - \int -4e^{2t} \cos t \, dt$$

$$\int e^{2t} \cos t \, dt = e^{2t} \sin t + 2e^{2t} \cos t - \int 4e^{2t} \cos t \, dt$$

$$u = \frac{4}{K} \left[\frac{1}{2} e^{2t} - \left(\frac{1}{5} e^{2t} \sin t + \frac{2}{5} e^{2t} \cos t \right) + C \right]$$

$$A = 4e^{-2t} \left(\frac{4}{K} \right) \left[\frac{1}{2} e^{2t} - \frac{1}{5} e^{2t} \sin t - \frac{2}{5} e^{2t} \cos t \right]$$

$$A = 4Ce^{-2t} + 2 - \frac{4}{5} \sin t - \frac{8}{5} \cos t$$

$$\frac{100}{75} = 4C + 2 - \frac{8}{5}$$

$$\frac{220}{75} = 4C + 2$$

$$\frac{20}{75} = \frac{4C}{4}$$

$$C = \frac{7}{30}$$

$$A = \frac{28}{30} e^{2t} + 2 - \frac{4}{5} \sin t - \frac{8}{5} \cos t$$

8. d.H equation. Need more

$$\Delta F = 4 - 2 = 2 \text{ L/min}$$
$$95 - 70 = 25 \text{ L}$$

$$\Delta V = \Delta f \cdot t$$

$$25 \text{ L} \cdot \frac{1 \text{ min}}{2 \text{ L}} = 12.5 \text{ min}$$

$$A = \frac{28}{30} e^{-2t} + 2 - \frac{4}{5} \sin t - \frac{8}{5} \cos t$$

$$A = -156.5202712 \text{ Hg/L}$$

$$A \cdot 95 \text{ L} = 43.38 \text{ Hg}$$

$$43.38 \text{ Hg}$$

7. (diff equation) New my friend

bag of ice

$$\frac{dT}{dt} = k(T - T_m)$$

The heat sink is
much hotter than

$$\frac{dT}{dt} = k(T - 24)$$

$$\int \frac{dt}{(T - 24)} = \int k dt$$

$$\ln(T - 24) = k + tC$$

$$T(t) = 24 + Ce^{kt}$$

$$T(t) = 24 + 3e^{-5 \ln(2/3)t}$$

$$T = 24.8^\circ\text{C}$$

$$.4 = 3e^{-5 \ln(2/3)t}$$

$$2 \ln\left(\frac{.4}{3}\right) = \ln\left(\frac{2}{3}\right)t$$

$$t = 9.94 \text{ min}$$

$$T(0) = 27^\circ\text{C}$$

$$T(2) = 26^\circ\text{C}$$

$$T_m = 24^\circ\text{C}$$

$$T_m = 75.2^\circ\text{F}$$

$$27 = 24 + C(1)$$

$$C = 3$$

$$26 = 24 + 3e^{k \cdot 2}$$

$$\frac{2}{3} = e^{2k}$$

$$\frac{1}{2} \ln\left(\frac{2}{3}\right) = k$$