1. Nour mustafa | Math 225 | exam 2

$x^3 y''' + xy' - y = 0 \qquad y = x^r$

$x^r (x^r)''' + x(x^r)' - (x^r) = 0$

$(x^r)''' = r x^{r-3}(r-1)(r-2)$

$(x^r)' = r x^{r-1}$

$x^3 r x^{r-3}(r-1)(r-2) + x r x^{r-1} - x^r = 0$

$r^3 x^r - 3r^2 x^r + 3r x^r - x^r = 0$

$x^r(r^3 - 3r^2 + 3r - 1) = 0$

$x^r(r^3 - 3r^2 + 3r - 1) = 0$

$x^r \neq 0$

$r^3 - 3r^2 + 3r - 1 = 0$

$(r-1)^3 = 0$

$r - 1 = 0$

$r = 1$ multiplicity of 3

$y = c_1 x + c_2 \ln(x) x + c_3 \ln^2(x) x$

1. Noor | MTH 225 | exam 2

$y(1) = 2 \rightarrow C_1 = 2$

$y'(1) = -1 \Rightarrow C_1 + C_2 = 0 \rightarrow C_2 = -3$

$y''(1) = 0 \rightarrow 2C_3 + C_2 = 0 \rightarrow C_3 = \frac{3}{2}$

$y = 2x - 3x\ln(x) + \frac{3x\ln^2(x)}{2}$

Noor mustafa | MTH 225 | exam 2

2. $3y'' - 6y' + 6y = e^x \sec(x)$

$y = y_h + y_p$

$y_h = \quad y = e^{yx}$

$3(e^{yx})'' - 6(e^{yx})' + 6(e^{yx}) = 0$

$(e^{yx})'' = y^2 e^{yx}$

$(e^{yx})' = y e^{yx}$

$3y^2 e^{yx} - 6e^{yx} y + 6e^{yx} = 0$

$e^{yx}(3y^2 - 6y + 6) = 0$

$e^{yx} \neq 0$

$3y^2 - 6y + 6 = 0$

$$X_{1,2} = \frac{6 \pm \sqrt{(-6)^2 - 4(3)(6)}}{6}$$

$y = 1 + i$

$y = 1 - i$

$y_h = e^{yx}(c_1 \cos(x) + c_2 \sin(x))$

Noor mustafa / exam 2 | MTH 225

2. $\boxed{y_p}$ $\dfrac{3y''-6y'+6y}{3}=\dfrac{e^x\sec(x)}{3}$

$y''-2y'+2y=\dfrac{e^x\sec(x)}{3}$

$y_p=u_1y_1+u_2y_2$

$$\left|\begin{array}{l} u_1'y_1+u_2'y_2=0 \\ u_1'y_1'+u_2'y_2'=g(x) \end{array}\right|$$

$u_1=\int\dfrac{-y_2\,g(x)}{w}\,dx$

$u_2\int\dfrac{y_1\,g(x)}{w}\,dx$

$y_1=e^x\cos(x)$

$y_2=e^x\sin(x)$

product rule

$y_1'=e^x\cos(x)-e^x\sin(x)$

$y_2'=e^x\sin(x)+\cos(x)e^x$

$w=e^x\cos(x)\left(e^x\sin(x)+\cos(x)e^x\right)$
$-\left(e^x\cos(x)-e^x\sin(x)\right)e^x\sin(x)$

Noor Mustafa | MTH 225 | exam 2

2. $w = e^{2x}$

$$u_1 = \int - \frac{e^x \sin(x) \frac{e^x \sec(x)}{3}}{e^{2x}} \, dx$$

$$u_1 = \frac{1}{3} \ln((\cos(x))) + C$$

$$u_2 = \int \frac{e^x \cos(x) \frac{e^x \sec(x)}{3}}{e^{2x}} \, dx$$

$$u_2 = \frac{1}{3} x$$

$$y_p = \frac{e^x \ln((\cos(x))) \cos(x) + e^x x \sin(x)}{3}$$

$$\boxed{y = e^x(c_1 \cos(x) + c_2 \sin(x)) + \frac{e^x \ln((\cos(x))) \cos(x) + e^x x \sin(x)}{3}}$$

Noor
Muath
exam 2

3. $4x^2y'' + y = 0 \; ; \; y_1 = \sqrt{x}\ln x$

$y = x^r$

$4x^2(x^r)'' + x^r = 0$

$(x^r)'' = rx^{r-2}(r-1)$

$4x^2rx^{r-2}(r-1) + x^r = 0$

$4rx^r(r-1) + x^r = 0$

$x^r(4r(r-1)+1) = 0$

$4r(r-1)+1 = 0$

$4r^2 - 4r + 1 = 0$ ← using quadratic formula

$$r_{1,2} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(1)}}{2 \cdot 4}$$

$(-4)^2 - 4(4)(1) = 0$

$$r = \frac{-(-4)}{8}$$

$r = \frac{1}{2}$ with multiplicity of 2

$y = c_1x^r + c_2\ln(x)\,x^r$

$y = c_1\sqrt{x} + c_2\ln(x)\sqrt{x}$

(Moar) Math
exam 2

4. $xy'' + y' = x$

$ax^2y'' + bxy' + cy = g(t)$

$y = y_h + y_p$

$y = x^r$

$x(x^r)'' + (x^r)' = 0$

$(x^r)'' = rx^{r-2}(r-1)$

$x\,r\,x^{r-2}(r-1) + (x^r)' = 0$

$(x^r)' = rx^{r-1}$ ← power rule

$xrx^{r-2}(r-1) + rx^{r-1} = 0$

$xrx^{r-2}(r-1) = rx^{r-1}(r-1)$

$= rx^{1+r-2}(r-1)$

$= rx^{r-1}(r-1)$

$= rx^{r-1}(r-1) + rx^{r-1}$

$= rx^{r-1}(r-1)$ $\quad a(b-c) = ab - ac$

$a = rx^{r-1},\ b = r,\ c = 1$

4.

$= r x^{r-1} r - r x^{r-1} \cdot 1$

$= r r x^{r-1} - 1 - r x^{r-1}$

$r r x^{r-1} = r^2 x^{r-1}$

$1 \cdot r x^{r-1} = r x^{r-1}$

$= r^2 x^{r-1} - r x^{r-1}$

$= r^2 x^{r-1} \cancel{- r x^{r-1}} \cancel{+ r x^{r-1}}$

$r^2 x^{r-1} = 0$

$x^{r-1}(r^2) = 0$

$r^2 = 0$

$r = 0$ with multiplicity of 2

$y_h = c_1 + c_2 \ln(x)$

---

y_p $\quad y = a_0 x^2 + a_1 x$

$x(a_0 x^2 + a_1 x)'' + (a_0 x^2 + a_1 x)' = x$

Noor math exam 2

4. $((a_0x^2+a_1x)'' = 2a_0$

$x \cdot 2a_0 + (a_0x^2+a_1x)' = x$

$(a_0x^2+a_1x)'$

$= (a_0x^2)' + (a_1x)'$

$(a_0x^2)' = 2a_0x$

$(a_1x)' = a_1$

$(a_0x^2+a_1x)' = 2a_0x + a_1$

$x \cdot 2a_0 + 2a_0x + a_1 = x$

$4a_0x + a_1 = x$

$0 = a_1$

$1 = 4a_0$

$a_0 = \frac{1}{4}$

$a_1 = 0$

$y = \frac{1}{4}x^2 \cancel{+0 \cdot x}$

$y_p = \frac{x^2}{4}$

---

general solution $y = y_h + y_p$

$y = c_1 + c_2 \ln(x) + \frac{x^2}{4}$

Noor

5. $y''' + 8y = 2x - 5 + 8e^{-2x}$ Nour

$y = y_h + y_p$

$\boxed{y_h}$ $y = e^{yx}$

$(e^{yx})''' + 8e^{yx} = 0$ $\qquad (e^{yx})' = ye^{yx}$

$(e^{yx})''' = y^3 e^{yx}$ $\qquad (e^{yx})'' = y^2 e^{yx}$

$y^3 e^{yx} + 8e^{yx} = 0$

$e^{yx}(y^3 + 8) = 0$

$y^3 = -8$

$y = \sqrt[3]{-8}$, $y = \sqrt[3]{-8}\,\dfrac{-1+\sqrt{3}i}{2}$, $y = \sqrt[3]{-8}\,\dfrac{-1-\sqrt{3}i}{2}$

$y = -2$

$-2 \cdot \dfrac{-1+\sqrt{3}i}{2} = 1 - \sqrt{3}i$

$-2 \cdot \dfrac{-1-\sqrt{3}i}{2} = 1 + \sqrt{3}i$

$y_1 = -2$, $y_2 = 1 - \sqrt{3}i$, $y_3 = 1 + \sqrt{3}i$

$y_1 = c_1 e^{yx}$ $\qquad y_2 = c_3 e^{x}\sin(\sqrt{3}x)$

$y_1 = c_1 e^{-2x}$ $\qquad y_3 = c_2 e^{x}\cos(\sqrt{3}x)$

5. $y_h = c_1 e^{-2x} + e^{x}\left(c_2 \cos(\sqrt{3}\,x) + c_3 \sin(\sqrt{3}\,x)\right)$

y_p $\quad y''' + 8y = 2x - 5 \leftarrow y_{p_1}$

$y = a_0 x + a_1$

$(a_0 x + a_1)''' + 8(a_0 x + a_1) = 2x + 5$

$(a_0 x + a_1)''' = 0$

$8(a_0 x + a_1) = 2x + 5$

$8a_0 x + 8a_1 = 2x + 5$

$-5 = 8a_1 \qquad a_0 = \frac{1}{4}$

$2 = 8a_0 \qquad a_1 = -\frac{5}{8}$

$y = \frac{1}{4}x - \frac{5}{8}$

$y = \frac{x}{4} - \frac{5}{8}$

$y_{p_2}: \; y''' + 8y = 8e^{-2x} \qquad y = a_0 x e^{-2x}$

5- Noor MTH 225 exam 2

$(a_0xe^{-2x})''' + 8a_0xe^{-2x} = 8e^{-2x}$

$(a_0xe^{-2x})'''$

$(a_0xe^{-2x})' = a_0(e^{-2x} - 2e^{-2x}x)$

$= (a_0(e^{-2x} - 2e^{-2x}x))''$

$(a_0(e^{-2x} - 2e^{-2x}x))' = a_0(4e^{-2x}x - 4e^{-2x})$

$= (a_0(4e^{-2x}x - 4e^{-2x}))'$

$(a_0(4e^{-2x}x - 4e^{-2x}))' = a_0(-8e^{-2x}x + 12e^{-2x})$

$a_0(-8e^{-2x}x + 12e^{-2x}) + 8a_0xe^{-2x} = 8e^{-2x}$

$$\frac{12a_0e^{-2x}}{12a_0e^{-2x}} = \frac{8e^{-2x}}{12a_0e^{-2x}}$$

$a_0 = \frac{2}{3}$

$y = \frac{2}{3}xe^{-2x}$

$y = \frac{2e^{-2x}x}{3}$

$y_p = \frac{x}{4} - \frac{5}{8} + \frac{2e^{-2x}x}{3}$

5. Nour MTH 225 exam 2

$y = c_1 e^{-2x} + e^{x}(c_2 \cos(\sqrt{3}x) +$
$c_3 \sin(\sqrt{3}x)) + \frac{x}{4} - \frac{5}{8} + \frac{2e^{-2x}x}{3}$

6. Noor Mustafa MTH 225 exam 2

$m_1 = 1 \quad m_2 = 1$

$F = Kx$

$24 = K_1(24) \Rightarrow K_1 = 1 \, N/m$

$24 = K_2(6) \rightarrow K_2 = 4 \, N/m$

$y_2^{(4)} + (K_1 + K_2) y_2'' + K_1 K_2 y_2 = e^{-2t}$

$y_2^{(4)} + (1+4) y_2'' + 4y = e^{-2t}$

$y_2^{(4)} + 5y_2'' + 4y_2 = e^{-2t}$

$\lambda^4 + 5\lambda^2 + 4 = 0$

$\lambda^4 + \lambda^2 + 4\lambda^2 + 4 = 0$

$\lambda^2(\lambda^2 + 1) + 4(\lambda^2 + 1) = 0$

$(\lambda^2 + 1)(\lambda^2 + 4)$

$\lambda = \pm i$

$\lambda = \pm 2i$

general solution

$y_x = (A\cos t + B\sin t) + (C\cos(2t) + D\sin 2t)$

$$y_{2p} = \frac{e^{-2t}}{D^4 + 5D^2 + 4}$$

$$= \frac{e^{-2t}}{(-2)^4 + 5(-2)^2 + 4}$$

6. Noor Mustafa MTH 225 exam 2

$$y_p = \frac{1}{40} e^{-2t}$$

$$y_2 = (A\cos t + B\sin t) + (C\cos 2t + D\sin 2t) + \frac{1}{40} e^{-2t}$$

Noor Mustafa MTH 225 | exam 2

7. (a) $y_2'' = -k_2(y_2 - y_1) \Rightarrow y_2'' = -4(y_2 - y_1)$

$y_2 = (A\cos t + B\sin t) + (C\cos 2t + D\sin 2t) + \frac{1}{40}e^{-2t}$

$y_2' = (-A\sin t + B\cos t) + (-2C\sin 2t + 2D\cos 2t)$
$- \frac{2}{40}e^{-2t}$

$y_2'' = (-A\cos t - B\sin t) + (-4C\cos 2t -$
$4D\sin 2t) + \frac{1}{10}e^{-2t}$

$= -4\left[(A\cos t + B\sin t) + (C\cos 2t + D\sin 2t)\right.$
$\left. + \frac{e^{-2t}}{40} - y_1\right]$

$y_1 = \frac{3}{4}(A\cos t + B\sin t) + \frac{1}{20}e^{-2t}$

B. $y_1 = -.1\,m \quad y_1(0) = -0.1\,m$

$-.1 = \frac{3}{4}(A) + \frac{1}{20} \Rightarrow \frac{3}{4}A = -.15$

$A = \frac{4}{3}(-.15)$

$= -4(.05)$

$A = -0.2$

$y_1(0) = -.1\,m \quad y_2(0) = -.3\,m$

$y_1'(0) = 0 \quad y_2'(0) = 0$

7. [Moor | Math225 | exam2]

$y_1' = 0 \quad y_1'(0) = 0$

$y_1' = \frac{3}{4}(-A\sin t + B\cos t) - \frac{1}{10}e^{-2t}$

$y_1' = 0 = \frac{3}{4}B\left(\frac{-1}{10}\right) \to B = \frac{1}{10} \times \frac{4}{3}$

$= \frac{4}{30}$

$y_1 = \frac{3}{4}\left(-\frac{1}{5}\cos t + \frac{4}{30}\sin t\right) + \frac{1}{20}e^{-2t}$

$y_2 = \frac{-1}{5}\cos t + \frac{4}{30}\sin t + C\cos 2t + D\sin 2t$
$+ \frac{1}{40}e^{-2t}$

$y_2 = 0.30\,m \quad y_2(0) = 0.30\,m$

$0.30 = -\frac{1}{5} + 0 + C + \frac{1}{40}$

$C = .475$

$y_2' = 0$ at $t = 0 \quad y_2'(0) = 0$

$0 = -\frac{1}{5}(0) + \frac{4}{30}(1) + 0 + 2D(1) - \frac{1}{20}e^0$

$2D = \frac{1}{20} - \frac{4}{30}$

$D = -.04166$

7- Noor | Math 225 | Exam 2

$$y_2 = \frac{-1}{5}\cos t + \frac{4}{30}\sin t + (.475\cos 2t) - (.04166)\sin 2t + \frac{1}{40}e^{-2t}$$