Quiz #14 | Noor Mustafa | Math 225

1) $\frac{dx}{dt} = 3x - y - z$

$\frac{dy}{dt} = x + y - z + t$

$\frac{dz}{dt} = x - y + z + 2e^t$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{bmatrix} 3 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 0 \\ t \\ 2e^t \end{pmatrix}$$

$x' = Ax + t$

$$A = \begin{pmatrix} 3 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$$

$$A - \lambda I = 0 \longrightarrow \begin{bmatrix} 3-\lambda & -1 & -1 \\ 1 & 1-\lambda & -1 \\ 1 & -1 & 1-\lambda \end{bmatrix} = 0$$

$(3-\lambda)((1-\lambda)^2 - 1) + 1(1 - \lambda + 1) - 1(-1 - 1 + \lambda)$
$= 0$

$\lambda = 1 \quad \lambda = 2 \quad \lambda = 2$

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1) $\lambda_1 = 1$:

$$\begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x - y = 0 \rightarrow x = y$

$x - z = 0 \rightarrow x = z$

$v_1 = [1, 1, 1]$

$\lambda_2 = 2$

$$\begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x - y - z = 0$

$z = 0 \rightarrow v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$y = 0 \rightarrow v_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$$x(t) = c_1 e^{t} v_1 + c_2 e^{2t} v_2 + c_3 t e^{2t} v_3$$

$$g(t) = c_1 e^{t} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_3 t e^{2t} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

general solution of homogeneous form

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$$\Phi = \begin{bmatrix} e^{t} & e^{2t} & te^{2t} \\ e^{t} & e^{2t} & 0 \\ e^{t} & 0 & te^{2t} \end{bmatrix}$$

$$F = \begin{bmatrix} 0 \\ t \\ 2e^{t} \end{bmatrix}$$

$$\Phi^{-1} \cdot F = 0$$

$$\Phi^{-1} = \begin{bmatrix} -\frac{1}{e^{t}} & \frac{1}{e^{t}} & \frac{1}{e^{t}} \\ \frac{1}{e^{2t}} & 0 & -\frac{1}{e^{2t}} \\ \frac{1}{te^{2t}} & -\frac{1}{te^{2t}} & 0 \end{bmatrix}$$

$$\Phi^{-1} \cdot F =$$

$$\begin{pmatrix} -\frac{1}{e^{t}} & \frac{1}{e^{t}} & \frac{1}{e^{t}} \end{pmatrix} \begin{pmatrix} 0 \\ t \\ 2e^{t} \end{pmatrix} = \left(-\frac{1}{e^{t}}\right) \cdot 0 + \frac{1}{e^{t}} t + \frac{1}{e^{t}} \cdot 2e^{t}$$

$$\begin{pmatrix} \frac{1}{2^{t}} & 0 & -\frac{1}{e^{2t}} \end{pmatrix} \begin{pmatrix} 0 \\ t \\ 2e^{t} \end{pmatrix} = \frac{1}{e^{2t}} \cdot 0 + 0 \cdot t + -\frac{1}{e^{2t}} \cdot 2e^{t}$$

$$\begin{pmatrix} \frac{1}{te^{2t}} & -\frac{1}{te^{2t}} & 0 \end{pmatrix} \begin{pmatrix} 0 \\ t \\ 2e^{t} \end{pmatrix} = \frac{1}{te^{2t}} \cdot 0 + \left(-\frac{1}{te^{2t}}\right) t + 0 \cdot 2e^{t}$$

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$$= \begin{pmatrix} -te^{-t} + e^{t} + 2^{t} \\ -2e^{-t} \\ -e^{-2t} \end{pmatrix} \leftarrow \Phi^{-1} \cdot F$$

$\int \Phi^{-1} \cdot F$

$$= \begin{bmatrix} -te^{-t} + e^{t} + 2t \\ 2e^{t} \\ \frac{1}{2}e^{2t} \end{bmatrix}$$

$$= \begin{bmatrix} e^{t} & e^{2t} & te^{2t} \\ e^{t} & e^{2t} & 0 \\ e^{t} & 0 & te^{2t} \end{bmatrix} \begin{bmatrix} -te^{-t} + e^{t} + 2t \\ 2e^{t} \\ \frac{1}{2}e^{2t} \end{bmatrix}$$

$$= \begin{pmatrix} e^{t}(-te^{-t} + e^{t} + 2t) + e^{2t} \cdot 2e^{t} + te^{2t} \frac{1}{2}e^{2t} \\ e^{t}(-te^{-t} + e^{t} + 2t) + e^{2t} \cdot 2e^{t} + 0 \cdot \frac{1}{2}e^{2t} \\ e^{t}(-te^{-t} + e^{t} + 2t) + 0 \cdot 2e^{t} + te^{2t} \cdot \frac{1}{2}e^{2t} \end{pmatrix}$$

$$= \begin{bmatrix} -t + e^{2t} + 2te^{t} + 2e^{3t} + \frac{1}{2}te^{4t} \\ -t + e^{2t} + 2te^{t} + 2e^{3t} \\ -t + e^{3t} + 2te^{t} + \frac{1}{2}te^{4t} \end{bmatrix}$$

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$$\Phi_p = t\begin{bmatrix}-1\\-1\\-1\end{bmatrix} + e^{2t}\begin{bmatrix}1\\1\\1\end{bmatrix} + te^{t}\begin{bmatrix}2\\2\\2\end{bmatrix} + e^{3t}\begin{bmatrix}2\\2\\0\end{bmatrix}$$
$$+ te^{4t}\begin{bmatrix}1/2\\0\\1/2\end{bmatrix}$$

$$X(t) = c_1 e^{t}\begin{pmatrix}1\\1\\1\end{pmatrix} + c_2 e^{2t}\begin{pmatrix}1\\1\\0\end{pmatrix} + c_3 te^{2t}\begin{pmatrix}1\\0\\1\end{pmatrix}$$
$$+ t\begin{pmatrix}-1\\-1\\-1\end{pmatrix} + e^{2t}\begin{bmatrix}1\\1\\1\end{bmatrix} + te^{t}\begin{bmatrix}2\\2\\2\end{bmatrix} + e^{3t}\begin{pmatrix}2\\2\\0\end{pmatrix}$$
$$+ te^{4t}\begin{bmatrix}1/2\\0\\1/2\end{bmatrix}$$

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(2) $\frac{\partial A}{\partial t}$ = tank A $\quad \frac{\partial B}{\partial t}$ = tank B $\quad \frac{\partial C}{\partial t}$ = tank C

$$\frac{\partial A}{\partial t} = 3(150) + 10\frac{B}{L_B} - 40\frac{A}{L_A} - 20\frac{A}{L_A}$$

$$\frac{\partial B}{\partial t} = 40\frac{A}{L_A} + 10\frac{C}{L_C} - 10\frac{B}{L_B} - 30\frac{B}{L_B} - 10\frac{B}{L_B}$$

$$\frac{\partial C}{\partial t} = 30\frac{B}{L_C} - 10\frac{C}{L_C} - 20\frac{C}{L_C}$$

$L_A = 200 + (50 + 10 - 40 - 20)t \quad t = 200$

$L_B = 100 + (40 + 10 - 10 - 30 - 10)t \quad t = 100$

$L_C = 100 + (30 - 10 - 20)t \quad t = 100$

$$\boxed{\begin{aligned} \frac{dA}{dt} &= 150 + \frac{B}{10} - \frac{6A}{20} \\ \frac{dB}{dt} &= \frac{A}{5} + \frac{C}{10} - \frac{B}{2} \\ \frac{\partial C}{\partial t} &= \frac{3B}{10} - \frac{3C}{10} \end{aligned}}$$

$A(0) = 20$

$B(0) = 20$

$C(0) = 0$

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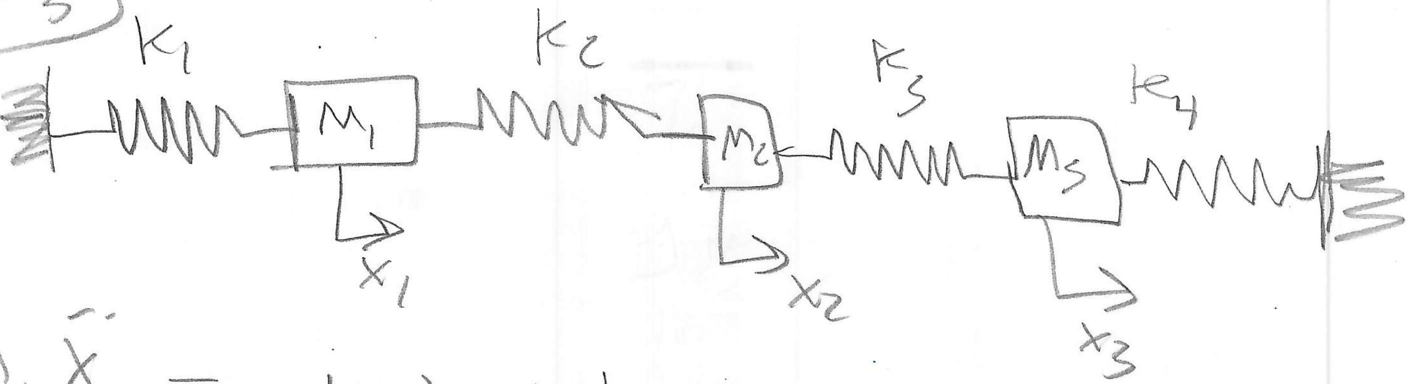
#3



$$m_1 \ddot{x}_1 = -k_1 x_1 + k_2 (x_2 - x_1)$$

$$m_2 \ddot{x}_2 = -k_2 (x_2 - x_1) + k_3 (x_3 - x_2)$$

$$m_3 \ddot{x}_3 = -k_3 (x_3 - x_2) + k_4 (x_3)$$

$$\ddot{x}_1 = \frac{-k_1 - k_2}{m_1} x_1 + k_2 x_2 = \dot{x}_4$$

$$\ddot{x}_2 = \frac{k_2}{m_2} x_1 + \frac{-k_2 - k_3}{m_2} x_2 + \frac{k_3}{m_2} x_3 = \dot{x}_5$$

$$\ddot{x}_3 = \frac{k_3}{m_3} x_2 + \frac{-k_3 + k_4}{m_3} x_3 = \dot{x}_6$$

$$\dot{x}_1 = x_4$$

$$\dot{x}_2 = x_5$$

$$\dot{x}_3 = x_6$$

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$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-k_1-k_2}{m_1} & k_2 & 0 & 0 & 0 & 0 \\ \frac{k_2}{m_2} & \frac{-k_2-k_1}{m_2} & \frac{k_3}{m_2} & 0 & 0 & 0 \\ 0 & \frac{k_3}{m_3} & \frac{-k_3+k_4}{m_3} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{Posit} \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{velociti} \end{matrix}$$

$$X(0) = 0$$

$\Uparrow$

column vector of X