Math 225 – Exam #2

Clearly and neatly show all work with proper notation for each problem. Solutions with no work will receive no credit. Simplify all answers completely.

1. (10) Solve the following third-order initial-value problem.

$$x^3y''' + xy' - y = 0, \qquad y(1) = 2, \qquad y'(1) = -1, \qquad y''(1) = 0$$

2. (10) Find the general solution for the following second-order differential equation.

$$3y'' - 6y' + 6y = e^x \sec x$$

3. (10) Use the given solution to find the general solution to the following second-order differential equation.

$$4x^2y'' + y = 0; \quad y_1 = \sqrt{x}\ln x$$

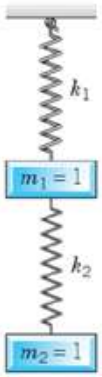
4. (10) Find a general solution for the following second-order differential equation.

$$xy'' + y' = x$$

5. (10) Find a general solution for the following third-order differential equation.

$$y''' + 8y = 2x - 5 + 8e^{-2x}$$

6. (18) Consider the spring-mass system shown, consisting of two unit masses m_1 and m_2 suspended from springs with constants k_1 and k_2, respectively. Assuming that there is no damping in the system, the displacement $y_2(t)$ of the bottom mass m_2 from its equilibrium positions satisfies the 4th-order equation



$$y_2^{(4)} + (k_1 + k_2)y_2'' + k_1 k_2 y_2 = e^{-2t},$$

where $f(t) = e^{-2t}$ is an outside force driving the motion of m_2.

If a 24 N weight would stretch the top spring by 24 m and the bottom spring by 6 m, find the general equation of motion for m_2.
**The downward direction is assumed to be the positive direction.*

7. (12) In the same setup as above, the displacement $y_1(t)$ of mass m_1 from its equilibrium positions satisfies the equation

$$y_2'' = -k_2(y_2 - y_1).$$

a) Using substitution with results from above, find the general equation of motion for m_1.
b) If m_1 starts from rest 0.10 m above its equilibrium position and m_2 starts from rest 0.30 m below its equilibrium position, write out initial conditions and find the exact equations of motion for **each** mass.

**You may assume that the motion of the spring-mass system will not distort the springs nor will it result in the masses colliding.*