به نام خدا

دانشگاه تهران

دانشکده مهندسی برق و کامپیوتر

دانشکده مهندسی مکانیک

رباتیک و مکاترونیک

مینی پروژه شماره ۲

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فهرست گزارش سوالات

Problem 1 – Kinematic and Dynamic Modeling

1.

i	a_i	b_i	α_i	θ_i
1	0	b_1	$\frac{\pi}{2}$	0
2	400	257.7	0	θ_2
3	250	0	0	θ_3
4	0	b_4	0	0

2.

$$\overrightarrow{a_1} = \begin{bmatrix} 0 \\ 0 \\ b_1 \end{bmatrix} \quad \overrightarrow{a_2} = \begin{bmatrix} 400\cos(\theta_2) \\ 400\sin(\theta_2) \\ 257.7 \end{bmatrix} \quad \overrightarrow{a_3} = \begin{bmatrix} 250\cos(\theta_3) \\ 250\sin(\theta_3) \\ 0 \end{bmatrix} \quad \overrightarrow{a_4} = \begin{bmatrix} 0 \\ 0 \\ b_4 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \quad Q_2 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 \\ \sin(\theta_2) & \cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Q_3 = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Q_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\vec{p} = \overrightarrow{a_1} + Q_1\overrightarrow{a_2} + Q_1Q_2\overrightarrow{a_2} + Q_1Q_2Q_3\overrightarrow{a_3} + Q_1Q_2Q_3Q_4\overrightarrow{a_4} = \begin{pmatrix} 250\ \cos(\theta_2+\theta_3) + 400\ \cos(\theta_2) \\ -b_4 - \frac{2577}{10} \\ b_1 + 250\ \sin(\theta_2+\theta_3) + 400\ \sin(\theta_2) \end{pmatrix}$$

$$Q = Q_1Q_2Q_3Q_4 = \begin{pmatrix} \cos(\theta_2+\theta_3) & -\sin(\theta_2+\theta_3) & 0 \\ 0 & 0 & -1 \\ \sin(\theta_2+\theta_3) & \cos(\theta_2+\theta_3) & 0 \end{pmatrix}$$

$$\begin{cases} x = 250\ \cos(\theta_2+\theta_3) + 400\ \cos(\theta_2) \\ y = -b_4 - \frac{2577}{10} \\ z = b_1 + 250\ \sin(\theta_2+\theta_3) + 400\ \sin(\theta_2) \\ \phi = \theta_2 + \theta_3 \end{cases}$$

3.

$$x - 250\cos(\phi) = 400\cos(\theta_2)$$
$$z - b_1 - 250\sin(\phi) = 400\sin(\theta_2)$$

$$x^2 - 500x\cos(\phi) + 62500\cos^2(\phi) + z^2 + b_1^2 + 62500\sin^2(\phi) - 2b_1z - 500zsin(\phi) + 500b_1\sin(\phi) = 160000\cos^2(\theta_2) + 160000\sin^2(\theta_2)$$
$$x^2 + z^2 + b_1^2 - 2b_1z - 97500 - 500xcos(\phi) + (500b_1 - 500z)\sin(\phi) = 0$$

$$b_1^2 + (500\sin(\phi) - 2z)b_1 + (x^2 + z^2 - 97500 - 500xcos(\phi) - 500zsin(\phi)) = 0$$

$$b_1 = \frac{(-500\sin(\phi) - 2z) \pm \sqrt{(-500\sin(\phi) - 2z)^2 - 4\left(x^2 + z^2 - 97500 - 500xcos(\phi) - 500zsin(\phi)\right)}}{2}$$

$$\theta_2 = atan2(x - 250\cos(\phi), z - b_1 - 250\sin(\phi))$$
$$\theta_3 = \phi - \theta_2$$

4.

$$J = \begin{bmatrix} 0 & 1 & 1 & 0 \\ \overrightarrow{e_1} & \overrightarrow{e_2} \times \overrightarrow{r_2} & \overrightarrow{e_3} \times \overrightarrow{r_3} & \overrightarrow{e_4} \times \overrightarrow{r_4} \end{bmatrix}$$

$$[\overrightarrow{e_1}]_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$[\overrightarrow{e_2}]_1 = Q_1[\overrightarrow{e_2}]_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$[\overrightarrow{e_3}]_1 = Q_1Q_2[\overrightarrow{e_3}]_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 \\ \sin(\theta_2) & \cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$[\overrightarrow{e_3}]_1 = Q_1Q_2Q_3[\overrightarrow{e_4}]_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 \\ \sin(\theta_2) & \cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$\overrightarrow{r_2} = [\overrightarrow{a_2}]_1 + [\overrightarrow{a_3}]_1 + [\overrightarrow{a_4}]_1 = Q_1\overrightarrow{a_2} + Q_1Q_2\overrightarrow{a_3} + Q_1Q_2Q_3\overrightarrow{a_4} = \begin{pmatrix} 250\ \cos(\theta_2 + \theta_3) + 400\ \cos(\theta_2) \\ -b_4 - \frac{2577}{10} \\ 250\ \sin(\theta_2 + \theta_3) + 400\ \sin(\theta_2) \end{pmatrix}$$

$$\overrightarrow{r_3} = [\overrightarrow{a_3}]_1 + [\overrightarrow{a_4}]_1 = Q_1Q_2\overrightarrow{a_3} + Q_1Q_2Q_3\overrightarrow{a_4} = \begin{pmatrix} 250\ \cos(\theta_2+\theta_3) \\ -b_4 \\ 250\ \sin(\theta_2+\theta_3) \end{pmatrix}$$

$$J = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & -250\ \sin(\theta_2+\theta_3) - 400\ \sin(\theta_2) & -250\ \sin(\theta_2+\theta_3) & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 250\ \cos(\theta_2+\theta_3) + 400\ \cos(\theta_2) & 250\ \cos(\theta_2+\theta_3) & 0 \end{pmatrix}$$

5. As it provided in the paper, we should calculate the dynamic of the robot as follow:

$$T_i = \frac{1}{2}m_i\left||\dot{c}_i|\right|^2 + \frac{1}{2}w_i^T I_i w_i\ , \qquad \dot{c}_i = N_i\dot{\theta}$$

$$[N_i]_1 = [e_1 \times r_{1i} \quad \dots \quad e_i \times r_{ii} \quad 0 \quad \dots \quad 0]$$

$$N_1 = [e_1 \quad 0 \quad 0 \quad 0]$$

$$N_2 = [e_1 \quad e_2 \times r_{22} \quad 0 \quad 0]$$

$$N_3 = [e_1 \quad e_2 \times r_{23} \quad e_3 \times r_{33} \quad 0]$$

$$N_4 = [e_1 \quad e_2 \times r_{24} \quad e_3 \times r_{34} \quad e_4]$$

So we have:

$$r_{11} = Q_z\left(\theta_1 - \theta_1^{init}\right)c_1$$

$$r_{ij} = \sum_{k=i}^{j-1}[a_k]_1 + \left[Q_z\left(\theta_j - \theta_j^{init}\right)c_j\right]_1$$

Then we should compute W_i in i^{th} joint DH frame:

$$[W_1]_1 = [z \quad 0 \quad 0 \quad 0]$$

$$[W_2]_2 = [Q_1^T z \quad z \quad 0 \quad 0]$$

$$[W_3]_3 = [Q_2^T Q_1^T z \quad Q_2^T z \quad z \quad 0]$$

$$[W_4]_4 = [Q_3^T Q_2^T Q_1^T z \quad Q_3^T Q_2^T z \quad Q_3^T z \quad z]$$

And because the first and the last joint in prismatic so:

$$[W_1]_1 = [0 \quad 0 \quad 0 \quad 0]$$

$$[W_2]_2 = [0 \quad z \quad 0 \quad 0]$$

$$[W_3]_3 = [0 \quad Q_2^T z \quad z \quad 0]$$

$$[W_4]_4 = [0 \quad Q_3^T Q_2^T z \quad Q_3^T z \quad 0]$$

$$M_i(\theta) = m_i N_i^T N_i + [W_1]_i^T Q_z\left(\theta_i - \theta_i^{init}\right) I_i Q_z\left(\theta_i - \theta_i^{init}\right)^T [W_i]_i$$

One can formulate kinetic energy and potential energy of each joint as:

$$T_i = \frac{1}{2}\dot{\theta}^T M_i \dot{\theta}$$

$$V_i = m_i g h_i, \;\; h_i = r_{1i}^T z$$

the Lagrangian term can be calculated and the dynamic model of manipulator can be expressed as follows:

$$\mathcal{L} = T - V$$

$$\tau = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) - \frac{\partial \mathcal{L}}{\partial \theta}$$

$$\tau = M\ddot{\theta} + \dot{M}\dot{\theta} - \frac{\partial T}{\partial \theta} + \frac{\partial V}{\partial \theta}$$

6. Based on the formulas and calculations performed in the previous section, enter them into MATLAB file to carry out the computations completely and accurately. Finally, we will plot the torque and force of the dynamic model on a graph.

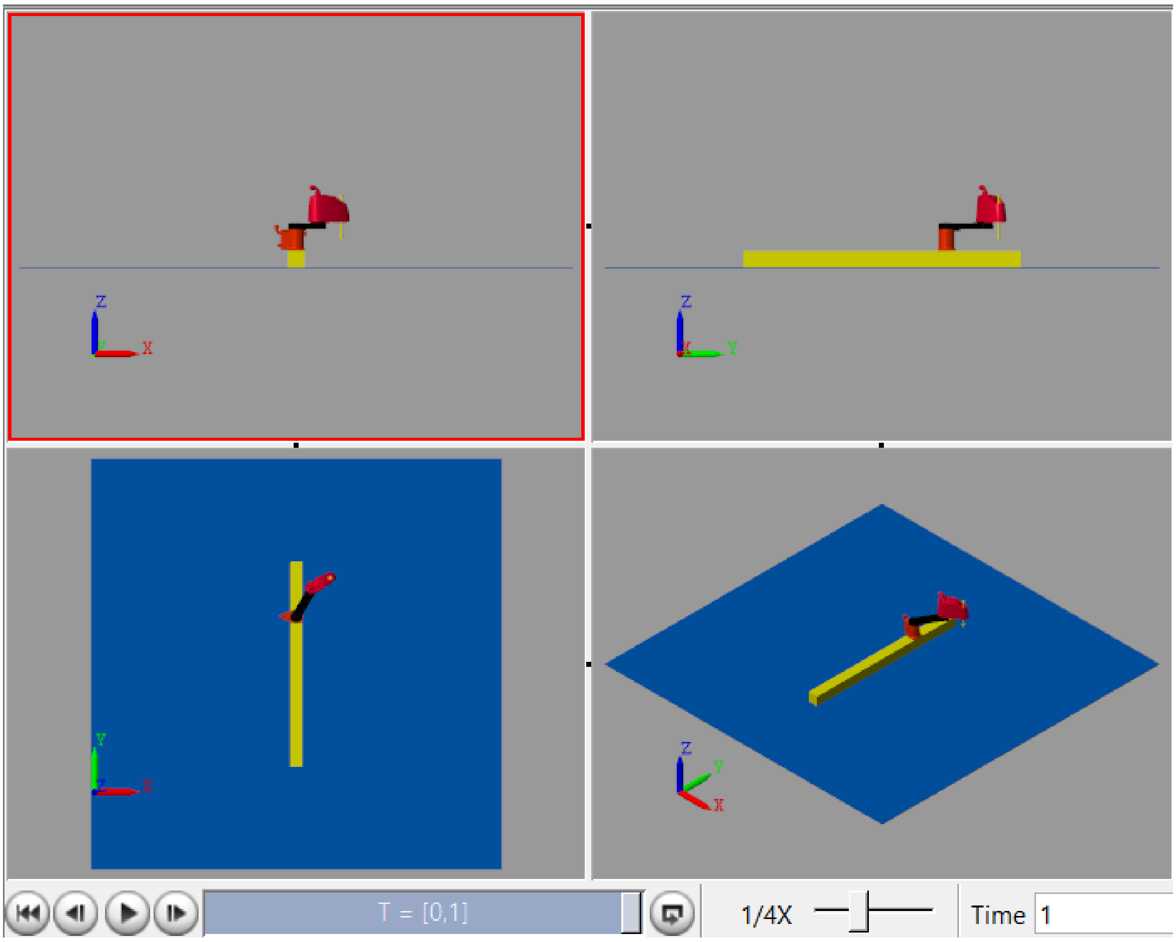

Figure 1 dynamic model simulation in simscape

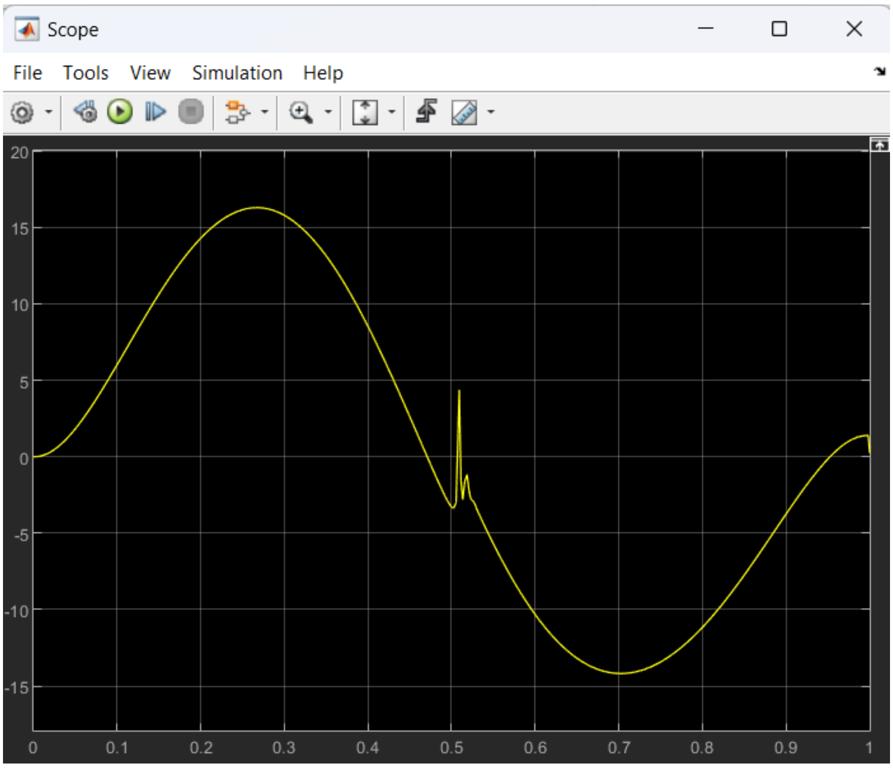

Figure 2 output of f1 (simulation)

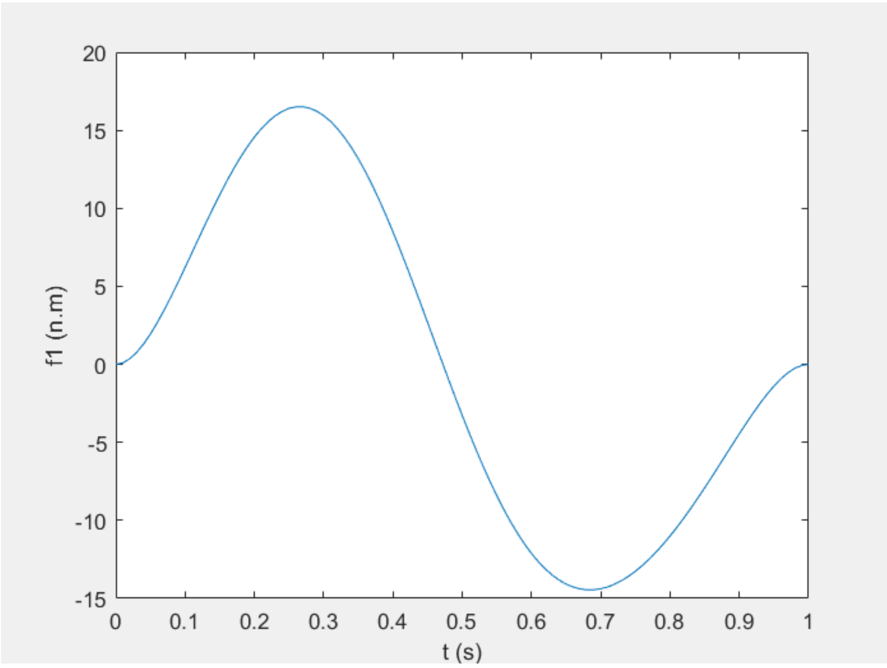

Figure 3 output of f1 (code)

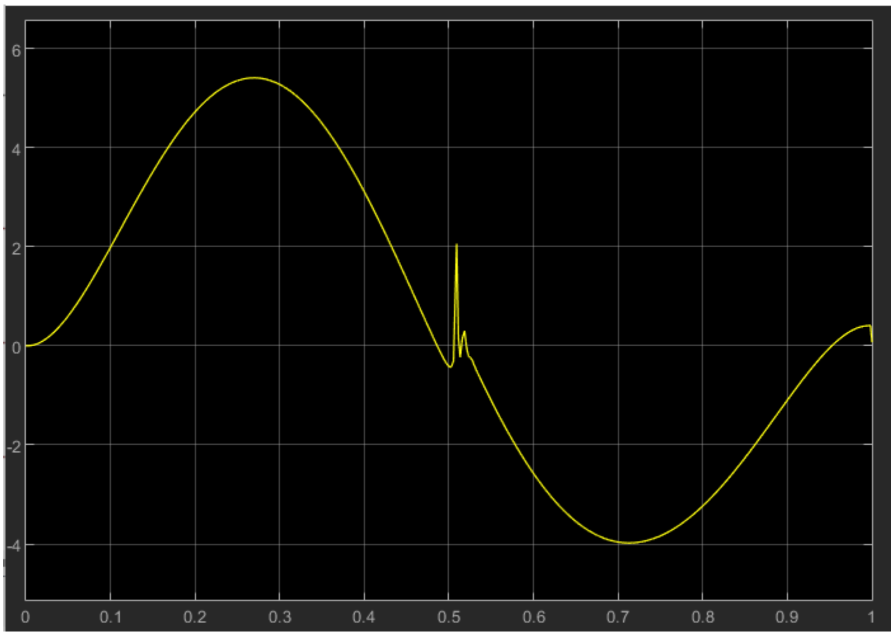

Figure 4 output of tau2 (simulation)

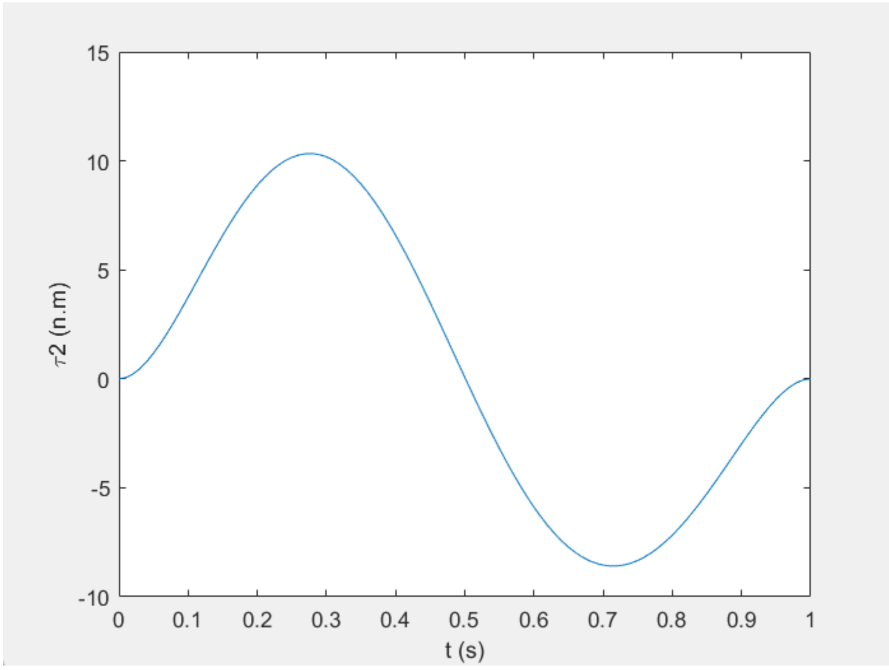

Figure 5 output of tau2 (code)

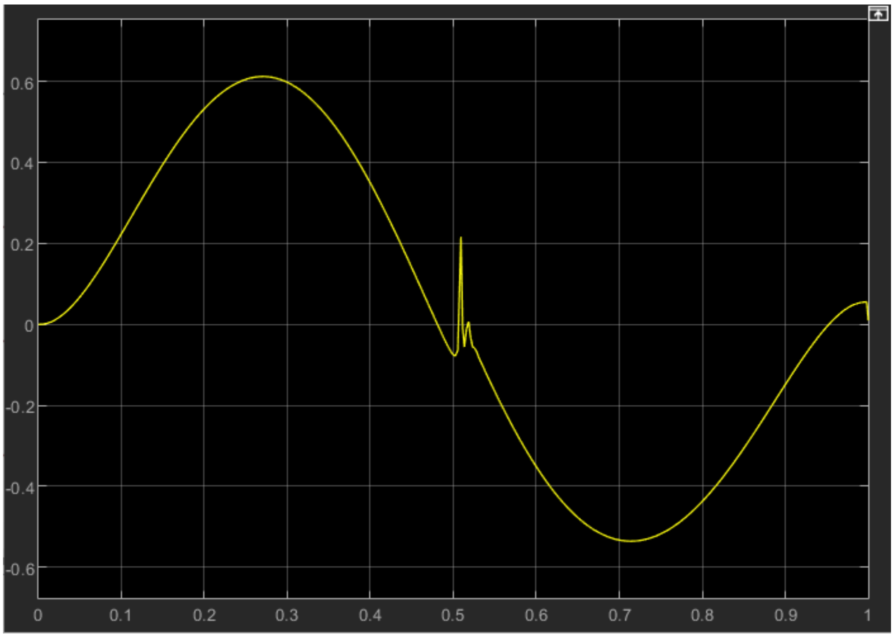

Figure 6 output of tau3 (simulation)

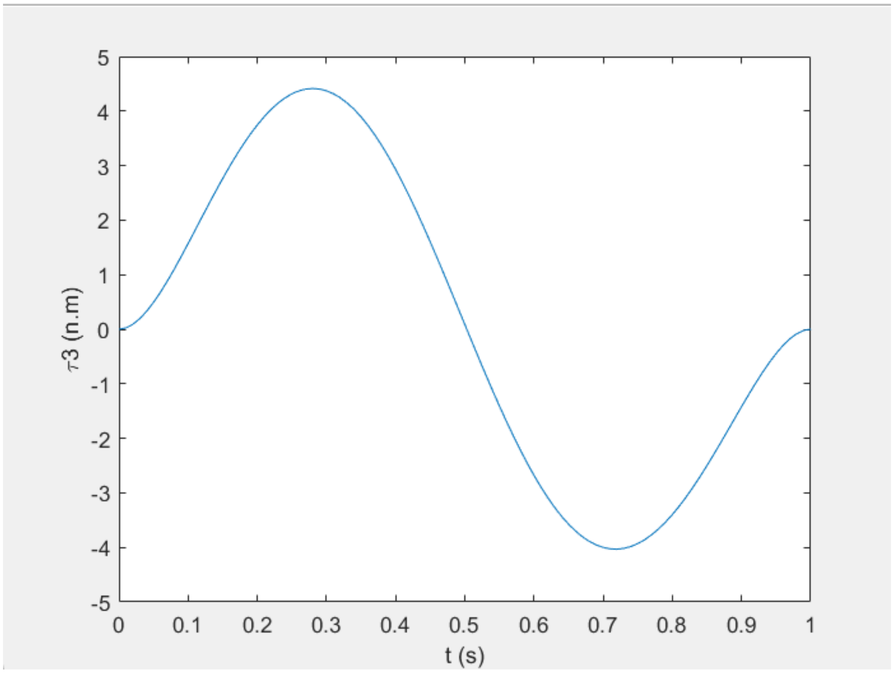

Figure 7 output of tau3 (code)

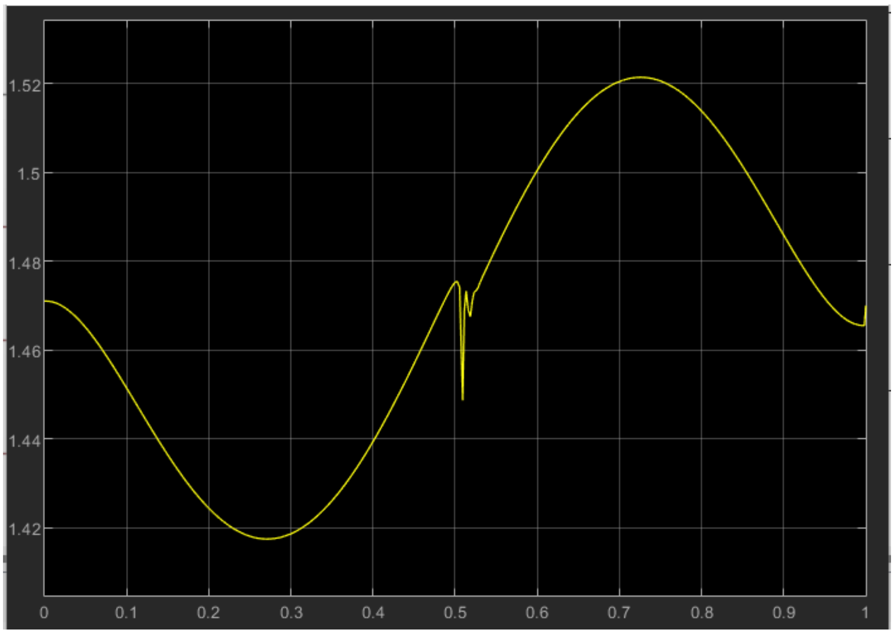

Figure 8 output of f4 (simulation)

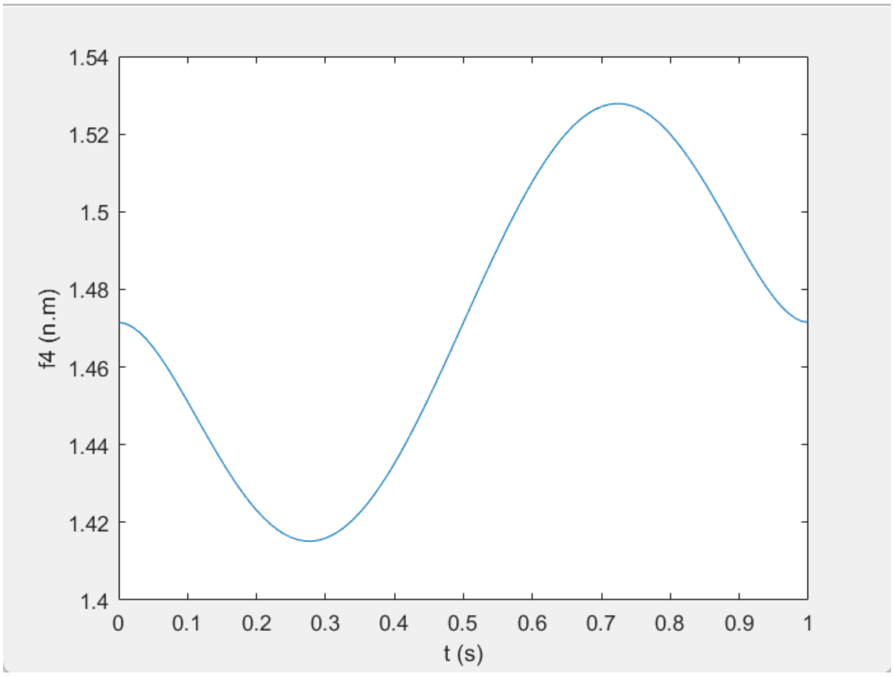

Figure 9 output of f4 (code)

As observed, the output of the code and the simulation have been obtained similarly.

Now, we plot the pick and place motion with the 4-5-6-7 interpolation.

$$s(\tau) = -20t^7 + 70t^6 - 84t^5 + 35t^4$$

$$s(\dot{\tau}) = -280t^6 + 420t^5 - 420t^4 + 140t^3$$

$$s(\ddot{\tau}) = -6180t^5 + 2100t^4 - 1680t^3 + 420t^2$$

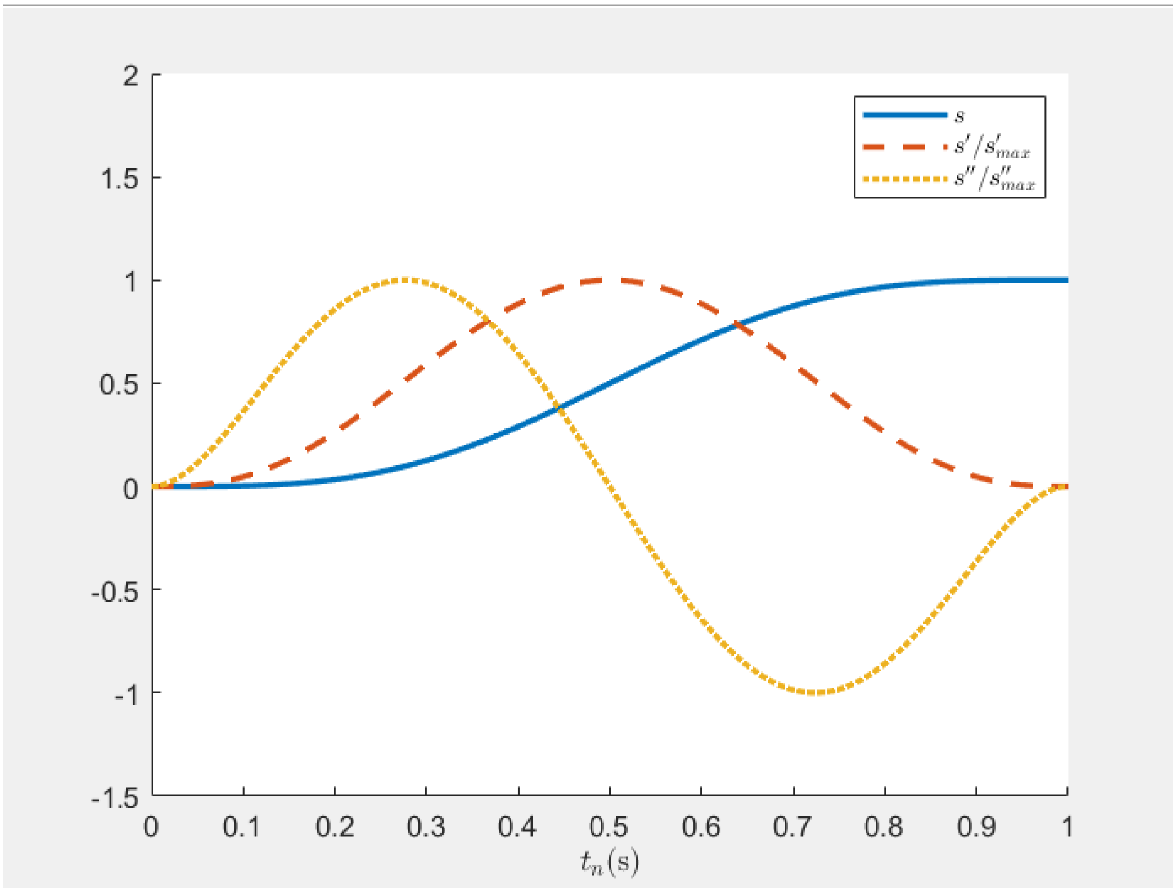

Figure 10 polynomial 4-5-6-7