Introduction to discrete Control

Frequency domain

Continuous

S-Domain transfer function

$\rightarrow \dfrac{S+1}{S^2+2S+1}$

Discrete ✓

Z-domain transfer function

$\dfrac{0.63z-0.23}{z^2-0.74z+0.14}$

→ Relation between z and S-domain

$z = e^{sT}$ where $\sum_{n=0}^{\infty} e^{-STn} = \sum_{n=0}^{\infty} z^{-n}$

$f^*(s) = \sum_{n=0}^{\infty} f(nT)\, e^{-nTS}$

$\Longrightarrow f(z) = \sum_{n=0}^{\infty} f(nT)\, z^{-n}$ ↳ function

Finally: $X(z) = z[(x(t))] = z[x(KT)] = \sum_{k=0}^{\infty} x(KT)\, z^{-K}$

$x(z) = z[x(K)] = \sum_{K=0}^{\infty} x(K)\, z^{-K}$

Example: ① unit step function: $x(t) = \begin{cases} 1(t) & 0 \leq t \\ 0 & t < 0 \end{cases}$

$X(z) = z[1(t)] = \sum_{K=0}^{\infty} 1 z^{-K} = \sum_{K=0}^{\infty} z^{-K}$

$= 1 + z^{-1} + z^{-2} + z^{-3} + \cdots = \dfrac{1}{1-z^{-1}} \dfrac{*z}{*z} = \boxed{\dfrac{z}{z-1}}$

Example: ② $x(t) = \begin{cases} t & 0 \leq t \\ 0 & t < 0 \end{cases}$

$x(z) = z[t] = \sum_{K=0}^{\infty} x(KT)\, z^{-K}$, $x(KT) = KT$

$\therefore X(z) = T \cdot \sum_{K=0}^{\infty} K z^{-K} = T(z^{-1} + 2z^{-2} + 3z^{-3} + \cdots)$

$X(z) = \dfrac{Tz^{-1}}{(1-z^{-1})^2} = \dfrac{Tz}{(z-1)^2}$

Example: ③ $x(t) = e^{at}$

$X(z) = \sum_{K=0}^{\infty} e^{aKT} z^{-K} = 1 + e^{aT} z^{-1} + e^{2aT} z^{-2} + \cdots = \dfrac{1}{1-e^{aT}z^{-1}} \dfrac{*z}{*z} = \dfrac{z}{z-e^{aT}}$

Example: ④ $z[e^{j\omega t}] = \dfrac{z}{z-e^{j\omega T}} = \dfrac{z}{z-[\cos\omega T + j\sin\omega T]}$

Example: ⑤ $z[\cos\omega t] = \dfrac{z^2 - z\cos\omega t}{z^2 - 2z\cos\omega t + 1}$

Example: ⑥ $z[\sin\omega t] = \dfrac{z\sin\omega t}{z^2 - 2z\cos\omega t + 1}$

Example: ① $Z(1-e^{-t}) = \frac{1}{1-z^{-1}} - \frac{1}{1-e^{-T}z^{-1}}$

$$= \frac{(1-e^{-T})z^{-1}}{(1-z^{-1})(1-e^{-T}z^{-1})} \frac{*z}{*z} = \frac{z(1-e^{-T})}{(z-1)(z-e^{-T})}$$

*** properties of Z-transform:**

① **multiplication by a Constant:**

$$Z[ax(t)] = a\,Z[x(t)] = aX(z)$$

② **Linearity:** Ex $x(K) = \alpha f(K) + \beta g(K)$

$$\therefore X(z) = \alpha f(z) + \beta g(z)$$

where f, g are z-transformable

α, β are scalar quantity.

③ **Multiplication by a^K**

$$Z[a^K x(K)] = X(a^{-1}z) = X(z/a)$$

④ **Shifting theorem**

$Z[x(t-nT)] = z^{-n}X(z) \rightarrow$ function shifting to the right

$Z[x(t+nT)] = z^{n}\left[X(z) - \sum_{K=0}^{n-1} x(KT)z^{-K}\right] \rightarrow$ function shifting to the left.

* $Z[x(K+1)] = zX(z) - zX(0)$

* $Z[x(K+2)] = z * Z[x(K+1)] - zX(1)$

$= z^2X(z) - z^2X(0) - zX(1)$

In general: $Z[x(K+n)] = z^nX(z) - z^nx(0) - z^{n-1}x(1) - z^{n-2}x(2) \text{ ------ } - zX(n-1)$

where n is positive integer

*** initial value theorem**

$$\boxed{X(0) = \lim_{z\to\infty} X(z)}$$

Example: find $X(0)$ if $X(z) = \frac{(1-e^{-T})z^{-1}}{(1-z^{-1})(1-e^{-T}z^{-1})}$

$$X(0) = \lim_{z\to\infty} \frac{(1-e^{-T})\,z^{-1} \to 0}{(1-z^{-1} \to 0)(1-e^{-T}z^{-1} \to 0)} = 0$$

* **final value theorem:**

$$\lim_{K\to\infty} x(K) = \lim_{z\to 1} (1-z^{-1})\, X(z)$$

Example: find $x(\infty)$ if $X(z) = \left(\frac{1}{1-z^{-1}} - \frac{1}{1-e^{-aT}z^{-1}}\right)$

$$x(\infty) = \lim_{z\to 1} (1-\cancel{z^{-1}})\left(\frac{1}{\cancel{1-z^{-1}}} - \frac{1}{1-e^{-aT}z^{-1}}\right) = \lim_{z\to 1}\left(1 - \frac{\cancel{(1-z^{-1})}}{1-\underbrace{e^{-aT}z^{-1}}_{0}}\right) = 1$$

* **Inversion of z-transform:**

a] Direct Division Method:

Example: $X(z) = \dfrac{10z+5}{(z-1)(z-0.2)}$

$$= \frac{10z+5}{z^2-1.2z+0.2} = \frac{10z^{-1}+5z^{-2}}{\underbrace{1}_{z^0} - 1.2z^{-1}+0.2z^{-2}}$$

using long division

$$\begin{array}{r|l}
 & 10z^{-1} + 17z^{-2} + 18.4z^{-3} + 18.68z^{-4} + \cdots \\
\hline
1 - 1.2z^{-1} + 0.2z^{-2} & \cancel{10z^{-1}} + 5z^{-2} \\
 & -\;\cancel{10z^{-1}} \pm 12z^{-2} \mp 2z^{-3} \\
\cline{2-2}
 & 17\cancel{z^{-2}} - 2z^{-3} \\
 & \cancel{17z^{-2}} \overset{+}{-} 20.4z^{-3} \overset{-}{+} 3.4z^{-4} \\
\cline{2-2}
 & 18.4z^{-3} - 3.4z^{-4}
\end{array}$$

$$X(z) = 10z^{-1} + 17z^{-2} + 18.4z^{-3} + 18.68z^{-4} + \cdots$$

and Compare this infinite series with $X(z) = \sum_{K=0}^{\infty} x(K)z^{-K} = x(0) + x(1)z^{-1} + x(2)z^{-2} + \cdots$

$x(0) = 0$, $x(1) = 10$, $x(2) = 17$ $x(3) = 18.4$, $x(4) = 18.68$

~~the method~~ &

b] Partial Fraction method:

Sheet One

$\boxed{12}$ $y(K+2) - 3/4\, y(K+1) + 1/8\, y(K) = e(K)$

$y(0) = y(1) = 0$, $e(0) = 0$, $e(k) = 1$. $K = 1, 2, \ldots$

8
10
2

$\boxed{a}$ $y(k+2) = z^2 y(z) - z^2 y(0) - zY(1)$

$y(k+1) = zy(z) - z(y(0)$

$$= [z^2 Y(z) - z^2 \underbrace{Y(0)}_{0} - z\underbrace{Y(1)}_{0}] - 3/4 [zY(z) - z\underbrace{Y(0)}_{0}] + 1/8 Y(z) = e(K)$$

→ unit step

$$e(K) \rightarrow E(z) = \frac{1}{1 - z^{-1}} \frac{*z}{*z} = \frac{z}{z-1}$$

$$[z^2 Y(z)] - 3/4 [zY(z)] + 1/8 Y(z) = z^2 u(z) - z^2 \underbrace{u(0)}_{0} - z\underbrace{u(1)}_{1}$$

$Y(z) [z^2 - 3/4 + 1/8] - z^2$

$\boxed{b}$ $y(K+2) = e(K) + 3/4\, y(K+1) - 1/8\, y(K)$

$y(2) = 0 + 3/4(0) - 1/8(0) = 0$

$y(3) = 1 + 3/4(0) - 1/8(0) = 1$

$y(4) = 1 + 3/4(1) - 1/8(0) = 7/4$

$\boxed{c}$ $\boxed{a}$ $y(K+2) - 3/4\, y(K+1) + 1/8\, y(K) = 0$

$z^2 [Y(z) - Y(0) - Y(1)] - 3/4\, z [Y(z) - Y(0)] + 1/8 Y(z) = 0$

$[z^2 - 3/4 + 1/8] Y(z) = z^2 - 2z - 3/4\, z$

$$Y(z) = \left[\frac{z - 1/4}{(z - 1/2)(z - 1/4)}\right] z = z \left[\frac{-9}{z - 1/2} + \frac{10}{z - 1/4}\right]$$

$Y(K) = -9 (1/2)^K + 10 (1/4)^K$

$y(0)=1$, $y(1)=-2$, $y(2)=-13/8$, $y(3)=-31/32$, $y(4)=-67/128$

b] $y(k+2) = 3/4\, y(k+1) - 1/8\, y(k)$

$y(2) = 3/4(-2) - 1/8(1) = -13/8$

$y(3) = 3/4\left(-13/8\right) - 1/8(-2) = -31/32$

$y(4) = 3/4\left(-31/32\right) - 1/8\left(-13/8\right) = -67/128$

4] $\underline{z\text{-transform of } te^{-at}}$

$e(t) = te^{-at}$

from table $\rightarrow e(t) = t \Rightarrow E(z) = \dfrac{Tz^{-1}}{(1-z^{-1})^2} \begin{matrix} *z^2 \\ *z^2 \end{matrix} = \dfrac{Tz}{(z-1)^2}$

Shifting by $\underline{a}$ From table

$$E(z) = \frac{T(ze^{aT})}{(ze^{aT}-1)^2}$$

$\underline{z\text{-transform of } t^2 e^{-at}}$

from table: $z(t^2) = \dfrac{T^2 z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$

$$z(t^2 e^{-at}) = \frac{T^2 (ze^{aT})^{-1}[1+(ze^{aT})^{-1}]}{(1-(ze^{aT})^{-1})^3}$$

$\boxed{\begin{matrix} \sin \ominus & \dfrac{e^{j\omega t} - e^{-j\omega t}}{2j} \\ \cos \oplus & \dfrac{e^{j\omega t} + e^{-j\omega t}}{2} \end{matrix}}$

5] $\sin(\omega t) + 5\cos(\omega t)$

$z[\sin\omega t + 5\cos\omega t] = z(\sin\omega t) + 5z(\cos\omega t)$

$z(\sin\omega t) = z\left(\dfrac{e^{j\omega t} - e^{-j\omega t}}{2j}\right) = \dfrac{1}{2j}\left[z(e^{j\omega t}) - z(e^{-j\omega t})\right]$

$= \dfrac{1}{2j}\left[\dfrac{z}{z-e^{j\omega t}} - \dfrac{z}{z-e^{-j\omega t}}\right] = \dfrac{1}{2j}\left[\dfrac{\cancel{z^2} - ze^{-j\omega t} - \cancel{z^2} + ze^{j\omega t}}{(z-e^{j\omega t})(z-e^{-j\omega t})}\right]$

$= \dfrac{\cancel{1}}{\cancel{2j}}\left[\dfrac{z(e^{j\omega t} - e^{-j\omega t})}{z^2 - z(e^{j\omega t} + e^{-j\omega t}) + 1}\right] = \boxed{\dfrac{z\sin\omega t}{z^2 - 2z\cos\omega t + 1}}$

$z(\cos\omega t) = z\left[\dfrac{e^{j\omega t} + e^{-j\omega t}}{2}\right] = \dfrac{1}{2}\left[z(e^{j\omega t} + e^{-j\omega t})\right]$

$= \dfrac{1}{2}\left[z(e^{j\omega t}) + z(e^{-j\omega t})\right] = \dfrac{1}{2}\left[\dfrac{z}{z-e^{j\omega t}} + \dfrac{z}{z-e^{-j\omega t}}\right]$

$= \dfrac{1}{2}\left[\dfrac{z^2 - ze^{-j\omega t} + z^2 - ze^{j\omega t}}{(z-e^{j\omega t})(z-e^{-j\omega t})}\right] = \dfrac{1}{\cancel{2}}\left[\dfrac{\cancel{2}z^2 - z(e^{j\omega t} + e^{-j\omega t})}{z^2 - z(e^{j\omega t} + e^{-j\omega t}) + 1}\right] = \boxed{\dfrac{z^2 - z\cos\omega t}{\dots}}$

7 $x(0) = ??$ $\quad X(z) = \dfrac{(1-e^{-T})z^{-1}}{(1-z^{-1})(1-e^{-T}z^{-1})}$

$$x(0) = \lim_{T\to 0} x(KT) = \lim_{z\to\infty} X(z) \qquad \frac{1}{\infty} = \infty^{-1} = 0$$

$$\lim_{z\to\infty} X(z) = \lim_{z\to\infty} \frac{(1-e^{-T})\cancel{z^{-1}} \to 0}{(1-\cancel{z^{-1}})(1-e^{-T}\cancel{z^{-1}})} = \frac{0}{1} = \boxed{0}$$

8 $x(\infty) = ??$ $\quad X(z) = \dfrac{1}{1-z^{-1}} - \dfrac{1}{1-e^{-aT}z^{-1}}$

$$x(\infty) = \lim_{T\to\infty} x(KT) = \lim_{z\to 1} X(z)(1-z^{-1})$$

$$\lim_{z\to 1} X(z) = \lim_{z\to 1} \cancel{(1-z^{-1})}\left(\frac{1}{\cancel{1-z^{-1}}} - \frac{1}{1-e^{-aT}z^{-1}}\right)$$

$$\lim_{z\to 1} 1 - \frac{1-z^{-1}}{1-e^{-aT}z^{-1}} = 1 - \frac{0}{1-e^{-aT}} = \boxed{1}$$

10 a $X(z) = \dfrac{2z^3+z}{(z-2)^2(z-1)}$ all in copybook

2 $y[n] + y[n-2] = 2x[n] - x[n-1]$

a $Y[z] + z^{-2}Y(z) = 2X[z] - z^{-1}X[z]$

$$Y[z]\left[1+z^{-2}\right] = X[z]\left[2-z^{-1}\right]$$

$$\frac{Y[z]}{X[z]} = \frac{2-z^{-1} * z^2}{1+z^{-2} * z^2} = \frac{\cancel{2z^2}}{\cancel{z^2}} = \frac{2z^2-z}{z^2+1}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2z^2-z}{z^2+1}$$

b $u[n]$ is input signal

$$X[z] = Z\{u[n]\} = \frac{z}{z-1}$$

$$Y[z] = \frac{2z^2-z}{z^2+1}\left[\frac{z}{z-1}\right]$$

$$= \frac{2z^3-z^2}{(z^2+1)(z-1)} = \frac{2z^3-z^2}{z^3-z^2+z-1}$$

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Sheet 1: / /

1) Obtain the inverse Z-transform of $X(Z) = \dfrac{2z^3+z}{(z-2)^2(z-1)}$

→ using partial fraction:

$$X_1(z) = \frac{X(z)}{z} = \frac{2z^2+1}{(z-2)^2(z-1)}$$

$$2z^2+1 = \frac{A}{(z-2)^2} + \frac{B}{(z-2)} + \frac{C}{(z-1)}$$

$$A(z-1) + B(z-2)(z-1) + C(z-2)^2 = 2z^2+1$$

$$Az - A + B(z^2-3z+2) + C(z^2-4z+4) = 2z^2+1$$

$$(B+C)z^2 + (A-3B-4C)z + (-A+2B+4C) = 2z^2+1$$

at $z=0$ → $B+C = 2$ → ①

at $z=$ → $A-3B-4C=0$ → ② $-31\ A = \quad 2+1=5$ $\boxed{5/3}$

at $z=$ → $-A+2B+4C = 1$ → ③

from ①, ②, ③ $\boxed{A=9}$, $\boxed{B=-1}$, $\boxed{C=3}$

$$X(z) = \frac{9z\ (\div z^2)}{(z-2)^2\ (\div z^2)} + \frac{-z\ (\div z)}{(z-2)\ (\div z)} + \frac{3z\ (\div z)}{(z-1)\ (\div z)}$$

$\frac{z}{z^2} + 2z^{-2}$

$$= \frac{9z^{-1}}{(1-2z^{-1})^2} - \frac{1}{(1-2z^{-1})} + \frac{3}{(1-z^{-1})}$$

• from tables: $z^{-1}\left[\dfrac{z^{-1}}{(1-az^{-1})^2}\right] = \boxed{Ka^{K-1}}$, $K = 0, 1, 2, \ldots$

$$z^{-1}\left[\frac{1}{1-az^{-1}}\right] = \boxed{a^K}, \quad K = 0, 1, 2, \ldots$$

$$\therefore X(z) = \frac{9z^{-1}}{(1-2z^{-1})^2} - \frac{1}{(1-2z^{-1})} + \frac{3}{(1-z^{-1})}$$

$$\therefore X(K) = 9K(2^{K-1}) - 2^K + 3(1^K) \quad ??$$

1, $K = 0, 1, 2, \ldots$

$$= 9K2^{K-1} - 2^K + 3$$

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Obtain the inverse z Transform of:- $X(z) = \dfrac{z+2}{(z-2)z^2}$ [2 Methods].

→ using partial fractions

$$\frac{X(z)}{z} = \frac{A}{z^2} + \frac{B}{z} + \frac{C}{z-2}$$

$$A(z-2) + Bz(z-2) + Cz^2 = z+2$$

at $z=2$ → $4C = 4$ → $\boxed{C=1}$ ✓

at $z=0$ → $-2A = 2$ → $\boxed{A=-1}$ ✓

at $z=1$ → $-A-B+C=3$ → $\boxed{B=-1}$ ✓

$-z^2$

$$X(z) = \frac{-1\ (\div z^2)}{z^2\ (\div z^2)} + \frac{-1\ (\div z)}{z\ (\div z)} + \frac{1\ (\div z)}{z-2\ (\div z)}$$

$$= -z^{-2} - z^{-1} + \frac{z^{-1}}{(1-2z^{-1})}$$

$-z^{-2}$

from table:

$$z^{-1}\left[\frac{z^{-1}}{1-az^{-1}}\right] = \begin{cases} \boxed{a^{K-1}} & K = 1,2,3,\ldots \\ \boxed{0} & K \leq 0 \end{cases}$$

$$z^{-1}\left[z^{-n}\right] = \begin{cases} \boxed{1} & n = K \\ \boxed{0} & n \neq K \end{cases}$$

$$\therefore X(K) = \begin{cases} -0-0+0 = 0 ✓ & K=0 ✓ \\ -0+(-1)+2^{1-1} = 0 ✓ & K=1 ✓ \\ -1-0+2^{2-1} = 1 ✓ & K=2 ✓ \\ -0-0+2^{K-1} = \boxed{2^{K-1}} ✓ & K=3,4,5,\ldots \end{cases}$$

2 Methods

$$x(K) = \begin{cases} 0 & K=0,1 ✓ \\ 1 ✓ & K=2 ✓ \\ 2^{K-1} ✓ & K=3,4,5,\ldots ✓ \end{cases}$$

→ using direct division method:

$$X(z) = \frac{z+2}{(z-2)z^2} = \frac{z+2 \quad (\div z^3)}{z^3 - 2z^2 \quad (\div z^3)}$$

$$= \frac{z^{-2} + 2z^{-3}}{1 - 2z^{-1}}$$

$$\begin{array}{r|l} & z^{-2} + 4z^{-3} + 8z^{-4} + 16 \\ \hline 1-2z^{-1} & z^{-2} + 2z^{-3} \\ & -z^{-2} \pm 2z^{-3} \\ \hline & 4z^{-3} \\ & -4z^{-3} \pm 8z^{-4} \\ \hline & 8z^{-4} \\ & -8z^{-4} \pm 16z^{-5} \\ \hline & \boxed{16z^{-5}} \end{array}$$

$X(z) = 1z^{-2} + 4z^{-3} + 8z^{-4} + 16z^{-5}$ 4 terms

by comparison to:

$$X(z) = \sum_{K=0}^{\infty} x(K) z^{-K}$$

$$= x(0) + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} + x(4)z^{-4} + \ldots$$

$x(0) = 0$, $x(1) = 0$, $x(2) = 1$

$x(3) = 4 = 2^2 = 2^{K-1}$ $K^3 = K^{2-1}$ 2^2

$x(4) = 8 = 2^3 = 2^{K-1} = 2^{4-1}$ $\boxed{2^{K-1}}$ 2^2

$x(5) = 16 = 2^4 = 2^{K-1} = 2^{5-1} = 2^4$ 2^3 2^4 $\boxed{2^{K-1}}$

$$x(K) = \begin{cases} 0 & K = 0, 1 \\ 1 & K = 2 \\ 2^{K-1} & K = 3, 4, 5, \ldots \end{cases}$$

3) obtain inverse z transform of $X(z) = \dfrac{z(z+2)}{(z-1)^2}$ by 2 methods:

→ using partial fraction:

$$X(z) = \frac{z^2 + 2z}{(z-1)^2} \implies X_1(z) = \frac{X(z)}{z} = \frac{z+2}{(z-1)^2}$$

$$X_1(z) = \frac{A}{(z-1)^2} + \frac{B}{(z-1)} \implies A + B(z-1) = z+2$$

$1 \to A$
$0 \to B$

$\boxed{B = 1}$, $\boxed{A = 3}$

$$X(z) = \frac{3z \quad (\div z^2)}{(z-1)^2 \quad (\div z^2)} + \frac{z \quad (\div z)}{(z-1) \quad (\div z)}$$

$$= \frac{3z^{-1}}{(1-z^{-1})^2} + \frac{1}{(1-z^{-1})}$$

From tables: $z^{-1}\left[\dfrac{z^{-1}}{(1-az^{-1})^2}\right] = \boxed{Ka^{K-1}}$, $z^{-1}\left[\dfrac{1}{1-az^{-1}}\right] = \boxed{a^K}$

$x(K) = 3K(1)^{K-1} + 1^K$

$= 3K + 1$

using the partial

Using Direct Division method:

$$X(z) = \frac{z^2 + 2z \;(\div z^2)}{z^2 - 2z + 1 \;(\div z^2)} = \frac{1 + 2z^{-1}}{1 - 2z^{-1} + z^{-2}}$$

$$X(z) = 1 + 4z^{-1} + 7z^{-2} + 10z^{-3} + \cdots$$

$$\begin{array}{l} \boxed{1 + 4z^{-1} + 7z^{-2} + 10z^{-3}} \\ 1 - 2z^{-1} + z^{-2} \,\big)\, 1 + 2z^{-1} \\ -\;1 \pm 2z^{-1} \mp z^{-2} \\ \quad 4z^{-1} - z^{-2} \\ -\;4z^{-1} \pm 8z^{-2} \mp 4z^{-3} \\ \quad 7z^{-2} - 4z^{-3} \\ -\;7z^{-2} \pm 14z^{-3} \mp 7z^{-4} \\ \quad 10z^{-3} - 7z^{-4} \end{array}$$

by comparing to:

$$X(z) = \sum_{K=0}^{\infty} x(KT)\, z^{-K} = x(0) + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} + \cdots$$

$x(0) = 1$

$x(1) = 4$

$x(2) = 7$

$x(3) = 10$

$z^{2}\, 0.5 - z^{-1}$

$z^{-1}\, 0.5 - z^{-2}$ $\qquad -1 + 2 = 1$

[4] Find inverse z Transform of $X(z) = \dfrac{z^{-1}(0.5 - z^{-1})}{(1 - 0.5z^{-1})(1 - 0.8z^{-1})^2}$ $(1-0.5z^{-1})(1 + 1.016z^{-2} - 1.6z^{-1})$

$0.5z^{-1} - z^{-2}$ $\quad = 0.5z^{2} - z^{1} = z(0.5z - 1)$ $\quad 1 - 0.16z^{2} - 1.6z^{-1} -$ [18]

$$X(z) = \frac{z^{-1}(0.5 - z^{-1})}{(1 - 0.5z^{-1})(1 - 0.8z^{-1})^2} \; \frac{(* z^3)}{(* z^3)} = \frac{z(0.5z - 1)}{(z - 0.5)(z - 0.8)^2}$$

$$X_1(z) = \frac{X(z)}{z} = \frac{(0.5z - 1)}{(z - 0.5)(z - 0.8)^2} = \frac{A}{z - 0.5} + \frac{B}{(z - 0.8)^2} + \frac{C}{z - 0.8}$$

$$A(z - 0.8)^2 + B(z - 0.5) + C(z - 0.5)(z - 0.8) = 0.5z - 1$$

at $z = 0.8 \longrightarrow \boxed{B = -2}$

$z = 0.5 \longrightarrow \boxed{A = -25/3 = -8.33}$

$z = 1 \longrightarrow \boxed{C = 25/3 = 8.33}$

$$X(z) = \frac{-8.33z}{z - 0.5} + \frac{-2z}{(z - 0.8)^2} + \frac{8.33z}{(z - 0.8)}$$

$$= \frac{-8.33}{(1 - 0.5z^{-1})} + \frac{-2z^{-1}}{(1 - 0.8z^{-1})^2} + \frac{8.33}{1 - 0.8z^{-1}}$$

$$-8.33(0.5^K) - 2(K(0.8)^{K-1}) + 8.33(0.8)^K$$

$$\frac{0.5z^{-1} - z^{-2}}{(1 - 0.5z^{-1})(1 - 0.8z^{-1})^2} = \frac{0.5z^2 - z}{(z - 0.5)(z - 0.8)^2}$$

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Obtain z transform of $X(s) = \dfrac{s}{(s+1)^2(s+2)}$ by using the partial Fraction Expansion method.

$$\rightarrow \; = \frac{A}{(s+1)^2} + \frac{B}{s+1} + \frac{C}{s+2} = X(s)$$

$$A(s+2) + B(s+1)(s+2) + C(s+1)^2 = s$$

at $s = -1 \rightarrow A = -1$

$s = -2 \rightarrow C = -2$

$s = 0 \rightarrow B = 2$

$$X(s) = \frac{-1}{(s+1)^2} + \frac{2}{(s+1)} + \frac{-2}{s+2}$$

From table: $z\left[\dfrac{1}{(s+a)^2}\right] = \dfrac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$

$$z\left[\frac{1}{s+a}\right] = \frac{1}{1-e^{-aT}z^{-1}}$$

$$X(z) = \frac{-Te^{-T}z^{-1}}{(1-e^{-T}z^{-1})^2} + \frac{2}{1-e^{-T}z^{-1}} - \frac{2}{1-e^{-2T}z^{-1}}$$

Solve following difference Equation: $x(k+2) - x(k+1) + 0.25x(k) = u(k+2)$. where $x(0) = 1$ & $x(1) = 2$. The input function $u(k)$ is given by.

Sol: $u(k) = 1$, $k = 0, 1, 2, \ldots$

$$\rightarrow x(k+2) - x(k+1) + 0.25x(k) = u(k+2)$$

$$[z^2X(z) - z^2\underbrace{x(0)}_{1} - z\underbrace{x(1)}_{2}] - [zX(z) - zx(0)] + 0.25X(z) = z^2U(z) - z^2u(0) - zu(1)$$

$$U(z) = \boxed{\frac{z}{z-1}}$$

From table: $z[u(k)] = \dfrac{1}{1-z^{-1}} = \dfrac{z}{z-1}$

$$(z^2 - z + 0.25)X(z) \cancel{-z^2} - 2z + \cancel{z} = \frac{z^2}{1-z^{-1}} \cancel{-z^2} \cancel{-z}$$

$$(z^2 - z + 0.25)X(z) = \frac{z^2}{1-z^{-1}} \rightarrow \frac{z^3}{z-1} = X(z)(z^2 - z + 0.25)$$

$$X(z) = \frac{z^2}{(1-z^{-1})(z^2-z+0.25)}\frac{(\times z)}{(\times z)} = \frac{z^3}{(z-1)(z-0.5)^2}.$$

$$X_1(z) = \frac{X(z)}{z} = \frac{z^2}{(z-1)(z-0.5)^2} = \frac{A}{(z-1)} + \frac{B}{(z-0.5)^2} + \frac{C}{z-0.5}$$

$$A(z-0.5)^2 + B(z-1) + C(z-0.5)(z-1) = z^2$$

at $z = 0.5 \longrightarrow B = -0.5$

$z = 1 \longrightarrow A = 4$

$z = 2 \longrightarrow C = -3$

$$X(z) = \frac{4z \ (\div z)}{z-1 \ (\div z)} - \frac{0.5z \ (\div z^2)}{(z-0.5)^2 \ (\div z^2)} + \frac{-3z \ (\div z)}{z-0.5 \ (\div z)}$$

$$= \frac{4}{1-z^{-1}} - \frac{0.5z^{-1}}{(1-0.5z^{-1})^2} - \frac{3}{1-0.5z^{-1}}$$

$$x(K) = 4(1^K) - 0.5K(0.5)^{K-1} - 3(0.5)^K \qquad K = 0, 1, 2, \ldots$$

$$4 - 0.5K(0.5)^{K-1} - 3(0.5)^K$$

7 Consider the difference equation system:

$y(K+2) + y(K) = x(K)$, where $\boxed{y(K) = 0}$ for $\boxed{K < 0}$. obtain the response $y(K)$ when input $x(K)$ is a unit step response. $\rightarrow K = -1$, $K = -2$

$$[z^2 Y(z) - z^2 \cancel{Y(0)} - z\cancel{Y(1)}] + Y(z) = X(z) \rightarrow z/z+1$$

$$y(K+2) + y(K) = x(K)$$

at $K = \boxed{-2} \longrightarrow y(0) + y(-2) = x(-2)$

$$\boxed{y(0) = 0}$$

at $K = \boxed{-1} \longrightarrow \boxed{y(1)} + y(-1) = x(-1)$

$$\boxed{y(1) = 0}$$

$$z^2 Y(z) + Y(z) = z/z-1 \quad \Leftarrow \text{unit response}$$

$$Y(z) = \frac{z}{(z^2+1)(z-1)} \ (\div z) \qquad Y(z)[(z^2+1)] = z/z+1$$

$$Y_1(z) = \frac{Y(z)}{z} = \frac{Az+B}{z^2+1} + \frac{C}{z-1} \qquad \boxed{Az+B}$$

$$(Az+B)(z-1) + C(z^2+1) = 1$$

at $z = 1 \longrightarrow C = 0.5$

$z = 0 \longrightarrow B = -0.5$

$z = 2 \longrightarrow A = -0.5$

$\boxed{11}$

$\boxed{12}$ $y(K+2) - 3/4\, y(K+1) + 1/8\, y(K) = e(\overset{1}{\cancel{K}})$

$$[z^2 y(z) - z^2 \underbrace{\cancel{y(0)}}_{0} - z \underbrace{\cancel{y(1)}}_{0}] - 3/4\,[z\, y(z) - z \underbrace{\cancel{y(0)}}_{0}] + 1/8\, y(z) = e$$

$$z^2 y(z) - 3/4\, z\, y(z) + 1/8\, y(z) = 1$$

$$y(z)[z^2 - 3/4\, z + 1/8] = 1$$

$$y(z) = \frac{1}{z^2 - 3/4\, z + 1/8} = \frac{A}{z - 1/2} + \frac{B}{z - 1/4}$$

$$1 = A(z - 1/4) + B(z - 1/2)$$

$z = 1/2 \longrightarrow A = 4$

$z = 1/4 \longrightarrow B = -4$

$$y(z) = \frac{4}{z - 1/2} - \frac{4}{z - 1/4}$$

$$y(K) = 4(0.5)^{K-1} - 4(0.25)^{K-1}$$

$y(0) = -8$

$y(1) = 0$

$y(2) = 1$

$y(3) = 3/4$

$y(4) = 7/16$

let's verify by taking $K = 0$

$$y(0+2) - 3/4\, y(0+1) + 1/8\, y(0) = e(0)$$

$$y(2) - 3/4\, y(1) + 1/8\, y(0) = 0$$

$$1 - 3/4 * 0 + 1/8\,(-8) = 0$$

4] $E(z) = \dfrac{0.9z}{(z-1)(z-0.5)} \rightarrow \dfrac{E(z)}{z} = \dfrac{0.9}{(z-1)(z-0.9)}$

$$\frac{E(z)}{z} = \frac{0.9}{(z-1)(z-0.9)} = \frac{A}{z-1} + \frac{B}{z-0.9}$$

$$0.9 = A(z-0.9) + B(z-1)$$

$z = 1 \Longrightarrow 0.5 = A * 0.5 \Longrightarrow A = 1$

$z = 0.5 \Longrightarrow 0.5 = B * -(0.5) \Longrightarrow B = -1$

$$E(z) = \frac{z}{z-1} - \frac{z}{z-0.5}$$

$$e(n) = u(n) - (0.5)^K$$

6] a] $E(z) = \dfrac{z}{z^2-1} \begin{matrix} \div z^2 \\ \div z^2 \end{matrix} \Longrightarrow \dfrac{1/z}{1-1/z^2}$

$$E(0) = \lim_{z \to \infty} E(z) = \frac{0}{1-0} = 0$$

$E(\infty) = \lim_{z \to 1} (z-1)\dfrac{z}{z^2-1} = \dfrac{0}{0}$ doesnot Exist

c] $E(z) = \dfrac{z^2+2}{(z-1)(z^2+1)} \begin{matrix} \div z^3 \\ \div z^3 \end{matrix} = \dfrac{1/z + 2/z^3}{(1-1/z)(1+1/z^2)}$

$$E(0) = \lim_{z \to \infty} E(z) = \frac{0+0}{(1-0)(1+0)} = 0$$

$$E(\infty) = \lim \cancel{(z-1)} \frac{z^2+2}{\cancel{(z-1)}(z^2+1)} = 1+2/1+1 = 3/2$$