Detailed Stability and Control Analysis of an Aircraft Pitch System

MATLAB and Simulink Based Study

Noor Ul Huda

Department of Computer Systems Engineering
University of Engineering and Technology, Peshawar

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Presentation Outline

- System Modeling
- Open-Loop Stability Analysis
- Controllability Analysis
- Observability Analysis
- Desired Eigenvalue Selection
- State Feedback Controller Design
- Observer Design
- Closed-Loop Response (MATLAB)
- Closed-Loop Response (Simulink)
- Steady-State Error Analysis
- PID Controller Design
- MATLAB and Simulink PID Results
- Performance Comparison
- Conclusion

System Mathematical Model

$$\dot{x} = Ax + Bu, \quad y = Cx$$

$$A = \begin{bmatrix} -0.313 & 56.7 & 0 \\ -0.0139 & -0.426 & 0 \\ 0 & 56.7 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0.232 \\ 0.0203 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$$

Open-Loop Stability Analysis

The open-loop stability is verified using:

- Transfer Function Poles
- Eigenvalue Analysis
- Step Response
- Root Locus
- Pole-Zero Map
- Routh-Hurwitz Criterion

Stability via Transfer Function Poles

- One pole at the origin
- Remaining poles in left-half plane
- Indicates marginal stability

```
The poles of transfer function are

poles_of_tf =

   0.0000 + 0.0000i
  -0.3695 + 0.8860i
  -0.3695 - 0.8860i
```

Eigenvalue Analysis

Eigenvalues of matrix A:

$$\lambda = \{0, \ -0.3695 \pm 0.8860i\}$$

- Zero eigenvalue $\Rightarrow$ marginal stability
- Complex poles $\Rightarrow$ oscillatory response

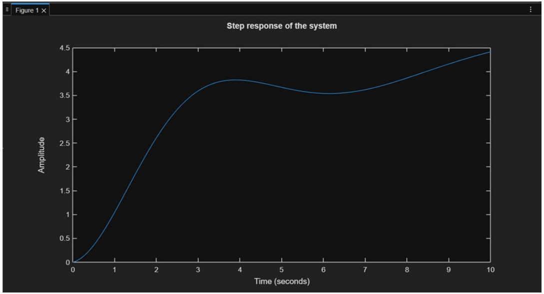
```
The eigen values of matrix A are

eigen_values =

   0.0000 + 0.0000i
  -0.3695 + 0.8860i
  -0.3695 - 0.8860i
```

Open-Loop Step Response

Generated using:

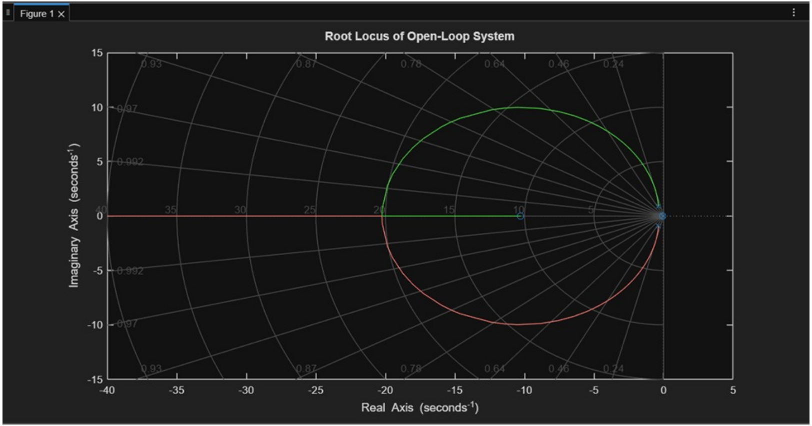
```
step(sys)
```

- Output does not settle
- Confirms instability for reference tracking



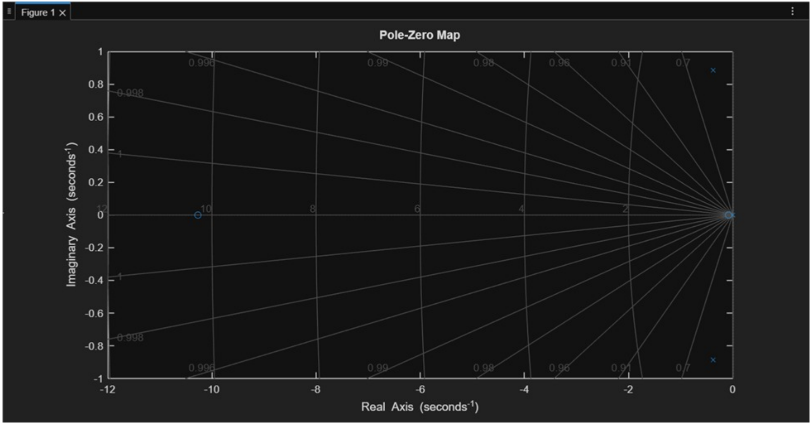
Root Locus Analysis

- Shows pole trajectories with gain variation
- Confirms marginal stability at low gain



Pole-Zero Map

- Pole at origin
- Remaining poles in left-half plane



Routh-Hurwitz Criterion

s^3	1	57.46
s^2	0.739	0
s^1	57.46	0
s^0	0	

- Zero in first column
- Indicates poles on imaginary axis
- System is marginally stable

Controllability Analysis

The controllability of the system determines whether all states can be driven to desired values using the control input.
The controllability matrix is defined as:

$$\mathcal{C} = [B\ AB\ A^2B]$$

Using MATLAB:

$$\texttt{rank(ctrb(A,B))} = 3$$

- System order $= 3$
- Rank($\mathcal{C}$) $= 3 \Rightarrow$ full rank
- All system states are controllable

Conclusion: The unstable and marginal modes of the system can be shifted to desired stable locations using state feedback.

Observability Analysis

Observability determines whether internal system states can be reconstructed from output measurements.
The observability matrix is:

$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$$

Using MATLAB:

$$\texttt{rank(obsv(A,C))} = 3$$

- Rank equals system order
- No hidden or unmeasurable states

Conclusion: The system is fully observable, allowing the design of a state observer for unmeasured states.

Desired Eigenvalue Selection

Since the system is fully controllable, closed-loop poles can be arbitrarily placed in the Left-Half Plane (LHP).

Pole selection criteria:

- All poles must lie in the LHP for stability
- Desired poles should be faster than open-loop poles
- Excessively fast poles increase control effort

The desired eigenvalues were selected using the last three digits of the registration number:

$$(4,\ 2,\ 14) \Rightarrow \lambda_d = \{-4,\ -2,\ -14\}$$

- $-2, -4$: dominant poles for smooth transient response
- -14: fast pole to improve settling time

State Feedback Controller

Control law:

$$u = -Kx$$

MATLAB command:

$$K = \texttt{place(A,B,[-4 -2 -14])}$$

```
Matrix K:
   1.0e+03 *

  -0.5259    6.9591    0.6313
```

Observer Design

Observer poles selected faster than controller:

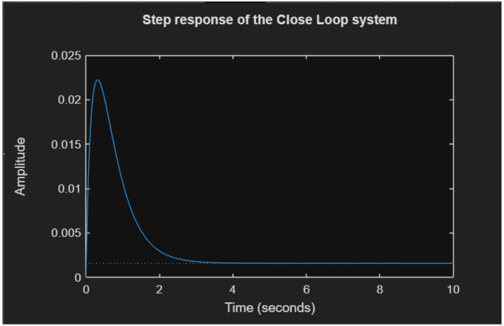
$$\{-20,\ -10,\ -70\}$$

$$L = \texttt{place(A',C',observer\_poles)'}$$

```
Matrix L:
   1.0e+03 *

   -8.1658
   -0.0029
    8.2650
```

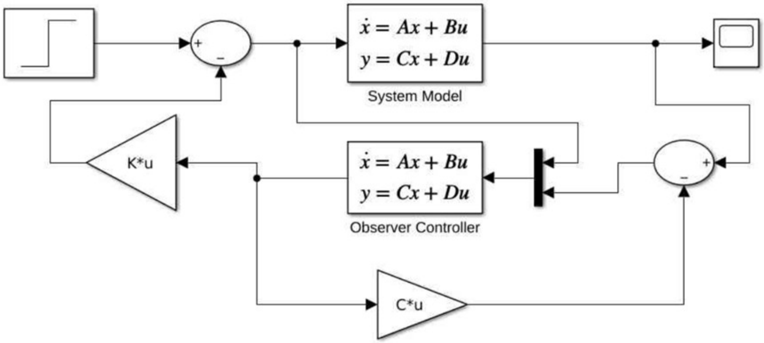
Closed-Loop Response (MATLAB)

- System stabilized
- Improved transient response



Closed-Loop Response (Simulink)

- Confirms MATLAB results
- Observer-based implementation



Closed-Loop Response (Simulink)

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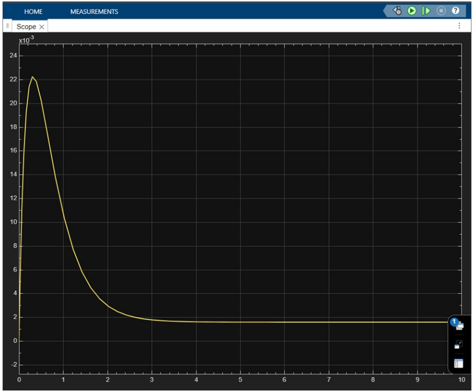


Steady-State Error Analysis

Steady-state error is defined as:

$$e_{ss} = \lim_{t \to \infty} [r(t) - y(t)]$$

For a unit step input ($r = 1$):

- Open-loop system: Marginally stable $\Rightarrow e_{ss} = \infty$
- State feedback system: Stable but $e_{ss} \neq 0$

Reason:

- State feedback does not change system type
- No integrator present in the control loop

Conclusion: Integral action is required to eliminate steady-state error.

PID Controller Design

The PID controller is defined as:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt}$$

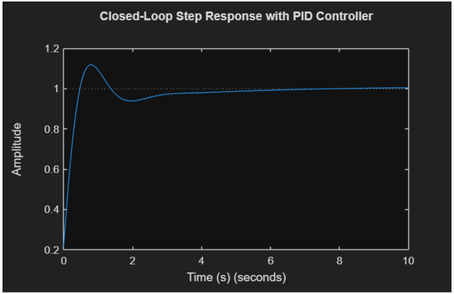
- K_p: Improves response speed
- K_i: Eliminates steady-state error
- K_d: Improves damping and reduces overshoot

Chosen gains:

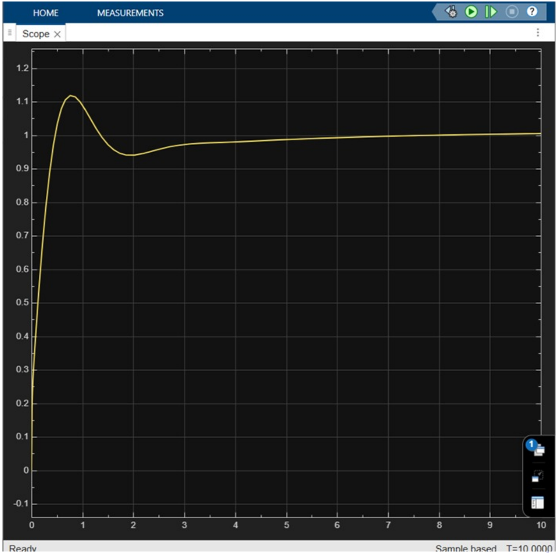
$$K_p = 5, \quad K_i = 1, \quad K_d = 1.2$$

Objective: Achieve zero steady-state error with acceptable transient performance.

PID Response (MATLAB)



PID Response (Simulink)



Performance Comparison

System	Stability	Steady-State Error
Open Loop	Marginal	∞
State Feedback	Stable	≈ 1
PID Controlled	Stable	0

Overall Understanding and Key Insights

- Open-loop aircraft pitch system is marginally stable
- Controllability ensures poles can be reassigned
- Observability allows reconstruction of internal states
- State feedback stabilizes the system but does not remove SSE
- PID controller introduces integral action to eliminate SSE
- MATLAB and Simulink results validate theoretical analysis

Final Outcome: A stable, well-controlled aircraft pitch system with zero steady-state error.

Thank You

Questions?