Homework 2

Student: Zhetessa Nur

Chapter 3.8., p. 58-61

Ex: 1, 4, 5, 9, 11.

1.

X_i	$2c$	$\frac{c}{2}$
P	$\frac{1}{2}$	$\frac{1}{2}$

$$E(X_i) = 2c \cdot \frac{1}{2} + \frac{c}{2} \cdot \frac{1}{2} = \frac{2c}{2} + \frac{c}{4} = \frac{5}{4}c$$

On each trial I expect to gain

$\frac{5}{4}$ of my current balance.

On $X_0 = c$, because I didn't do any trial of the game

On $X_1 = \frac{5}{4} \cdot c$, in the 1st trial I expect to gain this much of money $\cdot \frac{5}{4}$

On $X_2 = \frac{5}{4} \cdot X_1 = \frac{5}{4} \cdot \frac{5}{4} \cdot c$, then I update my balance by money I gained on X_1

Moving with this logic on

$$X_n = \left(\frac{5}{4}\right)^n \cdot c$$

$$E(X_n) = E(X_{n-1}) \cdot \frac{5}{4} = \left(\frac{5}{4}\right)^{n-1} \cdot \frac{5}{4} \cdot c =$$

$= \left(\frac{5}{4}\right)^n \cdot c$ - expected fortune after n trials.

2.

X_i	-1	$+1$
p_i	p	$1-p$

where distribution of X_i ~ Bernoulli (p), either left or right

$E(X_n) = n \cdot E(X_i) = n(1-2p)$ - expectation

$$E(X_i) = (-1 \cdot p) + (1 \cdot (1-p)) = -p + 1 - p =$$
$$= 1 - 2p$$

$$Var(X_n) = Var(X_1 + \dots X_n) = n \cdot Var(X_i) =$$
$= n \cdot 4p(1-p) = 4 \cdot n \cdot p \cdot (1-p)$ - variance

$$Var(X_i) = E(X^2) - [E(X)]^2 =$$
$$= [(-1)^2 \cdot p + (+1)^2 \cdot (1-p)] - (1-2p)^2 =$$
$$= \cancel{p} + 1 - \cancel{p} - (1 - 4p + 4p^2) = \cancel{1} - \cancel{1} + 4p - 4p^2 =$$

$= 4p(1-p)$

5. What is the E(X) of tosses until a head obtained?

$$E(X) = \sum_{i=1}^{n} x_i \cdot P(X = x_i) = P(X=1) \cdot 1 +$$
$$+ P(X=2) \cdot 2 + P(X=3) \cdot 3 + \ldots + P(X=n) \cdot n =$$
$$= 1p + 2p(1-p) + 3p(1-p)^2 + \ldots + np(1-p)^{n-1} -$$

$$(1-p)E(X) = 1p(1-p) + 2p(1-p)^2 + 3p(1-p)^3 + \ldots + np(1-p)^n$$

$$E(X) - (1-p)E(X) = 1p + (2p(1-p) - 1p(1-p)) +$$
$$+ (3p(1-p)^2 - 2p(1-p)^2) + \ldots$$

$$\cancel{E(X)} - \cancel{E(X)} + pE(X) = 1p + 1p(1-p) + 1p(1-p)^2 + \ldots$$

$$E(X) = 1 + (1-p) + (1-p)^2 + (1-p)^3 + \ldots + (1-p)^n =$$
$$= \frac{1}{1-(1-p)} = \frac{1}{\cancel{1}-\cancel{1}+p} = \frac{1}{p}$$

The expected value of geometric random variable is equal to $\frac{1}{p}$

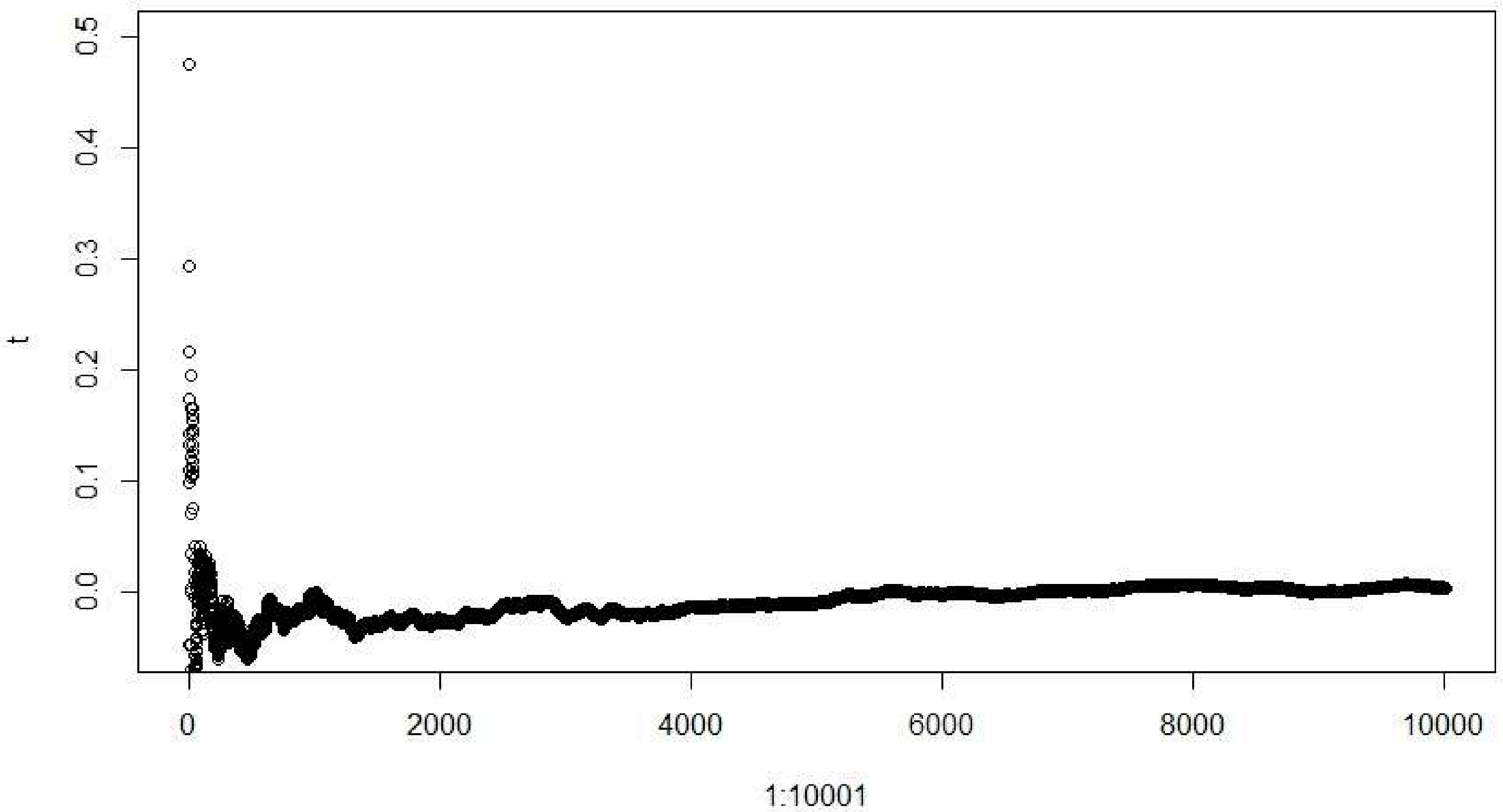
9Ex_Zhetessov.R

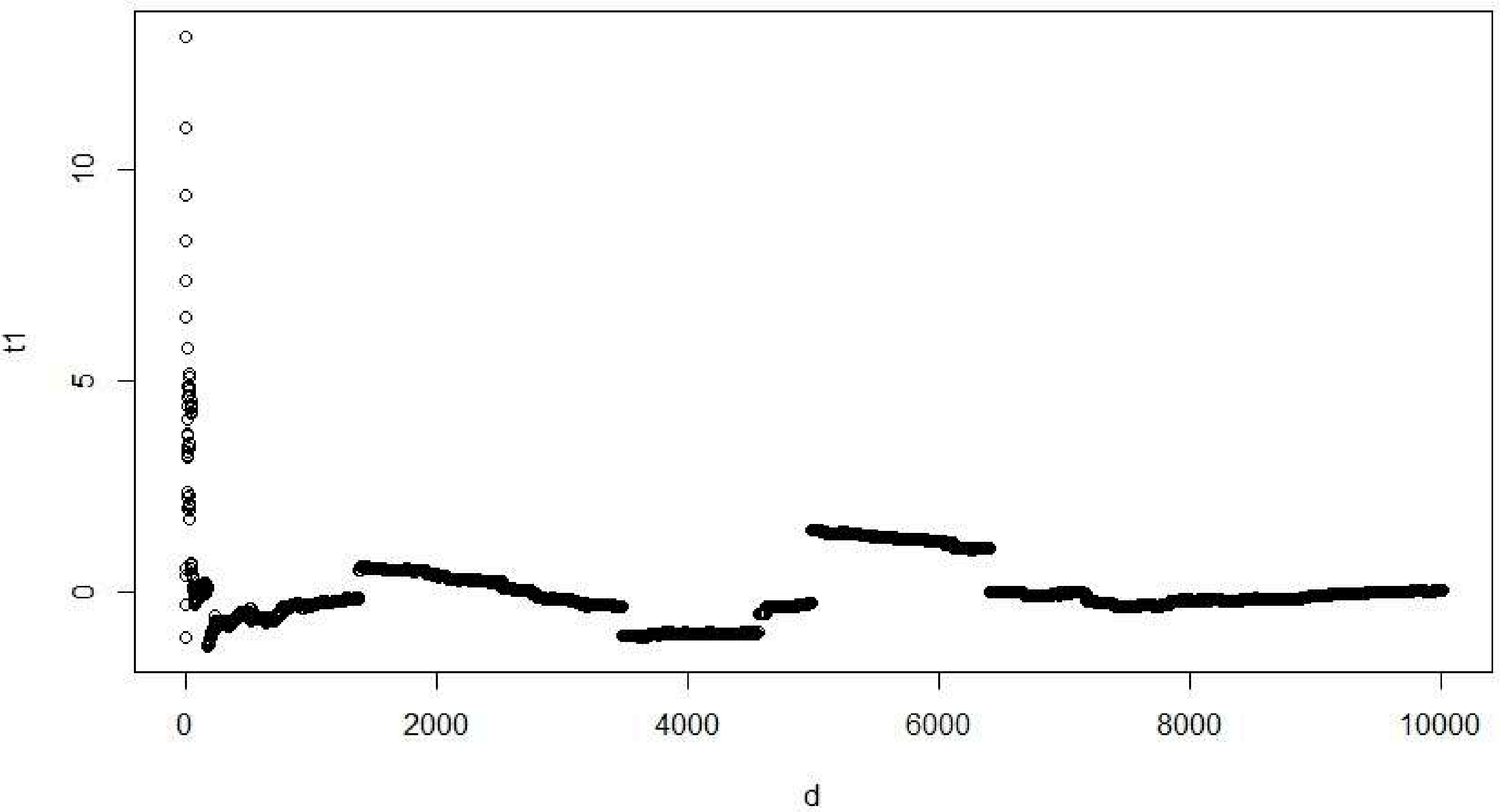
Source on Save

Run

Source

```r

t <- c()
x <- rnorm(10001, mean=0, sd=1)
for(val in 1:10001){
  t <- append(t, mean(x[1:val]))
}
plot(1:10001, t, ylim = c(-0.05,0.5))



x_cauchy <- rcauchy(10001)
t1 <- c()
for(val in 1:10001){
  t1 <- append(t1, sum(x_cauchy[1:val])/val)
}
d <- seq(1,10001)
plot(d, t1)



```


0.5
0.4
0.3
0.2
0.1
0.0
t
0
2000
4000
6000
8000
10000
1:10001



t1
0
5
10
0
2000
4000
6000
8000
10000
d

9. Explain why there is such a difference.
Initially I have created one array of normal random variables. In my loop the val $\in [1, 10\,000]$ was taking first val elements and calculating their sum. After each $\bar{X}_i$ was plotted versus i - the length of array.

The same was done for Cauchy distribution.

Conclusion:
In normal distribution $\sim N(0,1)$ the $\bar{X}_i$ was approaching 0 as n - the length of array or the size of sample increased. The bigger n - more precise and closer we can approach the true mean.
In Cauchy distribution the mean and standard deviation is not defined. It means that $\bar{X}_n$ is not going to

converge on specific value on the graph.

11.
Let $y_1, y_2, \ldots, y_n$ – I.R.V.

$P(y_i = 1) = P(y_i = -1) = \frac{1}{2}$

$E(y_i) = \sum_{i}^{n} y_i P(X_i = y_i) = 1 \cdot \frac{1}{2} + (-1) \cdot \frac{1}{2} =$
$= 0$ – expected value on only one day

a) Find $E(X_n)$

$X_n = \sum_{i=1}^{n} y_i$

$E(X_n) = E(\sum_{i=1}^{n} y_i) = \sum_{i=1}^{n} E(y_i) = \sum_{i=1}^{n} 0 = 0 \cdot n =$
$= 0$

Find $Var(X_n)$

$Var(X_n) = Var(\sum_{i=1}^{n} y_i) = E[(\sum_{i=1}^{n} y_i)^2] - (E[\sum_{i=1}^{n} y_i])^2 =$

$= n - 0^2 = n - 0 = n$

$$E\left[\left(\sum_{i=1}^{n} y_i\right)^2\right] = E\left[\left(\sum_{i=1}^{n} y_i\right)\left(\sum_{i=1}^{n} y_i\right)\right] =$$

$$= E\left[\sum_{i=1}^{n} y_i \cdot y_i + \sum_{i=1}^{n}\sum_{j=1}^{n} y_i \cdot y_j\right] =$$

$$= \sum_{i=1}^{n} E(y_i^2) + \sum_{i=1}^{n}\sum_{j=1}^{n} E(y_i) \cdot E(y_j) =$$

$$= \sum_{i=1}^{n} E(y_i^2) + \sum_{i=1}^{n}\sum_{j=1}^{n} 0 \cdot 0 =$$

$$= \sum_{i=1}^{n}\left(\frac{1}{2} \cdot 1^2 + \frac{1}{2} \cdot (-1)^2\right) + 0 =$$

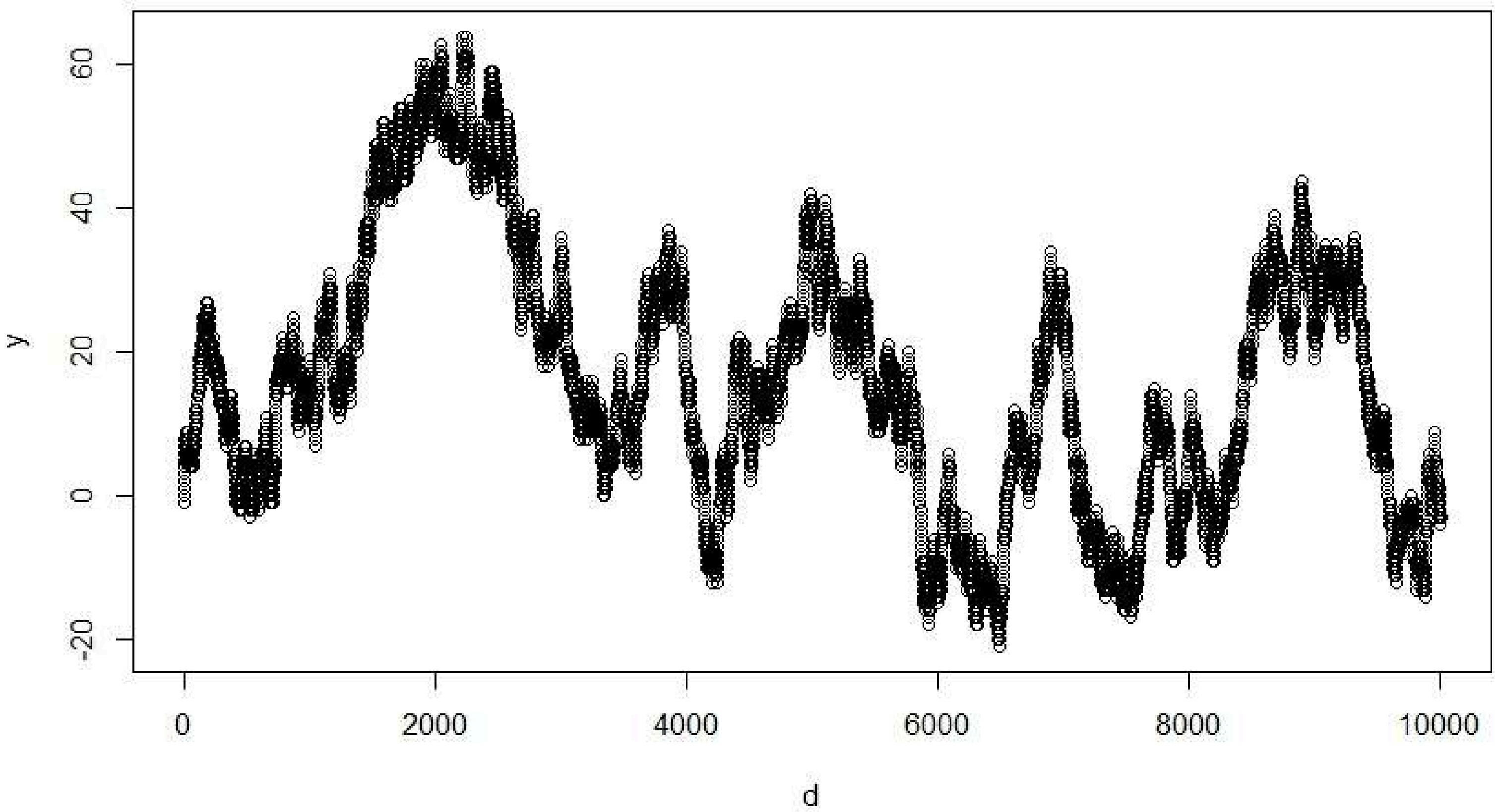
$$= \sum_{i=1}^{n}\left(\frac{1}{2} + \frac{1}{2}\right) = \sum_{i=1}^{n} 1 = 1 \cdot n = n$$

b) I have simulated the stock price change by taking a 10.000 samples of size 1 where values can be either 1 or -1 with equal probabilities 0.5. Then I summarized values of sizes $1 \ldots n$, which were plotted versus their sizes. (Sum versus n)

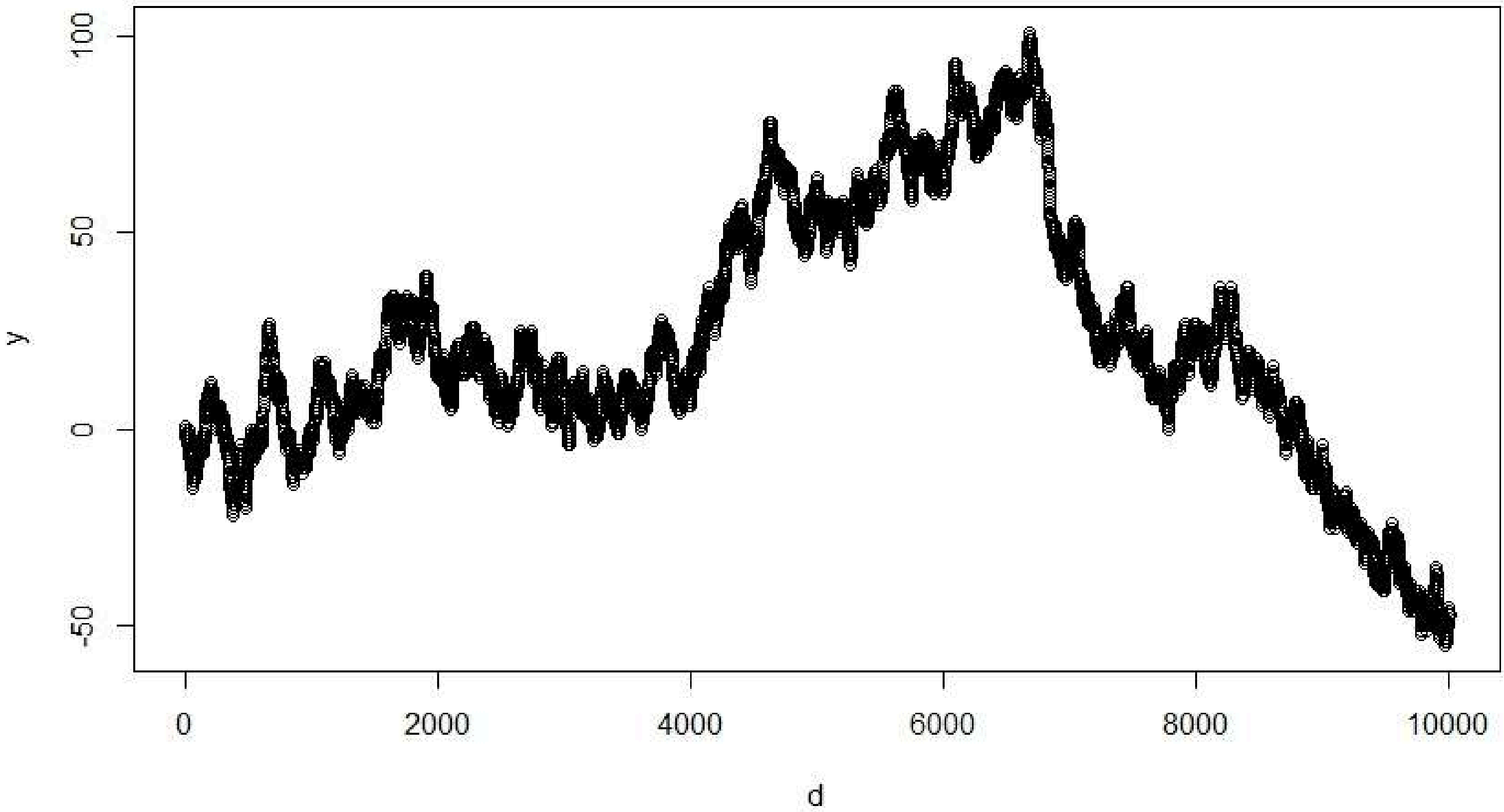
11Ex_Zhetessov.R*

Source on Save

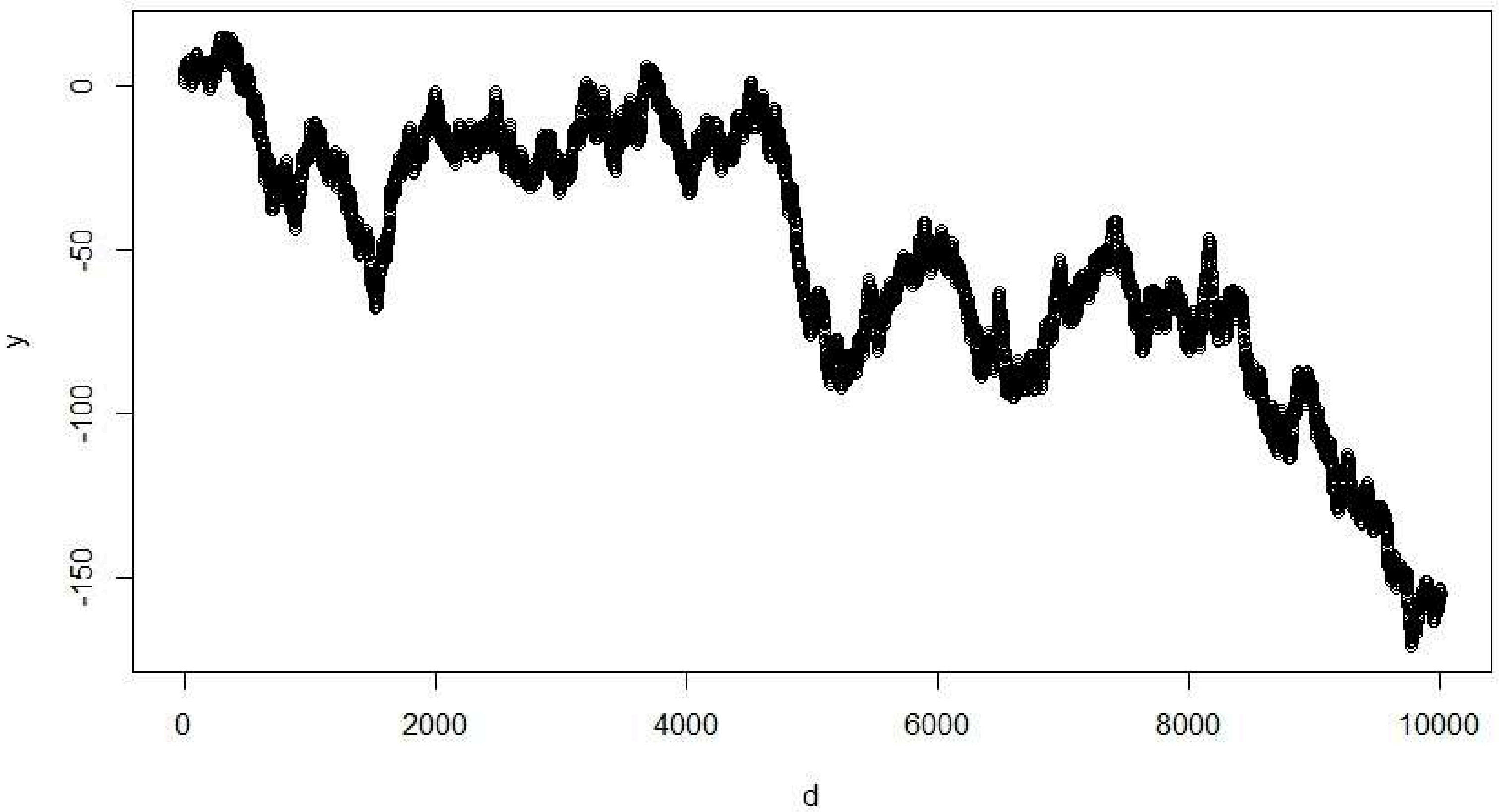
```r
x <- sample(c(1,-1), size=10001, prob = c(0.5,0.5), replace = TRUE)
y <- cumsum(x)

d <- seq(1,10001)

plot(d, y)

#zhetessov Nur
```



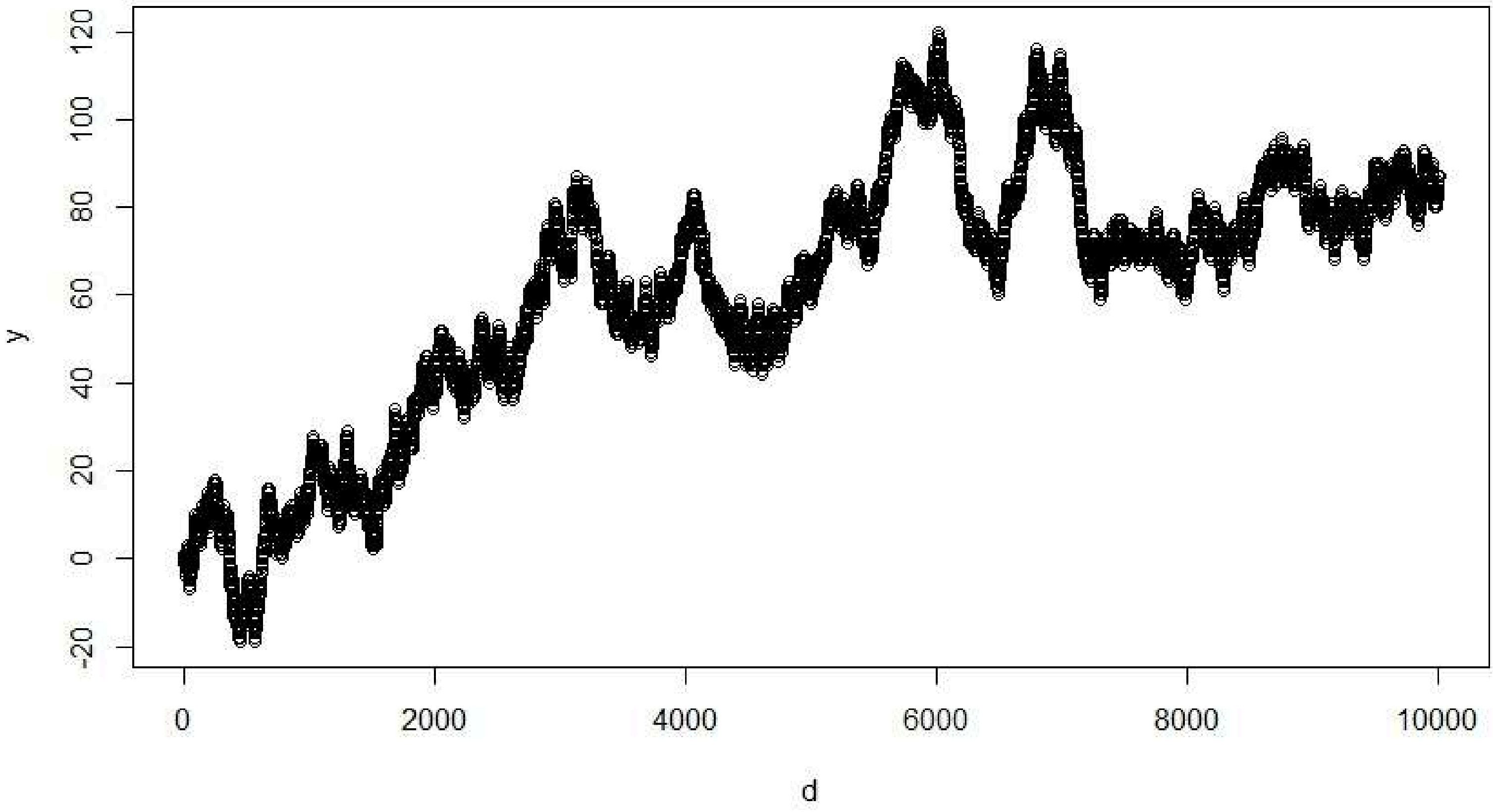
60
40
20
0
-20
y
0
2000
4000
6000
8000
10000
d



100
50
0
-50
y
0
2000
4000
6000
8000
10000
d



0
-50
-100
-150
y
0
2000
4000
6000
8000
10000
d



120
100
80
60
40
20
0
-20
y
0
2000
4000
6000
8000
10000
d

The expectation is zero. It means, that no matter how much trials we gonna do and no matter how high or low stock prices raise or decrease, the value is gonna to return the value of 0, which is a point of symmetry on the graph.

The standard deviation is $\sqrt{n}$. And variation is n. It increases proportionally to n increasement.

Chapter 4.5 p. 68 - 69 : ex. 2.

2) Let $X \sim Poisson(h)$. Show
$P(X \geq 2h) \leq \frac{1}{h}$
In Poisson distribution $\mu_x = h$ and $Var(X) = h$

By the formula: $P(|X - \mu| \geq t) \leq \frac{\sigma^2}{t^2}$

Therefor,

$$P(|X-h| \geq t) \leq \frac{h}{t^2},$$

$$P(|X-h| \geq 2h) \leq \frac{h}{(2h)^2},$$

$$P(|X-h| \geq 2h) \leq \frac{h}{4h^2},$$

$$P(|X-h| \geq 2h) \leq \frac{1}{4h}$$

Where $\frac{1}{4h} < \frac{1}{h}$, because if $h = 1$, then $\frac{1}{4} < 1$

then

$$P(|X-h| \geq 2h) \leq \frac{1}{4h} \leq \frac{1}{h}$$

$$P(X \geq 2h) \leq \frac{1}{h}$$

Chapter 5.8 p. 82-84: ex. 6, 8.

6. $\mu = 68$ inch.
$\sigma = 4$ inch.
$n = 100$

$P(\overline{X} \geq 68) - ?$

Since $n = 100$ and $n \geq 30$ we can hold CLT, where $\overline{X} \sim N(\mu, \frac{Var(X)}{n})$

$$P(\overline{X} \geq 68) = P\left(\frac{\overline{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}} \geq \frac{68 - \mu}{\sqrt{\frac{\sigma^2}{n}}}\right) =$$

$$= P\left(\frac{\sqrt{n}(\overline{X} - \mu)}{\sigma} \geq \frac{(68 - \mu)\sqrt{n}}{\sigma}\right) = P\left(Z \geq \frac{(68-68)\sqrt{100}}{4}\right) =$$

$$= P\left(Z \geq \frac{0 \cdot 10}{4}\right) = P(Z \geq 0) = 1 - P(Z < 0) =$$

$$= 1 - \frac{1}{2} = \frac{1}{2}$$

8.

$$y = \sum_{i=1}^{n} \overline{X_i} = n \cdot \overline{X_n}$$

$n = 100$
$\overline{X_i} = 1$
$P(y < 90)$

$$P(y < 90) = P(n \cdot \overline{X_n} < 90) =$$

$$= P\left(\overline{X_n} < \frac{90}{100}\right) = P(\overline{X_n} < 0.9)$$

In Poisson distribution, $\mu = h = 1$, and

$Var(X) = h = 1$

We use CLT, then $X_i \sim N(\mu, \frac{Var(X)}{n})$

$$P(y < 90) = P(\bar{X}_n < 0.9) = P\left(\frac{\sqrt{n}(\bar{X}_n - \mu)}{6} < \frac{\sqrt{100}(0.9-1)}{\sqrt{1}}\right) =$$

$$= P\left(Z < \frac{10 \cdot (-0,1)}{1}\right) = P(Z < -1) = 0.1587$$