chap2:-

⇒ Graphs and Their operations:

⇒ Union of two graphs:-

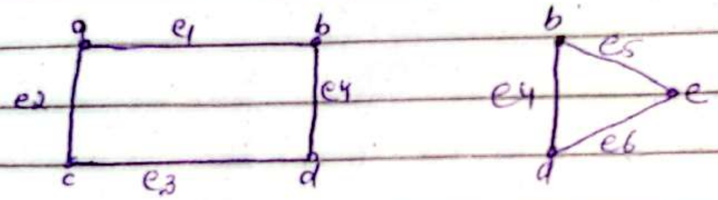
→ Given two graphs G_1 and G_2 their union will be a graph such that.

$$V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$$

and

$$E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$$

The union of G_1 and G_2 is denoted by $G_1 \cup G_2$



G_1 G_2

$V(G_1) = \{a, b, c, d\}$

$V(G_2) = \{b, d, e\}$

$V(G_1 \cup G_2) = \{a, b, c, d, e\}$

similarly for edges.

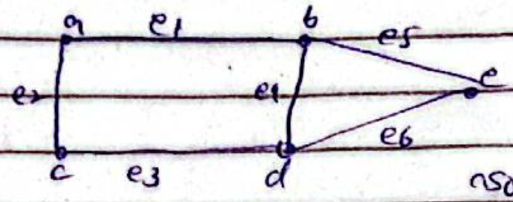
$E(G_1) = \{e_1, e_2, e_3, e_4\}$

$E(G_2) = \{e_4, e_5, e_6\}$

$E(G_1 \cup G_2) = \{e_1, e_2, e_3, e_4, e_5, e_6\}$

so e_1 edge in original graph is a to b

so and 2 edge ~~ a to c



so $G_1 G_2$

e_4 is in both graph b to d so write it one.
e_5 b to e
e_6 d to e.

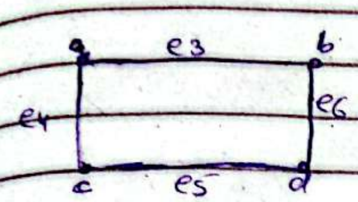
⇒ Intersection
Given two Graphs G_1 and G_2 at least one vertex is common then their intersection will be a graph such that

$$V(G_1 \cap G_2) = V(G_1) \cap V(G_2)$$

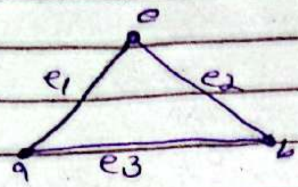
and $E(G_1 \cap G_2) = E(G_1) \cap E(G_2)$

The intersection of G_1 and G_2 denoted by $G_1 \cap G_2$

eg:- 1



G_1



G_2

$V(G_1) = \{a, b, c, d\}$ $\quad$ $E(G_1) = \{e_3, e_5, e_4, e_6\}$

$V(G_2) = \{a, b, e\}$ $\quad$ $E(G_2) = \{e_1, e_2, e_3\}$

$G_1 \cap G_2 = V\{a, b, \cancel{e}\}$

$G_1 \cap G_2 = E\{e_3,\}$

$G_1 \cap G_2$

⇒ Decomposition of graphs means breaking a graph into smaller subgraph in such way that.

- the subgraphs are edge-disjoint (they dont share edges)
- Together, they cover all the edges of the original graph.

ex:

$V = \{A, B, C, D\}$

$E = \{AB, BC, CD, DA\}$

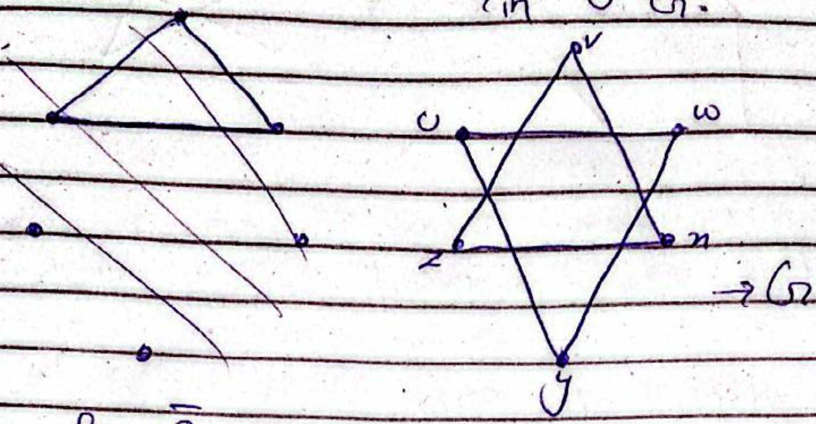
one decomposition couldbe

- subgraph H_1 : edge AB, CD
- subgraph H_2 : edge BC, DA

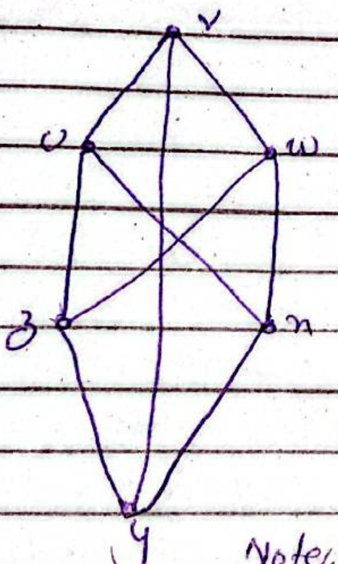
Together they form the original square, but subgraph has disjoint edges.

⇒ Complement of graph: The complement G' or $\bar{G}$ of G is defined as a simple graph with the same vertex set as G and where two vertices u and v are adjacent only when they are not adjacent ~~only when they~~ in G.

eg:-



so f $\bar{G}$ → write vertices as they are



write or connect those edge which are not adjecent with each other.

so this is complement of G

Note: for a graph G and its complement G'

(i) $G \cup G' = K_n$ → means complete graph we will get

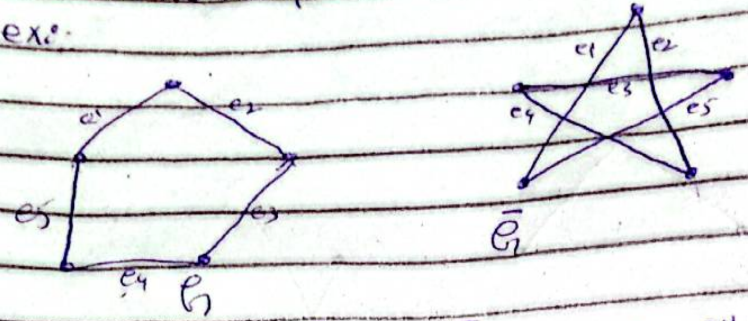
(ii) $V(G) = V(G')$ → vertex will remain same for both

(iii) $E(G) \cup E(\bar{G}) = E(K_n)$ → complete graph

→ Self - complementary graphs -

A graph G is self complementary if it is isomorphic to it its complement.

ex:-



so here G and $\bar{G}$ are ismopihic thats why it is called self-c graph.

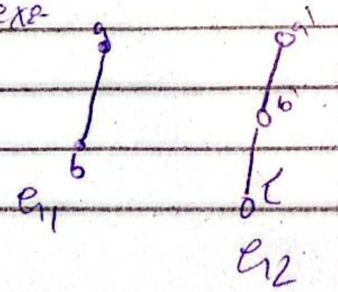
→ Def. 2:- A graph G is said to be self - complementary if G is isomorphic to its complement. ~~if G is self complementary~~

⇒ Join of graph/sum.

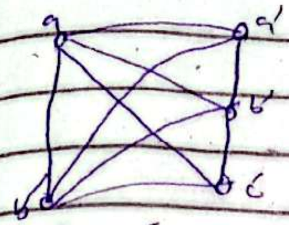
If the graphs G_1 and G_2 such that $V(G_1) \cap V(G_2) = \emptyset$ then the sum $G_1 + G_2$ is defined as the graph whose vertex set is $V(G_1) + V(G_2)$ and the edge set is consisting those edges, which are in G_1 and in G_2 and the edges obtained, by joining each vertex of G_1 to each vertex of G_2.

Note:- $G_1(V)$ and $G_2(V)$ should not be same.

exe-



so



conect a with all v of G. and same b.

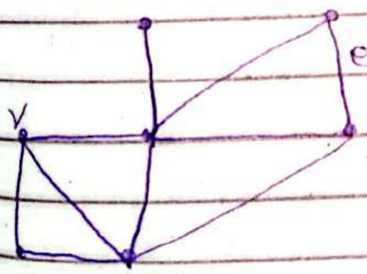
$G_1 + G_2$

→ Deletion of graphs

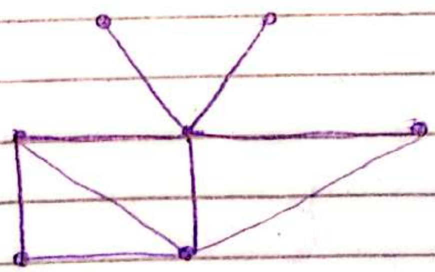
Def: Edge Deletion in graphs, if e is edge of G, then G-e is the graph obtained by removing the edge of G, The subgraph of G thus obtained is called edge-deleted subgraph of G.

→ vertex Deletion in graphs: if v is a vertex of G, then G-v is the graph obtained by removing the vertex v and all edges G that are incident on v. The subgraph of G thus obtained is called an vertex-deleted subgraph of G.

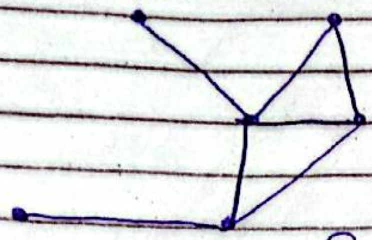
→ figure 2.7 illustrate edge deletion and the vertex deletion of a graph G.



a(G)



(b) G-e

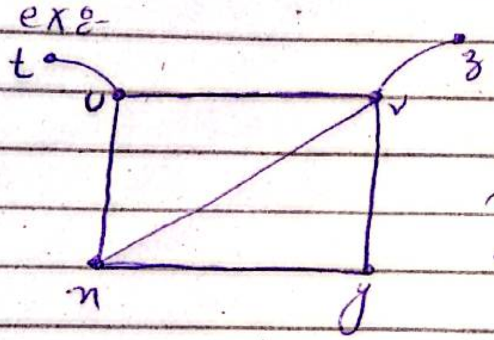


(c) $G - v$

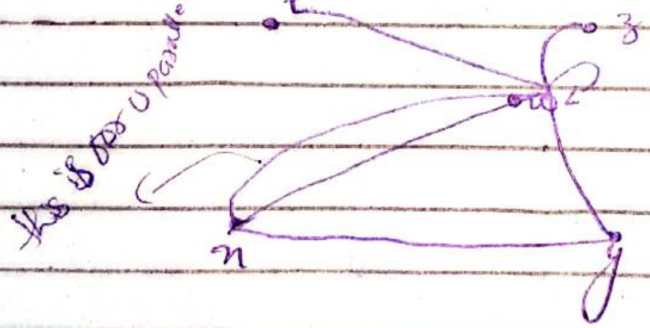
⇒ Fusion graphe-

A Pair of vertices v_1 and v_2 in graph G is said to be "fused" if these two vertices are replaced by a single new vertex v such that every edge that was adjacent to either v_1 or v_2 both is adjacent v.

ex:-



so Now we want to merge u and v so for that. u and v combine we create new vertex w so create self loop



Note: vertex will remain V-1 but edges will remain same.

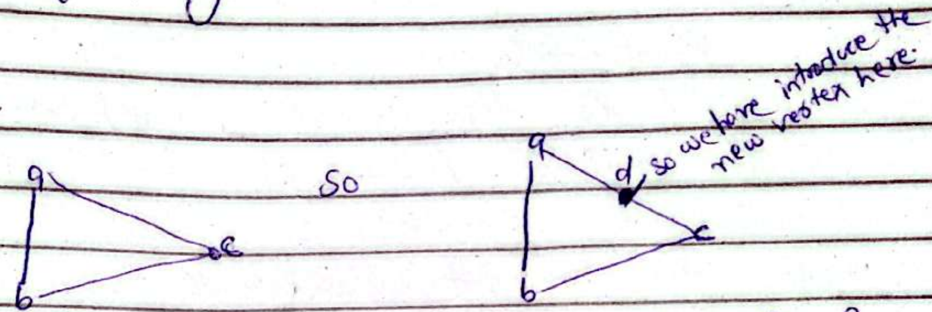
→ Edge construction:-

→ sub division of a graph:-

→ Edge subdivision:-

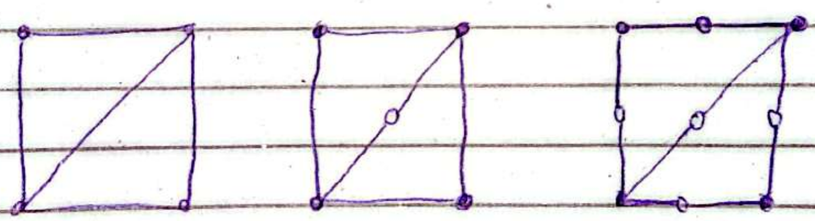
The edge subdivision operation for an edge {u,v} ∈ E is deleting of {u,v} edge from G and addition of two edges {u,w} and {w,v} along with new vertex.

ex:-



so ac → {ad}, {d,c}
same thing we can also for
ab → {af}, {fb}

ex: from books:- figure 2.11



a(G) (b) (c)

→ Homeomorphic Graphs: Two graphs are said to be homeomorphic if both can be obtained by the same subdivisions of edges.

in figure 2.11, the second and third graphs are homeomorphic, as they are obtained by subdividing the edges of the first in the figure

→ Smoothing a vertex:- is like a removing a "middle stop" on a straight path. (

imagine you are on a road trip:

- your path is Home → Gas station → work
- The gas station is a vertex degree 2 (only two roads connect to it; one from Home, one from work).

ex:

$A \to B \to C$

B is the middle stop, smoothing it makes $A \to C$)

Note: you can not smooth this.