

⇒ 14 A graph is connected if there is a walk  
between every pair of distinct vertices in the  
graph.

or  
A graph is connected if you can travel (walk)  
from any vertex to any other vertex by following  
the edges.

ex:-  
A-B-C-D

15 A subgraph is just a smaller graph  
taken from a bigger graph.

(16) Order of a Graph is simply the number  
of vertices (also called nodes) it contains.  
Notation  $(n)$

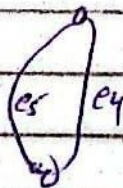
(17) Size of a graph

The size of a graph is the  
number of edges it contains.

18 A graph with a finite number of vertices  
as well as a finite number of edges  
is called a finite graph.

⇒ 19 Parallel edges: The edges connecting the same pair  
of vertices are called Multiple edges or parallel  
edges.

ex:-





20, Isolated vertex: A vertex having no incident edge is called an isolated vertex.

21, A vertex of degree 1, is called a pendant vertex or an end vertex.

22, Internal vertex: A vertex which is neither a pendant vertex nor an isolated vertex, is called an internal vertex or an intermediate vertex.

23, Degree sequence: is a list of the degrees of all vertices in a graph. The degree of a vertex is simply the number of edges connected to it. A degree seq. is typically written in non-increasing order (from largest to smallest) for consistency. ex:  $v_1$  has 3 edges,  $v_2$  has 2 edges,  $v_3$  has 3,  $v_4$  has 1,  $v_5$  has 1. So the degree seq. is  $(3, 3, 2, 1, 1)$ .

24, Graphical sequence: is a list of numbers  $(3, 3, 2, 1, 1)$  that can represent the degree seq. of a simple graph.

→ Not every sequence of numbers is a graphical seq. There are two that graphical seq. must follow.

1, The sum of the degrees must be an even number.

2, No number in the seq. can be larger than the total

25, ~~Neighbourhood of a vertex~~: number of vertices minus one.

For example: sequence  $(3, 3, 2, 1, 1)$

1,  $3 + 3 + 2 + 1 + 1 = 10$  sum of the degrees even.

2, Is any number larger than the number of vertices minus one?

So the seq. has 5 numbers, so there are 5 vertices. The largest number is 3, which is less than  $5 - 1 = 4$ .

Yes.

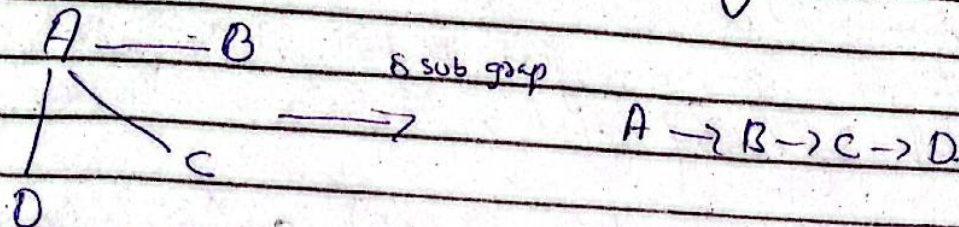


→ can we draw a graph for it? Yes we can draw with 5 vertices where are two vertices have a degree of 3 and one 2 and two 1. There for it is graphical seq.

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Spanning subgraphs.

Is a subgraph that includes all the vertices of the original graph but not necessary all of the edges. ex:



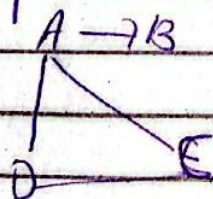
27 Induced subgraphs. It is a special type of graph you just choose a set of vertices from the big graph. then you keep all the edges that exist b/w those chosen vertices in the original graph. in other words

once you pick some vertices, the induced subgraph is automatically formed by taking all the edges that connect them in the original graph.

ex: Big graph  $G$

$V = \{A, B, C, D\}$

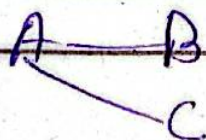
$E = \{A-B, B-C, C-D, A-D, A-C\}$



suppose we pick vertices  $\{A, B, C\}$

in the big graph, edges among them are  $\{A-B, A-C, B-C\}$

so the induced subgraph will be

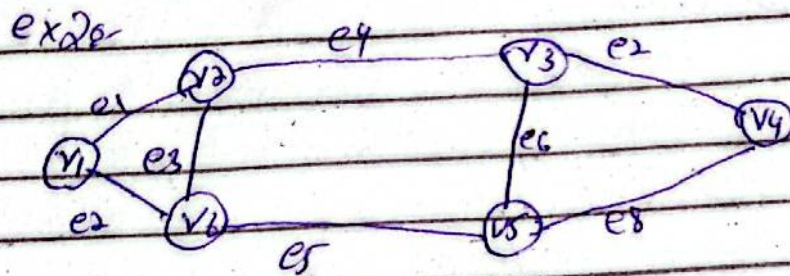




2)  $\Rightarrow$  Neighborhood of vertex  $x$  is the set of all vertices directly connected to it by a single edge. These vertices are called ~~single~~ its neighbours.

cont  
 Moreover if we just picked  $\{A, B, C\}$ , But only took  $\{A-B\}$  that would be a subgraph.

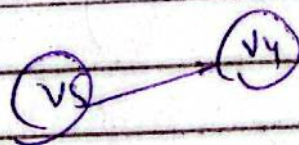
But if we take  $\{A, B, C\}$  and include all edges among them from the original graph, that is the induced graph.



$$V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$$

$$S = \{v_1, v_2, v_4, v_5\}$$

take those from  $S$  whose (sub graph) end points are present.





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Bi-Partite graph is a graph where the vertices can be split into two groups (say  $X$  and  $Y$ )

every edge always connects one vertex from group  $X$  to one vertex from group  $Y$ .

No edge connects two vertices within the same group.

But not every vertex in  $X$  needs to connect to every vertex in  $Y$ .

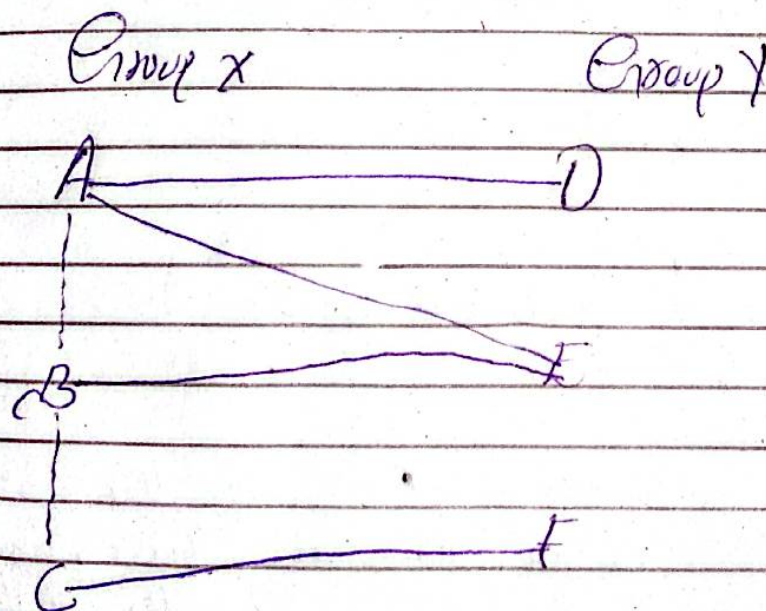
vertices  $(A, B, C, D, E, F)$

split into two groups

group  $X = \{A, B, C\}$

group  $Y = \{D, E, F\}$

Edges:  $\{A-D, A-E, B-E, C-F\}$



Note: in complete bipartite every vertex in  $X$  need

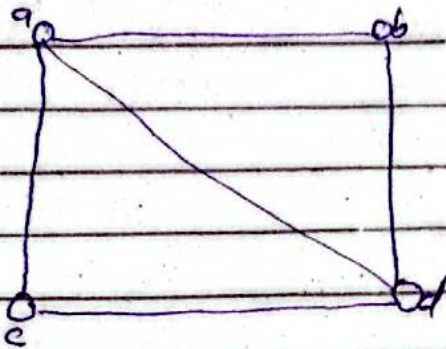


(29) Regular graphs: A graph  $G$  is said to be regular graph if all its vertices have the same degree.

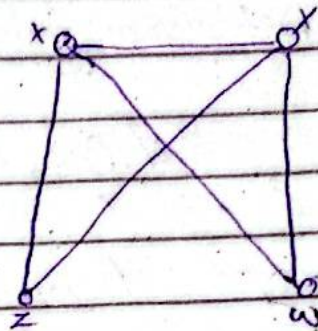
(30) Isomorphic graphs: Two graphs, which are seem to be different but in actual after looking some properties then they will seem to be isomorphism following graph.

ex:-

$G_1$



$G_2$



step 1:- check no of vertex.

$G_1 = 4$  vertex

$G_2 = 4$  vertex means both have equal

step 2:- check no of edges:  $5=5$

step 3:- Degree seq

$G_1 = a=3, b=2, c=2, d=3$

$G_2 = 3, 3, 2, 2$

step 4:- Mapping of vertices

$a = x$	$x$
$b = z$	$z$
$c = w$	$w$
$d = y$	$y$

After doing this check whether is there any edge b/w  $a$  and  $b$ , Yes, so it must be in  $x$  and  $z$  analog. Similarly  $b$  and  $c$  b/w does edge present?