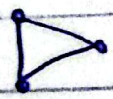
⇒ Handshaking theorem,

If a undirected Graph G then the sum of all the vertices in $G = 2$ (No of edges in 4)

means sum of no of vertices degree will become ~~ex:~~ twice the number of edges:-

ex:- w have undirected graph:-



wE have number of edges $= 3$

and it's twice $2 \times e = 6$

and we have three vertices and each has 2 degree

$2+2+2=6$ Hence proves.

OR

$G = (V, E) \rightarrow$ Undirected graph

$e \rightarrow$ number of edges

$$\boxed{\Rightarrow \sum_{v \in V} \deg(v) = 2e}$$

Proof:- Let $V = \{v_1, v_2, \ldots, v_n\}$ be the set of vertices in G.

• degree sum: $\deg(v_1) + \deg(v_2) + \ldots + \deg(v_n)$

$$= \sum_{v \in V} \deg(v)$$

• Every edge $v_i v_j$ contributes 2 [one for $\deg(v_i)$ and one for $\deg(v_j)$]

$$v_i \overset{e}{\text{———}} v_j$$

$\Rightarrow$ all the edges are counted twice in the degree sum.

$$\sum_{v \in V} \deg(v) = 2e.$$

Hence proved.

Conclusion:-

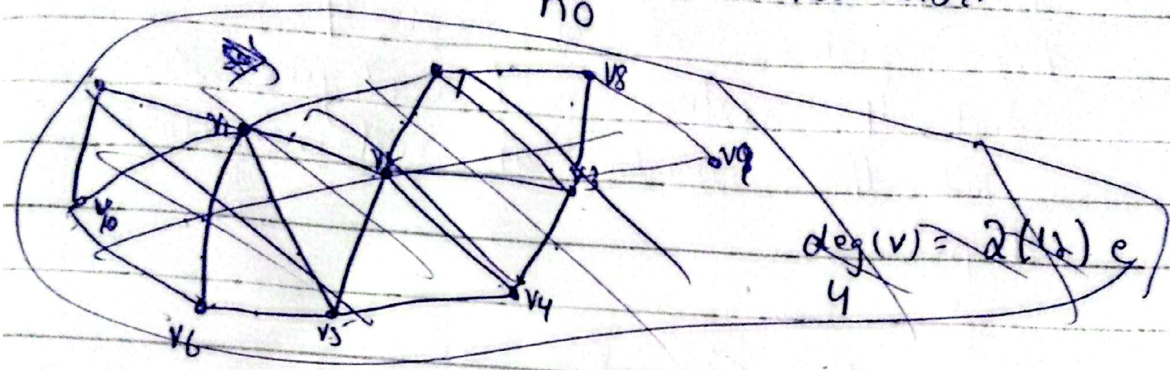
1) The sum of degree of all vertices is always even.

2) The sum of degree of all vertices with odd degree is always even.

3) The sum of vertices with odd degree are always even.

ex:-1

A graph G that has 24 edges and degree of each vertices is 4. Find the ~~sum~~ no of vertices:



$$e = 24$$

$$\deg(4)$$

let no of vertices = n.

$$4 \times n = 2(24)$$

$$n = \frac{\cancel{48}^{12}}{\cancel{4}}$$

$$\boxed{n = 12}$$

Q2,

⇒ A simple graph has 35 edges.
4 vertices of degree 5,
Five vertices of degree 4
and 4 vertices of degree 3.
Find the number of vertices with degree 2.

sol

$e = 35$

$4 \times n = 4 \times 5 = 20$ four vertice with deg 5.

$4 \times 5 = 20$

$4 \times 3 = 12$

$n \times 2 = ?$

$4 \times 5 + 4 \times 5 + 4 \times 3 + n \times 2 = 35 \times \underset{e}{2}$

$52 + 2n = 70$

$2n = 70 - 52$

$n = \frac{18}{2}$

$\boxed{n = 9}$

Q-3:

How many edges are there in a graph with 10 vertices each of which deg 6?

sol

$deg(v) = 2e$

$n = 10$

$deg = 6 = 2e$

$n \times v = 2e$

$10 \times 6 = 2e$

$\frac{\cancel{60}^{30}}{\cancel{2}} = e$

$e = 30$ Ans.