

Hall's Theorem.

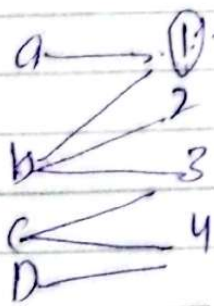
Def:

Let G be a bipartite graph with bipartition (X, Y) , then G contains a matching that saturates every vertex in X if and only if $|N(S)| \geq |S|$ for all $S \subseteq X$

Proof:-

Let us assume that G contains a Matching M which saturates every vertex in X .

To prove: $|N(S)| \geq |S|$
let S be a subset of X .



matching where no edge share the same vertex.

$$2 = 2, 8$$

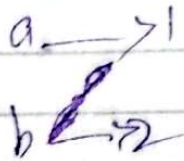


(8)

$$|S| = \{a\}$$

$$|S| = 1$$

$$N(S) = \{1\}$$



perfect matching

$$n = 4$$

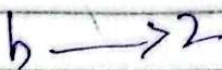
$$2 = 2 = 4$$

$$N(S) = 1$$

$$1 \geq 1$$



$$\{ \}, \{a\}, \{b\}, \{c\}$$



$$\{a, b\}, \{a, c\}, \{b, c\}$$

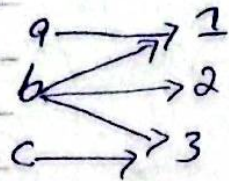


$$\{a, b, c\}$$



$$\begin{matrix} a & b \\ c & 3 \end{matrix}$$

\Rightarrow Hall's Marriage theorem



Now step 1

~~NO~~ it is Matching because initially humy Matching nh mili hue hogi phir humy identity karnay hoge ke either yai Hall's Marriage theorem ko satisfy ker sta hai ya nh.

so step 2. \rightarrow make the subset

$$2^1 = 2^3 = 8$$

$$\{ \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\} \}$$

so now step 3:

$$|N(S)| \geq |S|$$

for that

take

$$S = \{a\} \text{ means having vertex 1}$$

Now

$$S(N) = 1 \text{ ta ura a connected karta}$$

seen ahy. 1 sa hai.

so

$$\boxed{S(N) \geq |S|} \\ \boxed{1 \geq 1 \text{ satisfied}}$$

Now

$(S) = \{b\}$ which is vertex 1.

For $s(n) = 3$

So

$s(n) \geq 5$

$3 \geq 1$

hence proved and same goes for other.

