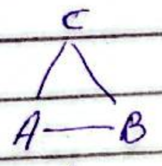
→ Graph theory: Basic Definitions and Theorems:

→ Def: A graph $G = (V, E)$ consist of a set V of vertices (also called Nodes) and a set E of edges.

(2) → If an edge connects to a vertex we say the edge is incident to the vertex and say the vertex is an endpoint of edge.

- → incident describes the relationship b/w ~~and~~ an edge and a vertex it touches.
- Think of it like a handshake: the edge "shakes hands" (is incident) with both of its ends points vertices.

ex:



ex:- A–C is incident to A and C.

(3) If an edge has only one endpoint then it is called a loop edge.

ex $A \to A$

(4) If two or more edges have the same endpoints then they are called multiple or parallell edges.

(5) Two vertices that are joined by an edge are called adjacent vertices.

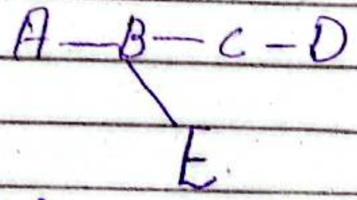
ex:-

A — B — C

- A and B and are adjacent (connected by an edge)
- B and C are adjacent (connected by an edge)
- A and C are Not adjacent (no direct edge b/w them.

(6) A pedant vertex is a vertex that is connected to exactly one other vertex by a single edge.

ex:-



- D is a pendant vertex (only connected to C)
- E is a Pendant vertex (only connected to B)
- A is a Pendant vertex (only connected to B)
- B and C are not pendant because they each have Multiple connections.

(7). A walk in a graph is a sequence of alternating vertices and edges $v_1 e_1 v_2 e_2 \ldots v_n e_n v_{n+1}$ with $n \geq 0$. if $v_1 = v_{n+1}$ then the walk is closed.

The length of is the number of edges in the walk.

A walk of length zero is a trival walk.

→ simple:
A walk is a seq of alternating vertices (nodes) and edges where you "travel" from one vertex to another by following edges.

ex:- $v_1 - (e_1) \rightarrow v_2 - (e_2) \rightarrow v_3 - (e_3) \rightarrow \ldots - (e_n)$

- start at v_1, move along edge e_1 to v_2, then e_2 to v_3, etc.

length = Number of edges (n)

- A trival walk has zero edges (just a single vertex, no movement)

(8) Trial → No edge repeated
Path → No vertex repeated.
Circuit → Trial that comes back to start.
Eulerial - covers every edge exactly one.

(9) A cycle is a nontrival circuit in which only repeated vertex is the first/last.
ex:-
Graph: A—B—C—A
Cycle: A→B→C→A
- Edges not repeated.
- only vertex A is repeated (start = end)

loop
→ An edge that starts and ends at the same vertex.
ex:- An edge from A back to A
it has only one vertex and one edge.

cycle :-
A closed Path with at least 3 vertices.
Ex:- A→B→C→A
- only start/end vertex repeats, others are unique.

simple Diff
- Loop = 1 vertex, 1 edge (small self-connection.
- cycle = 3 or more vertices connected in a closed chain.

(10) A simple graph is a graph with no loop edges. Edges in a simple graph may be specified by a set $\{v_i, v_j\}$ of the two vertices that the edge makes adjacent.

A graph with more than one edge b/w a pair of vertices is called a multigraph while a graph with loop edges is called a pseudograph.

(11) A directed graph is a graph in which the edges may only be traversed in one direction.

(12) The degree of a vertex is the number of edges incident to the vertex and is denoted deg (v).

example: A–B, A–C, B–C, C–D

- deg (A) = 2 (edges A-B, A-C)
- deg (B) = 2 (edges B-A, A-C)
- deg (C) = 3 (edges C-A, C-B, C-D)
- deg (D) = 1 (edge D-C)

(13) In a directed graph, the in-degree of a vertex is the number of edges

14 A graph is connected if there is a walk b/w every pair of distinct vertices in the graph.

or

A graph is connected if you can travel (walk) from any vertex to any other vertex by following the edges.

ex:

A — B — C — D

15. A subgraph is just a smaller graph taken from a bigger graph.

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