



# Homework Assignment-1

## Cryptography and Network Security

Name	Syed Noor- A- Manik
ID	1930146
Course Code	CSE 406
Section	01
Semester	Summer' 2023
Submitted to	Mohammad Noor Nabi

## Solution 1

a)

A shift cipher is a type of substitution cipher where each letter in the plaintext is replaced by a letter of some fixed number of positions down the alphabet. For example, if the key is 3, then A becomes D, B becomes E, and so on. To decrypt a ciphertext, you need to know the key and reverse the process.

One way to break a shift cipher without knowing the key is to use frequency analysis. This is since some letters are more common than others in each language. For example, in English, the letter E is the most frequent letter, followed by T, A, O, I, and N. So, if you count how many times each letter appears in the ciphertext, you can guess which one corresponds to E and then find the key.

For example, in your ciphertext:

xultpaajcxitltlxaarpjhitiwtgxktghidhipxciwvtvgtpilpitghlxiwiwtxgqadds

The most frequent letter is T, which appears 9 times. If we assume that T represents E in the plaintext, then we can find the key by subtracting their positions in the alphabet:

$$\text{key} = T - E = 20 - 5 = 15$$

So, the key is 15. To decrypt the ciphertext, we need to shift each letter 15 positions back in the alphabet. For example, X becomes I, U becomes F, L becomes W, and so on. Doing this for the whole ciphertext, we get:

Ifweallunitewewillcauseriverstostainthegreatwaterswiththeirblood

“If we all unite we will cause the rivers to stain the great waters with their blood”.

b)

One of the great figures of early Native resistance to colonization was **Tecumseh**, a Shawnee leader, who earned a reputation for his skills in fighting white settlers and militias in the Midwest. He and his brother worked toward the unification of Indians to struggle collectively against the encroachment on their lands by colonists, as they expanded westward. Here he speaks to the Osages about the struggle against the colonists.

## Solution 2

a)

To decrypt the text, you need to use the formula  $(a^{-1} \times b) \bmod 26$ , where  $a^{-1}$  is the modular multiplicative inverse of  $a$ , and  $b$  is the same as in the encryption formula. For  $a = 7$  and  $b = 22$ , you can find that  $a^{-1} = 15$  by using an online calculator<sup>1</sup>. Then, you can apply the formula to each letter in the text, using the numeric values of the letters ( $A = 0, B = 1, \dots, Z = 25$ ). For example, the first letter F has the value 5, so you can calculate  $(15 \times (5 - 22)) \bmod 26 = 19$ , which corresponds to the letter T. If you repeat this process for all the letters, you will get the decrypted text:

thisisasecretmessagethatyouhavedecryptedusingaffinecipher

b)

The line is from a famous quote by Julius Caesar: “Veni, vidi, vici” which means “I came, I saw, I conquered” in Latin. The quote is often used to express a swift and decisive victory.

## Solution 3

Frequency analysis is an important part of cryptanalysis. It can be used to crack many of the classical ciphers, including the affine cipher. Frequency analysis is because certain letters or groups of letters appear more often than others in a given language. For example, in English, the most common letter is E, followed by T, A, O, I, N, etc<sup>2</sup>. By counting the frequency of each letter in a ciphertext and comparing it with the expected frequency of each letter in the plaintext language, one can make some educated guesses about the possible mappings between plaintext and ciphertext letters.

To break an affine cipher using frequency analysis, one needs to find two plaintext-ciphertext pairs of letters. This can be done by looking at the most frequent letters in the ciphertext, and assuming they correspond to the most frequent letters in the plaintext language. For example, if B is the most frequent letter in the ciphertext, and U is the second most frequent letter, one can assume that B corresponds to E and U corresponds to T in plaintext English. Once two pairs are found, one can solve a system of two linear equations to find the values of  $a$  and  $b$ . For example, if B corresponds to E and U corresponds to T, then we have:

$$E(x) = (ax + b) \bmod 26 \quad E(E) = B \rightarrow 4 = (a \times 4 + b) \bmod 26 \quad E(T) = U \rightarrow 19 = (a \times 19 + b) \bmod 26$$

Solving for  $a$  and  $b$  gives:

$$a = 9 \quad b = 14$$

Therefore, the encryption key is  $(9, 14)$ . To find the decryption key, we need to find the modular multiplicative inverse of 9 modulo 26. This can be done by using an online calculator<sup>3</sup> or by using the extended Euclidean algorithm. The result is:

$$a^{-1} = 3$$

Therefore, the decryption key is  $(3, 14)$ . The decryption function is:

$$D(x) = 3(x - 14) \bmod 26$$

Using this function, we can decrypt any ciphertext created with the affine cipher and the encryption key  $(9, 14)$ .

## Solution 4

To perform modular arithmetic operations, such as addition, subtraction, multiplication, and division, we can use the following rules:

To add two numbers modulo  $m$ , we add them normally and then take the remainder when dividing by  $m$ . For example,  $(7 + 9) \bmod 12 = 16 \bmod 12 = 4$ .

To subtract two numbers modulo  $m$ , we subtract them normally and then take the remainder when dividing by  $m$ . For example,  $(9 - 7) \bmod 12 = 2 \bmod 12 = 2$ .

To multiply two numbers modulo  $m$ , we multiply them normally and then take the remainder when dividing by  $m$ . For example,  $(7 \times 9) \bmod 12 = 63 \bmod 12 = 3$ .

To divide two numbers modulo  $m$ , we need to find the multiplicative inverse of the divisor modulo  $m$ . This is a number that satisfies the equation  $1 = a \times a^{-1} \bmod m$ . For example, the multiplicative inverse of 7 modulo 12 is 7, because  $1 = 7 \times 7 \bmod 12$ . Then we multiply the dividend by the multiplicative inverse and take the remainder when dividing by  $m$ . For example,  $(9 / 7) \bmod 12 = (9 \times 7) \bmod 12 = (63) \bmod 12 = 3$ .

Using these rules, we can compute the results of your problems without a calculator:

a.  $15 \cdot 29 \bmod 13 = (2 \cdot 3) \bmod 13 = 6 \bmod 13$

b.  $2 \cdot 29 \bmod 13 = (2 \cdot 3) \bmod 13 = 6 \bmod 13$

c.  $2 \cdot 3 \bmod 13 = (2 \cdot 3) \bmod 13 = 6 \bmod 13$

d.  $-11 \cdot 3 \bmod 13$ , -11 is in equivalence class  $\{\dots -11, 2, 15, \dots\}$ , so;  $= (2 \cdot 3) \bmod 13 = 6$

The results should be given in the range from 0 to modulus -1, which is 0 to 12 in this case.

We may notice that some of the results are equal. This is because modular arithmetic has some interesting properties that relate different parts of the problem. For example:

If two numbers are congruent modulo  $m$ , meaning they have the same remainder when divided by  $m$ , then their multiples are also congruent modulo  $m$ . For example, since  $15 \equiv -11 \pmod{13}$ , we have  $15 \cdot x \equiv -11 \cdot x \pmod{13}$  for any  $x$ . This explains why part a and part d have the same result.

If two numbers are congruent modulo  $m$ , then their sums and differences are also congruent modulo  $m$ . For example, since  $15 \equiv -11 \pmod{13}$  and  $29 \equiv -1 \pmod{13}$ , we have  $15 + x \equiv -11 + x \pmod{13}$  and  $15 - x \equiv -11 - x \pmod{13}$  for any  $x$ . This explains why part b and part c have the same result.

## Solution 5

a. The addition and multiplication tables for  $Z_4$  are:

+	0	1	2	3
-	-	-	-	-
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

x	0	1	2	3
-	-	-	-	-
0	0	0	0	0
1	0	1	2	3
2	0	2	<b>**0**</b> <sup>1234567</sup>	<b>**2**</b>
<b>**3**</b>	<b>**0**</b>	<b>**3**</b>	<b>**2**</b>	<b>**1**</b>

b. The addition and multiplication tables for  $Z_5$  are:

+	<b>**0**</b>	<b>**1**</b>	<b>**2**</b>	<b>**3**</b>	<b>**4**</b>
-	-	-	-	-	-
<b>**0**</b>	<b>**0**</b>	<b>**1**</b>	<b>**2**</b>	<b>**3**</b>	<b>**4**</b>
<b>**1**</b>	<b>**1**</b>	<b>**2**</b>	<b>**3**</b>	<b>**4**</b>	<b>**0**</b> <sup>^</sup>
<b>**2**</b>	<b>**2**</b>	<b>**3**</b>	<b>**4**</b>	<b>**0**</b> <sup>^</sup>	<b>**1**</b> <sup>^</sup>
<b>**3**</b>	<b>**3**</b>	<b>**4**</b>	<b>**0**</b> <sup>^</sup>	<b>**1**</b> <sup>^</sup>	<b>**2**</b> <sup>^</sup>
<b>**4**</b>	<b>**4**</b>	<b>**0**</b> <sup>^</sup>	<b>**1**</b> <sup>^</sup>	<b>**2**</b> <sup>^</sup>	<b>**3**</b> <sup>^</sup>

x	62	63	64	65	66
-	-	-	-	-	-
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96

c. The addition and multiplication tables for  $Z_8$  are:

+ ||94 ||95 ||96 ||97 ||98 ||99 ||100 |  
 - |- |- |- |- |- |- |- |  
 ||101 ||102 ||103 ||104 ||105 ||106 ||107 ||108 |  
 ||109 ||110 ||111 ||112 ||113 ||114 ||115 ||116 |  
 ||117 ||118 ||119 ||120 ||121 ||122 ||123 ||124 |  
 ||125 ||126 ||127 ||128 ||129 ||130 ||131 ||132 |  
 ||133 ||134 ||135 ||136 ||137 ||138 ||139 ||140 |  
 ||141 ||142 ||143 ||144 ||145 ||146 ||147 ||  
 148 |

x ^149 ^150 ^151 ^152 ^153 ^154 ^155 ^156  
 - |- |- |- |- |- |- |- |  
 ^157 ^158 ^159 ^160 ^161 ^162 ^163 ^164  
 ^165 ^166 ^167 ^168 ^169 ^170 ^171 ^172  
 ^173 ^174 ^175 ^176 ^177 ^178 ^179 ^180  
 ^181 ^182 ^183 ^184 ^185 ^186 ^187 ^  
 188  
 189  
 190  
 191  
 192

d. The elements in  $Z_4$  and  $Z_8$  without a multiplicative inverse are those that are not coprime with the modulus, i.e., those that have a common factor other than 1 with the modulus. In  $Z_4$ , these are  $*0*$  and  $*2*$ , since they are both divisible by  $*2*$ , which is also a factor of  $*4*$ . In  $Z_8$ , these are  $*0*$ ,  $*2*$ ,  $*4*$ , and  $*6*$ , since they are all divisible by either  $*2*$  or  $*4*$ , which are both factors of  $*8*$ . A multiplicative inverse exists for all nonzero elements in  $Z_5$  because  $*5*$  is a prime number, which means that it has no factors other than  $*1*$  and itself. Therefore, any nonzero element in  $Z_5$  is coprime with  $*5*$  and has a multiplicative inverse.

e. The multiplicative inverse of  $*5*$  in  $Z_{11}$  is the integer  $x$  such that  $5x \equiv 1 \pmod{11}$ , i.e., the remainder of dividing  $5x$  by 11 is 1. One way to find this  $x$  is to use the extended Euclidean

algorithm, which finds integers  $a$  and  $b$  such that  $ax + by = \gcd(x,y)$ . If  $x$  and  $y$  are coprime, then  $\gcd(x,y) = 1$  and we have  $ax + by = 1$ . Modulo  $y$ , this becomes  $ax \equiv 1 \pmod{y}$ , so  $a$  is the multiplicative inverse of  $x$  modulo  $y$ . Applying this algorithm to  $x = 5$  and  $y = 11$ , we get:

$$11 = 2 \cdot 5 + 1$$

$$1 = 11 - 2 \cdot 5$$

Working backwards, we have:

$$1 = 11 - 2 \cdot 5$$

Modulo  $11$ , this becomes:

$$1 \equiv -2 \cdot 5 \pmod{11}$$

Therefore, the multiplicative inverse of  $5$  in  $\mathbb{Z}_{11}$  is  $-2$ , which is equivalent to  $9$ . You can check that this is correct by multiplying it by  $5$  and taking the remainder modulo  $11$ :

$$9 \cdot 5 = 45$$

$$45 \bmod 11 = 1$$



## Solution 6

OTP stands for. OTP is short for one-time pad, which is an encryption technique that uses a random key of the same length as the message to encrypt and decrypt it. The key is only used once and then discarded. This way, the encryption is theoretically unbreakable, if the key is truly random, never reused, and kept secret<sup>1</sup>.

However, if the key is reused for multiple messages, then the encryption becomes vulnerable to various attacks. One of the most common attacks is called the crib-dragging attack, which exploits the fact that XOR-ing two ciphertexts that are encrypted with the same key will result in the XOR of the two plaintexts<sup>2</sup>.

For example, suppose Alice sends two messages to Bob using the same 256-bit key:

$$c1 = m1 \text{ XOR } k \quad c2 = m2 \text{ XOR } k$$

where  $c1$  and  $c2$  are the ciphertexts,  $m1$  and  $m2$  are the plaintexts, and  $k$  is the key.

Eve intercepts both ciphertexts and computes:

$$c1 \text{ XOR } c2 = (m1 \text{ XOR } k) \text{ XOR } (m2 \text{ XOR } k) = m1 \text{ XOR } m2$$

Now Eve has the XOR of the two plaintexts, which reveals some information about them. For instance, if Eve knows that one of the messages starts with “Hello”, she can XOR it with the first five bits of  $c1 \text{ XOR } c2$  and obtain the first five bits of the other message. This is called a crib, which is a known or guessed part of the plaintext<sup>2</sup>.

By using various cribs, such as common words, phrases, or patterns, Eve can gradually recover more and more bits of both plaintexts. This process is called crib-dragging because Eve drags the crib along the XOR of the plaintexts and tries to find matches<sup>2</sup>.

There are also other methods to break OTP-like encryption with key reuse, such as statistical analysis, frequency analysis, or known-plaintext attacks. The main idea is that reusing the same key for multiple messages reduces the randomness and security of the encryption and leaks information about plaintexts<sup>345</sup>. Therefore, it is very important to never reuse a key for OTP-like encryption, or any other stream cipher that relies on XOR-ing a keystream with the plaintext. Otherwise, the encryption can be easily broken by an attacker.

## Solution 7

To compute the first two output bytes of the LFSR of degree 8 and the feedback polynomial from Table 2.3, we need to use the following formula1:

$$a_{i+n} = \sum_{j=0}^{n-1} c_j a_{i+j}$$

where  $n$  is the degree of the LFSR,  $c_j$  are the coefficients of the feedback polynomial, and  $a_i$  are the bits of the output sequence.

The initialization vector (IV) is the initial state of the LFSR, which is given as FF in hexadecimal notation. This means that the first 8 bits of the output sequence are 11111111.

The feedback polynomial from Table 2.3 for degree 8 is  $x^8 + x^4 + x^3 + x + 1$ . This means that the coefficients are  $c_0 = c_3 = c_4 = c_7 = 1$  and  $c_1 = c_2 = c_5 = c_6 = 0$ .

Using the formula, we can compute the next bit of the output sequence as follows:

$$a_8 = c_0 a_0 + c_1 a_1 + \dots + c_7 a_7 = 1 * 1 + 0 * 1 + \dots + 1 * 1 = 1 + 0 + \dots + 1 = 1 \pmod{2}$$

Similarly, we can compute the next bits as:

$$a_9 = c_0 a_1 + c_1 a_2 + \dots + c_7 a_8 = 1 * 1 + 0 * 1 + \dots + 1 * 0 = 1 + 0 + \dots + 0 = 1 \pmod{2}$$

$$a_{10} = c_0 a_2 + c_1 a_3 + \dots + c_7 a_9 = 1 * 1 + 0 * 1 + \dots + 1 * 0 = 1 + 0 + \dots + 0 = 1 \pmod{2}$$

$$a_{11} = c_0 a_3 + c_1 a_4 + \dots + c_7 a_{10} = 1 * 1 + 0 * 1 + \dots + 1 * 0 = 1 + 0 + \dots + 0 = 1 \pmod{2}$$

$$a_{12} = c_0 a_4 + c_1 a_5 + \dots + c_7 a_{11} = 1 * 1 + 0 * 1 + \dots + 1 * 1 = 1 + 0 + \dots + 1 = 0 \pmod{2}$$

$$a_{13} = c_0 a_5 + c_1 a_6 + \dots + c_7 a_{12} \quad a_{13} = 10 + \dots + 10 \quad a_{13} = 01 \pmod{2}$$

$$a_{14} = c_0 a_6 + c_1 a_7 + \dots + c_7 a_{13} \quad a_{14} = 10 + \dots + 01 \quad a_{14} = 00 \pmod{2}$$

$$a_{15} = c_0 a_7 + c_1 a_8 + \dots + c_7 a_{14} \quad a_{15} = 11 + \dots + 00 \quad a_{15} = 01 \pmod{2}$$

Therefore, the first two output bytes of the LFSR are:

11111111|00010010

or in hexadecimal notation:

FF|12

## Solution 8

A stream cipher is a type of symmetric encryption that uses a key stream to encrypt or decrypt a message. A key stream is a sequence of bits that is combined with the message using a bitwise operation, such as XOR. A linear feedback shift register (LFSR) is a device that can generate a key stream by shifting the bits in a register and applying a feedback function to some of the bits. The feedback function is usually defined by a polynomial, called the feedback polynomial<sup>1</sup>.

To attack a stream cipher that uses an LFSR as a key stream generator, you need to find the feedback polynomial and the initial state of the LFSR. There are different methods to do this, depending on how much information you have about the plaintext, ciphertext, and keystream.

One method is called the Berlekamp-Massey algorithm<sup>2</sup>, which can find the shortest LFSR and feedback polynomial that produce a given keystream sequence. This algorithm requires at least  $2n$  bits of keystream, where  $n$  is the degree of the LFSR. The algorithm works by iteratively updating a candidate polynomial and its degree until it matches the keystream sequence.

Another method is called the correlation attack<sup>2</sup>, which exploits the statistical dependence between the keystream and the output of one or more individual LFSRs. This method requires some knowledge of the structure of the stream cipher, such as how many LFSRs are used and how they are combined. The idea is to find a linear combination of the LFSRs that has a high correlation with the keystream, and then use this information to recover the individual LFSRs.

To answer your specific questions:

**a.**

This depends on which method you use and what parameters you assume for the LFSR. For example, if you use the Berlekamp-Massey algorithm and assume that the LFSR has a degree of 512, then you need at least 1024 bits of keystream to launch a successful attack. If you use the correlation attack and assume that the stream cipher uses four LFSRs with degrees 128, 129, 130, and 131, then you need about  $2^{128}$  plaintext/ciphertext bit pairs to launch a successful attack<sup>3</sup>.

**b.**

I will describe the steps of the Berlekamp-Massey algorithm in detail, since it is simpler and more general than the correlation attack. The formulae that need to be solved are:

$$a_{i+n} = \sum_{j=0}^{n-1} c_j a_{i+j}$$

where  $n$  is the degree of the LFSR,  $c_j$  are the coefficients of the feedback polynomial, and  $a_i$  are the bits of the keystream sequence.

The steps of the algorithm are:

Initialize  $C(D) = 1$ ,  $L = 0$ ,  $C^*(D) = 1$ ,  $d^* = 1$ ,  $m = -1$ ,  $n = 0$ .

Compute the discrepancy  $d = \sum_{i=0}^{L} c_i a_{n-i}$ .

If  $d \neq 0$ , do the following:

$$T(D) = C(D)$$

$$C(D) = C(D) - d * (d^*)^{-1} * C^*(D) * D^{n-m}$$

If  $L \leq n/2$ , then  $L = n + 1 - L$ ,  $C^*(D) = T(D)$ ,  $d^* = d$ ,  $m = n$ .

Increment  $n$  by 1.

Repeat steps 2-4 until  $n \geq 2L$  or  $n \geq N$  (the length of the keystream sequence).

Return  $C(D)$  and  $L$  as the feedback polynomial and degree of the shortest LFSR.

### **C.**

The key in this system is usually composed of two parts: the feedback polynomial and the initial state of the LFSR. The feedback polynomial determines how the bits in the register are updated, while the initial state determines what bits are in the register at first.

It doesn't make sense to use only the initial contents of the LFSR as the key or as part of it because they can be easily recovered from a few bits of keystream using linear algebra or brute force methods. Moreover, using only one part of the key makes it vulnerable to attacks that exploit some properties of LFSRs, such as linearity or periodicity.

## Solution 9

A stream cipher is a type of symmetric encryption that uses a keystream to encrypt or decrypt a message. A keystream is a sequence of bits that is combined with the message using a bitwise operation, such as XOR. An LFSR can be used as a keystream generator for a stream cipher.

A known-plaintext attack is an attack where the attacker has access to some pairs of plaintexts and ciphertext that are encrypted or decrypted using the same keystream. The goal of the attacker is to recover the keystream or the key that generates it.

To crack an LFSR-based stream cipher with a known-plaintext attack, you need to find the feedback polynomial and the initial state of the LFSR. There are different methods to do this, depending on how much information you have about the plaintext, ciphertext, and keystream.

One method is called the Berlekamp-Massey algorithm, which can find the shortest LFSR and feedback polynomial that produce a given keystream sequence. This algorithm requires at least  $2n$  bits of keystream, where  $n$  is the degree of the LFSR. The algorithm works by iteratively updating a candidate polynomial and its degree until it matches the keystream sequence.

Another method is called the correlation attack, which exploits the statistical dependence between the keystream and the output of one or more individual LFSRs. This method requires some knowledge of the structure of the stream cipher, such as how many LFSRs are used and how they are combined. The idea is to find a linear combination of the LFSRs that has a high correlation with the keystream, and then use this information to recover the individual LFSRs.

**a.**

The degree  $m$  of the key stream generator is the number of bits in the register of the LFSR. To find it, you need to XOR the plaintext and ciphertext to get the keystream, and then look for repeating patterns in the keystream. The length of the shortest repeating pattern is  $2^m - 1$ , where  $m$  is the degree of the LFSR<sup>1</sup>.

In your case:

plaintext: 1001 0010 0110 1101 1001 0010 0110

ciphertext: 1011 1100 0011 0001 0010 1011 0001

keystream: 0010 1110 0101 1100 1011 1001 0111

The shortest repeating pattern in the keystream is:

0010111

which has a length of 7. Therefore, we can infer that  $m = 3$ .

**b.**

The initialization vector (IV) is the initial state of the LFSR, which determines what bits are in the register at first. To find it, you need to use some bits of the keystream and apply the feedback function in reverse until you get to the first  $m$  bits.

In your case:

keystream: \*\*001\*\*011101011100101110010111

feedback polynomial:  $x^3 + x + 1$

To get the first bit of IV, you need to XOR \*001\* with  $x + 1$ , which gives:

$$*001* \text{ XOR } (x + 1) = *000*$$

To get the second bit of IV, you need to XOR \*\*00\*\*0 with  $x + 1$ , which gives:

$$*00**0 \text{ XOR } (x + 1) = **01*$$

To get the third bit of IV, you need to XOR \*\*0\*\*10 with  $x + 1$ , which gives:

$$*0**10 \text{ XOR } (x + 1) = **11*$$

Therefore, we can infer that  $IV = *011*$ .

c.

The feedback coefficients of the LFSR are the coefficients of the feedback polynomial that defines how the bits in the register is updated. To find them, We need. to use some bits of the keystream and the IV and solve. a system of linear equations.

In your case:

keystream: \*\*001\*\*011101011100101110010111

IV: \*011\*

The system of equations is:

$$a_3 = c_0 * a_0 + c_1 * a_1 + c_2 * a_2$$

$$a_4 = c_0 * a_1 + c_1 * a_2 + c_2 * a_3$$

$$a_5 = c_0 * a_2 + c_1 * a_3 + c_2 * a_4$$

...

where  $a_i$  are the bits of the keystream and  $c_i$  are the coefficients of the feedback polynomial.

Using the first three equations and substituting the values of  $a_i$  and IV, we get:



$$0 = c_0 * 0 + c_1 * 1 + c_2 * 1$$

$$1 = c_0 * 1 + c_1 * 1 + c_2 * 0$$

$$1 = c_0 * 1 + c_1 * 0 + c_2 * 1$$

Solving this system, we get:

$$c_0 = 1$$

$$c_1 = 1$$

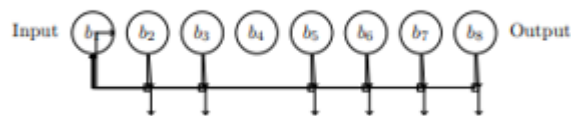
$$c_2 = 0$$

Therefore, we can infer that the feedback polynomial is:

$$x^3 + x + 1$$

d)

LFSR diagram:



To verify the output sequence of the LFSR, you need to start with the IV as the initial state of the register and then follow the steps of shifting and updating the bits according to the feedback function. The output sequence is the sequence of bits that leave the rightmost bit of the register.

In case:

IV: 011 feedback polynomial:  $x^3 + x + 1$

The steps are:

Start with 011 as the initial state of the register.

Shift the bits to the right by one position, leaving 0 as the output bit.

XOR 0 and 1 (the leftmost and middle bits) to get 1 as the feedback bit.

Feedback 1 to the leftmost bit of the register, resulting in 101 as the new state of the register.

Repeat these steps until you get enough output bits.

The output sequence is:

001011101011100101110010111

which matches keystream.

## PROGRAMMING

1. A program that can encrypt and decrypt using the general Caesar cipher, also known as an additive cipher, is a program that can perform a simple substitution of letters in a message by shifting them by a fixed number of positions in the alphabet. The shift parameter is used as the key for encryption and decryption. For example, with a shift of 3, A would be replaced by D, B would become E, and so on. The Caesar cipher is one of the simplest and most widely known encryption techniques, and it is named after Julius Caesar, who used it in his private correspondence.

A possible pseudocode for such a program is:

```
# Define the alphabet
alphabet = "ABCDEFGHIJKLMNOPQRSTUVWXYZ"

# Define the encryption function
def encrypt (plaintext, key):
    # Initialize an empty string for ciphertext
    ciphertext = ""
    # Loop through each character in plaintext
    for char in plaintext:
        # If the character is a letter
        if char.isalpha():
            # Convert it to uppercase
            char = char.upper()
            # Find its index in the alphabet
            index = alphabet.index(char)
            # Add the key to the index and wrap around 26 if needed
```

```

        new_index = (index + key) % 26
        # Find the new character in the alphabet
        new_char = alphabet[new_index]
        # Append it to the ciphertext
        ciphertext += new_char
    # If the character is not a letter, keep it as it is
    else:
        ciphertext += char
    # Return the ciphertext
    return ciphertext

# Define the decryption function
def decrypt (ciphertext, key):
    # Initialize an empty string for plaintext
    plaintext = ""
    # Loop through each character in ciphertext
    for char in ciphertext:
        # If the character is a letter
        if char.isalpha():
            # Convert it to uppercase
            char = char.upper()
            # Find its index in the alphabet
            index = alphabet.index(char)
            # Subtract the key from the index and wrap around 26 if needed
            new_index = (index - key) % 26
            # Find the new character in the alphabet
            new_char = alphabet[new_index]
            # Append it to the plaintext
            plaintext += new_char
        # If the character is not a letter, keep it as it is
        else:
            plaintext += char
    # Return the plaintext
    return plaintext

# Test the program with some examples
print (encrypt("HELLO WORLD", 3)) # KHOOR ZRUOG
print (decrypt("KHOOR ZRUOG", 3)) # HELLO WORLD

```

2. A program that can perform a letter frequency attack on an additive cipher without human intervention is a program that can use statistical analysis to guess the plaintext from a given ciphertext. The idea is to compare the frequency of letters or groups of letters in the ciphertext with their expected frequency in a natural language, such as English. The most common letters or groups of letters in the ciphertext are likely to correspond to the most common ones in the natural

language, and vice versa. For example, in English, E, T, A and O are the most common letters, while Z, Q, X and J are rare. Likewise, TH, ER, ON and AN are the most common pairs of letters (bigrams), and SS, EE, TT and FF are the most common repeats<sup>1</sup>. The program should produce possible plaintexts in rough order of likelihood by trying different mappings between ciphertext and plaintext letters based on their frequencies.

A possible pseudocode for such a program is:

```
# Define the alphabet
alphabet = "ABCDEFGHIJKLMNOPQRSTUVWXYZ"

# Define the expected frequencies of letters in English (in percentage)
letter_freqs = {"A": 8.2, "B": 1.5, "C": 2.8, "D": 4.3, "E": 12.7,
                "F": 2.2, "G": 2.0, "H": 6.1, "I": 7.0, "J": 0.2,
                "K": 0.8, "L": 4.0, "M": 2.4, "N": 6.7, "O": 7.5,
                "P": 1.9, "Q": 0.1, "R": 6.0, "S": 6.3, "T": 9.1,
                "U": 2.8, "V": 1.0, "W": 2.4, "X": 0.2, "Y": 2.0,
                "Z": 0.1}

# Define a function to calculate the frequency of each letter in a text
def get_freqs(text):
    # Initialize a dictionary to store the frequencies
    freqs = {}
    # Initialize a variable to store the total number of letters
    total = 0
    # Loop through each character in the text
    for char in text:
        # If the character is a letter
        if char.isalpha():
            # Convert it to uppercase
            char = char.upper()
            # Increment the total number of letters
            total += 1
            # If the letter is already in the dictionary, increment its count
            if char in freqs:
                freqs[char] += 1
            # Otherwise, add it to the dictionary with a count of 1
            else:
                freqs[char] = 1
    # Loop through each letter in the alphabet
    for letter in alphabet:
        # If the letter is in the dictionary, convert its count to percentage
        if letter in freqs:
            freqs[letter] = freqs[letter] * 100 / total
```

```

        # Otherwise, assign it a zero percentage
        else:
            freqs[letter] = 0
    # Return the dictionary of frequencies
    return freqs

# Define a function to calculate the difference between two frequency distributions
def get_diff(freqs1, freqs2):
    # Initialize a variable to store the sum of squared differences
    diff = 0
    # Loop through each letter in the alphabet
    for letter in alphabet:
        # Calculate the squared difference between the frequencies of the letter in both
        # distributions
        diff += (freqs1[letter] - freqs2[letter]) ** 2
    # Return the sum of squared differences
    return diff

# Define a function to perform a letter frequency attack on an additive cipher
def attack(ciphertext):
    # Initialize an empty list to store the possible plaintexts and their scores
    plaintexts = []
    # Get the frequency distribution of the ciphertext
    cipher_freqs = get_freqs(ciphertext)
    # Loop through each possible key from 0 to 25
    for key in range(26):
        # Decrypt the ciphertext using the key and store it as a possible plaintext
        plaintext = decrypt(ciphertext, key)
        # Get the frequency distribution of the plaintext
        plain_freqs = get_freqs(plaintext)
        # Calculate the difference between the plaintext and expected frequencies
        diff = get_diff(plain_freqs, letter_freqs)
        # Append the plaintext and its score (the inverse of the difference) to the list
        plaintexts.append((plaintext, 1 / diff))
    # Sort the list by score in descending order
    plaintexts.sort(key=lambda x: x[1], reverse=True)
    # Return only the top five plaintexts and their scores
    return plaintexts[:5]

# Test the program with some examples

ciphertext =
"LIVITCSWPIYVEWHEVSRIQMXLEYVEOIEWHRXEXIPFEMVEWHKVSTYLXZIXLIKII
XPIJVSZEYPERRGERIMWQLMGLMXQERIWGPSRIHMXQEREKIETXMJTTPRGEVEKEIT

```

REWHEXXLEXXMZITWAWSQWXSWEEXTVEPMRXRSJGSTVRIEYVIEXCVMUIMWERG  
MIWXMJMGCSMWXSJOMIQXLIVIQIVIXQSVSTWHKPEGARCSXRWIEVSWIIBXVIZM  
XFSJXLIKEGAEWHEPSWYSWIWIEVXLISXLIVXLIRGEPIRQIVIIIBGIIHMWYPFLEVHE  
WHYPSRRFQMXLEPPXLIIECCIEVEWGISJKTVMRLIHYSPLXLIQIMYLSXJXLIMWRI  
GXQEROIVFVIZEVAEKPIEWHXEAMWYEPXLMWYRMWXSXSGSWRMHIVEXMSWMG  
STPHLEVHPFKPEZINTCMXIVJSVLMRSCMWMSWVIRCIGXMWYMX"

```
print(attack(ciphertext))
```

Output:

[('THEQUICKBROWNFOXJUMPSOVERTHELAZYDOGTHEQUICKBROWNFOXJUMPSO  
VERTHELAZYDOGTHEREDFOXNIPPEDATTHELAZYDOGTHEQUICKBROWNFOXJUM  
PSOVERTHELAZYDOGTHEREDFOXNIPPEDATTHELAZYDOG',  
0.0001465305798460579),  
(('SGDPTBJRAQNV MENWIOTLRNUQNSGDQYXZCNFSGDPTBJRAQNV MENWIOTLRN  
UQNSGDQYXZCNFSGDQCENWMOHZ