

$$\begin{aligned}
& \int \tan x \, dx \\
& \int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3-x^2} \, dx \\
& \int \log x \, dx \\
& \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\log(\sin x)}{\tan x} \, dx \\
& \int e^x \cos x \, dx \\
& \int \frac{dx}{\sin^3 x} \\
& \int \tan^4 x \, dx \\
& \int \frac{dx}{\sin^2 x} \\
& \int \sqrt{1-x} \, dx \\
& \int_0^{\frac{\pi}{2}} \cos^2 x \, dx \\
& \int x^2 \sin x \, dx \\
& \int_0^{2\pi} \sqrt{1+\cos x} \, dx \\
& \int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\cos x}{1+\cos x}} \, dx \\
& \int_0^{\frac{\pi}{3}} \sqrt{1+\sin x} \, dx \\
& \int_0^1 \frac{dx}{1+x^2}
\end{aligned}$$

$$\begin{aligned}
& \int_0^1 \frac{dx}{3+x^2} \\
& \int_0^1 \frac{dx}{x^2+x+1} \\
& \int x^x(1+\log x) dx \\
& \int \frac{dx}{\sqrt{1+x^2}} \\
& \int_2^3 \frac{dx}{\sqrt{x^2-1}} \\
& \int \sqrt{1-e^{-2x}} dx \\
& \int_0^1 \frac{dx}{1+x^3} \\
& \int \frac{dx}{\sin^4 x} \\
& \int \frac{dx}{\sqrt{x}+\sqrt{x+2}} \\
& \int_0^e \frac{e^x}{e^{e-x}+e^x} dx \\
& \int_{-1}^1 \sqrt{1-x^2} dx \\
& \int \sqrt{1-e^{-2x}} dx \\
& \int \tan^2 x dx \\
& \int \tan^3 x dx \\
& \int \tan^5 x dx
\end{aligned}$$

KingProperty

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$t = a + b - x$ と置換する

$$dx = -dt$$

$$\therefore \int_b^a -f(t) dt = \int_a^b f(t) dt$$
$$= \int_a^b f(x) dx \quad (\because \text{定積分では変数を変更してもよい})$$

因って、 $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

x	$a \rightarrow b$
t	$b \rightarrow a$

$$\begin{aligned}
& \int \tan x \, dx \quad \tan \text{ の一次式} \rightarrow \frac{\sin x}{\cos x} \text{ で一次式に} \\
& \int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx \quad \text{分数関数はまず最初に} \rightarrow \text{分母の微分を考える} \\
& \quad = -\log|\cos x| + C
\end{aligned}$$

$$\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3-x^2} dx \quad \sqrt{a^2-x^2} \text{を含む} \rightarrow x = a \sin t \text{とおく}$$

$$x = \sqrt{3} \sin t \text{ とする}$$

$$dx = \sqrt{3} \cos t dt$$

$$\begin{aligned} \int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3-x^2} dx &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{3 - (\sqrt{3} \sin t)^2} \sqrt{3} \cos t dt \\ &= 3 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 - \sin^2 t} \cos t dt \\ &= 3 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\cos t| \cos t dt \\ &= 3 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t dt \left(\because -\frac{\pi}{2} \leq t \leq \frac{\pi}{2} \Rightarrow 0 \leq \forall \cos t \right) \\ &= \frac{3}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 + \cos 2t dt \left(\because \cos^2 x = \frac{1 + \cos 2x}{2} \quad c.f. \sin^2 x = \frac{1 - \cos 2x}{2} \right) \\ &= \frac{3}{2} \left[t + \frac{1}{2} \sin 2t \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\ &= 3 \left[t + \frac{1}{2} \sin 2t \right]_0^{\frac{\pi}{2}} \\ &= \frac{3}{2} \pi \end{aligned}$$

x	$-\sqrt{3} \rightarrow \sqrt{3}$
t	$-\frac{\pi}{2} \rightarrow \frac{\pi}{2}$

$\int \frac{dx}{\sin^2 x} dx$ sinがcosだったら都合がいい $\rightarrow \frac{\pi}{2}$ で cos に

$$\begin{aligned}\int \frac{dx}{\sin^2 x} &= \int \frac{dx}{\cos^2\left(\frac{\pi}{2} - x\right)} \\ &= -\tan\left(\frac{\pi}{2} - x\right) + C \\ &= -\frac{1}{\tan x} + C\end{aligned}$$

$$\int_{-1}^1 \sqrt{1-x^2} dx \quad \sqrt{(\quad)^2} = |\quad| \text{ としてほしい.}$$

ここで, $1 - \sin^2 t = \cos^2 t$ である.

また, $1 - x^2 \geq 0 \Leftrightarrow -1 \leq x \leq 1$ であるから, $x = \sin t$ とおける.

$x = \sin t$ とする

$$dx = \cos t dt$$

$$\begin{aligned} \int_{-1}^1 \sqrt{1-x^2} dx &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1-\sin^2 t} \cos t dt \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\cos t| \cos t dt \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t dt \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1 + \cos 2t}{2} dt \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{\cos 2t}{2} \right) dt \\ &= \frac{1}{2} \left[t + \frac{1}{2} \sin 2t \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\ &= \frac{\pi}{2} \end{aligned}$$

x	$-1 \rightarrow 1$
t	$-\frac{\pi}{2} \rightarrow \frac{\pi}{2}$

$$\int_0^1 \frac{dx}{1+x^2} \quad \text{分母を単項式にしたい.}$$

$$\text{ここで, } 1 + \tan^2 t = \frac{1}{\cos^2 t} \text{ である.}$$

$$\text{また, } (\tan t)' = \frac{1}{\cos^2 t} \text{ であるから, } x = \tan t \text{ とする}$$

$$x = \tan t \text{ とおく}$$

$$1 + x^2 = \frac{1}{\cos^2 t}$$

$$dx = \frac{dt}{\cos^2 t}$$

$$\begin{aligned} \int_0^1 \frac{dx}{1+x^2} &= \int_0^{\frac{\pi}{4}} \frac{\frac{1}{\cos^2 t}}{\frac{1}{\cos^2 t}} dt \\ &= \int_0^{\frac{\pi}{4}} dt \\ &= \frac{\pi}{4} \end{aligned}$$

x	$0 \rightarrow 1$
t	$0 \rightarrow \frac{\pi}{4}$

$$\begin{aligned}
& \int \tan^4 x \, dx \quad \text{三角関数は2次ごとに分割} \\
& \int \tan^4 x \, dx = \int \tan^2 x \tan^2 x \, dx \quad \tan^2 \text{ は } \cos^2 \\
& \quad = \int \tan^2 x \left(\frac{1}{\cos^2 x} - 1 \right) dx \\
& \quad = \int \frac{\tan^2 x}{\cos^2 x} dx - \int \tan^2 x \, dx \\
& \quad = \frac{1}{3} \tan^3 x - \tan x + x + C
\end{aligned}$$

$\int \log x \, dx$ \log を含む \rightarrow 部分積分

$$\begin{aligned}\int \log x \, dx &= \int 1 \cdot \log x \, dx \\ &= x \log x - \int x \cdot \frac{1}{x} \, dx \\ &= x \log x - x + C\end{aligned}$$

$$\int_0^e \frac{e^x}{e^{e-x} + e^x} dx \quad a+b-x \text{ を含む, 下端が } 0, \text{ 指数関数を含む式} \rightarrow \text{KingProperty}$$

$$\int_0^e \frac{e^x}{e^{e-x} + e^x} dx = \int_0^e \frac{e^{e-x}}{e^x + e^{e-x}} dx$$

$$I = \int_0^e \frac{e^x}{e^{e-x} + e^x} dx \text{ とすると}$$

$$2I = \int_0^e \frac{e^x}{e^{e-x} + e^x} dx + \int_0^e \frac{e^{e-x}}{e^x + e^{e-x}} dx$$

$$= \int_0^e \frac{e^x + e^{e-x}}{e^x + e^{e-x}} dx$$

$$= \int_0^e dx$$

$$= e$$

$$\therefore I = \frac{e}{2}$$

$$\int_0^e \frac{e^x}{e^{e-x} + e^x} dx \quad \text{指数関数の定数部分は分離}$$

$$\int_0^e \frac{e^x}{e^{e-x} + e^x} dx = \int_0^e \frac{e^x}{e^e \cdot e^{-x} + e^x} dx$$

指数の符号は揃える, 指数関数の分数関数→分母分子に e^x をかける

指数関数の分数関数→微分接触をつくる

$$= \frac{1}{2} \int_0^e \frac{2e^{2x}}{e^e + e^{2x}} dx$$

$$= \frac{1}{2} [\log(e^e + e^{2x})]_0^e \quad (\because e^e + e^{2x} \geq 0)$$

$$= \frac{1}{2} \log \frac{e^e + e^{2e}}{e^e + 1}$$

$$= \frac{1}{2} \log \frac{e^e(1 + e^e)}{e^e + 1}$$

$$= \frac{e}{2}$$

$$\begin{aligned}
& \int \frac{dx}{\sin^4 x} \quad f(\sin^2 x, \cos^2 x, \tan x) \rightarrow g(\tan x) \cdot \frac{1}{\cos^2 x} \text{ へ} \\
& \int \frac{dx}{\sin^4 x} = \int \frac{dx}{\tan^4 x \cos^4 x} \\
& \quad = \int \frac{1}{\tan^4 x} (1 + \tan^2 x) \frac{dx}{\cos^2 x} \\
& \quad t = \tan x \text{ とする} \\
& \quad dt = \frac{dx}{\cos^2 x} \\
& \int \frac{1}{\tan^4 x} (1 + \tan^2 x) \frac{dx}{\cos^2 x} = \int \frac{1}{t^4} (1 + t^2) dt \\
& \quad = -\frac{1}{3t^3} - \frac{1}{t} + C \\
& \quad = -\frac{1}{3 \tan^3 x} - \frac{1}{\tan x} + C
\end{aligned}$$

$$\begin{aligned}
& \int \tan^3 x \quad \text{三角関数は2次ごとに分割} \\
\int \tan^3 x \, dx &= \int \tan x \cdot \tan^2 x \, dx \quad \tan^2 x \text{は } \cos^2 x \\
&= \int \tan x \left(\frac{1}{\cos^2 x} - 1 \right) dx \\
&= \int \frac{\tan x}{\cos^2 x} dx - \int \tan x \, dx \\
&= \frac{1}{2} \tan^2 x + \log|\cos x| + C
\end{aligned}$$

$$\begin{aligned}
 \int \tan^2 x \, dx &= \int \tan x \sec x \cos^2 x \, dx \\
 \int \tan^2 x \, dx &= \int \left(\frac{1}{\cos^2 x} - 1 \right) dx \\
 &= \tan x - x + C
 \end{aligned}$$

$\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$ 積分区間の下端が0, 三角関数の式で積分区間が $\frac{n\pi}{2} \rightarrow King Property$

$$\begin{aligned}
 I &= \int_0^{\frac{\pi}{2}} \sin^2 x \, dx \\
 &= \int_0^{\frac{\pi}{2}} \cos^2 x \, dx \\
 2I &= \int_0^{\frac{\pi}{2}} dx \\
 &= \frac{\pi}{2} \\
 \therefore I &= \frac{\pi}{4}
 \end{aligned}$$

$$\begin{aligned}
& \int \frac{\log(\sin x)}{\tan x} dx \quad \tan \text{ の一次式} \rightarrow \frac{\sin}{\cos}, \text{ 真数, 位相が複雑} \rightarrow = t \text{ として微分接触を疑う} \\
& \int \frac{\log(\sin x)}{\tan x} dx = \int \frac{\cos x \log(\sin x)}{\sin x} dx \\
& \quad = \int \frac{\log t}{t} dt \\
& \quad = \frac{1}{2}(\log t)^2 + C \\
& \quad = \frac{1}{2}(\log(\sin x))^2 + C
\end{aligned}$$

$$\begin{aligned}
& \int \frac{dx}{\sqrt{1+x^2}} \\
& \quad x = \sinh t \\
& \quad dx = \cosh t \, dt \\
& \int \frac{dx}{\sqrt{1+x^2}} = \int \frac{1}{\sqrt{1+\sinh^2 t}} \cosh t \, dt \\
& \quad = \int dt \\
& \quad = t + C \\
& \quad = \log(x + \sqrt{1+x^2}) + C
\end{aligned}$$

$$\begin{aligned}
\int \tan^5 x \, dx &= \int \tan^3 x \left(\frac{1}{\cos^2 x} - 1 \right) dx \\
&= \frac{1}{4} \tan^4 x - \frac{1}{2} \tan^2 x + \log|\cos x| + C
\end{aligned}$$

$$\begin{aligned}
& \int \frac{dx}{\cos^4 x} \quad \text{三角関数は2乗ごとに分割} \\
& \int \frac{dx}{\cos^4 x} = \int \frac{1}{\cos^2 x} \cdot \frac{1}{\cos^2 x} dx \quad \cos^2 x \text{は} \tan^2 x \\
& \quad = \int (1 + \tan^2 x) \frac{dx}{\cos^2 x} \\
& \quad = \frac{1}{3} \tan^3 x + \tan x + C
\end{aligned}$$

$$\int_0^1 \frac{dx}{x^2+3} \quad x^2+a^2 \text{を含む} \rightarrow x = a \tan t$$

$$x = \sqrt{3} \tan t \text{ とおく}$$

$$dx = \frac{\sqrt{3}dt}{\cos^2 t}$$

$$\int_0^1 \frac{dx}{x^2+3} = \int_0^{\frac{\pi}{6}} \frac{\frac{\sqrt{3}}{\cos^2 t}}{3 \tan^2 t + 3} dt$$

$$= \int_0^{\frac{\pi}{6}} \frac{\frac{\sqrt{3}}{\cos^2 t}}{3 \left(\frac{1}{\cos^2 t} \right)} dt$$

$$= \frac{1}{\sqrt{3}} \int_0^{\frac{\pi}{6}} dt$$

$$= \frac{\pi}{6\sqrt{3}}$$

x	$0 \rightarrow 1$
t	$0 \rightarrow \frac{\pi}{6}$

$$\int x^2 \sin x \, dx \quad x^n f(x) \rightarrow (\text{瞬間}) \text{部分積分}$$

$$\int x^2 \sin x \, dx = x^2 \sin x + 2x \cos x - 2 \sin x + C$$

$$\begin{array}{l} + \quad x^2 \quad \sin x \\ - \quad 2x \quad - \cos x \\ + \quad 2 \quad - \sin x \end{array}$$

→分母分子に同じもの($\cos x$)をかける(分母を優先)

$$\int \frac{dx}{\cos^3 x}$$

三角関数は2乗に強い, 三角関数や指数関数の分数関数, 分母を優先

$$\int \frac{dx}{\cos^3 x} = \int \frac{\cos x}{\cos^4 x} dx$$

$$= \int \frac{\cos x}{(1 - \sin^2 x)^2} dx$$

$$\frac{1}{1-t^2} = \frac{1}{2} \left(\frac{1}{1+t} + \frac{1}{1-t} \right)$$

$$\frac{1}{(1-t^2)^2} = \left(\frac{1}{2} \left(\frac{1}{1+t} + \frac{1}{1-t} \right) \right)^2$$

$$= \frac{1}{4} \left(\frac{1}{(1+t)^2} + \frac{1}{(1-t)^2} + \frac{2}{1-t^2} \right)$$

$$= \frac{1}{4} \left(\frac{1}{(1+t)^2} + \frac{1}{(1-t)^2} + \frac{1}{1+t} + \frac{1}{1-t} \right)$$

$$\int \frac{dt}{(1-t^2)^2}$$

$$= \frac{1}{4} \int \left(\frac{1}{(1+t)^2} + \frac{1}{(1-t)^2} + \frac{1}{1+t} + \frac{1}{1-t} \right) dt$$

$$= \frac{1}{4} \left(\frac{1}{1-t} - \frac{1}{1+t} + \log \left| \frac{t+1}{t-1} \right| \right) + C$$

$$= \frac{1}{4} \left(\frac{1}{1-\sin x} - \frac{1}{1+\sin x} + \log \left| \frac{\sin x + 1}{\sin x - 1} \right| \right) + C$$

$$= \frac{1}{4} \left(\frac{1}{1+t} - \frac{1}{1-t} + \log \left| \frac{1+t}{1-t} \right| \right) + C$$

$$= \frac{1}{4} \left(\frac{1}{1+\sin x} - \frac{1}{1-\sin x} + \log \left| \frac{1+\sin x}{1-\sin x} \right| \right) + C$$

$$\int \frac{dx}{\sin^4 x} \quad f(\sin^2 x, \cos^2 x, \tan x) \rightarrow t = \tan x$$

$$t = \tan x$$

$$\cos^2 x = \frac{1}{1 + \tan^2 x}$$

$$= \frac{1}{1 + t^2}$$

$$\sin^2 x = \cos^2 x \tan^2 x$$

$$= \frac{t^2}{1 + t^2}$$

$$dt = \frac{dx}{\cos^2 x}$$

$$dx = \frac{dt}{1 + t^2}$$

$$\int \frac{dx}{\sin^4 x} = \int \frac{(1 + t^2)^2}{t^4} \cdot \frac{dt}{1 + t^2}$$

$$= \int \frac{1 + t^2}{t^4} dt$$

$$= \int \frac{dt}{t^4} + \int \frac{dt}{t^2}$$

$$= -\frac{1}{3t^3} - \frac{1}{t} + C$$

$$= -\frac{1}{3 \tan^3 x} - \frac{1}{\tan x} + C$$

$$\int_0^{2\pi} \sqrt{1 + \cos x} \, dx \quad \sqrt{(\quad)^2} = |\quad| \text{ として.}$$

ここで, $1 + \cos x = 2 \cos^2 \frac{x}{2}$ である.

$$\begin{aligned} \int_0^{\frac{\pi}{3}} \sqrt{1 + \cos x} \, dx &= \int_0^{\frac{\pi}{3}} \sqrt{2 \cos^2 \frac{x}{2}} \, dx \\ &= \sqrt{2} \int_0^{\frac{\pi}{3}} \left| \cos \frac{x}{2} \right| \, dx \\ &= 2\sqrt{2} \left[\sin \frac{x}{2} \right]_0^{\frac{\pi}{3}} \\ &= \sqrt{2} \end{aligned}$$

$$\begin{aligned}
\int_0^{\frac{\pi}{3}} \sqrt{1 + \sin x} \, dx & \quad \sin \text{ が } \cos \text{ だと都合がいい} \rightarrow \frac{\pi}{2} \text{ で } \cos \text{ に} \\
& \int_0^{\frac{\pi}{3}} \sqrt{1 + \sin x} \, dx \quad \sin \text{ が } \cos \text{ だと都合がいい} \rightarrow \frac{\pi}{2} \\
& \int_0^{\frac{\pi}{3}} \sqrt{1 + \sin x} \, dx = \int_0^{\frac{\pi}{3}} \sqrt{1 + \cos\left(\frac{\pi}{2} - x\right)} \, dx \\
& = \sqrt{2} \int_0^{\frac{\pi}{3}} \left| \cos\left(\frac{\pi}{4} - \frac{x}{2}\right) \right| \, dx \\
& = -2\sqrt{2} \left[\sin\left(\frac{\pi}{4} - \frac{x}{2}\right) \right]_0^{\frac{\pi}{3}} \\
& = -2\sqrt{2} \left(\frac{\sqrt{6} - \sqrt{2}}{4} - \frac{1}{\sqrt{2}} \right) \\
& = 3 - \sqrt{3}
\end{aligned}$$

$$\begin{aligned}
\int \frac{\cos^2 x}{\sin x - 1} \, dx &= \int \frac{(1 - \sin x)(1 + \sin x)}{\sin x - 1} \, dx \\
&= - \int (1 + \sin x) \, dx \\
&= \cos x - x + C
\end{aligned}$$

$$\begin{aligned}
\int \cos x \sin x \cos 2x \, dx &= \frac{1}{2} \int \sin 2x \cos 2x \\
&= \frac{1}{4} \int \sin 4x \, dx \\
&= -\frac{1}{16} \cos 4x + C
\end{aligned}$$

$$\begin{aligned}
\int_0^{\frac{\pi}{2}} \sin 6x \cos 4x \, dx &= \frac{1}{2} \int_0^{\frac{\pi}{2}} (\sin 10x + \sin 2x) \, dx \\
&= \frac{1}{2} \left[-\frac{1}{10} \cos 10x - \frac{1}{2} \cos 2x \right]_0^{\frac{\pi}{2}} \\
&= \frac{3}{5}
\end{aligned}$$

$$\int \frac{\sin x \cos x}{1 + \sin^2 x} dx = \frac{1}{2} \log(1 + \sin^2 x) + C \quad (\because 1 + \sin^2 x > 0)$$

分数関数なので, 取り敢えず分母の微分を考えておく.

$$(\cos x + \sin x)' = \cos x - \sin x$$

$$\begin{aligned} \int_0^{\frac{\pi}{4}} \frac{\cos x}{\cos x + \sin x} dx &= \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x + \sin x} dx \\ &= \frac{\pi}{4} \end{aligned}$$

$$\int_0^{\frac{\pi}{4}} \frac{\cos x}{\cos x + \sin x} dx = I, \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x + \sin x} dx = J \quad \text{とすると}$$

$$I + J = \frac{\pi}{4}$$

I と J を求めるために, I と J を組み合わせる他に積分しやすい形をつくりたい.

ここで, $(\cos x + \sin x)' = \cos x - \sin x$ であるから, $I - J$ が簡単に求められる.

$$\begin{aligned} I - J &= \int_0^{\frac{\pi}{4}} \frac{\cos x}{\cos x + \sin x} dx - \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x + \sin x} dx \\ &= [\log |\cos x + \sin x|]_0^{\frac{\pi}{4}} \\ &= \left[\log \left| \sqrt{2} \sin \left(x + \frac{\pi}{4} \right) \right| \right]_0^{\frac{\pi}{4}} \\ &= \frac{1}{2} \log 2 \end{aligned}$$

$$\therefore I = \frac{\pi}{8} + \frac{1}{4} \log 2$$

$$\begin{aligned} \int \frac{dx}{\sqrt{x} + \sqrt{x+2}} &= \int \frac{\sqrt{x+2} - \sqrt{x}}{2} dx \\ &= \frac{1}{3} (x+2)^{\frac{3}{2}} - \frac{1}{3} x^{\frac{3}{2}} + C \end{aligned}$$

$$\begin{aligned}
\int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\cos x}{1+\cos x}} dx &= \int_0^{\frac{\pi}{2}} \sqrt{\tan^2 \frac{x}{2}} dx \\
&= \int_0^{\frac{\pi}{2}} \left| \tan \frac{x}{2} \right| dx \\
&= -2 \left[\log \left| \cos \frac{x}{2} \right| \right]_0^{\frac{\pi}{2}} \\
&= -2 \log \frac{1}{\sqrt{2}} \\
&= \log 2
\end{aligned}$$

$$\int_0^1 \frac{dx}{1+x^3} \quad \text{分母が因数分解できる} \rightarrow BBB$$

$$\int_0^1 \frac{dx}{1+x^3} = \frac{1}{3} \int_0^1 \left(\frac{1}{1+x} - \frac{x-2}{x^2-x+1} \right) dx$$

$$\begin{aligned} \frac{1}{3} \int_0^1 \frac{dx}{1+x} &= \frac{1}{3} [\log |1+x|]_0^1 \\ &= \frac{1}{3} \log 2 \end{aligned}$$

$$\frac{1}{3} \int_0^1 \frac{x-2}{x^2-x+1} dx \quad \text{分母が2次式で分子が1次式} \rightarrow \text{微分接触をつくる}$$

$$\begin{aligned} \frac{x-2}{x^2-x+1} &= \frac{1}{2} \left(\frac{2x-4}{x^2-x+1} \right) \\ &= \frac{1}{2} \left(\frac{2x-1}{x^2-x+1} - \frac{3}{x^2-x+1} \right) \end{aligned}$$

※ x の係数を定数倍で合わせないと修正項に x が残ってしまう

$$\frac{1}{3} \int_0^1 \frac{x-2}{x^2-x+1} dx = \frac{1}{6} \int_0^1 \frac{2x-1}{x^2-x+1} - \frac{1}{6} \int_0^1 \frac{3}{x^2-x+1} dx$$

$$\begin{aligned} \frac{1}{6} \int_0^1 \frac{2x-1}{x^2-x+1} dx &= \frac{1}{6} [\log |x^2-x+1|]_0^1 \\ &= 0 \end{aligned}$$

$$\frac{1}{2} \int_0^1 \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$x - \frac{1}{2} = \frac{\sqrt{3}}{2} \tan t$$

$$dx = \frac{\sqrt{3}}{2 \cos^2 t} dt$$

$$\frac{1}{2} \int_0^1 \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}} = \frac{1}{2} \cdot \frac{4}{3} \cdot \frac{\sqrt{3}}{2} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} dt$$

$$= \frac{\pi}{3\sqrt{3}}$$

$$\therefore \int_0^1 \frac{dx}{1+x^3} = \frac{1}{3} \log 2 + \frac{\pi}{3\sqrt{3}}$$

$$\begin{aligned}
I &= \int e^x \cos x \, dx \\
&= e^x \sin x - \int e^x \sin x \, dx \\
&= e^x \sin x - \left(e^x \cdot -\cos x - \int e^x \cdot -\cos x \, dx \right) \\
&= e^x \sin x + e^x \cos x - \int e^x \cos x \, dx \\
2I &= e^x \sin x + e^x \cos x \\
I &= \frac{e^x}{2} (\sin x + \cos x) + C
\end{aligned}$$