$$\tan x \, dx$$

$$\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3 - x^2} \, dx$$

$$\int \log x \, dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\log(\sin x)}{\tan x} \, dx$$

$$\int e^x \cos x \, dx$$

$$\int \frac{dx}{\sin^3 x}$$

$$\int \tan^4 x \, dx$$

$$\int \frac{dx}{\sin^2 x}$$

$$\int \sqrt{1 - x} \, dx$$

$$\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$

$$\int x^2 \sin x \, dx$$

$$\int_0^{2\pi} \sqrt{1 + \cos x} \, dx$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\frac{1 - \cos x}{1 + \cos x}} \, dx$$

$$\int_0^{\frac{\pi}{3}} \sqrt{1 + \sin x} \, dx$$

$$\int_0^1 \frac{dx}{1 + x^2}$$

$$\int_0^1 \frac{dx}{3+x^2}$$
$$\int_0^1 \frac{dx}{x^2+x+1}$$
$$\int x^x\,(1+\log x)\;dx$$
$$\int \frac{dx}{\sqrt{1+x^2}}$$
$$\int_2^3 \frac{dx}{\sqrt{x^2-1}}$$
$$\int \sqrt{1-e^{-2x}}\,dx$$
$$\int_0^1 \frac{dx}{1+x^3}$$
$$\int \frac{dx}{\sin^4 x}$$
$$\int \frac{dx}{\sqrt{x}+\sqrt{x+2}}$$
$$\int_0^e \frac{e^x}{e^{e-x}+e^x}\,dx$$
$$\int_{-1}^1 \sqrt{1-x^2}\,dx$$
$$\int \sqrt{1-e^{-2x}}\,dx$$
$$\int \tan^2 x\,dx$$
$$\int \tan^3 x\,dx$$
$$\int \tan^5 x\,dx$$

King Property

$\int_a^b f(x)\ dx = \int_a^b f(a+b-x)\ dx$

$t = a+b-x$

$dx = -dt$

$$\int_b^a -f(t)\ dt = \int_a^b f(t)\ dt$$

$$= \int_a^b f(x)\ dx\ \ ()$$

$$\int_a^b f(x)\ dx = \int_a^b f(a+b-x)\ dx$$

$xa \to b$

$tb \to a$

$\int \tan x \, dx \qquad \tan \frac{\sin x}{\cos x}$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$$

$= -\log \cos x + C$

$\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3-x^2}\,dx \quad \sqrt{a^2-x^2} x = a\sin t$

$x = \sqrt{3}\sin t$

$dx = \sqrt{3}\cos t\,dt$

$$\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3-x^2}\,dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{3-\left(\sqrt{3}\sin t\right)^2}\sqrt{3}\cos t\,dt$$

$$= 3\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1-\sin^2 t}\cos t\,dt$$

$$= 3\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t\ \cos t\,dt$$

$$= 3\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t\,dt\ \left(-\frac{\pi}{2} \le t \le \frac{\pi}{2} \Rightarrow 0 \le \forall \cos t\right)$$

$$= \frac{3}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1+\cos 2t\,dt\ \left(\cos^2 x = \frac{1+\cos 2x}{2} \quad c.f. \sin^2 x = \frac{1-\cos 2x}{2}\right)$$

$$= \frac{3}{2}\left[t+\frac{1}{2}\sin 2t\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= 3\left[t+\frac{1}{2}\sin 2t\right]_{0}^{\frac{\pi}{2}}$$

$$= \frac{3}{2}\pi$$

$x -\sqrt{3} \to \sqrt{3}$

$t \quad -\frac{\pi}{2} \to \frac{\pi}{2}$

$$\int\frac{dx}{\sin^2 x}\,dx \quad \sin \; \cos \frac{\pi}{2} \cos$$

$$\int \frac{dx}{\sin^2 x} = \int \frac{dx}{\cos^2\left(\frac{\pi}{2} - x\right)}$$

$$= -\tan\left(\frac{\pi}{2} - x\right) + C$$

$$= -\frac{1}{\tan x} + C$$

$\int_{-1}^{1} \sqrt{1-x^2}\,dx \quad \sqrt{(\quad)^2} = |\quad|.$

$, 1-\sin^2 t = \cos^2 t.$

$, 1-x^2 \geq 0 -1 \leq x \leq 1, x = \sin t.$

$x = \sin t$

$dx = \cos t\,dt$

$$\int_{-1}^{1} \sqrt{1-x^2}\,dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1-\sin^2 t}\cos t\,dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t\,\cos t\,dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t\,dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2t}{2}\,dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{\cos 2t}{2}\right)\,dt$$

$$= \frac{1}{2}\left[t + \frac{1}{2}\sin 2t\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

$x \; -1 \to 1$

$t -\frac{\pi}{2} \to \frac{\pi}{2}$

$\int_0^1 \frac{dx}{1+x^2}$.

, $1+\tan^2 t = \frac{1}{\cos^2 t}$., $(\tan t)' = \frac{1}{\cos^2 t}$, $x = \tan t$

$x = \tan t$

$1 + x^2 = \frac{1}{\cos^2 t}$

$dx = \frac{dt}{\cos^2 t}$

$\int_0^1 \frac{dx}{1+x^2} = \int_0^{\frac{\pi}{4}} \frac{\frac{1}{\cos^2 t}}{\frac{1}{\cos^2 t}}\, dt$

$= \int_0^{\frac{\pi}{4}} dt$

$= \frac{\pi}{4}$

$x\, 0 \to 1$

$t\, 0 \to \frac{\pi}{4}$

$$\int \tan^4 x\,dx \quad 2$$

$$= \int \tan^2 x \tan^2 x\,dx \quad \tan^2 \ \cos^2$$

$$= \int \tan^2 x \left(\frac{1}{\cos^2 x} - 1 \right) dx$$

$$= \int \frac{\tan^2 x}{\cos^2 x}\,dx - \int \tan^2 x\,dx$$

$$= \frac{1}{3}\tan^3 x - \tan x + x + C$$

$\int \log x \, dx$ $\log$

$$
\begin{aligned}
&= \int 1 \cdot \log x \, dx \\
&= x \log x - \int x \cdot \frac{1}{x} \, dx \\
&= x \log x - x + C
\end{aligned}
$$

$$\int_0^e \frac{e^x}{e^{e-x}+e^x}\,dx \quad a+b-x, 0, King\ Property$$

$$\int_0^e \frac{e^x}{e^{e-x}+e^x}\,dx \quad a+b-x, 0, KingProperty$$

$$\int_0^e \frac{e^x}{e^{e-x}+e^x}\,dx = \int_0^e \frac{e^{e-x}}{e^x+e^{e-x}}\,dx$$

$$I = \int_0^e \frac{e^x}{e^{e-x}+e^x}\,dx$$

$$2I = \int_0^e \frac{e^x}{e^{e-x}+e^x}\,dx + \int_0^e \frac{e^{e-x}}{e^x+e^{e-x}}\,dx$$

$$= \int_0^e \frac{e^x+e^{e-x}}{e^x+e^{e-x}}\,dx$$

$$= \int_0^e dx$$

$$= e$$

$$I = \frac{e}{2}$$

$$\int_0^e \frac{e^x}{e^{e-x}+e^x}\,dx$$

$$= \int_0^e \frac{e^x}{e^e\cdot e^{-x}+e^x}\,dx$$

$, e^x$

$$= \frac{1}{2}\int_0^e \frac{2e^{2x}}{e^e+e^{2x}}$$

$$= \frac{1}{2}\left[\log(e^e+e^{2x})\right]_0^e \quad (e^e+e^{2x}\geq 0)$$

$$= \frac{1}{2}\log\frac{e^e+e^{2e}}{e^e+1}$$

$$= \frac{1}{2}\log\frac{e^e(1+e^e)}{e^e+1}$$

$$= \frac{e}{2}$$

$$\int \frac{dx}{\sin^4 x} \quad f(\sin^2 x, \cos^2 x, \tan x) g(\tan x) \cdot \frac{1}{\cos^2 x}$$

$$\int \frac{dx}{\sin^4 x} = \int \frac{dx}{\tan^4 x \cos^4 x}$$

$$= \int \frac{1}{\tan^4 x} \left(1 + \tan^2 x\right) \frac{dx}{\cos^2 x}$$

$$t = \tan x$$

$$dt = \frac{dx}{\cos^2 x}$$

$$\int \frac{1}{\tan^4 x} \left(1 + \tan^2 x\right) \frac{dx}{\cos^2 x} = \int \frac{1}{t^4} \left(1 + t^2\right) \, dt$$

$$= -\frac{1}{3t^3} - \frac{1}{t} + C$$

$$= -\frac{1}{3 \tan^3 x} - \frac{1}{\tan x} + C$$

$\int \tan^3 x \quad 2\,dx$

$$\int \tan^3 x\,dx = \int \tan x \cdot \tan^2 x\,dx \quad \tan^2 x \cos^2 x$$

$$= \int \tan x \left(\frac{1}{\cos^2 x} - 1 \right) dx$$

$$= \int \frac{\tan x}{\cos^2 x}\,dx - \int \tan x\,dx$$

$$= \frac{1}{2} \tan^2 x + \log \cos x \; + C$$

$\int \tan^2 x\,dx \quad \tan^2 x \cos^2 x$

$$\int \tan^2 x\,dx = \int \left(\frac{1}{\cos^2 x} - 1\right) dx$$
$$= \tan x - x + C$$

$$\int_0^{\frac{\pi}{2}} \cos^2 x\, dx \quad 0, \frac{n\pi}{2} King\ Property$$

$$I = \int_0^{\frac{\pi}{2}} \sin^2 x\, dx$$

$$= \int_0^{\frac{\pi}{2}} \cos^2 x\, dx$$

$$2I = \int_0^{\frac{\pi}{2}} \, dx$$

$$= \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

$$\int \frac{\log(\sin x)}{\tan x}\,dx \quad \tan\frac{\sin}{\cos},\,, = t$$

$$\int \frac{\log(\sin x)}{\tan x}\,dx = \int \frac{\cos x \log(\sin x)}{\sin x}\,dx$$

$$= \int \frac{\log t}{t}\,dt$$

$$= \frac{1}{2}(\log t)^2 + C$$

$$= \frac{1}{2}(\log(\sin x))^2 + C$$

$$\int \frac{dx}{\sqrt{1+x^2}}$$

$x = \sinh t$

$dx = \cosh t \, dt$

$$\int \frac{dx}{\sqrt{1+x^2}} = \int \frac{1}{\sqrt{1+\sinh^2 t}} \cosh t \, dt$$

$$= \int dt$$

$$= t + C$$

$$= \log\left(x + \sqrt{1+x^2}\right) + C$$

$$\int \tan^5 x \, dx = \int \tan^3 x \left(\frac{1}{\cos^2 x} - 1 \right) dx$$

$$= \frac{1}{4} \tan^4 x - \frac{1}{2} \tan^2 x + \log \cos x + C$$

$\int\frac{dx}{\cos^4 x}$ 2

$$\int \frac{dx}{\cos^4 x} = \int \frac{1}{\cos^2 x} \cdot \frac{1}{\cos^2 x}\, dx \quad \cos^2 x \tan^2 x$$

$$= \int \left(1 + \tan^2 x\right) \frac{dx}{\cos^2 x}$$

$$= \frac{1}{3} \tan^3 x + \tan x + C$$

$$\int_0^1 \frac{dx}{x^2+3} \quad x^2 + a^2 x = a\tan t$$

$$x = \sqrt{3}\tan t$$

$$dx = \frac{\sqrt{3}dt}{\cos^2 t}$$

$$\int_0^1 \frac{dx}{x^2+3} = \int_0^{\frac{\pi}{6}} \frac{\frac{\sqrt{3}}{\cos^2 t}}{3\tan^2 t + 3}\,dt$$

$$= \int_0^{\frac{\pi}{6}} \frac{\frac{\sqrt{3}}{\cos^2 t}}{3\left(\frac{1}{\cos^2 t}\right)}\,dt$$

$$= \frac{1}{\sqrt{3}}\int_0^{\frac{\pi}{6}} dt$$

$$= \frac{\pi}{6\sqrt{3}}$$

$$x\,0 \to 1$$

$$t\,0 \to \frac{\pi}{6}$$

$\int x^2 \sin x \, dx \quad x^n f(x)()$

$$\int x^2 \sin x \, dx = x^2 \sin x + 2x \cos x - 2 \sin x + C$$

$+x^2 \quad \sin x$
$-2x - \cos x$
$+\ 2 \ - \sin x$

$$\overrightarrow{\quad} \qquad (\cos x)()$$

$$\int \frac{dx}{\cos^3 x} 2,,$$

$$\int \frac{dx}{\cos^3 x} = \int \frac{\cos x}{\cos^4 x}\,dx$$

$$= \int \frac{\cos x}{\left(1-\sin^2 x\right)^2}\,dx$$

$$\frac{1}{1-t^2} = \frac{1}{2}\left(\frac{1}{1+t} + \frac{1}{1-t}\right)$$

$$\frac{1}{\left(1-t^2\right)^2} = \left(\frac{1}{2}\left(\frac{1}{1+t} + \frac{1}{1-t}\right)\right)^2$$

$$= \frac{1}{4}\left(\frac{1}{(1+t)^2} + \frac{1}{(1-t)^2} + \frac{2}{1-t^2}\right)$$

$$= \frac{1}{4}\left(\frac{1}{(1+t)^2} + \frac{1}{(1-t)^2} + \frac{1}{1+t} + \frac{1}{1-t}\right)$$

$$\int \frac{dt}{\left(1-t^2\right)^2}$$

$$= \frac{1}{4}\int \left(\frac{1}{(1+t)^2} + \frac{1}{(1-t)^2} + \frac{1}{1+t} + \frac{1}{1-t}\right) dt$$

$$= \frac{1}{4}\left(\frac{1}{1-t} - \frac{1}{1+t} + \log\frac{t+1}{t-1}\right) + C$$

$$= \frac{1}{4}\left(\frac{1}{1-\sin x} - \frac{1}{1+\sin x} + \log\frac{\sin x+1}{\sin x-1}\right) + C$$

$$= \frac{1}{4}\left(\frac{1}{1+t} - \frac{1}{1-t} + \log\frac{1+t}{1-t}\right) + C$$

$$= \frac{1}{4}\left(\frac{1}{1+\sin x} - \frac{1}{1-\sin x} + \log\frac{1+\sin x}{1-\sin x}\right. + C$$

$$\int\frac{dx}{\sin^4 x} \quad f(\sin^2 x, \cos^2 x, \tan x) t = \tan x$$

$t = \tan x$

$\cos^2 x = \dfrac{1}{1+\tan^2 x}$

$= \dfrac{1}{1+t^2}$

$\sin^2 x = \cos^2 x \tan^2 x$

$= \dfrac{t^2}{1+t^2}$

$dt = \dfrac{dx}{\cos^2 x}$

$dx = \dfrac{dt}{1+t^2}$

$$\int \frac{dx}{\sin^4 x} = \int \frac{\left(1+t^2\right)^2}{t^4} \cdot \frac{dt}{1+t^2}$$

$$= \int \frac{1+t^2}{t^4}\, dt$$

$$= \int \frac{dt}{t^4} + \int \frac{dt}{t^2}$$

$$= -\frac{1}{3t^3} - \frac{1}{t} + C$$

$$= -\frac{1}{3\tan^3 x} - \frac{1}{\tan x} + C$$

$\int_0^{2\pi} \sqrt{1+\cos x}\, dx \quad \sqrt{(\quad)^2} = |\quad|.$

$, 1+\cos x = 2\cos^2 \dfrac{x}{2}.$

$$\int_0^{\frac{\pi}{3}} \sqrt{1+\cos x}\, dx = \int_0^{\frac{\pi}{3}} \sqrt{2\cos^2 \frac{x}{2}}\, dx$$

$$= \sqrt{2} \int_0^{\frac{\pi}{3}} \cos \frac{x}{2} \; dx$$

$$= 2\sqrt{2} \left[\sin \frac{x}{2}\right]_0^{\frac{\pi}{3}}$$

$$= \sqrt{2}$$

$\int_0^{\frac{\pi}{3}} \sqrt{1+\sin x}\,dx \quad \sin\ \cos\frac{\pi}{2}\cos$

$$\int_0^{\frac{\pi}{3}} \sqrt{1+\sin x}\,dx \quad \cos\frac{\pi}{2}$$

$$\int_0^{\frac{\pi}{3}} \sqrt{1+\sin x}\,dx = \int_0^{\frac{\pi}{3}} \sqrt{1+\cos\left(\frac{\pi}{2}-x\right)}\,dx$$

$$= \sqrt{2}\int_0^{\frac{\pi}{3}} \cos\left(\frac{\pi}{4}-\frac{x}{2}\right)\ dx$$

$$= -2\sqrt{2}\left[\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]_0^{\frac{\pi}{3}}$$

$$= -2\sqrt{2}\left(\frac{\sqrt{6}-\sqrt{2}}{4}-\frac{1}{\sqrt{2}}\right)$$

$$= 3-\sqrt{3}$$

$$\int \frac{\cos^2 x}{\sin x - 1}\,dx = \int \frac{(1-\sin x)(1+\sin x)}{\sin x - 1}dx$$

$$= -\int (1+\sin x)dx$$

$$= \cos x - x + C$$

$$\int \cos x \sin x \cos 2x\,dx = \frac{1}{2}\int \sin 2x \cos 2x$$

$$= \frac{1}{4}\int \sin 4x\,dx$$

$$= -\frac{1}{16}\cos 4x + C \quad \int_0^{\frac{\pi}{2}} \sin 6x \cos 4x\,dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} (\sin 10x + \sin 2x)\,dx$$

$$= \frac{1}{2}\left[-\frac{1}{10}\cos 10x - \frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}}$$

$$= \frac{3}{5}$$

$$\int\frac{\sin x\cos x}{1+\sin^2 x}\,dx=\frac{1}{2}\log(1+\sin^2 x)+C\quad(1+\sin^2 x\geq 0)\int_0^{\frac{\pi}{4}}\frac{\cos x}{\cos x+\sin x}\,dx$$
, .
$(\cos x+\sin x)'=\cos x-\sin x$

$$\int_0^{\frac{\pi}{4}}\frac{\cos x}{\cos x+\sin x}\,dx\int_0^{\frac{\pi}{4}}\frac{\sin x}{\cos x+\sin x}\,dx=\frac{\pi}{4}$$
$$\int_0^{\frac{\pi}{4}}\frac{\cos x}{\cos x+\sin x}\,dx=I,\int_0^{\frac{\pi}{4}}\frac{\sin x}{\cos x+\sin x}\,dx=J$$
$$I+J=\frac{\pi}{4}$$
$IJ, IJ.$
$, (\cos x+\sin x)'=\cos x-\sin x, I-J.$
$$I-J=\int_0^{\frac{\pi}{4}}\frac{\cos x}{\cos x+\sin x}\,dx-\int_0^{\frac{\pi}{4}}\frac{\sin x}{\cos x+\sin x}\,dx$$
$$=[\log|\cos x+\sin x|]_0^{\frac{\pi}{4}}$$
$$=\left[\log|\sqrt{2}\sin(x+\frac{\pi}{4})|\right]_0^{\frac{\pi}{4}}$$
$$=\frac{1}{2}\log 2$$
$$I=\frac{\pi}{8}+\frac{1}{4}\log 2$$
$$\int\frac{dx}{\sqrt{x}+\sqrt{x+2}}=\int\frac{\sqrt{x+2}-\sqrt{x}}{2}\,dx$$
$$=\frac{1}{3}(x+2)^{\frac{3}{2}}-\frac{1}{3}x^{\frac{3}{2}}+C$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\cos x}{1+\cos x}}\, dx = \int_0^{\frac{\pi}{2}} \sqrt{\tan^2 \frac{x}{2}}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \tan \frac{x}{2} \ dx$$

$$= -2 \left[\log \cos \frac{x}{2}\right]_0^{\frac{\pi}{2}}$$

$$= -2 \log \frac{1}{\sqrt{2}}$$

$$= \log 2$$

$\int_0^1 \frac{dx}{1+x^3}$ $\quad BBB$

$$\int_0^1 \frac{dx}{1+x^3} = \frac{1}{3}\int_0^1 \left(\frac{1}{1+x} - \frac{x-2}{x^2-x+1}\right) dx$$

$$\frac{1}{3}\int_0^1 \frac{dx}{1+x} = \frac{1}{3}\left[\log|1+x|\right]_0^1$$

$$= \frac{1}{3}\log 2$$

$$\frac{1}{3}\int_0^1 \frac{x-2}{x^2-x+1}\,dx \quad 21$$

$$\frac{x-2}{x^2-x+1} = \frac{1}{2}\left(\frac{2x-4}{x^2-x+1}\right)$$

$$= \frac{1}{2}\left(\frac{2x-1}{x^2-x+1} - \frac{3}{x^2-x+1}\right)$$

xx

$$\frac{1}{3}\int_0^1 \frac{x-2}{x^2-x+1}\,dx = \frac{1}{6}\int_0^1 \frac{2x-1}{x^2-x+1} - \frac{1}{6}\int_0^1 \frac{3}{x^2-x+1}\,dx$$

$$\frac{1}{6}\int_0^1 \frac{2x-1}{x^2-x+1}\,dx = \frac{1}{6}\left[\log x^2-x+1\right]_0^1$$

$$= 0$$

$$\frac{1}{2}\int_0^1 \frac{dx}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$x - \frac{1}{2} = \frac{\sqrt{3}}{2}\tan t$$

$$dx = \frac{\sqrt{3}}{2\cos^2 t}\,dt$$

$$\frac{1}{2}\int_0^1 \frac{dx}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}} = \frac{1}{2}\cdot\frac{4}{3}\cdot\frac{\sqrt{3}}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} dt$$

$$= \frac{\pi}{3\sqrt{3}}$$

$$\int_0^1 \frac{dx}{1+x^3} = \frac{1}{3}\log 2 + \frac{\pi}{3\sqrt{3}}$$

$$
\begin{aligned}
\mathrm{I} &= \int e^x \cos x \, dx \\
&= e^x \sin x - \int e^x \sin x \, dx \\
&= e^x \sin x - \left(e^x \cdot - \cos x - \int e^x \cdot - \cos x \, dx \right) \\
&= e^x \sin x + e^x \cos x - \int e^x \cos x \, dx \\
2I &= e^x \sin x + e^x \cos x \\
I &= \frac{e^x}{2} (\sin x + \cos x) + C
\end{aligned}
$$