

$$\begin{aligned}
& \int \tan x \, dx \\
& \int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3-x^2} \, dx \\
& \int \log x \, dx \\
& \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\log(\sin x)}{\tan x} \, dx \\
& \int e^x \cos x \, dx \\
& \int \frac{dx}{\sin^3 x} \\
& \int \tan^4 x \, dx \\
& \int \frac{dx}{\sin^2 x} \\
& \int \sqrt{1-x} \, dx \\
& \int_0^{\frac{\pi}{2}} \cos^2 x \, dx \\
& \int x^2 \sin x \, dx \\
& \int_0^{2\pi} \sqrt{1+\cos x} \, dx \\
& \int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\cos x}{1+\cos x}} \, dx \\
& \int_0^{\frac{\pi}{3}} \sqrt{1+\sin x} \, dx \\
& \int_0^1 \frac{dx}{1+x^2}
\end{aligned}$$

$$\begin{aligned}
& \int_0^1 \frac{dx}{3+x^2} \\
& \int_0^1 \frac{dx}{x^2+x+1} \\
& \int x^x(1+\log x) \, dx \\
& \int \frac{dx}{\sqrt{1+x^2}} \\
& \int_2^3 \frac{dx}{\sqrt{x^2-1}} \\
& \int \sqrt{1-e^{-2x}} \, dx \\
& \int_0^1 \frac{dx}{1+x^3} \\
& \int \frac{dx}{\sin^4 x} \\
& \int \frac{dx}{\sqrt{x}+\sqrt{x+2}} \\
& \int_0^e \frac{e^x}{e^{e-x}+e^x} \, dx \\
& \int_{-1}^1 \sqrt{1-x^2} \, dx \\
& \int \sqrt{1-e^{-2x}} \, dx \\
& \int \tan^2 x \, dx \\
& \int \tan^3 x \, dx \\
& \int \tan^5 x \, dx
\end{aligned}$$

KingProperty

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$t = a + b - x$ と置換する

$$dx = -dt$$

$$\therefore \int_b^a -f(t) dt = \int_a^b f(t) dt$$
$$= \int_a^b f(x) dx \quad (\because \text{定積分では変数を変更してもよい})$$

因って、 $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

| | |
|-----|-------------------|
| x | $a \rightarrow b$ |
| t | $b \rightarrow a$ |

$$\begin{aligned}
\int \tan x \, dx &= \int \frac{\sin x}{\cos x} \, dx \left(\because \tan x = \frac{\sin x}{\cos x} \right) \\
&= - \int \frac{-\sin x}{\cos x} \, dx \\
&= -\log|\cos x| + C \left(\because \int \frac{f'(x)}{f(x)} \, dx = \log|f(x)| + C \right)
\end{aligned}$$

$$\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3-x^2} dx$$

$$x = \sqrt{3} \sin t \text{ とおく}$$

$$dx = \sqrt{3} \cos t dt$$

$$\begin{aligned} \int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3-x^2} dx &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{3 - (\sqrt{3} \sin t)^2} \sqrt{3} \cos t dt \\ &= 3 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 - \sin^2 t} \cos t dt \\ &= 3 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\cos t| \cos t dt \\ &= 3 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t dt \left(\because -\frac{\pi}{2} \leq t \leq \frac{\pi}{2} \Rightarrow 0 \leq \forall \cos t \right) \\ &= \frac{3}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 + \cos 2t dt \left(\because \cos^2 x = \frac{1 + \cos 2x}{2} \quad c.f. \sin^2 x = \frac{1 - \cos 2x}{2} \right) \\ &= \frac{3}{2} \left[t + \frac{1}{2} \sin 2t \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\ &= 3 \left[t + \frac{1}{2} \sin 2t \right]_0^{\frac{\pi}{2}} \\ &= \frac{3}{2} \pi \end{aligned}$$

| | |
|-----|--|
| x | $-\sqrt{3} \rightarrow \sqrt{3}$ |
| t | $-\frac{\pi}{2} \rightarrow \frac{\pi}{2}$ |

$$\begin{aligned}
\int \frac{dx}{\sin^2 x} &= \int \frac{dx}{\cos^2\left(\frac{\pi}{2} - x\right)} \left(\because \sin x = \cos\left(\frac{\pi}{2} - x\right)\right) \\
&= -\tan\left(\frac{\pi}{2} - x\right) + C \\
&= -\frac{1}{\tan x} + C \left(\because \tan x = \tan\left(\frac{\pi}{2} - x\right)\right)
\end{aligned}$$

$$\int_{-1}^1 \sqrt{1-x^2} dx$$

$$x = \sin t \text{ とおく}$$

$$dx = \cos t dt$$

$$\int_{-1}^1 \sqrt{1-x^2} dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1-\sin^2 t} \cos t dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\cos t| \cos t dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t dt \left(\because -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \Rightarrow 0 \leq \forall \cos x \right)$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2t}{2} dt \left(\because \cos^2 x = \frac{1+\cos 2x}{2} \quad c.f. \sin^2 x = \frac{1-\cos 2x}{2} \right)$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{\cos 2t}{2} \right) dt$$

$$= \frac{1}{2} \left[t + \frac{1}{2} \sin 2t \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

| | |
|-----|--|
| x | $-1 \rightarrow 1$ |
| t | $-\frac{\pi}{2} \rightarrow \frac{\pi}{2}$ |

$$\int_0^1 \frac{dx}{1+x^2}$$

$$x = \tan t \text{ とおく}$$

$$\frac{1}{1+x^2} = \frac{1}{\cos^2 t}$$

$$dx = \frac{dt}{\cos^2 t}$$

$$\int_0^1 \frac{dx}{1+x^2} = \int_0^{\frac{\pi}{4}} \frac{\frac{1}{\cos^2 t}}{\frac{1}{\cos^2 t}} dt$$

$$= \int_0^{\frac{\pi}{4}} dt$$

$$= \frac{\pi}{4}$$

| | |
|-----|-------------------------------|
| x | $0 \rightarrow 1$ |
| t | $0 \rightarrow \frac{\pi}{4}$ |

$$\begin{aligned}
\int \tan^4 x \, dx &= \int \tan^2 x \tan^2 x \, dx \\
&= \int \tan^2 x \left(\frac{1}{\cos^2 x} - 1 \right) dx \\
&= \int \frac{\tan^2 x}{\cos^2 x} dx - \int \tan^2 x \, dx \\
&= \frac{1}{3} \tan^3 x - \tan x + x + C
\end{aligned}$$

$$\begin{aligned}
\int \log x \, dx &= \int 1 \cdot \log x \, dx \\
&= x \log x - \int x \cdot \frac{1}{x} \, dx \\
&= x \log x - x + C
\end{aligned}$$

$$\begin{aligned}
\int_0^e \frac{e^x}{e^{e-x} + e^x} dx &= \int_0^e \frac{e^{e-x}}{e^x + e^{e-x}} dx \quad (\because KingProperty) \\
I &= \int_0^e \frac{e^x}{e^{e-x} + e^x} dx \quad \text{とすると} \\
2I &= \int_0^e \frac{e^x}{e^{e-x} + e^x} dx + \int_0^e \frac{e^{e-x}}{e^x + e^{e-x}} dx \\
&= \int_0^e \frac{e^x + e^{e-x}}{e^x + e^{e-x}} dx \\
&= \int_0^e dx \\
&= e \\
\therefore I &= \frac{e}{2}
\end{aligned}$$

$$\begin{aligned}
\int \frac{dx}{\sin^4 x} &= \int \frac{dx}{\tan^4 x \cos^4 x} \\
&= \int \frac{1}{\tan^4 x} (1 + \tan^2 x) \frac{dx}{\cos^2 x} \\
&\quad t = \tan x \text{ とおく} \\
&\quad dt = \frac{dx}{\cos^2 x} \\
\int \frac{1}{\tan^4 x} (1 + \tan^2 x) \frac{dx}{\cos^2 x} &= \int \frac{1}{t^4} (1 + t^2) dt \\
&= -\frac{1}{3t^3} - \frac{1}{t} + C \\
&= -\frac{1}{3 \tan^3 x} - \frac{1}{\tan x} + C
\end{aligned}$$

$$\begin{aligned}
\int \tan^3 x \, dx &= \int \tan x \cdot \tan^2 x \, dx \\
&= \int \tan x \left(\frac{1}{\cos^2 x} - 1 \right) dx \\
&= \int \frac{\tan x}{\cos^2 x} dx - \int \tan x \, dx \\
&= \frac{1}{2} \tan^2 x + \log|\cos x| + C
\end{aligned}$$

$$\begin{aligned}\int \tan^2 x \, dx &= \int \left(\frac{1}{\cos^2 x} - 1 \right) dx \\ &= \tan x - x + C\end{aligned}$$

$$\begin{aligned}
\int_0^{\frac{\pi}{2}} \cos^2 x \, dx &= \int_0^{\frac{\pi}{2}} \sin^2 x \, dx \\
I &= \int_0^{\frac{\pi}{2}} \cos^2 x \, dx \\
2I &= \int_0^{\frac{\pi}{2}} \cos^2 x + \sin^2 x \, dx \\
&= \int_0^{\frac{\pi}{2}} dx \\
&= \frac{\pi}{2} \\
\therefore I &= \frac{\pi}{4}
\end{aligned}$$

$$\begin{aligned}
\int \frac{\log(\sin x)}{\tan x} dx &= \int \frac{\cos x \log(\sin x)}{\sin x} dx \\
&= \int \frac{\log t}{t} dt \\
&= \frac{1}{2}(\log t)^2 + C \\
&= \frac{1}{2}(\log(\sin x))^2 + C
\end{aligned}$$

$$\begin{aligned}
& \int \frac{dx}{\sqrt{1+x^2}} \\
& \quad x = \sinh t \\
& \quad dx = \cosh t \, dt \\
& \int \frac{dx}{\sqrt{1+x^2}} = \int \frac{1}{\sqrt{1+\sinh^2 t}} \cosh t \, dt \\
& \quad = \int dt \\
& \quad = t + C \\
& \quad = \log\left(x + \sqrt{1+x^2}\right) + C
\end{aligned}$$

$$\begin{aligned}
\int \tan^5 x \, dx &= \int \tan^3 x \left(\frac{1}{\cos^2 x} - 1 \right) dx \\
&= \frac{1}{4} \tan^4 x - \frac{1}{2} \tan^2 x + \log|\cos x| + C
\end{aligned}$$

$$\begin{aligned}
\int \frac{dx}{\cos^4} &= \int \frac{1}{\cos^2 x} \cdot \frac{1}{\cos^2 x} dx \\
&= \int (1 + \tan^2 x) \frac{dx}{\cos^2 x} \\
&= \frac{1}{3} \tan^3 x + \tan x + C
\end{aligned}$$

$$\int_0^1 \frac{dx}{x^2 + 3}$$

$$x = \sqrt{3} \tan t \text{ とおく}$$

$$dx = \frac{dt}{\cos^2 t}$$

$$\int_0^1 \frac{dx}{x^2 + 3} = \int_0^{\frac{\pi}{6}} \frac{\frac{\sqrt{3}}{\cos^2 t}}{3 \tan^2 t + 3} dt$$

$$= \int_0^{\frac{\pi}{6}} \frac{\frac{\sqrt{3}}{\cos^2 t}}{3 \left(\frac{1}{\cos^2 t} \right)} dt$$

$$= \frac{1}{\sqrt{3}} \int_0^{\frac{\pi}{6}} dt$$

$$= \frac{\pi}{6\sqrt{3}}$$

| | |
|-----|-------------------------------|
| x | $0 \rightarrow 1$ |
| t | $0 \rightarrow \frac{\pi}{6}$ |

$$\int x^2 \sin x \, dx$$

$$\int x^2 \sin x \, dx = x^2 \sin x + 2x \cos x - 2 \sin x + C$$

$$+ \quad x^2 \quad \sin x$$

$$- \quad 2x \quad - \cos x$$

$$+ \quad 2 \quad - \sin x$$

$$\begin{aligned}
\int \frac{dx}{\cos^3 x} &= \int \frac{\cos x}{\cos^4 x} dx \\
&= \int \frac{\cos x}{(1 - \sin^2 x)^2} dx \\
&= \int \frac{dt}{(1 - t^2)^2} \quad (t = \sin x) \\
\frac{1}{(1 - t^2)} &= \frac{1}{2} \left(\frac{1}{1 + t} + \frac{1}{1 - t} \right) \\
\left(\frac{1}{(1 - t^2)} \right)^2 &= \left(\frac{1}{2} \left(\frac{1}{1 + t} + \frac{1}{1 - t} \right) \right)^2 \\
&= \frac{1}{4} \left(\frac{1}{(1 + t)^2} + \frac{1}{(1 - t)^2} + \frac{2}{1 - t^2} \right) \\
&= \frac{1}{4} \left(\frac{1}{(1 + t)^2} + \frac{1}{(1 - t)^2} + \frac{1}{1 + t} + \frac{1}{1 - t} \right) \\
\int \frac{dt}{(1 - t^2)^2} &= \frac{1}{4} \int \left(\frac{1}{(1 + t)^2} + \frac{1}{(1 - t)^2} + \frac{1}{1 + t} + \frac{1}{1 - t} \right) dt \\
&= \frac{1}{4} \left(\frac{1}{1 + t} - \frac{1}{1 - t} + \log \left| \frac{1 + t}{1 - t} \right| \right) + C \\
&= \frac{1}{4} \left(\frac{1}{1 + \sin x} - \frac{1}{1 - \sin x} + \log \left| \frac{1 + \sin x}{1 - \sin x} \right| \right) + C
\end{aligned}$$

$$\begin{aligned}
& \int \frac{dx}{\sin^4 x} \\
& \quad t = \tan x \\
& \cos^2 x = \frac{1}{1 + \tan^2 x} \\
& \quad = \frac{1}{1 + t^2} \\
& \sin^2 x = \cos^2 x \tan^2 x \\
& \quad = \frac{t^2}{1 + t^2} \\
& dt = \frac{dx}{\cos^2 x} \\
& dx = \frac{dt}{1 + t^2} \\
& \int \frac{dx}{\sin^4 x} = \int \frac{(1 + t^2)^2}{t^4} \cdot \frac{dt}{1 + t^2} \\
& \quad = \int \frac{1 + t^2}{t^4} dt \\
& \quad = \int \frac{dt}{t^4} + \int \frac{dt}{t^2} \\
& \quad = -\frac{1}{3t^3} - \frac{1}{t} + C \\
& \quad = -\frac{1}{3 \tan^3 x} - \frac{1}{\tan x} + C
\end{aligned}$$

$$\begin{aligned}
\int_0^{2\pi} \sqrt{1 + \cos x} \, dx &= \int_0^{2\pi} \sqrt{2 \cos^2 \frac{x}{2}} \, dx \\
&= \sqrt{2} \int_0^{2\pi} \left| \cos \frac{x}{2} \right| \, dx \\
&= 2\sqrt{2} \left[\sin \frac{x}{2} \right]_0^{2\pi} \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\int_0^{\frac{\pi}{3}} \sqrt{1 + \sin x} \, dx &= \int_0^{\frac{\pi}{3}} \sqrt{1 + \cos\left(\frac{\pi}{2} - x\right)} \, dx \\
&= \sqrt{2} \int_0^{\frac{\pi}{3}} \left| \cos\left(\frac{\pi}{4} - \frac{x}{2}\right) \right| \, dx \\
&= -2\sqrt{2} \left[\sin\left(\frac{\pi}{4} - \frac{x}{2}\right) \right]_0^{\frac{\pi}{3}} \\
&= -2\sqrt{2} \left(\frac{\sqrt{6} - \sqrt{2}}{4} - \frac{1}{\sqrt{2}} \right) \\
&= 3 - \sqrt{3}
\end{aligned}$$

$$\begin{aligned}\int \frac{dx}{\sqrt{x} + \sqrt{x+2}} &= \int \frac{\sqrt{x+2} - \sqrt{x}}{2} dx \\ &= \frac{1}{3}(x+2)^{\frac{3}{2}} - \frac{1}{3}x^{\frac{3}{2}} + C\end{aligned}$$

$$\begin{aligned}
\int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\cos x}{1+\cos x}} dx &= \int_0^{\frac{\pi}{2}} \sqrt{\tan^2 \frac{x}{2}} dx \\
&= \int_0^{\frac{\pi}{2}} \left| \tan \frac{x}{2} \right| dx \\
&= -2 \left[\log \left| \cos \frac{x}{2} \right| \right]_0^{\frac{\pi}{2}} \\
&= -2 \log \frac{1}{\sqrt{2}} \\
&= \log 2
\end{aligned}$$

$$\begin{aligned}
\int_0^1 \frac{dx}{1+x^3} &= \frac{1}{3} \int_0^1 \left(\frac{1}{1+t} - \frac{x-2}{x^2-x+1} \right) dx \\
\frac{1}{3} \int_0^1 \frac{dx}{1+x} &= \frac{1}{3} \log 2 \\
\frac{1}{3} \int_0^1 \frac{x-2}{x^2-x+1} dx &= \frac{1}{6} \int_0^1 \frac{2x-1}{x^2-x+1} - \frac{3}{x^2-x+1} dx \\
\frac{1}{6} \int_0^1 \frac{2x-1}{x^2-x+1} dx &= 0 \\
\frac{1}{2} \int_0^1 \frac{dx}{\left(x-\frac{1}{2}\right)^2 + \frac{3}{4}} &= \frac{1}{2} \int_0^1 \frac{dx}{\left(x-\frac{1}{2}\right)^2 + \frac{3}{4}} \\
x - \frac{1}{2} &= \frac{\sqrt{3}}{2} \tan t \\
dx &= \frac{\sqrt{3}}{2 \cos^2 t} dt \\
\frac{1}{2} \int_0^1 \frac{dx}{\left(x-\frac{1}{2}\right)^2 + \frac{3}{4}} &= \frac{1}{2} \cdot \frac{4}{3} \cdot \frac{\sqrt{3}}{2} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} dt \\
&= \frac{\pi}{3\sqrt{3}} \\
\therefore \int_0^1 \frac{dx}{1+x^3} &= \frac{1}{3} \log 2 + \frac{\pi}{3\sqrt{3}}
\end{aligned}$$

$$\begin{aligned}
I &= \int e^x \cos x \, dx \\
&= e^x \sin x - \int e^x \sin x \, dx \\
&= e^x \sin x - \left(e^x \cdot -\cos x - \int e^x \cdot -\cos x \, dx \right) \\
&= e^x \sin x + e^x \cos x - \int e^x \cos x \, dx \\
2I &= e^x \sin x + e^x \cos x \\
I &= \frac{e^x}{2} (\sin x + \cos x) + C
\end{aligned}$$