$$\int \tan x \, dx$$

$$\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3 - x^2} \, dx$$

$$\int \log x \, dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\log(\sin x)}{\tan x} \, dx$$

$$\int e^x \cos x \, dx$$

$$\int \frac{dx}{\sin^3 x}$$

$$\int \tan^4 x \, dx$$

$$\int \frac{dx}{\sin^2 x}$$

$$\int \sqrt{1 - x} \, dx$$

$$\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$

$$\int x^2 \sin x \, dx$$

$$\int_0^{2\pi} \sqrt{1 + \cos x} \, dx$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\frac{1 - \cos x}{1 + \cos x}} \, dx$$

$$\int_0^{\frac{\pi}{3}} \sqrt{1 + \sin x} \, dx$$

$$\int_0^1 \frac{dx}{1 + x^2}$$

$$\int_0^1 \frac{dx}{3+x^2}$$

$$\int_0^1 \frac{dx}{x^2+x+1}$$

$$\int x^x (1+\log x)\,dx$$

$$\int \frac{dx}{\sqrt{1+x^2}}$$

$$\int_2^3 \frac{dx}{\sqrt{x^2-1}}$$

$$\int \sqrt{1-e^{-2x}}\,dx$$

$$\int_0^1 \frac{dx}{1+x^3}$$

$$\int \frac{dx}{\sin^4 x}$$

$$\int \frac{dx}{\sqrt{x}+\sqrt{x+2}}$$

$$\int_0^e \frac{e^x}{e^{e-x}+e^x}\,dx$$

$$\int_{-1}^1 \sqrt{1-x^2}\,dx$$

$$\int \sqrt{1-e^{-2x}}\,dx$$

$$\int \tan^2 x\,dx$$

$$\int \tan^3 x\,dx$$

$$\int \tan^5 x\,dx$$

$KingProperty$

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$t = a+b-x \text{ と置換する}$$

$$dx = -\,dt$$

$$\therefore \int_b^a -f(t)\,dt = \int_a^b f(t)\,dt$$

$$= \int_a^b f(x)\,dx \ (\because \text{定積分では変数を変更してもよい})$$

$$因って、\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

x	$a \to b$
t	$b \to a$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx \left(\because \tan x = \frac{\sin x}{\cos x} \right)$$

$$= -\int \frac{-\sin x}{\cos x} \, dx$$

$$= -\log|\cos x| + C \left(\because \int \frac{f'(x)}{f(x)} \, dx = \log|f(x)| + C \right)$$

$$\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3-x^2} \, dx$$

$$x = \sqrt{3}\sin t \text{ とおく}$$

$$dx = \sqrt{3}\cos t \, dt$$

$$\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3-x^2} \, dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{3 - \left(\sqrt{3}\sin t\right)^2} \sqrt{3}\cos t \, dt$$

$$= 3\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 - \sin^2 t} \cos t \, dt$$

$$= 3\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\cos t| \cos t \, dt$$

$$= 3\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t \, dt \left(\because -\frac{\pi}{2} \le t \le \frac{\pi}{2} \Rightarrow 0 \le \forall \cos t\right)$$

$$= \frac{3}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 + \cos 2t \, dt \left(\because \cos^2 x = \frac{1 + \cos 2x}{2} \quad c.f. \sin^2 x = \frac{1 - \cos 2x}{2}\right)$$

$$= \frac{3}{2}\left[t + \frac{1}{2}\sin 2t\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= 3\left[t + \frac{1}{2}\sin 2t\right]_{0}^{\frac{\pi}{2}}$$

$$= \frac{3}{2}\pi$$

x	$-\sqrt{3} \to \sqrt{3}$
t	$-\dfrac{\pi}{2} \to \dfrac{\pi}{2}$

$$\int \frac{dx}{\sin^2 x} = \int \frac{dx}{\cos^2\left(\frac{\pi}{2} - x\right)} \quad \left(\because \sin x = \cos\left(\frac{\pi}{2} - x\right)\right)$$

$$= -\tan\left(\frac{\pi}{2} - x\right) + C$$

$$= -\frac{1}{\tan x} + C \quad \left(\because \tan x = \tan\left(\frac{\pi}{2} - x\right)\right)$$

$$\int_{-1}^{1} \sqrt{1-x^2}\,dx$$

$$x = \sin t \ \text{とおく}$$

$$dx = \cos t\,dt$$

$$\int_{-1}^{1} \sqrt{1-x^2}\,dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1-\sin^2 t}\,\cos t\,dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\cos t|\cos t\,dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t\,dt \ \left(\because -\frac{\pi}{2} \le x \le \frac{\pi}{2} \Rightarrow 0 \le \forall \cos x \right)$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2t}{2}\,dt \ \left(\because \cos^2 x = \frac{1+\cos 2x}{2} \quad c.f. \sin^2 x = \frac{1-\cos 2x}{2} \right)$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{\cos 2t}{2} \right) dt$$

$$= \frac{1}{2} \left[t + \frac{1}{2}\sin 2t \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

x	$-1 \to 1$
t	$-\dfrac{\pi}{2} \to \dfrac{\pi}{2}$

$$\int_0^1 \frac{dx}{1+x^2}$$

$$x = \tan t \text{ とおく}$$

$$\frac{1}{1+x^2} = \frac{1}{\cos^2 t}$$

$$dx = \frac{dt}{\cos^2 t}$$

$$\int_0^1 \frac{dx}{1+x^2} = \int_0^{\frac{\pi}{4}} \frac{\frac{1}{\cos^2 t}}{\frac{1}{\cos^2 t}} \, dt$$

$$= \int_0^{\frac{\pi}{4}} dt$$

$$= \frac{\pi}{4}$$

x	$0 \to 1$
t	$0 \to \dfrac{\pi}{4}$

$$\int \tan^4 x \, dx = \int \tan^2 x \tan^2 x \, dx$$

$$= \int \tan^2 x \left(\frac{1}{\cos^2 x} - 1 \right) dx$$

$$= \int \frac{\tan^2 x}{\cos^2 x} \, dx - \int \tan^2 x \, dx$$

$$= \frac{1}{3} \tan^3 x - \tan x + x + C$$

$$\int \log x \, dx = \int 1 \cdot \log x \, dx$$

$$= x \log x - \int x \cdot \frac{1}{x} \, dx$$

$$= x \log x - x + C$$

$$\int_0^e \frac{e^x}{e^{e-x} + e^x}\,dx = \int_0^e \frac{e^{e-x}}{e^x + e^{e-x}}\,dx \ (\because KingProperty)$$

$$I = \int_0^e \frac{e^x}{e^{e-x} + e^x}\,dx \ \text{とすると}$$

$$2I = \int_0^e \frac{e^x}{e^{e-x} + e^x}\,dx + \int_0^e \frac{e^{e-x}}{e^x + e^{e-x}}\,dx$$

$$= \int_0^e \frac{e^x + e^{e-x}}{e^x + e^{e-x}}\,dx$$

$$= \int_0^e dx$$

$$= e$$

$$\therefore I = \frac{e}{2}$$

$$\int \frac{dx}{\sin^4 x} = \int \frac{dx}{\tan^4 x \cos^4 x}$$

$$= \int \frac{1}{\tan^4 x} \left(1 + \tan^2 x\right) \frac{dx}{\cos^2 x}$$

$$t = \tan x \ \text{とおく}$$

$$dt = \frac{dx}{\cos^2 x}$$

$$\int \frac{1}{\tan^4 x} \left(1 + \tan^2 x\right) \frac{dx}{\cos^2 x} = \int \frac{1}{t^4} \left(1 + t^2\right) dt$$

$$= -\frac{1}{3t^3} - \frac{1}{t} + C$$

$$= -\frac{1}{3\tan^3 x} - \frac{1}{\tan x} + C$$

$$\int \tan^3 x \, dx = \int \tan x \cdot \tan^2 x \, dx$$

$$= \int \tan x \left(\frac{1}{\cos^2 x} - 1 \right) dx$$

$$= \int \frac{\tan x}{\cos^2 x} \, dx - \int \tan x \, dx$$

$$= \frac{1}{2} \tan^2 x + \log|\cos x| + C$$

$$\int \tan^2 x \, dx = \int \left(\frac{1}{\cos^2 x} - 1 \right) dx$$
$$= \tan x - x + C$$

$$\int_0^{\frac{\pi}{2}} \cos^2 x\, dx = \int_0^{\frac{\pi}{2}} \sin^2 x\, dx$$

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x\, dx$$

$$2I = \int_0^{\frac{\pi}{2}} \cos^2 x + \sin^2 x\, dx$$

$$= \int_0^{\frac{\pi}{2}} dx$$

$$= \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{4}$$

$$\int \frac{\log(\sin x)}{\tan x}\,dx = \int \frac{\cos x \log(\sin x)}{\sin x}\,dx$$
$$= \int \frac{\log t}{t}\,dt$$
$$= \frac{1}{2}(\log t)^2 + C$$
$$= \frac{1}{2}(\log(\sin x))^2 + C$$

$$\int \frac{dx}{\sqrt{1+x^2}}$$

$$x = \sinh t$$

$$dx = \cosh t \, dt$$

$$\int \frac{dx}{\sqrt{1+x^2}} = \int \frac{1}{\sqrt{1+\sinh^2 t}} \cosh t \, dt$$

$$= \int dt$$

$$= t + C$$

$$= \log\left(x + \sqrt{1+x^2}\right) + C$$

$$\int \tan^5 x \, dx = \int \tan^3 x \left(\frac{1}{\cos^2 x} - 1 \right) dx$$
$$= \frac{1}{4} \tan^4 x - \frac{1}{2} \tan^2 x + \log|\cos x| + C$$

$$\int \frac{dx}{\cos^4} = \int \frac{1}{\cos^2 x} \cdot \frac{1}{\cos^2 x} \, dx$$
$$= \int \left(1 + \tan^2 x\right) \frac{dx}{\cos^2 x}$$
$$= \frac{1}{3} \tan^3 x + \tan x + C$$

$$\int_0^1 \frac{dx}{x^2+3}$$

$$x = \sqrt{3}\tan t \text{ とおく}$$

$$dx = \frac{dt}{\cos^2 t}$$

$$\int_0^1 \frac{dx}{x^2+3} = \int_0^{\frac{\pi}{6}} \frac{\frac{\sqrt{3}}{\cos^2 t}}{3\tan^2 t + 3}\, dt$$

$$= \int_0^{\frac{\pi}{6}} \frac{\frac{\sqrt{3}}{\cos^2 t}}{3\left(\frac{1}{\cos^2 t}\right)}\, dt$$

$$= \frac{1}{\sqrt{3}} \int_0^{\frac{\pi}{6}} dt$$

$$= \frac{\pi}{6\sqrt{3}}$$

x	$0 \to 1$
t	$0 \to \dfrac{\pi}{6}$

$$\int x^2 \sin x \, dx$$

$$\int x^2 \sin x \, dx = x^2 \sin x + 2x \cos x - 2 \sin x + C$$

$+$	x^2	$\sin x$
$-$	$2x$	$-\cos x$
$+$	2	$-\sin x$

$$\int \frac{dx}{\cos^3 x} = \int \frac{\cos x}{\cos^4 x}\,dx$$

$$= \int \frac{\cos x}{\left(1 - \sin^2 x\right)^2}\,dx$$

$$= \int \frac{dt}{\left(1 - t^2\right)^2} \quad (t = \sin x)$$

$$\frac{1}{(1 - t^2)} = \frac{1}{2}\left(\frac{1}{1 + t} + \frac{1}{1 - t}\right)$$

$$\left(\frac{1}{(1 - t^2)}\right)^2 = \left(\frac{1}{2}\left(\frac{1}{1 + t} + \frac{1}{1 - t}\right)\right)^2$$

$$= \frac{1}{4}\left(\frac{1}{(1 + t)^2} + \frac{1}{(1 - t)^2} + \frac{2}{1 - t^2}\right)$$

$$= \frac{1}{4}\left(\frac{1}{(1 + t)^2} + \frac{1}{(1 - t)^2} + \frac{1}{1 + t} + \frac{1}{1 - t}\right)$$

$$\int \frac{dt}{(1 - t^2)^2} = \frac{1}{4}\int \left(\frac{1}{(1 + t)^2} + \frac{1}{(1 - t)^2} + \frac{1}{1 + t} + \frac{1}{1 - t}\right)dt$$

$$= \frac{1}{4}\left(\frac{1}{1 + t} - \frac{1}{1 - t} + \log\left|\frac{1 + t}{1 - t}\right|\right) + C$$

$$= \frac{1}{4}\left(\frac{1}{1 + \sin x} - \frac{1}{1 - \sin x} + \log\left|\frac{1 + \sin x}{1 - \sin x}\right|\right) + C$$

$$\int \frac{dx}{\sin^4 x}$$

$$t = \tan x$$

$$\cos^2 x = \frac{1}{1 + \tan^2 x}$$

$$= \frac{1}{1 + t^2}$$

$$\sin^2 x = \cos^2 x \tan^2 x$$

$$= \frac{t^2}{1 + t^2}$$

$$dt = \frac{dx}{\cos^2 x}$$

$$dx = \frac{dt}{1 + t^2}$$

$$\int \frac{dx}{\sin^4 x} = \int \frac{\left(1 + t^2\right)^2}{t^4} \cdot \frac{dt}{1 + t^2}$$

$$= \int \frac{1 + t^2}{t^4} \, dt$$

$$= \int \frac{dt}{t^4} + \int \frac{dt}{t^2}$$

$$= -\frac{1}{3t^3} - \frac{1}{t} + C$$

$$= -\frac{1}{3 \tan^3 x} - \frac{1}{\tan x} + C$$

$$\int_0^{2\pi} \sqrt{1 + \cos x}\, dx = \int_0^{2\pi} \sqrt{2 \cos^2 \frac{x}{2}}\, dx$$

$$= \sqrt{2} \int_0^{2\pi} \left| \cos \frac{x}{2} \right| dx$$

$$= 2\sqrt{2} \left[\sin \frac{x}{2} \right]_0^{2\pi}$$

$$= 0$$

$$\int_0^{\frac{\pi}{3}} \sqrt{1 + \sin x}\, dx = \int_0^{\frac{\pi}{3}} \sqrt{1 + \cos\left(\frac{\pi}{2} - x\right)}\, dx$$

$$= \sqrt{2} \int_0^{\frac{\pi}{3}} \left|\cos\left(\frac{\pi}{4} - \frac{x}{2}\right)\right| dx$$

$$= -2\sqrt{2} \left[\sin\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]_0^{\frac{\pi}{3}}$$

$$= -2\sqrt{2}\left(\frac{\sqrt{6} - \sqrt{2}}{4} - \frac{1}{\sqrt{2}}\right)$$

$$= 3 - \sqrt{3}$$

$$\int \frac{dx}{\sqrt{x} + \sqrt{x+2}} = \int \frac{\sqrt{x+2} - \sqrt{x}}{2}\, dx$$

$$= \frac{1}{3}(x+2)^{\frac{3}{2}} - \frac{1}{3}x^{\frac{3}{2}} + C$$