$$\int \tan x \, dx$$

$$\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3 - x^2} \, dx$$

$$\int \log x \, dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\log(\sin x)}{\tan x} \, dx$$

$$\int e^x \cos x \, dx$$

$$\int \frac{dx}{\sin^3 x}$$

$$\int \tan^4 x \, dx$$

$$\int \frac{dx}{\sin^2 x}$$

$$\int \sqrt{1 - x} \, dx$$

$$\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$

$$\int x^2 \sin x \, dx$$

$$\int_0^{2\pi} \sqrt{1 + \cos x} \, dx$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\frac{1 - \cos x}{1 + \cos x}} \, dx$$

$$\int_0^{\frac{\pi}{3}} \sqrt{1 + \sin x} \, dx$$

$$\int_0^1 \frac{dx}{1 + x^2}$$

$$\int_0^1 \frac{dx}{3 + x^2}$$

$$\int_0^1 \frac{dx}{x^2 + x + 1}$$

$$\int x^x (1 + \log x) \, dx$$

$$\int \frac{dx}{\sqrt{1 + x^2}}$$

$$\int_2^3 \frac{dx}{\sqrt{x^2 - 1}}$$

$$\int \sqrt{1 - e^{-2x}} \, dx$$

$$\int_0^1 \frac{dx}{1 + x^3}$$

$$\int \frac{dx}{\sin^4 x}$$

$$\int \frac{dx}{\sqrt{x}+\sqrt{x+2}}$$

$$\int_0^e \frac{e^x}{e^{e-x}+e^x}\,dx$$

$$\int_{-1}^1 \sqrt{1-x^2}\,dx$$

$$\int \sqrt{1-e^{-2x}}\,dx$$

$$\int \tan^2 x\,dx$$

$$\int \tan^3 x\,dx$$

$$\int \tan^5 x\,dx$$

$KingProperty$

$$\begin{aligned}
\int_a^b f(x)\,dx &= \int_a^b f(a+b-x)\,dx \\
t &= a+b-x \text{ と置換する} \\
dx &= -\,dt \\
\therefore \int_b^a -f(t)\,dt &= \int_a^b f(t)\,dt \\
&= \int_a^b f(x)\,dx \ \left(\because \text{定積分では変数を変更してもよい}\right) \\
\text{因って、}\int_a^b f(x)\,dx &= \int_a^b f(a+b-x)\,dx
\end{aligned}$$

x	$a \to b$
t	$b \to a$

$$
\begin{aligned}
\int \tan x \, dx &= \int \frac{\sin x}{\cos x} \, dx \left(\because \tan x = \frac{\sin x}{\cos x} \right) \\
&= - \int \frac{-\sin x}{\cos x} \, dx \\
&= -\log|\cos x| + C \left(\because \int \frac{f'(x)}{f(x)} \, dx = \log|f(x)| + C \right)
\end{aligned}
$$

$$\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3-x^2}\,dx$$

$$\begin{aligned}
x &= \sqrt{3}\sin t \text{ とおく} \\
dx &= \sqrt{3}\cos t\,dt \\
\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3-x^2}\,dx &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{3-\left(\sqrt{3}\sin t\right)^2}\sqrt{3}\cos t\,dt \\
&= 3\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1-\sin^2 t}\cos t\,dt \\
&= 3\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\cos t|\cos t\,dt \\
&= 3\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t\,dt \left(\because -\frac{\pi}{2} \le t \le \frac{\pi}{2} \Rightarrow 0 \le \forall \cos t\right) \\
&= \frac{3}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1+\cos 2t\,dt \left(\because \cos^2 x = \frac{1+\cos 2x}{2} \quad c.f. \sin^2 x = \frac{1-\cos 2x}{2}\right) \\
&= \frac{3}{2}\left[t+\frac{1}{2}\sin 2t\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= 3\left[t+\frac{1}{2}\sin 2t\right]_{0}^{\frac{\pi}{2}} \\
&= \frac{3}{2}\pi
\end{aligned}$$

x	$-\sqrt{3} \to \sqrt{3}$
t	$-\frac{\pi}{2} \to \frac{\pi}{2}$

$$
\begin{aligned}
\int \frac{dx}{\sin^2 x} &= \int \frac{dx}{\cos^2\left(\frac{\pi}{2} - x\right)} \left(\because \sin x = \cos\left(\frac{\pi}{2} - x\right)\right) \\
&= -\tan\left(\frac{\pi}{2} - x\right) + C \\
&= -\frac{1}{\tan x} + C \left(\because \tan x = \tan\left(\frac{\pi}{2} - x\right)\right)
\end{aligned}
$$

$$\int_{-1}^{1} \sqrt{1-x^2}\,dx$$

$$x = \sin t \text{ とおく}$$

$$dx = \cos t\,dt$$

$$\begin{aligned}
\int_{-1}^{1} \sqrt{1-x^2}\,dx &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1-\sin^2 t}\cos t\,dt \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\cos t| \cos t\,dt \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t\,dt \left(\because -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \Rightarrow 0 \leq \forall \cos x\right) \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2t}{2}\,dt \left(\because \cos^2 x = \frac{1+\cos 2x}{2} \quad c.f. \sin^2 x = \frac{1-\cos 2x}{2}\right) \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{\cos 2t}{2}\right) dt \\
&= \frac{1}{2}\left[t + \frac{1}{2}\sin 2t\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= \frac{\pi}{2}
\end{aligned}$$

x	$-1 \to 1$
t	$-\frac{\pi}{2} \to \frac{\pi}{2}$

$$\int_0^1 \frac{dx}{1+x^2}$$

$$x = \tan t \text{ とおく}$$

$$\frac{1}{1+x^2} = \frac{1}{\cos^2 t}$$

$$dx = \frac{dt}{\cos^2 t}$$

$$\begin{aligned}\int_0^1 \frac{dx}{1+x^2} &= \int_0^{\frac{\pi}{4}} \frac{\frac{1}{\cos^2 t}}{\frac{1}{\cos^2 t}}\,dt \\ &= \int_0^{\frac{\pi}{4}} dt \\ &= \frac{\pi}{4}\end{aligned}$$

x	$0 \to 1$
t	$0 \to \frac{\pi}{4}$

$$
\begin{aligned}
\int \tan^4 x \, dx &= \int \tan^2 x \tan^2 x \, dx \\
&= \int \tan^2 x \left(\frac{1}{\cos^2 x} - 1 \right) dx \\
&= \int \frac{\tan^2 x}{\cos^2 x} \, dx - \int \tan^2 x \, dx \\
&= \frac{1}{3} \tan^3 x - \tan x + x + C
\end{aligned}
$$

$$
\begin{aligned}
\int \log x \, dx &= \int 1 \cdot \log x \, dx \\
&= x \log x - \int x \cdot \frac{1}{x} \, dx \\
&= x \log x - x + C
\end{aligned}
$$

$$
\begin{aligned}
\int_0^e \frac{e^x}{e^{e-x}+e^x}\,dx &= \int_0^e \frac{e^{e-x}}{e^x+e^{e-x}}\,dx \,(\because KingProperty)\\
I &= \int_0^e \frac{e^x}{e^{e-x}+e^x}\,dx \text{ とすると}\\
2I &= \int_0^e \frac{e^x}{e^{e-x}+e^x}\,dx + \int_0^e \frac{e^{e-x}}{e^x+e^{e-x}}\,dx\\
&= \int_0^e \frac{e^x+e^{e-x}}{e^x+e^{e-x}}\,dx\\
&= \int_0^e dx\\
&= e\\
\therefore I &= \frac{e}{2}
\end{aligned}
$$

$$
\begin{aligned}
\int \frac{dx}{\sin^4 x} &= \int \frac{dx}{\tan^4 x \cos^4 x} \\
&= \int \frac{1}{\tan^4 x}\left(1+\tan^2 x\right)\frac{dx}{\cos^2 x} \\
t &= \tan x \text{ とおく} \\
dt &= \frac{dx}{\cos^2 x} \\
\int \frac{1}{\tan^4 x}\left(1+\tan^2 x\right)\frac{dx}{\cos^2 x} &= \int \frac{1}{t^4}\left(1+t^2\right) dt \\
&= -\frac{1}{3t^3} - \frac{1}{t} + C \\
&= -\frac{1}{3\tan^3 x} - \frac{1}{\tan x} + C
\end{aligned}
$$

$$
\begin{aligned}
\int \tan^3 x \, dx &= \int \tan x \cdot \tan^2 x \, dx \\
&= \int \tan x \left(\frac{1}{\cos^2 x} - 1 \right) dx \\
&= \int \frac{\tan x}{\cos^2 x} \, dx - \int \tan x \, dx \\
&= \frac{1}{2} \tan^2 x + \log|\cos x| + C
\end{aligned}
$$

$$\begin{aligned}\int \tan^2 x\, dx &= \int \left(\frac{1}{\cos^2 x} - 1 \right) dx \\ &= \tan x - x + C\end{aligned}$$

$$
\begin{aligned}
\int_0^{\frac{\pi}{2}} \cos^2 x \, dx &= \int_0^{\frac{\pi}{2}} \sin^2 x \, dx \\
I &= \int_0^{\frac{\pi}{2}} \cos^2 x \, dx \\
2I &= \int_0^{\frac{\pi}{2}} \cos^2 x + \sin^2 x \, dx \\
&= \int_0^{\frac{\pi}{2}} \, dx \\
&= \frac{\pi}{2} \\
\therefore I &= \frac{\pi}{4}
\end{aligned}
$$

$$
\begin{aligned}
\int \frac{\log(\sin x)}{\tan x}\,dx &= \int \frac{\cos x \log(\sin x)}{\sin x}\,dx \\
&= \int \frac{\log t}{t}\,dt \\
&= \frac{1}{2}(\log t)^2 + C \\
&= \frac{1}{2}(\log(\sin x))^2 + C
\end{aligned}
$$

$$
\begin{aligned}
\sin^2 x + \cos^2 x &= 1 \\
\tan x &= \frac{\sin x}{\cos x} \\
\tan^2 x + 1 &= \frac{1}{\cos^2 x} \\
\sin 2x &= 2 \sin x \cos x \\
\cos 2x &= \sin^2 x - \cos^2 x \\
&= 1 - 2\cos^2 x \\
&= 2\sin^2 x - 1 \\
\sin^2 x &= \frac{1 - \cos 2x}{2} \\
\cos^2 x &= \frac{1 + \cos 2x}{2} \\
\sin^{-1} x &= \arcsin x \\
\cos^{-1} x &= \arccos x \\
\tan^{-1} x &= \arctan x \\
\sinh x &= \frac{e^x - e^{-x}}{2} \\
\cosh x &= \frac{e^x + e^{-x}}{2} \\
\tanh x &= \frac{e^x - e^{-x}}{e^x + e^{-x}} \\
\cosh^2 x - \sinh^2 x &= 1 \\
x &= \sinh t \\
e^t - e^{-t} - 2x &= 0 \\
e^{2t} - 2e^t x - 1 &= 0 \\
e^t &= x + \sqrt{1 + x^2} \; (\because e^t > 0) \\
t &= \log\left(x + \sqrt{1 + x^2}\right) \\
x &= \cosh t \\
e^t + e^{-t} - 2x &= 0 \\
e^{2t} + 2e^t x - 1 &= 0 \\
e^t &= -x \pm \sqrt{1 + x^2} \\
t &= \log\left(-x \pm \sqrt{1 + x^2}\right)
\end{aligned}
$$

$$\int \frac{dx}{\sqrt{1+x^2}}$$

$$\begin{aligned}
x &= \sinh t \\
dx &= \cosh t \, dt \\
\int \frac{dx}{\sqrt{1+x^2}} &= \int \frac{1}{\sqrt{1+\sinh^2 t}} \cosh t \, dt \\
&= \int dt \\
&= t + C \\
&= \log\left(x + \sqrt{1+x^2}\right) + C
\end{aligned}$$

$$\begin{aligned}\int \tan^5 x\,dx &= \int \tan^3 x\left(\frac{1}{\cos^2 x} - 1\right)dx \\ &= \frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x + \log|\cos x| + C\end{aligned}$$

$$
\begin{aligned}
\int \frac{dx}{\cos^4} &= \int \frac{1}{\cos^2 x} \cdot \frac{1}{\cos^2 x}\, dx \\
&= \int \bigl(1 + \tan^2 x\bigr) \frac{dx}{\cos^2 x} \\
&= \frac{1}{3} \tan^3 x + \tan x + C
\end{aligned}
$$

$$\int_0^1 \frac{dx}{x^2+3}$$

$$x = \sqrt{3}\tan t \text{ とおく}$$

$$dx = \frac{dt}{\cos^2 t}$$

$$\begin{aligned}
\int_0^1 \frac{dx}{x^2+3} &= \int_0^{\frac{\pi}{6}} \frac{\frac{\sqrt{3}}{\cos^2 t}}{3\tan^2 t + 3}\,dt \\
&= \int_0^{\frac{\pi}{6}} \frac{\frac{\sqrt{3}}{\cos^2 t}}{3\left(\frac{1}{\cos^2 t}\right)}\,dt \\
&= \frac{1}{\sqrt{3}} \int_0^{\frac{\pi}{6}} dt \\
&= \frac{\pi}{6\sqrt{3}}
\end{aligned}$$

x	$0 \to 1$
t	$0 \to \frac{\pi}{6}$

$$\int x^2 \sin x \, dx$$
$$\int x^2 \sin x \, dx = x^2 \sin x + 2x \cos x - 2 \sin x + C$$

$$\begin{array}{ccc} + & x^2 & \sin x \\ - & 2x & -\cos x \\ + & 2 & -\sin x \end{array}$$

$$
\begin{aligned}
\int \frac{dx}{\cos^3 x} &= \int \frac{\cos x}{\cos^4 x}\,dx \\
&= \int \frac{\cos x}{\left(1-\sin^2 x\right)^2}\,dx \\
&= \int \frac{dt}{\left(1-t^2\right)^2}\ (t=\sin x) \\
\frac{1}{(1-t^2)} &= \frac{1}{2}\left(\frac{1}{1+t}+\frac{1}{1-t}\right) \\
\left(\frac{1}{(1-t^2)}\right)^2 &= \left(\frac{1}{2}\left(\frac{1}{1+t}+\frac{1}{1-t}\right)\right)^2 \\
&= \frac{1}{4}\left(\frac{1}{\left(1+t\right)^2}+\frac{1}{\left(1-t\right)^2}+\frac{2}{1-t^2}\right) \\
&= \frac{1}{4}\left(\frac{1}{\left(1+t\right)^2}+\frac{1}{\left(1-t\right)^2}+\frac{1}{1+t}+\frac{1}{1-t}\right) \\
\int \frac{dt}{\left(1-t^2\right)^2} &= \frac{1}{4}\int\left(\frac{1}{\left(1+t\right)^2}+\frac{1}{\left(1-t\right)^2}+\frac{1}{1+t}+\frac{1}{1-t}\right)dt \\
&= \frac{1}{4}\left(\frac{1}{1+t}-\frac{1}{1-t}+\log\left|\frac{1+t}{1-t}\right|\right)+C \\
&= \frac{1}{4}\left(\frac{1}{1+\sin x}-\frac{1}{1-\sin x}+\log\left|\frac{1+\sin x}{1-\sin x}\right|\right)+C
\end{aligned}
$$

$$\int \frac{dx}{\sin^4 x}$$

$$\begin{aligned}
t &= \tan x \\
\cos^2 x &= \frac{1}{1+\tan^2 x} \\
&= \frac{1}{1+t^2} \\
\sin^2 x &= \cos^2 x \tan^2 x \\
&= \frac{t^2}{1+t^2} \\
dt &= \frac{dx}{\cos^2 x} \\
dx &= \frac{dt}{1+t^2}
\end{aligned}$$

$$\begin{aligned}
\int \frac{dx}{\sin^4 x} &= \int \frac{\left(1+t^2\right)^2}{t^4} \cdot \frac{dt}{1+t^2} \\
&= \int \frac{1+t^2}{t^4}\, dt \\
&= \int \frac{dt}{t^4} + \int \frac{dt}{t^2} \\
&= -\frac{1}{3t^3} - \frac{1}{t} + C \\
&= -\frac{1}{3\tan^3 x} - \frac{1}{\tan x} + C
\end{aligned}$$