$$\int \tan x \, dx$$

$$\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3-x^2} \, dx$$

$$\int \log x \, dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\log(\sin x)}{\tan x} \, dx$$

$$\int e^x \cos x \, dx$$

$$\int \frac{dx}{\sin^3 x}$$

$$\int \tan^4 x \, dx$$

$$\int \frac{dx}{\sin^2 x}$$

$$\int \sqrt{1-x} \, dx$$

$$\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$

$$\int x^2 \sin x \, dx$$

$$\int_0^{2\pi} \sqrt{1+\cos x} \, dx$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\cos x}{1+\cos x}} \, dx$$

$$\int_0^{\frac{\pi}{3}} \sqrt{1+\sin x} \, dx$$

$$\int_0^1 \frac{dx}{1+x^2}$$

$$\int_0^1 \frac{dx}{3+x^2}$$

$$\int_0^1 \frac{dx}{x^2+x+1}$$

$$\int x^x(1+\log x)\,dx$$

$$\int \frac{dx}{\sqrt{1+x^2}}$$

$$\int_2^3 \frac{dx}{\sqrt{x^2-1}}$$

$$\int \sqrt{1-e^{-2x}}\,dx$$

$$\int_0^1 \frac{dx}{1+x^3}$$

$$\int \frac{dx}{\sin^4 x}$$

$$\int \frac{dx}{\sqrt{x}+\sqrt{x+2}}$$

$$\int_0^e \frac{e^x}{e^{e-x}+e^x}\,dx$$

$$\int_{-1}^1 \sqrt{1-x^2}\,dx$$

$$\int \sqrt{1-e^{-2x}}\,dx$$

$$\int \tan^2 x\,dx$$

$$\int \tan^3 x\,dx$$

$$\int \tan^5 x\,dx$$

$$
\begin{aligned}
&KingProperty\\
\int_a^b f(x)\,dx &= \int_a^b f(a+b-x)\,dx\\
t &= a+b-x \text{ と置換する}\\
dx &= -\,dt\\
\therefore \int_b^a -f(t)\,dt &= \int_a^b f(t)\,dt\\
&= \int_a^b f(x)\,dx \ \left(\because \text{定積分では変数を変更してもよい}\right)\\
\text{因って、}\int_a^b f(x)\,dx &= \int_a^b f(a+b-x)\,dx
\end{aligned}
$$

x	$a \to b$
t	$b \to a$

$$\int \tan x \, dx \quad \rightarrow \frac{\sin x}{\cos x}\text{で一次式に}$$

$$\begin{aligned}
\int \tan x \, dx &= \int \frac{\sin x}{\cos x} \, dx \quad \text{分数関数→分母の微分を考える} \\
&= -\int \frac{-\sin x}{\cos x} \, dx \\
&= -\log|\cos x| + C
\end{aligned}$$

$$\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3-x^2}\,dx \quad \sqrt{a^2-x^2}\text{を含む}\to x=a\sin t\text{とおく}$$

$$x=\sqrt{3}\sin t \text{ とする}$$

$$dx=\sqrt{3}\cos t\,dt$$

$$\begin{aligned}
\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3-x^2}\,dx &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{3-\left(\sqrt{3}\sin t\right)^2}\sqrt{3}\cos t\,dt \\
&= 3\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1-\sin^2 t}\cos t\,dt \\
&= 3\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\cos t|\cos t\,dt \\
&= 3\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t\,dt \left(\because -\frac{\pi}{2}\le t\le\frac{\pi}{2} \Rightarrow 0\le\forall\cos t\right) \\
&= \frac{3}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1+\cos 2t\,dt \left(\because \cos^2 x=\frac{1+\cos 2x}{2} \quad c.f.\sin^2 x=\frac{1-\cos 2x}{2}\right) \\
&= \frac{3}{2}\left[t+\frac{1}{2}\sin 2t\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= 3\left[t+\frac{1}{2}\sin 2t\right]_{0}^{\frac{\pi}{2}} \\
&= \frac{3}{2}\pi
\end{aligned}$$

x	$-\sqrt{3}\to\sqrt{3}$
t	$-\frac{\pi}{2}\to\frac{\pi}{2}$

$$\int \frac{dx}{\sin^2 x}\,dx \rightarrow \frac{\pi}{2}\text{で}cos\text{に}$$

$$\begin{aligned}
\int \frac{dx}{\sin^2 x} &= \int \frac{dx}{\cos^2\left(\frac{\pi}{2}-x\right)} \\
&= -\tan\left(\frac{\pi}{2}-x\right)+C \\
&= -\frac{1}{\tan x}+C
\end{aligned}$$

$$\int_{-1}^{1}\sqrt{1-x^2}\,dx \quad \sqrt{|\qquad|^2} = |\qquad| \text{としたい.}$$

ここで, $1-\sin^2 t = \cos^2 t$である. また, $1-x^2 \geq 0$より, $-1 \leq x \leq 1$であるから, $x = \sin x$とおける.

$$\begin{aligned}
x &= \sin t \text{ とする} \\
dx &= \cos t\,dt \\
\int_{-1}^{1}\sqrt{1-x^2}\,dx &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1-\sin^2 t}\,\mathrm{c} \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\cos t| \cos t\,dt \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t\,dt \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2t}{2}\,dt \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{\cos 2t}{2}\right. \\
&= \frac{1}{2}\left[t + \frac{1}{2}\sin 2t\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= \frac{\pi}{2}
\end{aligned}$$

x	$-1 \to 1$
t	$-\frac{\pi}{2} \to \frac{\pi}{2}$

$$\int_0^1 \frac{dx}{1+x^2}.$$

ここで, $1+\tan^2 t = \dfrac{1}{\cos^2 t}$ である. また, $(\tan t)' = \dfrac{1}{\cos^2 t}$ であるから, $x = \tan t$ とする

$$x = \tan t \text{ とおく}$$

$$\frac{1}{1+x^2} = \frac{1}{\cos^2 t}$$

$$dx = \frac{dt}{\cos^2 t}$$

$$\int_0^1 \frac{dx}{1+x^2} = \int_0^{\frac{\pi}{4}} \frac{\frac{1}{\cos^2 t}}{\frac{1}{\cos^2 t}}\, dt$$

$$= \int_0^{\frac{\pi}{4}} dt$$

$$= \frac{\pi}{4}$$

x	$0 \to 1$
t	$0 \to \dfrac{\pi}{4}$

$\int \tan^4 x\,dx$　三角関数は2次ごとに分割

$$
\begin{aligned}
&= \int \tan^2 x \tan^2 x\,dx \qquad \tan^2 \text{は} \cos^2 \\
&= \int \tan^2 x \left(\frac{1}{\cos^2 x} - 1 \right) dx \\
&= \int \frac{\tan^2 x}{\cos^2 x}\,dx - \int \tan^2 x\,dx \\
&= \frac{1}{3} \tan^3 x - \tan x + x + C
\end{aligned}
$$

$\displaystyle\int \log x\,dx$ 　logを含む→部分積分

$$
\begin{aligned}
&= \int 1 \cdot \log x\,dx \\
&= x\log x - \int x \cdot \frac{1}{x}\,dx \\
&= x\log x - x + C
\end{aligned}
$$

$$\int_0^e \frac{e^x}{e^{e-x}+e^x}\,dx \quad a+b-x\text{を含む}, \text{下端が}0, \text{指数関数の式}\to KingProperty$$

$$\int_0^e \frac{e^x}{e^{e-x}+e^x}\,dx = \int_0^e \frac{e^{e-x}}{e^x+e^{e-x}}\,dx$$

$$I = \int_0^e \frac{e^x}{e^{e-x}+e^x}\,dx \text{ とすると}$$

$$2I = \int_0^e \frac{e^x}{e^{e-x}+e^x}\,dx + \int_0^e \frac{e^{e-x}}{e^x+e^{e-x}}\,dx$$

$$= \int_0^e \frac{e^x+e^{e-x}}{e^x+e^{e-x}}\,dx$$

$$= \int_0^e dx$$

$$= e$$

$$\therefore I = \frac{e}{2}$$

$$\int_0^e \frac{e^x}{e^{e-x}+e^x}\,dx = \int_0^e \frac{e^x}{e^e\cdot e^{-x}+e^x}\,dx$$

指数関数の定数部分は分離

指数の符号は揃える, 指数関数の分数関数→分母分子にe^xをかける

指数関数の分数関数→微分接触をつくる

$$= \frac{1}{2}\int_0^e \frac{2e^{2x}}{e^e+e^{2x}}\,dx$$

$$= \frac{1}{2}\left[\log(e^e+e^{2x})\right]_0^e \quad (\because e^e+e^{2x}\geq 0)$$

$$= \frac{1}{2}\log\frac{e^e+e^{2e}}{e^e+1}$$

$$= \frac{1}{2}\log\frac{e^e(1+e^e)}{e^e+1}$$

$$= \frac{e}{2}$$

$$\int_0^e \frac{e^x}{e^{e-x}+e^x}\,dx \quad \text{指数関数の定数部分は分離}$$

$$= \int_0^e \frac{e^x}{e^e\cdot e^{-x}+e^x}\,dx \quad \text{指数の符号は揃える, 指数関数の分数関数}\to\text{分母分子に}e^x\text{をかける}$$

指数関数の分数関数→微分接触をつくる

$$= \frac{1}{2}\int_0^e \frac{2e^{2x}}{e^e+e^{2x}}$$

$$= \frac{1}{2}\left[\log(e^e+e^{2x})\right]_0^e \quad (\because e^e+e^{2x}\geq 0)$$

$$= \frac{1}{2}\log\frac{e^e+e^{2e}}{e^e+1}$$

$$= \frac{1}{2}\log\frac{e^e(1+e^e)}{e^e+1}$$

$$= \frac{e}{2}$$

$$
\begin{aligned}
\int \frac{dx}{\sin^4 x} &= \int \frac{dx}{\tan^4 x \cos^4 x} \quad f(\sin^2 x, \cos^2 x, \tan x) \to g(\tan x) \text{に} \\
&= \int \frac{1}{\tan^4 x}\left(1+\tan^2 x\right)\frac{dx}{\cos^2 x} \\
t &= \tan x \text{ とする} \\
dt &= \frac{dx}{\cos^2 x} \\
\int \frac{1}{\tan^4 x}\left(1+\tan^2 x\right)\frac{dx}{\cos^2 x} &= \int \frac{1}{t^4}\left(1+t^2\right) dt \\
&= -\frac{1}{3t^3} - \frac{1}{t} + C \\
&= -\frac{1}{3\tan^3 x} - \frac{1}{\tan x} + C
\end{aligned}
$$

$$\int \tan^3 x \, dx \quad \text{三角関数は2次ごとに分割}$$

$$\begin{aligned}
\int \tan^3 x \, dx &= \int \tan x \cdot \tan^2 x \, dx \quad \tan^2 x \text{は} \cos^2 x \\
&= \int \tan x \left(\frac{1}{\cos^2 x} - 1 \right) dx \\
&= \int \frac{\tan x}{\cos^2 x} \, dx - \int \tan x \, dx \\
&= \frac{1}{2} \tan^2 x + \log|\cos x| + C
\end{aligned}$$

$\int \tan^2 x \, dx$　$\tan^2 x$は$\cos^2 x$

$$\begin{aligned}\int \tan^2 x \, dx &= \int \left(\frac{1}{\cos^2 x} - 1 \right) dx \\ &= \tan x - x + C\end{aligned}$$

$\int_0^{\frac{\pi}{2}} \cos^2 x\, dx 0,$三角関数の式で積分区間が$\frac{n\pi}{2} \rightarrow King Property$

$$\begin{aligned}
\int_0^{\frac{\pi}{2}} \cos^2 x\, dx &= \int_0^{\frac{\pi}{2}} \sin^2 x\, dx \\
I &= \int_0^{\frac{\pi}{2}} \cos^2 x\, dx \\
2I &= \int_0^{\frac{\pi}{2}} \cos^2 x + \sin^2 x\, dx \\
&= \int_0^{\frac{\pi}{2}} \, dx \\
&= \frac{\pi}{2} \\
\therefore I &= \frac{\pi}{4}
\end{aligned}$$

$$
\begin{aligned}
\int \frac{\log(\sin x)}{\tan x}\,dx &= \int \frac{\cos x \log(\sin x)}{\sin x}\,dx \\
&= \int \frac{\log t}{t}\,dt \\
&= \frac{1}{2}(\log t)^2 + C \\
&= \frac{1}{2}(\log(\sin x))^2 + C
\end{aligned}
$$

$$\int \frac{dx}{\sqrt{1+x^2}}$$

$$\begin{aligned}
x &= \sinh t \\
dx &= \cosh t \, dt \\
\int \frac{dx}{\sqrt{1+x^2}} &= \int \frac{1}{\sqrt{1+\sinh^2 t}} \cosh t \, dt \\
&= \int dt \\
&= t + C \\
&= \log\left(x + \sqrt{1+x^2}\right) + C
\end{aligned}$$

$$
\begin{aligned}
\int \tan^5 x \, dx &= \int \tan^3 x \left(\frac{1}{\cos^2 x} - 1 \right) dx \\
&= \frac{1}{4} \tan^4 x - \frac{1}{2} \tan^2 x + \log|\cos x| + C
\end{aligned}
$$

$\int \frac{dx}{\cos^4 x}$　三角関数は2乗ごとに分割

$$\begin{aligned}\int \frac{dx}{\cos^4 x} &= \int \frac{1}{\cos^2 x} \cdot \frac{1}{\cos^2 x}\, dx \qquad \cos^2 x \text{は} \tan^2 x \\ &= \int \left(1 + \tan^2 x\right) \frac{dx}{\cos^2 x} \\ &= \frac{1}{3} \tan^3 x + \tan x + C\end{aligned}$$

$$\int_0^1 \frac{dx}{x^2+3} \quad x^2+a^2\text{を含む}\rightarrow x=a\tan t$$

$$x=\sqrt{3}\tan t\text{ とおく}$$

$$dx=\frac{dt}{\cos^2 t}$$

$$\begin{aligned}
\int_0^1 \frac{dx}{x^2+3} &= \int_0^{\frac{\pi}{6}} \frac{\frac{\sqrt{3}}{\cos^2 t}}{3\tan^2 t+3}\,dt \\
&= \int_0^{\frac{\pi}{6}} \frac{\frac{\sqrt{3}}{\cos^2 t}}{3\left(\frac{1}{\cos^2 t}\right)}\,dt \\
&= \frac{1}{\sqrt{3}}\int_0^{\frac{\pi}{6}} dt \\
&= \frac{\pi}{6\sqrt{3}}
\end{aligned}$$

x	$0 \rightarrow 1$
t	$0 \rightarrow \frac{\pi}{6}$

$\int x^2 \sin x \, dx \quad x^n f(x)$→(瞬間)部分積分

$$\int x^2 \sin x \, dx = x^2 \sin x + 2x \cos x - 2 \sin x + C$$

$+$	x^2	$\sin x$
$-$	$2x$	$-\cos x$
$+$	2	$-\sin x$

$\displaystyle\int \frac{dx}{\cos^3 x}$三角関数は2乗に強い, 三角関数や指数関数の分数関数, 分母を優先

→分母分子に同じもの$(\cos x)$をかける

$$
\begin{aligned}
\int \frac{dx}{\cos^3 x} &= \int \frac{\cos x}{\cos^4 x}\,dx \\
&= \int \frac{\cos x}{\left(1-\sin^2 x\right)^2}\,dx \\
&= \int \frac{dt}{\left(1-t^2\right)^2}\ (t=\sin x) \\
\frac{1}{(1-t^2)} &= \frac{1}{2}\left(\frac{1}{1+t}+\frac{1}{1-t}\right) \\
\left(\frac{1}{(1-t^2)}\right)^2 &= \left(\frac{1}{2}\left(\frac{1}{1+t}+\frac{1}{1-t}\right)\right)^2 \\
&= \frac{1}{4}\left(\frac{1}{(1+t)^2}+\frac{1}{(1-t)^2}+\frac{2}{1-t^2}\right) \\
&= \frac{1}{4}\left(\frac{1}{(1+t)^2}+\frac{1}{(1-t)^2}+\frac{1}{1+t}+\frac{1}{1-t}\right) \\
\int \frac{dt}{\left(1-t^2\right)^2} &= \frac{1}{4}\int\left(\frac{1}{(1+t)^2}+\frac{1}{(1-t)^2}+\frac{1}{1+t}+\frac{1}{1-}\right. \\
&= \frac{1}{4}\left(\frac{1}{1+t}-\frac{1}{1-t}+\log\left|\frac{1+t}{1-t}\right|\right)+C \\
&= \frac{1}{4}\left(\frac{1}{1+\sin x}-\frac{1}{1-\sin x}+\log\left|\frac{1+\sin x}{1-\sin x}\right.\right.
\end{aligned}
$$

$$\int \frac{dx}{\sin^4 x} \quad f(sin^2 x, \cos^2 x, \tan x) \rightarrow t = \tan x$$

$$\begin{aligned}
t &= \tan x \\
\cos^2 x &= \frac{1}{1+\tan^2 x} \\
&= \frac{1}{1+t^2} \\
\sin^2 x &= \cos^2 x \tan^2 x \\
&= \frac{t^2}{1+t^2} \\
dt &= \frac{dx}{\cos^2 x} \\
dx &= \frac{dt}{1+t^2} \\
\int \frac{dx}{\sin^4 x} &= \int \frac{\left(1+t^2\right)^2}{t^4} \cdot \frac{dt}{1+t^2} \\
&= \int \frac{1+t^2}{t^4}\, dt \\
&= \int \frac{dt}{t^4} + \int \frac{dt}{t^2} \\
&= -\frac{1}{3t^3} - \frac{1}{t} + C \\
&= -\frac{1}{3\tan^3 x} - \frac{1}{\tan x} + C
\end{aligned}$$

$\int_0^{2\pi} \sqrt{1+\cos x}\,dx \quad \sqrt{|\qquad|^2} = |\qquad|$としたい.

ここで, $1+\cos x = 2\cos^2\dfrac{x}{2}$である.

$$\begin{aligned}
&\int_0^{\frac{\pi}{3}} \sqrt{1+\cos x}\,dx \\
&\qquad = \int_0^{\frac{\pi}{3}} \sqrt{2\cos^2\frac{x}{2}}\,dx \\
&\qquad = \sqrt{2}\int_0^{\frac{\pi}{3}} \left|\cos\frac{x}{2}\right| dx \\
&\qquad = 2\sqrt{2}\left[\sin\frac{x}{2}\right]_0^{\frac{\pi}{3}} \\
&\qquad = \sqrt{2}
\end{aligned}$$

$\displaystyle\int_0^{\frac{\pi}{3}} \sqrt{1+\sin x}\,dx \quad \cos$だと都合がいい$\to\dfrac{\pi}{2}$

$$\begin{aligned}\int_0^{\frac{\pi}{3}} \sqrt{1+\sin x}\,dx &= \int_0^{\frac{\pi}{3}} \sqrt{1+\cos\left(\frac{\pi}{2}-x\right)}\,dx \\ &= \sqrt{2}\int_0^{\frac{\pi}{3}} \left|\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)\right| dx \\ &= -2\sqrt{2}\left[\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]_0^{\frac{\pi}{3}} \\ &= -2\sqrt{2}\left(\frac{\sqrt{6}-\sqrt{2}}{4}-\frac{1}{\sqrt{2}}\right) \\ &= 3-\sqrt{3}\end{aligned}$$

$$\begin{aligned}\int \frac{\cos^2 x}{\sin x-1}\,dx &= \int \frac{(1-\sin x)(1+\sin x)}{\sin x-1}\,dx \\ &= -\int (1+\sin x)\,dx \\ &= \cos x - x + C\end{aligned}$$

$$\begin{aligned}\int \cos x\sin x\cos 2x\,dx &= \frac{1}{2}\int \sin 2x\cos 2x \\ &= \frac{1}{4}\int \sin 4x\,dx \\ &= -\frac{1}{16}\cos 4x + C\end{aligned}$$

$$\begin{aligned}\int_0^{\frac{\pi}{2}} \sin 6x\cos 4x\,dx &= \frac{1}{2}\int_0^{\frac{\pi}{2}} (\sin 10x+\sin 2x)\,dx \\ &= \frac{1}{2}\left[-\frac{1}{10}\cos 10x-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}} \\ &= \frac{3}{5}\end{aligned}$$

$$\int \frac{\sin x \cos x}{1+\sin^2 x}\,dx = \frac{1}{2}\log(1+\sin^2 x) + C \quad (\because 1+\sin^2 x \geq 0)$$

$$\int_0^1 \frac{dx}{1+x^3}\text{分母が因数分解できる}\to BBB$$

$$\frac{1}{1+x^3} = a$$

$$\begin{aligned}\int \frac{dx}{\sqrt{x}+\sqrt{x+2}} &= \int \frac{\sqrt{x+2}-\sqrt{x}}{2}\,dx \\ &= \frac{1}{3}(x+2)^{\frac{3}{2}} - \frac{1}{3}x^{\frac{3}{2}} + C\end{aligned}$$

$$
\begin{aligned}
\int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\cos x}{1+\cos x}}\, dx &= \int_0^{\frac{\pi}{2}} \sqrt{\tan^2 \frac{x}{2}}\, dx \\
&= \int_0^{\frac{\pi}{2}} \left|\tan \frac{x}{2}\right| dx \\
&= -2\left[\log\left|\cos \frac{x}{2}\right|\right]_0^{\frac{\pi}{2}} \\
&= -2 \log \frac{1}{\sqrt{2}} \\
&= \log 2
\end{aligned}
$$

$$\int_0^1 \frac{dx}{1+x^3} \quad \text{分母が因数分解できる} \rightarrow BBB$$

$$\begin{aligned}
\int_0^1 \frac{dx}{1+x^3} &= \frac{1}{3}\int_0^1 \left(\frac{1}{1+x} - \frac{x-2}{x^2-x+1}\right) dx \\
\frac{1}{3}\int_0^1 \frac{dx}{1+x} &= \frac{1}{3}[\log|1+x|]_0^1 \\
&= \frac{1}{3}\log 2
\end{aligned}$$

$$\frac{1}{3}\int_0^1 \frac{x-2}{x^2-x+1}\,dx \quad \text{分母が2次式で分子が1次式→微分接触をつくる}$$

$$\begin{aligned}\frac{x-2}{x^2-x+1} &= \frac{1}{2}\left(\frac{2x-4}{x^2-x+1}\right)\\ &= \frac{1}{2}\left(\frac{2x-1}{x^2-x+1}-\frac{3}{x^2-x+1}\right)\end{aligned}$$

※　xの係数を定数倍で合わせないと修正項にxが残ってしまう

$$\frac{1}{3}\int_0^1 \frac{x-2}{x^2-x+1}\,dx = \frac{1}{6}\int_0^1 \frac{2x-1}{x^2-x+1} - \frac{1}{6}\int_0^1 \frac{3}{x^2-x+1}\,dx$$

$$\begin{aligned}\frac{1}{6}\int_0^1 \frac{2x-1}{x^2-x+1}\,dx &= \frac{1}{6}\big[\log\big|x^2-x+1\big|\big]_0^1\\ &= 0\end{aligned}$$

$$\frac{1}{2}\int_0^1 \frac{dx}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$\begin{aligned}x-\frac{1}{2} &= \frac{\sqrt{3}}{2}\tan t\\ dx &= \frac{\sqrt{3}}{2\cos^2 t}\,dt\end{aligned}$$

$$\begin{aligned}\frac{1}{2}\int_0^1 \frac{dx}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}} &= \frac{1}{2}\cdot\frac{4}{3}\cdot\frac{\sqrt{3}}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} dt\\ &= \frac{\pi}{3\sqrt{3}}\end{aligned}$$

$$\therefore \int_0^1 \frac{dx}{1+x^3} = \frac{1}{3}\log 2 + \frac{\pi}{3\sqrt{3}}$$

$$\begin{aligned}\int \frac{\cos^2 x}{\sin x - 1}\,dx &= \int \frac{(1-\sin x)(1+\sin x)}{\sin x - 1}\,dx\\ &= -\int (1+\sin x)\,dx\\ &= \cos x - x + C\end{aligned}$$

$$\begin{aligned}\int \cos x \sin x \cos 2x\,dx &= \frac{1}{2}\int \sin 2x \cos 2x\\ &= \frac{1}{4}\int \sin 4x\,dx\\ &= -\frac{1}{16}\cos 4x + C\end{aligned}$$

$$\begin{aligned}\int_0^{\frac{\pi}{2}} \sin 6x \cos 4x\,dx &= \frac{1}{2}\int_0^{\frac{\pi}{2}} (\sin 10x + \sin 2x)\,dx\\ &= \frac{1}{2}\left[-\frac{1}{10}\cos 10x - \frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}}\\ &= \frac{3}{5}\end{aligned}$$

$$\int_0^{\frac{\pi}{4}} \frac{\cos x}{\cos x + \sin x}\,dx$$

分数関数なので, 取り敢えずまず分母の微分をかんがえてみる.

$$(\cos x + \sin x)' = \cos x - \sin x$$

$$\int_0^{\frac{\pi}{4}} \frac{\cos x}{\cos x + \sin x}\,dx + \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x + \sin x}\,dx = \frac{\pi}{4}$$

$$\int_0^{\frac{\pi}{4}} \frac{\cos x}{\cos x + \sin x}\,dx = I, \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x + \sin x}\,dx = J \quad \text{とすると}$$

$$I + J = \frac{\pi}{4}$$

IとJを求めるために, IとJを組み合わせて他に積分しやすい形をつくりたい.

ここで, $(\cos x + \sin x)' = \cos x - \sin x$であるから, $I - J$が簡単に求められる.

$$I - J = \int_0^{\frac{\pi}{4}} \frac{\cos x}{\cos x + \sin x}\,dx - \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x + \sin x}\,dx$$

$$= [\log|\cos x + \sin x|]_0^{\frac{\pi}{4}}$$

$$= \left[\log|\sqrt{2}\sin(x + \frac{\pi}{4})|\right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2}\log 2$$

$$\therefore I = \frac{\pi}{8} + \frac{1}{4}\log 2$$

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$$\int \frac{dx}{\sqrt{x} + \sqrt{x+2}} = \int \frac{\sqrt{x+2} - \sqrt{x}}{2}\,dx$$

$$= \frac{1}{3}(x+2)^{\frac{3}{2}} - \frac{1}{3}x^{\frac{3}{2}} + C$$

$$
\begin{aligned}
\int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\cos x}{1+\cos x}}\,dx &= \int_0^{\frac{\pi}{2}} \sqrt{\tan^2 \frac{x}{2}}\,dx \\
&= \int_0^{\frac{\pi}{2}} \left|\tan \frac{x}{2}\right| dx \\
&= -2\left[\log\left|\cos \frac{x}{2}\right|\right]_0^{\frac{\pi}{2}} \\
&= -2\log \frac{1}{\sqrt{2}} \\
&= \log 2
\end{aligned}
$$

$\int_0^1 \frac{dx}{1+x^3}$　分母が因数分解できる→BBB

$$
\begin{aligned}
\int_0^1 \frac{dx}{1+x^3} &= \frac{1}{3}\int_0^1 \left(\frac{1}{1+x} - \frac{x-2}{x^2-x+1}\right) dx \\
\frac{1}{3}\int_0^1 \frac{dx}{1+x} &= \frac{1}{3}[\log|1+x|]_0^1 \\
&= \frac{1}{3}\log 2
\end{aligned}
$$

$-\frac{1}{3}\int_0^1 \frac{x-2}{x^2-x+1}\,dx$　分母が2次式で分子が1次式→微分接触をつくる

$$
\begin{aligned}
\frac{x-2}{x^2-x+1} &= \frac{1}{2}\left(\frac{2x-4}{x^2-x+1}\right) \\
&= \frac{1}{2}\left(\frac{2x-1}{x^2-x+1} - \frac{3}{x^2-x+1}\right)
\end{aligned}
$$

※ 定数項ではなくxの係数を定数倍で合わせる

$$-\frac{1}{3}\int_0^1 \frac{x-2}{x^2-x+1}\,dx = -\frac{1}{6}\int_0^1 \frac{2x-1}{x^2-x+1} + \frac{1}{6}\int_0^1 \frac{3}{x^2-x+1}\,dx$$

$$\frac{1}{6}\int_0^1 \frac{2x-1}{x^2-x+1}\,dx = \frac{1}{6}\left[\log|x^2-x+1|\right]_0^1$$

$$= 0$$

$$※\int_0^1 \frac{2x-1}{x^2-x+1}\,dx = \int_0^1 \frac{2(1-x)-1}{(1-x)^2-(1-x)+1}\,dx$$

$$= \int_0^1 \frac{-2x+1}{x^2-x+1}\,dx$$

$$\therefore I = -I$$

$$2I = 0$$

$$\therefore \int_0^1 \frac{2x-1}{x^2-x+1}\,dx = 0$$

申し訳ないが解説不能.$f(x) = x^2 - x + 1 = f(1-x)$であることを

$\frac{1}{6}\int \frac{3}{x^2-x+1}$　分子が定数で, 分母が$D<0$の2次式→平方完成&$(x-p)=a\tan t$

$$
\begin{aligned}
\frac{1}{2}\int_0^1 \frac{dx}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}} \\
x-\frac{1}{2} &= \frac{\sqrt{3}}{2}\tan t \\
dx &= \frac{\sqrt{3}}{2\cos^2 t}\,dt \\
\frac{1}{2}\int_0^1 \frac{dx}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}} &= \frac{1}{2}\cdot\frac{4}{3}\cdot\frac{\sqrt{3}}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} dt \\
&= \frac{\pi}{3\sqrt{3}} \\
\therefore \int_0^1 \frac{dx}{1+x^3} &= \frac{1}{3}\log 2+\frac{\pi}{3\sqrt{3}}
\end{aligned}
$$

$$
\begin{aligned}
I &= \int e^x \cos x \, dx \\
&= e^x \sin x - \int e^x \sin x \, dx \\
&= e^x \sin x - \left(e^x \cdot - \cos x - \int e^x \cdot - \cos x \, dx \right) \\
&= e^x \sin x + e^x \cos x - \int e^x \cos x \, dx \\
2I &= e^x \sin x + e^x \cos x \\
I &= \frac{e^x}{2} (\sin x + \cos x) + C
\end{aligned}
$$