

$$\begin{aligned}
& \int \tan x \, dx \\
& \int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3-x^2} \, dx \\
& \int \log x \, dx \\
& \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\log(\sin x)}{\tan x} \, dx \\
& \int e^x \cos x \, dx \\
& \int \frac{dx}{\sin^3 x} \\
& \int \tan^4 x \, dx \\
& \int \frac{dx}{\sin^2 x} \\
& \int \sqrt{1-x} \, dx \\
& \int_0^{\frac{\pi}{2}} \cos^2 x \, dx \\
& \int x^2 \sin x \, dx \\
& \int_0^{2\pi} \sqrt{1+\cos x} \, dx \\
& \int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\cos x}{1+\cos x}} \, dx \\
& \int_0^{\frac{\pi}{3}} \sqrt{1+\sin x} \, dx \\
& \int_0^1 \frac{dx}{1+x^2}
\end{aligned}$$

$$\begin{aligned}
& \int_0^1 \frac{dx}{3+x^2} \\
& \int_0^1 \frac{dx}{x^2+x+1} \\
& \int x^x(1+\log x) dx \\
& \int \frac{dx}{\sqrt{1+x^2}} \\
& \int_2^3 \frac{dx}{\sqrt{x^2-1}} \\
& \int \sqrt{1-e^{-2x}} dx \\
& \int_0^1 \frac{dx}{1+x^3} \\
& \int \frac{dx}{\sin^4 x} \\
& \int \frac{dx}{\sqrt{x}+\sqrt{x+2}} \\
& \int_0^e \frac{e^x}{e^{e-x}+e^x} dx \\
& \int_{-1}^1 \sqrt{1-x^2} dx \\
& \int \sqrt{1-e^{-2x}} dx \\
& \int \tan^2 x dx \\
& \int \tan^3 x dx \\
& \int \tan^5 x dx
\end{aligned}$$

*KingProperty*

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$t = a + b - x$  と置換する

$$dx = -dt$$

$$\therefore \int_b^a -f(t) dt = \int_a^b f(t) dt$$
$$= \int_a^b f(x) dx \quad (\because \text{定積分では変数を変更してもよい})$$

因って、 $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

$x$	$a \rightarrow b$
$t$	$b \rightarrow a$

$$\begin{aligned}
& \int \tan x \, dx \quad \tan \text{ の一次式} \rightarrow \frac{\sin x}{\cos x} \text{ で一次式に} \\
& \int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx \quad \text{分数関数はまず最初に} \rightarrow \text{分母の微分を考える} \\
& \quad = - \int \frac{-\sin x}{\cos x} \, dx \\
& \quad = -\log|\cos x| + C
\end{aligned}$$

$$\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3-x^2} dx \quad \sqrt{a^2-x^2} \text{を含む} \rightarrow x = a \sin t \text{とおく}$$

$$x = \sqrt{3} \sin t \text{ とする}$$

$$dx = \sqrt{3} \cos t dt$$

$$\begin{aligned} \int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3-x^2} dx &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{3 - (\sqrt{3} \sin t)^2} \sqrt{3} \cos t dt \\ &= 3 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 - \sin^2 t} \cos t dt \\ &= 3 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\cos t| \cos t dt \\ &= 3 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t dt \left( \because -\frac{\pi}{2} \leq t \leq \frac{\pi}{2} \Rightarrow 0 \leq \forall \cos t \right) \\ &= \frac{3}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 + \cos 2t dt \left( \because \cos^2 x = \frac{1 + \cos 2x}{2} \quad c.f. \sin^2 x = \frac{1 - \cos 2x}{2} \right) \\ &= \frac{3}{2} \left[ t + \frac{1}{2} \sin 2t \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\ &= 3 \left[ t + \frac{1}{2} \sin 2t \right]_0^{\frac{\pi}{2}} \\ &= \frac{3}{2} \pi \end{aligned}$$

$x$	$-\sqrt{3} \rightarrow \sqrt{3}$
$t$	$-\frac{\pi}{2} \rightarrow \frac{\pi}{2}$

$$\begin{aligned}
& \int \frac{dx}{\sin^2 x} dx \quad \sin \text{か} \cos \text{だったら都合がいい} \rightarrow \frac{\pi}{2} \text{で} \cos \text{に} \\
& \int \frac{dx}{\sin^2 x} = \int \frac{dx}{\cos^2\left(\frac{\pi}{2} - x\right)} \\
& \quad = -\tan\left(\frac{\pi}{2} - x\right) + C \\
& \quad = -\frac{1}{\tan x} + C
\end{aligned}$$

$$\int_{-1}^1 \sqrt{1-x^2} dx \quad \sqrt{(\quad)^2} = |\quad| \text{ として }$$

ここで,  $1 - \sin^2 t = \cos^2 t$  である

また,  $1 - x^2 \geq 0 \Leftrightarrow -1 \leq x \leq 1$  であるから,  $x = \sin t$  とおける

$$x = \sin t \text{ とする}$$

$$dx = \cos t dt$$

$$\begin{aligned} \int_{-1}^1 \sqrt{1-x^2} dx &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1-\sin^2 t} \cos t dt \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\cos t| \cos t dt \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t dt \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1 + \cos 2t}{2} dt \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left( \frac{1}{2} + \frac{\cos 2t}{2} \right) dt \\ &= \frac{1}{2} \left[ t + \frac{1}{2} \sin 2t \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\ &= \frac{\pi}{2} \end{aligned}$$

$x$	$-1 \rightarrow 1$
$t$	$-\frac{\pi}{2} \rightarrow \frac{\pi}{2}$

$$\int_0^1 \frac{dx}{1+x^2} \quad \text{分母を単項式にしたい}$$

$$\text{ここで, } 1 + \tan^2 t = \frac{1}{\cos^2 t} \text{ である}$$

$$\text{また, } (\tan t)' = \frac{1}{\cos^2 t} \text{ であるから, } x = \tan t \text{ とする}$$

$$1 + x^2 = \frac{1}{\cos^2 t}$$

$$dx = \frac{dt}{\cos^2 t}$$

$$\int_0^1 \frac{dx}{1+x^2} = \int_0^{\frac{\pi}{4}} \frac{\frac{1}{\cos^2 t}}{\frac{1}{\cos^2 t}} dt$$

$$= \int_0^{\frac{\pi}{4}} dt$$

$$= \frac{\pi}{4}$$

$x$	$0 \rightarrow 1$
$t$	$0 \rightarrow \frac{\pi}{4}$



$$\begin{aligned}
\int \tan^4 x \, dx & \quad \text{三角関数は2次ごとに分割} \\
&= \int \tan^2 x \tan^2 x \, dx \quad \tan^2 \text{ は } \cos^2 \\
&= \int \tan^2 x \left( \frac{1}{\cos^2 x} - 1 \right) dx \\
&= \int \frac{\tan^2 x}{\cos^2 x} dx - \int \tan^2 x \, dx \\
&= \frac{1}{3} \tan^3 x - \tan x + x + C
\end{aligned}$$

$$\begin{aligned}
\int \log x \, dx & \quad \log \text{ を含む} \rightarrow \text{部分積分} \\
&= \int 1 \cdot \log x \, dx \\
&= x \log x - \int x \cdot \frac{1}{x} \, dx \\
&= x \log x - x + C
\end{aligned}$$

$$\int_0^e \frac{e^x}{e^{e-x} + e^x} dx = \int_0^e \frac{e^x}{e^{e-x} + e^x} dx$$

$a + b - x$ を含む, 下端が0, 指数関数の式→KingProperty

$$\begin{aligned} &= \int_0^e \frac{e^{e-x}}{e^x + e^{e-x}} dx \\ I &= \int_0^e \frac{e^x}{e^{e-x} + e^x} dx \text{ とすると} \\ 2I &= \int_0^e \frac{e^x}{e^{e-x} + e^x} dx + \int_0^e \frac{e^{e-x}}{e^x + e^{e-x}} dx \\ &= \int_0^e \frac{e^x + e^{e-x}}{e^x + e^{e-x}} dx \\ &= \int_0^e dx \\ &= e \\ \therefore I &= \frac{e}{2} \end{aligned}$$

$$\int_0^e \frac{e^x}{e^{e-x} + e^x} dx = \int_0^e \frac{e^x}{e^e \cdot e^{-x} + e^x} dx$$

指数関数の定数部分は分離

指数の符号は揃える, 指数関数の分数関数→分母分子に $e^x$ をかける

指数関数の分数関数→微分接触をつくる

$$\begin{aligned} &= \frac{1}{2} \int_0^e \frac{2e^{2x}}{e^e + e^{2x}} dx \\ &= \frac{1}{2} [\log(e^e + e^{2x})]_0^e \quad (\because e^e + e^{2x} \geq 0) \\ &= \frac{1}{2} \log \frac{e^e + e^{2e}}{e^e + 1} \\ &= \frac{1}{2} \log \frac{e^e(1 + e^e)}{e^e + 1} \\ &= \frac{e}{2} \end{aligned}$$

$$\begin{aligned}
\int \frac{dx}{\sin^4 x} &= \int \frac{dx}{\sin^2 x \cos^2 x} = \int \frac{dx}{\tan^2 x \cos^4 x} \\
&= \int \frac{1}{\tan^4 x} (1 + \tan^2 x) \frac{dx}{\cos^2 x} \\
&\quad t = \tan x \text{ とする} \\
&\quad dt = \frac{dx}{\cos^2 x} \\
\int \frac{1}{\tan^4 x} (1 + \tan^2 x) \frac{dx}{\cos^2 x} &= \int \frac{1}{t^4} (1 + t^2) dt \\
&= -\frac{1}{3t^3} - \frac{1}{t} + C \\
&= -\frac{1}{3 \tan^3 x} - \frac{1}{\tan x} + C
\end{aligned}$$

$$\begin{aligned}
& \int \tan^3 x \, dx \quad \text{三角関数は2次ごとに分割} \\
& \int \tan^3 x \, dx = \int \tan x \cdot \tan^2 x \, dx \quad \tan^2 x \text{は } \cos^2 x \\
& \quad = \int \tan x \left( \frac{1}{\cos^2 x} - 1 \right) dx \\
& \quad = \int \frac{\tan x}{\cos^2 x} dx - \int \tan x \, dx \\
& \quad = \frac{1}{2} \tan^2 x + \log|\cos x| + C
\end{aligned}$$

$$\int \tan^2 x \, dx = \int \frac{\tan^2 x}{\cos^2 x} \, dx$$

$$\int \tan^2 x \, dx = \int \left( \frac{1}{\cos^2 x} - 1 \right) dx$$

$$= \tan x - x + C$$

$\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$  積分区間の下端が0, 三角関数の式で積分区間が $\frac{n\pi}{2} \rightarrow KingProperty$

$$\begin{aligned}
 I &= \int_0^{\frac{\pi}{2}} \sin^2 x \, dx \\
 &= \int_0^{\frac{\pi}{2}} \cos^2 x \, dx \\
 2I &= \int_0^{\frac{\pi}{2}} dx \\
 &= \frac{\pi}{2} \\
 \therefore I &= \frac{\pi}{4}
 \end{aligned}$$

$$\int \frac{\log(\sin x)}{\tan x} dx$$

$\tan$  の一次式  $\rightarrow \frac{\sin}{\cos}$ , 真数, 位相が複雑

$\rightarrow = t$  として微分接触を疑う

$$\int \frac{\log(\sin x)}{\tan x} dx = \int \frac{\cos x \log(\sin x)}{\sin x} dx$$

$$= \int \frac{\log t}{t} dt$$

$$= \frac{1}{2}(\log t)^2 + C$$

$$= \frac{1}{2}(\log(\sin x))^2 + C$$



$$\begin{aligned}
& \int \frac{dx}{\sqrt{1+x^2}} \\
& \quad x = \sinh t \\
& \quad dx = \cosh t \, dt \\
& \int \frac{dx}{\sqrt{1+x^2}} = \int \frac{1}{\sqrt{1+\sinh^2 t}} \cosh t \, dt \\
& \quad = \int dt \\
& \quad = t + C \\
& \quad = \log(x + \sqrt{1+x^2}) + C
\end{aligned}$$

$$\begin{aligned}
\int \tan^5 x \, dx &= \int \tan^3 x \left( \frac{1}{\cos^2 x} - 1 \right) dx \\
&= \frac{1}{4} \tan^4 x - \frac{1}{2} \tan^2 x + \log|\cos x| + C
\end{aligned}$$

$$\int \frac{dx}{\cos^4 x} \quad \text{三角関数は2乗ごとに分割}$$

$$\begin{aligned} \int \frac{dx}{\cos^4 x} &= \int \frac{1}{\cos^2 x} \cdot \frac{1}{\cos^2 x} dx \quad \cos^2 x \text{は } \tan^2 x \\ &= \int (1 + \tan^2 x) \frac{dx}{\cos^2 x} \\ &= \frac{1}{3} \tan^3 x + \tan x + C \end{aligned}$$

$$\int_0^1 \frac{dx}{x^2 + 3} \quad x^2 + a^2 \text{ を含む } \rightarrow x = a \tan t$$

$$x = \sqrt{3} \tan t \text{ とおく}$$

$$dx = \frac{\sqrt{3}dt}{\cos^2 t}$$

$$\int_0^1 \frac{dx}{x^2 + 3} = \int_0^{\frac{\pi}{6}} \frac{\frac{\sqrt{3}}{\cos^2 t}}{3 \tan^2 t + 3} dt$$

$$= \int_0^{\frac{\pi}{6}} \frac{\frac{\sqrt{3}}{\cos^2 t}}{3 \left( \frac{1}{\cos^2 t} \right)} dt$$

$$= \frac{1}{\sqrt{3}} \int_0^{\frac{\pi}{6}} dt$$

$$= \frac{\pi}{6\sqrt{3}}$$

$x$	$0 \rightarrow 1$
$t$	$0 \rightarrow \frac{\pi}{6}$

$$\int x^2 \sin x \, dx \quad x^n f(x) \rightarrow (\text{瞬間}) \text{部分積分}$$

$$\int x^2 \sin x \, dx = x^2 \sin x + 2x \cos x - 2 \sin x + C$$

$$\begin{array}{rcl} + & x^2 & \sin x \\ - & 2x & - \cos x \\ + & 2 & - \sin x \end{array}$$

$\int \frac{dx}{\cos^3 x}$  三角関数は2乗に強い, 三角関数や指数関数の分数関数

→分母分子に同じもの( $\cos x$ )をかける(分母を優先)

$$\begin{aligned}
 \int \frac{dx}{\cos^3 x} &= \int \frac{\cos x}{\cos^4 x} dx \\
 &= \int \frac{\cos x}{(1 - \sin^2 x)^2} dx \\
 &= \int \frac{dt}{(1 - t^2)^2} \quad (t = \sin x) \\
 \frac{1}{1 - t^2} &= \frac{1}{2} \left( \frac{1}{1 + t} + \frac{1}{1 - t} \right) \\
 \frac{1}{(1 - t^2)^2} &= \left( \frac{1}{2} \left( \frac{1}{1 + t} + \frac{1}{1 - t} \right) \right)^2 \\
 &= \frac{1}{4} \left( \frac{1}{(1 + t)^2} + \frac{1}{(1 - t)^2} + \frac{2}{1 - t^2} \right) \\
 &= \frac{1}{4} \left( \frac{1}{(1 + t)^2} + \frac{1}{(1 - t)^2} + \frac{1}{1 + t} + \frac{1}{1 - t} \right) \\
 \int \frac{dt}{(1 - t^2)^2} &= \frac{1}{4} \int \left( \frac{1}{(1 + t)^2} + \frac{1}{(1 - t)^2} + \frac{1}{1 + t} + \frac{1}{1 - t} \right) dt \\
 &= \frac{1}{4} \left( \frac{1}{1 - t} - \frac{1}{1 + t} + \log \left| \frac{t + 1}{t - 1} \right| \right) + C \\
 &= \frac{1}{4} \left( \frac{1}{1 - \sin x} - \frac{1}{1 + \sin x} + \log \left| \frac{\sin x + 1}{\sin x - 1} \right| \right) + C
 \end{aligned}$$

$$\int \frac{dx}{\sin^4 x} \quad f(\sin^2 x, \cos^2 x, \tan x) \rightarrow t = \tan x$$

$$t = \tan x$$

$$\cos^2 x = \frac{1}{1 + \tan^2 x}$$

$$= \frac{1}{1 + t^2}$$

$$\sin^2 x = \cos^2 x \tan^2 x$$

$$= \frac{t^2}{1 + t^2}$$

$$dt = \frac{dx}{\cos^2 x}$$

$$dx = \frac{dt}{1 + t^2}$$

$$\int \frac{dx}{\sin^4 x} = \int \frac{(1 + t^2)^2}{t^4} \cdot \frac{dt}{1 + t^2}$$

$$= \int \frac{1 + t^2}{t^4} dt$$

$$= \int \frac{dt}{t^4} + \int \frac{dt}{t^2}$$

$$= -\frac{1}{3t^3} - \frac{1}{t} + C$$

$$= -\frac{1}{3 \tan^3 x} - \frac{1}{\tan x} + C$$

$$\int_0^{2\pi} \sqrt{1 + \cos x} \, dx = \int_0^{2\pi} \sqrt{2 \cos^2 \frac{x}{2}} \, dx = 2 \int_0^{2\pi} \left| \cos \frac{x}{2} \right| \, dx \quad \text{としたい.}$$

ここで,  $1 + \cos x = 2 \cos^2 \frac{x}{2}$  である.

$$\begin{aligned} \int_0^{\frac{\pi}{3}} \sqrt{1 + \cos x} \, dx &= \int_0^{\frac{\pi}{3}} \sqrt{2 \cos^2 \frac{x}{2}} \, dx \\ &= \sqrt{2} \int_0^{\frac{\pi}{3}} \left| \cos \frac{x}{2} \right| \, dx \\ &= 2\sqrt{2} \left[ \sin \frac{x}{2} \right]_0^{\frac{\pi}{3}} \\ &= \sqrt{2} \end{aligned}$$



$$\int_0^{\frac{\pi}{3}} \sqrt{1 + \sin x} \, dx \quad \sin \text{が} \cos \text{だと都合がいい} \rightarrow \frac{\pi}{2} \text{で} \cos \text{に}$$

$$\begin{aligned} \int_0^{\frac{\pi}{3}} \sqrt{1 + \sin x} \, dx &= \int_0^{\frac{\pi}{3}} \sqrt{1 + \cos\left(\frac{\pi}{2} - x\right)} \, dx \\ &= \sqrt{2} \int_0^{\frac{\pi}{3}} \left| \cos\left(\frac{\pi}{4} - \frac{x}{2}\right) \right| \, dx \\ &= -2\sqrt{2} \left[ \sin\left(\frac{\pi}{4} - \frac{x}{2}\right) \right]_0^{\frac{\pi}{3}} \\ &= -2\sqrt{2} \left( \frac{\sqrt{6} - \sqrt{2}}{4} - \frac{1}{\sqrt{2}} \right) \\ &= 3 - \sqrt{3} \end{aligned}$$

$$= - \int (1 + \sin x) \, dx$$

$$= \cos x - x + C$$

$$= \frac{1}{4} \int \sin 4x \, dx$$

$$= -\frac{1}{16} \cos 4x + C$$

$$\begin{aligned} \int_0^{\frac{\pi}{2}} \sin 6x \cos 4x \, dx &= \frac{1}{2} \int_0^{\frac{\pi}{2}} (\sin 10x + \sin 2x) \, dx \\ &= \frac{1}{2} \left[ -\frac{1}{10} \cos 10x - \frac{1}{2} \cos 2x \right]_0^{\frac{\pi}{2}} \\ &= \frac{3}{5} \end{aligned}$$

$$\int \frac{\sin x \cos x}{1 + \sin^2 x} dx = \frac{1}{2} \log(1 + \sin^2 x) + C \quad (\because 1 + \sin^2 x \geq 0)$$

$$\int_0^1 \frac{dx}{1+x^3} \text{ 分母が因数分解できる} \rightarrow BBB$$

$$\frac{1}{1+x^3} = a$$

$$\begin{aligned} \int \frac{dx}{\sqrt{x} + \sqrt{x+2}} &= \int \frac{\sqrt{x+2} - \sqrt{x}}{2} dx \\ &= \frac{1}{3}(x+2)^{\frac{3}{2}} - \frac{1}{3}x^{\frac{3}{2}} + C \end{aligned}$$

$$\begin{aligned}
\int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\cos x}{1+\cos x}} dx &= \int_0^{\frac{\pi}{2}} \sqrt{\tan^2 \frac{x}{2}} dx \\
&= \int_0^{\frac{\pi}{2}} \left| \tan \frac{x}{2} \right| dx \\
&= -2 \left[ \log \left| \cos \frac{x}{2} \right| \right]_0^{\frac{\pi}{2}} \\
&= -2 \log \frac{1}{\sqrt{2}} \\
&= \log 2
\end{aligned}$$

$$\int_0^1 \frac{dx}{1+x^3} \quad \text{分母が因数分解できる} \rightarrow BBB$$

$$\begin{aligned} \int_0^1 \frac{dx}{1+x^3} &= \frac{1}{3} \int_0^1 \left( \frac{1}{1+x} - \frac{x-2}{x^2-x+1} \right) dx \\ \frac{1}{3} \int_0^1 \frac{dx}{1+x} &= \frac{1}{3} [\log|1+x|]_0^1 \\ &= \frac{1}{3} \log 2 \end{aligned}$$

$$\frac{1}{3} \int_0^1 \frac{x-2}{x^2-x+1} dx \quad \text{分母が2次式で分子が1次式} \rightarrow \text{微分接触をつくる}$$

$$\frac{x-2}{x^2-x+1} = \frac{1}{2} \left( \frac{2x-4}{x^2-x+1} \right) = \frac{1}{2} \left( \frac{2x-1}{x^2-x+1} - \frac{3}{x^2-x+1} \right)$$

※ $x$ の係数を定数倍で合わせないと修正項に $x$ が残ってしまう

$$\begin{aligned} \frac{1}{3} \int_0^1 \frac{x-2}{x^2-x+1} dx &= \frac{1}{6} \int_0^1 \frac{2x-1}{x^2-x+1} - \frac{1}{6} \int_0^1 \frac{3}{x^2-x+1} dx \\ \frac{1}{6} \int_0^1 \frac{2x-1}{x^2-x+1} dx &= \frac{1}{6} [\log|x^2-x+1|]_0^1 \\ &= 0 \end{aligned}$$

$$\frac{1}{2} \int_0^1 \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$x - \frac{1}{2} = \frac{\sqrt{3}}{2} \tan t$$

$$dx = \frac{\sqrt{3}}{2 \cos^2 t} dt$$

$$\frac{1}{2} \int_0^1 \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}} = \frac{1}{2} \cdot \frac{4}{3} \cdot \frac{\sqrt{3}}{2} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} dt$$

$$= \frac{\pi}{3\sqrt{3}}$$

$$\therefore \int_0^1 \frac{dx}{1+x^3} = \frac{1}{3} \log 2 + \frac{\pi}{3\sqrt{3}}$$

$$\begin{aligned}
I &= \int e^x \cos x \, dx \\
&= e^x \sin x - \int e^x \sin x \, dx \\
&= e^x \sin x - \left( e^x \cdot -\cos x - \int e^x \cdot -\cos x \, dx \right) \\
&= e^x \sin x + e^x \cos x - \int e^x \cos x \, dx \\
2I &= e^x \sin x + e^x \cos x \\
I &= \frac{e^x}{2} (\sin x + \cos x) + C
\end{aligned}$$