Ficha 11

1.

$$\begin{tikzcd} A^* \arrow[r, shift right, "out"'] & 1 + A \times A^* \arrow[l, shift right, "in"'] \\ A^* \arrow[u, "r"] \arrow[r, "g"'] & 1 + A \times A^* \arrow[u, "Fr"', "id + id \times r"] \end{tikzcd}$$

$\boxed{r = reverse}$

2.

$(3 \ominus 2) \ominus 3$

$= (1 + (3 \ominus (2+1))) \ominus 3$

$= (1 + (3 \ominus 3)) \ominus 3$

$= (1 + 0) \ominus 3$

$= 1 \ominus 3$

$= 0$

$(3 \ominus 4) + 4$

$= 0 + 4$

$= 4$

$$\begin{tikzcd} \mathbb{N}_0 \arrow[r, shift right, "out"'] & 1 + \mathbb{N}_0 \arrow[l, shift right, "in"'] \\ \mathbb{N}_0 \times \mathbb{N}_0 \arrow[u, "\hat{\ominus}"] \arrow[r, "g"'] & 1 + \mathbb{N}_0 \times \mathbb{N}_0 \arrow[u, "F\hat{\ominus}"', "id + \hat{\ominus}"] \end{tikzcd}$$

$x \ominus y = \text{if } x \leq y \text{ then } 0 \text{ else } 1 + x \ominus (y+1)$

$\Leftrightarrow \hat{\ominus}(x,y) = \text{if } (\hat{\leq})(x,y) \text{ then } \underline{0}(x,y) \text{ else } succ(\hat{\ominus}(x, succ\, y))$ {86, 75}

$\Leftrightarrow \hat{\ominus}(x,y) = \text{if } (\hat{\leq})(x,y) \text{ then } \underline{0}(x,y) \text{ else } succ \cdot \hat{\ominus} \cdot (id \times succ)(x,y)$ {78, 74, 73}

$\Leftrightarrow \hat{\ominus} = (\hat{\leq}) \rightarrow \underline{0}, succ \cdot \hat{\ominus} \cdot (id \times succ)$ {82, 72}

$\Leftrightarrow \hat{\ominus} = [\underline{0}, succ \cdot \hat{\ominus} \cdot (id \times succ)] \cdot (\hat{\leq})?$ {30}

$\Leftrightarrow \hat{\ominus} = [\underline{0} \cdot !, succ \cdot \hat{\ominus} \cdot (id \times succ)] \cdot (\hat{\leq})?$ {3}

$\Leftrightarrow \hat{\ominus} = in \cdot (! + \cancel{succ}\ \hat{\ominus} \cdot (id \times succ)) \cdot (\hat{\leq})?$ {22}

$\Leftrightarrow out \cdot \hat{\ominus} = (id \cdot !) + (\hat{\ominus} \cdot (id \times succ)) \cdot (\hat{\leq})?$ {34, 1}

$\Leftrightarrow out \cdot \hat{\ominus} = F\hat{\ominus} \cdot (! + (id \times succ)) \cdot (\hat{\leq})?$ {25}

$\Leftrightarrow out \cdot \hat{\ominus} = F\hat{\ominus} \cdot (\hat{\leq} \rightarrow i_1 \cdot !, i_2 \cdot (id \times succ))$ {30, 21}

$\Leftrightarrow \hat{\ominus} = [\!(\hat{\leq} \rightarrow i_1 \cdot !, i_2 \cdot (id \times succ))\!]$ {55}

$\boxed{g(x,y) = \text{if } x \leq y \text{ then } i_1() \text{ else } i_2(x, y+1)}$

3.

$$\begin{tikzcd} TA & B(A, TA) \arrow[l, "in"'] \arrow[d, "B(f, Tf)"] \\ TA' \arrow[from=u, "Tf"'] & B(A', TA') \arrow[l, "in"] \end{tikzcd}$$

$\boxed{Tf \cdot in = in \cdot B(f, Tf)}$

Prop. Natural in

$T f \cdot in = in \cdot B(f, T f)$

Para listas, temos:

$(map\ f) \cdot [nil, cons] = [nil, cons] \cdot (id + f \times (map\ f))$

$\Rightarrow \begin{cases} (map\ f) \cdot nil = nil \\ (map\ f) \cdot cons = cons \cdot (f \times (map\ f)) \end{cases}$

{20, 22, 1, 27}

$\Leftrightarrow \begin{cases} map\ f\ [\,] = [\,] \\ map\ f\ (h:t) = (f\ h) : (map\ f\ t) \end{cases}$

Obtemos, portanto, a definição de map f em listas.

4.

$B\_tree\ A \cong B(A, B\_tree\ A)$

$B\_tree\ A \cong 1 + B\_tree\ A \times (A \times B\_tree\ A)^*$

$B(A, Y) = 1 + Y \times (A \times Y)^*$

$B(f, g) = id + g \times (f \times g)^*$

$in = [\underline{Nil}, \hat{Block}]$

5.

Aplicando a heurística, temos:

divide $(x : y : xs)$

$\begin{cases} x > y = i_2\,(y, (x : xs)) \\ \text{otherwise} = i_2\,(x, (y : xs)) \end{cases}$

divide $x = i_1\, x$

conquer $= [id, cons]$

bubble $= [\![\text{conquer}, \text{divide}]\!]$

~~[illegible]~~

~~[illegible]~~.

$$\begin{tikzcd}
A^* \arrow[r, "divide"] \arrow[d, "{[\!(divide)\!]}"'] & A^* + A \times A^* \arrow[d, "{id + id \times [\!(divide)\!]}"] \\
A^* \arrow[d, "{(\!|conquer|\!)}"'] & A^* + A \times A^* \arrow[l, "{[id, cons]}"] \arrow[d, "{id + id \times (\!|conquer|\!)}"] \\
A^* & A^* + A \times A^* \arrow[l, "{[id, cons]}"]
\end{tikzcd}$$

$$A^* \underset{out}{\overset{in = [id, cons]}{\rightleftarrows}} A^* + A \times A^*$$

out $(h:t)$

$\begin{cases} \text{length } t > 0 = i_2\,(h, t) \\ \text{otherwise} = i_1\,(h:t) \end{cases}$

6.

$$\text{while } p\ f\ g = \text{tailr}\ ((g+f)\cdot(\neg\cdot p)?) \quad \{F1\}$$
$$= [\![\text{join}, ((g+f)\cdot(\neg\cdot p)?)]\!] \quad \{F2\}$$
$$= \text{join}\cdot F(\text{while } p\ f\ g)\cdot((g+f)\cdot(\neg\cdot p)?) \quad \{\text{Enunciado}\}$$
$$= [id, id]\cdot(id + (\text{while } p\ f\ g))\cdot((g+f)\cdot(\neg\cdot p)?) \quad \{\text{def } F\}$$
$$= [id, \text{while } p\ f\ g]\cdot((g+f)\cdot(\neg\cdot p)?) \quad \{22, 1\}$$
$$= [g, (\text{while } p\ f\ g)\cdot f]\cdot(\neg\cdot p)? \quad \{22, 1\}$$
$$= (\neg\cdot p) \to g, (\text{while } p\ f\ g)\cdot f \quad \{30\}$$

Introduzindo variáveis, temos: $\{72, 82, 73\}$

$$\boxed{\text{while } p\ f\ g\ x = \text{if not}(p\,x) \text{ then } g\,x \text{ else while } p\ f\ g\ (f\,x)}$$

7.

$$(\text{tailr}\ g)\cdot f = \text{tailr}\ h$$
$$\equiv \{\text{Def tailr } f = [\![\text{join}, f]\!]\ (F2),\ \text{join} = \nabla,\ [\![f, g]\!] = (\!| f |\!)\cdot[\![g]\!]\}$$
$$(\!| \nabla |\!)\cdot[\![g]\!]\cdot f = (\!| \nabla |\!)\cdot[\![h]\!]$$
$$\Leftarrow \{5\}$$
$$[\![g]\!]\cdot f = [\![h]\!]$$
$$\Leftarrow \{58\}$$
$$g\cdot f = F f\cdot h$$
$$\equiv \{F f = id + f\}$$
$$g\cdot f = (id + f)\cdot h$$