

1. { Def connect, sum }

{ 52 }

{ 49 }

{ 20, 22, 1, 27 }

{ 72, 73, 75, length [ ] = 0, 14, 1 }

{ 72, 73, 75, length (x+y) = length x + length y }

2.

$$Tf = \text{in} \cdot B(f, \text{id})$$

$$\Rightarrow \text{fmap } f = \text{in} \cdot B(f, \text{id})$$

{ fmap f = Tf }

$$\Rightarrow (\text{fmap } f) \cdot \text{in} = \text{in} \cdot B(f, \text{id}) \cdot F(\text{fmap } f)$$

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Introduzindo por BTree, temos:

$$(\text{fmap } f) \cdot [\text{Empty}, \text{Node}] = [\text{Empty}, \text{Node}] \cdot (\text{id} + f \times \text{id}^2) \cdot (\text{id} + \text{id} \times \text{fmap } f)$$

$$\Rightarrow \begin{cases} (\text{fmap } f) \cdot \text{Empty} = \text{Empty} & \{ 20, 22, 1, 27 \} \\ (\text{fmap } f) \cdot \text{Node} = \text{Node} \cdot (f \times \text{id}^2) \cdot (\text{id} \times (\text{fmap } f)^2) \end{cases}$$

$$\Rightarrow \begin{cases} (\text{fmap } f) \cdot \text{Empty} = \text{Empty} & \{ 14, 1 \} \\ (\text{fmap } f) \cdot \text{Node} = \text{Node} \cdot (f \times (\text{id}^2 \cdot (\text{fmap } f)^2)) \end{cases}$$

$$\Rightarrow \begin{cases} (\text{fmap } f) \cdot \text{Empty} = \text{Empty} & \{ 14, 1 \} \\ (\text{fmap } f) \cdot \text{Node} = \text{Node} \cdot (f \times (\text{fmap } f)^2) \end{cases}$$

$$\Rightarrow \begin{cases} (\text{fmap } f) \cdot \text{Empty} = \text{Empty} & \{ 72, 73, 75, 78 \} \\ (\text{fmap } f) \cdot \text{Node } (a, (x, y)) = \text{Node } (f a, (\text{fmap } f x, \text{fmap } f y)) \end{cases}$$

3. length = sum . (map 1)

$$\Rightarrow \text{length} = \text{in} [0, \text{add}] \cdot (\text{map } 1)$$

{ Def sum }

$$\Rightarrow \text{length} = \text{in} [0, \text{add}] \cdot (\text{id} + 1 \times \text{id})$$

{ 52 }

$$\Rightarrow \text{length} = \text{in} [0, \text{add} \cdot (1 \times \text{id})]$$

{ 22, 1 }

$$\Rightarrow \text{length} \cdot \text{in} = [0, \text{add} \cdot (1 \times \text{id})] \cdot (\text{id} + \text{id} \times \text{length})$$

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$$\Rightarrow \begin{cases} \text{length} \cdot \text{nil} = 0 \\ \text{length} \cdot \text{cons} = \text{add} \cdot (1 \times \text{id}) \cdot (\text{id} \times \text{length}) \end{cases}$$

{ 20, 22, 1, 27 }

$$\Rightarrow \begin{cases} \text{length} \cdot \text{nil} = 0 \\ \text{length} \cdot \text{cons} = \text{add} \cdot (1 \times \text{length}) \end{cases}$$

{ 22, 1 }



$$\text{length } [] = 0$$

{72, 73, 75, 78}

$$\text{length } (h:t) = 1 + \text{length } t$$

$$= \text{True} \quad \{ \text{def length} \}$$

$$\text{length} = \text{length} \cdot (\text{map } f)$$

$$\Rightarrow \text{length} = ([0, \text{succ} \cdot \text{Tr}] \cdot (\text{map } f))$$

{def length}

$$\Rightarrow \text{length} = ([0, \text{succ} \cdot \text{Tr}] \cdot (\text{id} + f \cdot \text{id}))$$

{52}

$$\Rightarrow \text{length} = ([0, \text{succ} \cdot \text{Tr}] \cdot \text{id})$$

{22, 1, 13}

$$\Rightarrow \text{True}$$

{def length}

$$\textcircled{4} \text{ depth} \cdot \text{Tree } f = \text{depth}$$

$$\Rightarrow \text{depth} = ([\text{one}, \text{succ} \cdot \text{umax}] \cdot \text{Tree } f)$$

{def depth}

$$\Rightarrow \text{depth} = ([\text{one}, \text{succ} \cdot \text{umax}] \cdot B(f, \text{id}))$$

{52}

$$\Rightarrow \text{depth} = ([\text{one}, \text{succ} \cdot \text{umax}] \cdot (f + \text{id}^2))$$

{def B Tru}

$$\Rightarrow \text{depth} = ([\text{one} \cdot f, \text{succ} \cdot \text{umax} \cdot \text{id}])$$

{22, 15}

$$\Rightarrow \text{depth} = ([\text{one}, \text{succ} \cdot \text{umax}])$$

{3, 1}

$$\Rightarrow \text{True}$$

{def depth}

$$\textcircled{5} K = [(\text{id} + \langle f, \text{id} \rangle) \cdot \text{out}_{\text{in}_0}]$$

$$\Rightarrow \text{out}_{\text{out}} \cdot K = FK \cdot ((\text{id} + \langle f, \text{id} \rangle) \cdot \text{out}_{\text{in}_0})$$

{55}

$$\Rightarrow K = \text{in}_{\text{list}} \cdot FK \cdot ((\text{id} + \langle f, \text{id} \rangle) \cdot \text{out}_{\text{in}_0})$$

{32}

$$\Rightarrow K \cdot \text{in}_{\text{in}_0} = \text{in}_{\text{list}} \cdot FK \cdot (\text{id} + \langle f, \text{id} \rangle)$$

{33}

~~XXXXXXXXXX~~

$$\Rightarrow K \cdot [0, \text{succ}] = [\text{nil}, \text{cons}] \cdot (\text{id} + \text{id} \times K) \cdot (\text{id} + \langle f, \text{id} \rangle)$$

$$\Rightarrow [K \cdot 0, K \cdot \text{succ}] = [\text{nil}, \text{cons} \cdot (\text{id} \times K) \cdot \langle f, \text{id} \rangle]$$

{20, 22, 1}

$$\Rightarrow \begin{cases} K \cdot 0 = \text{nil} \\ K \cdot \text{succ} = \text{cons} \cdot \langle f, K \rangle \end{cases}$$

{27, 11, 1}

$$\Rightarrow \begin{cases} K 0 = [] \\ K(n+1) = (fn) : (Kn) \end{cases}$$

{72, 73, 75, 77}

$$\Rightarrow \begin{cases} K 0 = [] \\ K(n+1) = (fn+1) : (Kn) \end{cases}$$

{def f}



$$\begin{aligned}K_0 &= [1] \\K_1 &= [1] \\K_2 &= [3, 1] \\K_3 &= [5, 3, 1] \\K_4 &= [7, 5, 3, 1]\end{aligned}$$

Logo,  $K$  é a função que gera a lista dos  $n$  primeiros números ímpares, por ordem decrescente.

$$(6) \quad \text{suffixes} = [(id + \langle \text{cons}, \pi_2 \rangle) \cdot \text{out}]$$

$$\Rightarrow \text{out} \cdot \text{suffixes} = (F \text{ suffixes}) \cdot ((id + \langle \text{cons}, \pi_2 \rangle) \cdot \text{out}) \quad \{55\}$$

$$\Rightarrow \text{suffixes} = \text{in} \cdot (id + id \times \text{suffixes}) \cdot ((id + \langle \text{cons}, \pi_2 \rangle) \cdot \text{out}) \quad \{34\}$$

$$\Rightarrow \text{suffixes} \cdot \text{in} = \text{in} \cdot (id + \langle \text{cons}, \text{suffixes} \cdot \pi_2 \rangle) \quad \{33, 25, 1, 11\}$$

$$\Rightarrow \begin{cases} \text{suffixes} \cdot \text{nil} = \text{nil} \\ \text{suffixes} \cdot \text{cons} = \text{cons} \cdot \langle \text{cons}, \text{suffixes} \cdot \pi_2 \rangle \end{cases} \quad \{20, 22, 1, 27\}$$

$$\Rightarrow \begin{cases} \text{suffixes} [] = [] \\ \text{suffixes} (h:t) = (h:t) : (\text{suffixes } t) \end{cases} \quad \{72, 73, 75, 77, 79\}$$

(7) ~~466~~

$$Tf = (id \cdot B(f, id))$$

$$\equiv \{46\}$$

$$Tf \cdot \text{in} = \text{in} \cdot B(f, id) \cdot F(Tf)$$

$$\equiv \{50\}$$

$$Tf \cdot \text{in} = \text{in} \cdot B(f, id) \cdot B(id, Tf)$$

$$\equiv \{B(f, g) \cdot B(k, l) = B(f \cdot k, g \cdot l)\}$$

$$Tf \cdot \text{in} = \text{in} \cdot B(f \cdot id, id \cdot Tf)$$

$$\equiv \{1(x2)\}$$

$$Tf \cdot \text{in} = \text{in} \cdot B(id \cdot f, Tf \cdot id)$$

$$\equiv \{B(f, g) \cdot B(k, l) = B(f \cdot k, g \cdot l)\}$$

$$Tf \cdot \text{in} = \text{in} \cdot B(id, Tf) \cdot B(f, id)$$

$$\equiv \{33, 34, 50\}$$

$$\text{out} \cdot Tf = F(Tf) \cdot B(f, id) \cdot \text{out}$$

$$\equiv \{55\}$$

$$Tf = [(B(f, id) \cdot \text{out})]$$



$$\begin{aligned}
 \textcircled{8} \quad & \text{length} = \langle [zero, svec \cdot \pi_2] \rangle \\
 \Rightarrow & \text{length} \cdot in_1 = [zero, svec \cdot \pi_2] \cdot F_1 \text{length} \quad \{46\} \\
 \Rightarrow & \text{length} = [zero \cdot id, svec \cdot \pi_2] \cdot F_1 \text{length} \cdot out_2 \quad \{33, 14\} \\
 \Rightarrow & out_{in_0} \cdot \text{length} = out_{in_0} \cdot in_{in_0} \cdot (id + \pi_2) \cdot (id + id \times \text{length}) \cdot out_2 \quad \{22\} \\
 \Rightarrow & out_{in_0} \cdot \text{length} = (id + (\text{length} \cdot \pi_2)) \cdot out_2 \quad \{25, 1, 13\} \\
 \Rightarrow & out_{in_0} \cdot \text{length} = (id + \text{length}) \cdot (id + \pi_2) \cdot out_2 \quad \{1, 25\} \\
 \Rightarrow & out_{in_0} \cdot \text{length} = F_{in_0} \text{length} \cdot ((id + \pi_2) \cdot out_2) \\
 \Rightarrow & \text{length} = \langle (id + \pi_2) \cdot out_2 \rangle \quad \{55\}
 \end{aligned}$$

$$\textcircled{9} \quad \{46\}$$

$$\{33, 34, F8\}$$

$$\{55\}$$

$$\boxed{\text{mirror} = \langle (id + \text{swap}) \cdot out \rangle}$$