

Ex 12 | 23/24

①  $\mu = id \circ id$

$f \circ \mu = f \wedge f = \mu \circ f$

$\Rightarrow \mu = \mu \cdot Tid \cdot id$  {F6}

$\Rightarrow \mu \cdot Tf \cdot \mu = f \wedge f = \mu \cdot T\mu \cdot f$  {F6}

$\Rightarrow \mu = \mu \cdot id$  {45, 1}

$\Rightarrow \mu \cdot \mu \cdot f = f \wedge f = id \cdot f$  {F3, F2}

$\Rightarrow \mu = \mu$  {1}

$\Rightarrow id \cdot f = f \wedge f = id \cdot f$  {F2}

$\Rightarrow True$  {PR1}

$\Rightarrow true$  {1}

$(f \cdot g) \cdot h = \mu \cdot T(f \cdot g) \cdot h$  {F6}

$Tf = (\mu \cdot f) \cdot id$

$= \mu \cdot Tf \cdot (Tg \cdot h)$  {44}

$\wedge Tf = \mu \cdot T(\mu \cdot f) \cdot id$  {F6}

$= f \cdot (Tg \cdot h)$  {F6}

$\Rightarrow Tf = \mu \cdot T\mu \cdot Tf$  {44, 1}

$\Rightarrow Tf = id \cdot Tf$  {F2}

$\Rightarrow Tf = Tf$  {1}

$\Rightarrow True$  {PR1}

②  $f \cdot [g, h]$

$= \mu \cdot Tf \cdot [g, h]$  {66}

$= [\mu \cdot Tf \cdot g, \mu \cdot Tf \cdot h]$  {20}

$= [f \cdot g, f \cdot h]$  {66}

③

$discollect = lstr \cdot id$

$\Rightarrow discollect = econcat \cdot Tlstr \cdot id$

{66}

$\Rightarrow discollect = ([nil, econc]) \cdot Tlstr$

{def econc, 1}

$\Rightarrow discollect = ([nil, econc] \cdot B(lstr, id))$

{52}

$\Rightarrow discollect = ([nil, econc] \cdot id + lstr \times id)$

{def B lists}

$\Rightarrow discollect = ([nil, econc \cdot (lstr \times id)])$

{22, 1}

$\Rightarrow discollect \cdot in = [nil, econc \cdot (lstr \times id)] \cdot f \cdot discollect$

{46}

$\Rightarrow discollect \cdot [nil, econc] = [nil, econc \cdot (lstr \times id)] \cdot (id + id \times discollect)$

{def f in lstr}

$\Rightarrow \left\{ \begin{array}{l} discollect \cdot nil = nil \\ discollect \cdot econc = econc \cdot (lstr \times id) \cdot (id \times discollect) \end{array} \right.$

{20, 22, 1, 23}



$$\omega \left\{ \begin{array}{l} \text{discollect} \cdot \text{nil} = \text{nil} \\ \text{discollect} \cdot \text{cons} = \text{cons} \cdot (\text{fst} \times \text{discollect}) \end{array} \right. \quad \{72, 73, 75, 78\}$$

$$\omega \left\{ \begin{array}{l} \text{discollect} [] = [] \\ \text{discollect} ((a, x) : y) = [(a, b) \mid b \leftarrow x] \# \text{discollect } y \end{array} \right.$$

$$(F1) \quad A \xrightarrow{\mu} TA \xleftarrow{\mu} T(TA) \quad , \quad \mu((x, t_1), t_2) = (x, t_1 + t_2)$$

$$[F2] \quad \mu \cdot \mu = \text{id} = \mu \cdot T\mu$$

$$\Rightarrow \mu((x, t_1), t_2) = (\text{id } x, \hat{t}(t_1, t_2)) \quad \{74, 76\}$$

$$[F3] \quad \mu \cdot \mu = \mu \cdot T\mu$$

$$\Rightarrow \mu((x, t_1), t_2) = (\text{id} \times \hat{t})(x, (t_1, t_2)) \quad \{77\}$$

$$\mu x = (x, 0)$$

$$\Rightarrow \mu((x, t_1), t_2) = (\text{id} \times \hat{t})(\text{asser}(x, (t_1, t_2))) \quad \{78\}$$

$$\Rightarrow \mu x = (\text{id } x, \underline{0} x)$$

$$\{74, 75\}$$

$$\Rightarrow \mu = (\text{id} \times \hat{t}) \cdot \text{asser} \quad \{73, 72\}$$

$$\Rightarrow \mu x = \langle \text{id}, \underline{0} \rangle x$$

$$\{77\}$$

$$\text{asser}: (A \times B) \times e \longrightarrow A \times (B \times e)$$

$$\Rightarrow \mu = \langle \text{id}, \underline{0} \rangle$$

$$\{72\}$$

$$(F2) \quad \mu \cdot \mu = \text{id} = \mu \cdot T\mu$$

$$\Rightarrow \forall a, t \mid (\mu \cdot \mu)(a, t) = \text{id}(a, t) = (\mu \cdot T\mu)(a, t) \quad \{72\}$$

$$\Rightarrow \mu(\mu(a, t)) = (a, t) = \mu(\mu \times \text{id})(a, t) \quad \{73, 74\}$$

$$\Rightarrow \mu((a, t), 0) = (a, t) = \mu((a, 0), t) \quad \{73, 74\}$$

$$\Rightarrow (a, t + 0) = (a, t) = (a, 0 + t)$$

$$\Rightarrow (a, t) = (a, t) = (a, t)$$

$$\Rightarrow \text{True}$$

$$(F3) \quad \mu \cdot \mu = \mu \cdot T\mu$$

$$\Rightarrow (\mu \cdot \mu)((a, t_3), t_2, t_1) = (\mu \cdot T\mu)((a, t_3), t_2, t_1) \quad \{72\}$$

$$\Rightarrow \mu((a, t_3), t_2 + t_1) = (\mu \cdot (\mu \times \text{id}))((a, t_3), t_2, t_1) \quad \{73\}$$

$$\Rightarrow (a, t_3 + t_2 + t_1) = \mu((a, t_3 + t_2), t_1) \quad \{73, 74\}$$

$$\Rightarrow (a, t_3 + t_2 + t_1) = (a, t_3 + t_2 + t_1)$$

$$\Rightarrow \text{True}$$



⑤

$$\boxed{F2} \quad \mu \cdot \mu = id = \mu \cdot Tu$$

$$\mu \cdot \mu = id$$

$$\Rightarrow \Delta[id, in \cdot i_2]D \cdot in \cdot i_2 = id$$

$$\Rightarrow [id, in \cdot i_2] \cdot F\mu \cdot i_1 = id \quad \{47\}$$

$$\Rightarrow [id, in \cdot i_2] \cdot (id + G\mu) \cdot i_1 = id \quad \{50\}$$

$$\Rightarrow [id, in \cdot i_2] \cdot (i_1 \cdot id) = id \quad \{23\}$$

$$\Rightarrow id = id \quad \{1, 18\}$$

$$\Rightarrow True \quad \{P.R.I.\}$$

$$\mu \cdot Tu = id$$

$$\Rightarrow \Delta[id, in \cdot i_2]D \cdot T(in \cdot i_1) = id \quad \{22\} \mu \cdot \mu$$

$$\Rightarrow \Delta[id, in \cdot i_2] \cdot B(in \cdot i_1, id)D = id \quad \{52\}$$

$$\Rightarrow \Delta[id, in \cdot i_2] \cdot ((in \cdot i_1) + id)D = id \quad \{45\}$$

$$\Rightarrow \Delta[in \cdot i_1, in \cdot i_2]D = id \quad \{22, 1\}$$

$$\Rightarrow \Delta[in \cdot [i_1, i_2]D = id \quad \{20\}$$

$$\Rightarrow \Delta[in \cdot id]D = id \quad \{19\}$$

$$\Rightarrow \Delta[in]D = id \quad \{1\}$$

$$\Rightarrow True \quad \{48\}$$

$$\boxed{F3} \quad \mu \cdot \mu = \mu \cdot Tu$$

$$\Rightarrow \mu \cdot \mu = \Delta[id, in \cdot i_2]D \cdot T\Delta[id, in \cdot i_2]D$$

$$\Rightarrow \mu \cdot \mu = \Delta[id, in \cdot i_2] \cdot (\Delta[id, in \cdot i_2]D + id)D \quad \{52, 45\}$$

$$\Rightarrow \mu \cdot \mu = \Delta[\Delta[id, in \cdot i_2]D, in \cdot i_2]D \quad \{22, 1\}$$

$$\Rightarrow \mu \cdot [id, in \cdot i_2] = [\mu, in \cdot i_2] \cdot F\mu \quad \{49\}$$

$$\Rightarrow \begin{cases} \mu = \mu \\ \mu \cdot in \cdot i_2 = in \cdot i_2 \cdot G\mu \end{cases} \quad \{20, 22, 1, 27\}$$

$$\Rightarrow [id, in \cdot i_2] \cdot (id + G\mu) \cdot i_2 = in \cdot i_2 \cdot G\mu \quad \{47, 50\}$$

$$\Rightarrow [id, in \cdot i_2] \cdot i_2 \cdot G\mu = in \cdot i_2 \cdot G\mu \quad \{24\}$$

$$\Rightarrow in \cdot i_2 \cdot G\mu = in \cdot i_2 \cdot G\mu \quad \{18\}$$

$$\Rightarrow True \quad \{P.R.I.\}$$

- ⑥ a) No caso de LTree,  $GY = Y^2$   
b) Obtém-se o Word Maybe  
c) Obtém-se o Word das Árvores de Expressão.



⑦.

$$\text{sequence} = ([\text{return}, \text{id}] \cdot (\text{nil} + \text{L cons}))$$

$$\Rightarrow \text{sequence} \cdot \text{in} = [\text{return}, \text{id}] \cdot (\text{nil} + \text{L cons}) \cdot \text{fsequence} \{4, 6\}$$

$$\Rightarrow \text{sequence} \cdot \text{in} = [\text{return} \cdot \text{nil}, \text{L cons}] \cdot (\text{id} + \text{id} \times \text{sequence}) \{22, 1\}$$

$$\Rightarrow \begin{cases} \text{sequence} \cdot \text{nil} = \text{return} \cdot \text{nil} \\ \text{sequence} \cdot \text{cons} = \text{L cons} \cdot (\text{id} \times \text{sequence}) \end{cases} \{22, 1, 27\}$$

$$\Rightarrow \begin{cases} \text{sequence} [] = \text{return} [] \\ \text{sequence} (h:t) = \text{do } \{ a \leftarrow h; b \leftarrow \text{sequence } t; \text{return} (\text{cons } (a, b)) \} \end{cases} \{72, 73(\times 2), 75(\times 2)\}$$

$$\{72, 78, 73(\times 2)\}$$