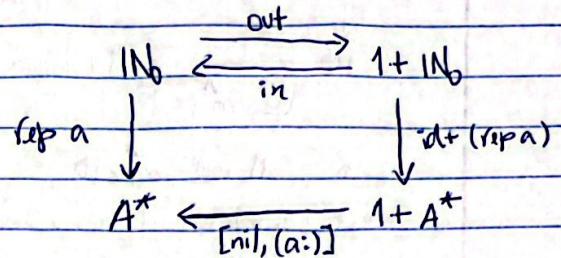


(1) $\text{cata } g = g \cdot (\text{id} - 1 - (\text{cata } g)) \cdot \text{out}$

$\text{rep } a$ é uma função que devolve uma lista com o elemento a n vezes, em que n é recebido como parâmetro.



(2) $f = \text{p2} \cdot \text{aux}$ where $\text{aux} = \text{for } (\text{split } (\text{succ} \cdot \text{p1}) \text{ mul}) (1, 1)$

$\text{for } b \ i = \text{ID}[i, b] \text{D}$ (nota anterior)

$\text{for } \langle \text{succ} \cdot \pi_1, \text{mul} \rangle (1, 1) = \text{ID}[(1, 1), \langle \text{succ} \cdot \pi_1, \text{mul} \rangle \text{D}$

$\text{aux } 0 = (1, 1); f 0 = 1$

$\text{aux } 1 = (2, 1); f 1 = 1$

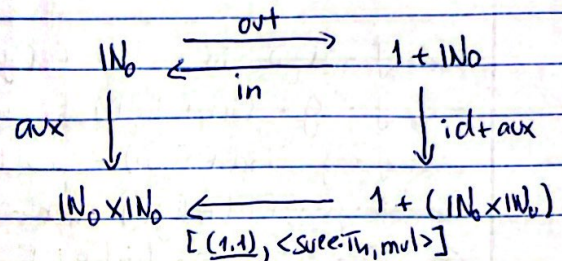
$\text{aux } 2 = (3, 2); f 2 = 2$

$\text{aux } 3 = (4, 6); f 3 = 6$

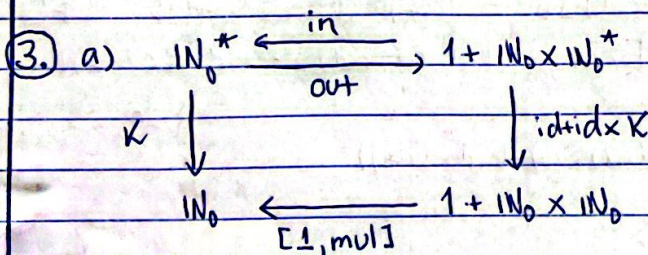
$\text{aux } 4 = (5, 24); f 4 = 24$

$\text{aux } 5 = (6, 120); f 5 = 120$

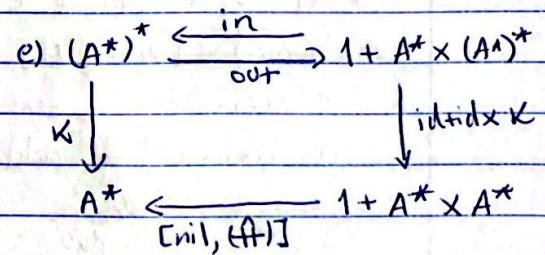
$\text{aux } 6 = (7, 720); f 6 = 720$



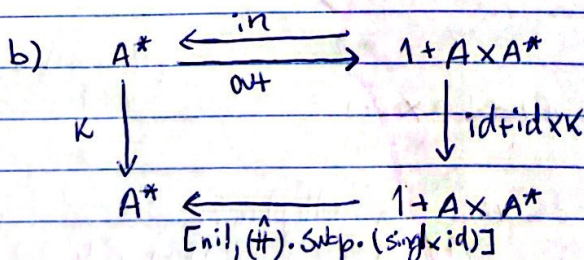
$f \pi = \pi!$



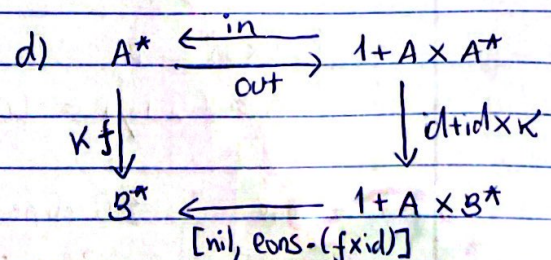
$K = \text{ID}[1, \text{mul}] \text{D}$



$K = \text{ID}[\text{nil}, (\hat{+})] \text{D}$



$K = \text{ID}[\text{nil}, (\hat{+}) \cdot \text{subp} \cdot (\text{sing} \times \text{id})] \text{D}$



$Kf = \text{ID}[\text{nil}, \text{cons} \cdot (fx \text{id})] \text{D}$

$$e) \quad \begin{array}{ccc} IN_0^* & \xleftarrow[\text{out}]{in} & 1 + IN_0 \times IN_0^* \\ \downarrow \kappa & & \downarrow \text{id} + \text{id} \times \kappa \\ IN_0 & \xleftarrow[\text{[0, max]}]{\text{[0, max]}} & 1 + IN_0 \times IN_0 \end{array}$$

$$\kappa = \text{[0, max]}D$$

$$\max(x, y) = \text{if } x > y \text{ then } x \text{ else } y$$

$$f) \quad \begin{array}{ccc} A^* & \xleftarrow[\text{out}]{in} & 1 + A \times A^* \\ \downarrow \kappa & & \downarrow \text{id} + \text{id} \times \kappa \\ A^* & \xleftarrow[\text{[nil, (*)]}]{\text{[nil, (*)]}} & 1 + A \times A^* \\ & & \uparrow \text{[}\pi_2, \text{cons}\text{]} \\ & & (A \times A^*) + (A \times A^*) \\ & & \uparrow \text{dist} \\ & & (A + A) \times A^* \\ & & \uparrow p? \times \text{id} \\ & & A \times A^* \end{array}$$

4.)

$$f \cdot \text{[gd]} = \text{[hd]}$$

$$\text{[hd]} = \text{[}\pi_2, \text{cons}\text{]} \cdot \text{dist} \cdot (p? \times \text{id})$$

$$p? x = \text{if } p x \text{ then Right } x \text{ else Left } x$$

$$\Rightarrow f \cdot \text{[gd]} \cdot \text{in} = h \cdot F(f \cdot \text{[gd]}) \quad \{46\}$$

$$\Rightarrow f \cdot g \cdot (\text{id} + \text{[gd]}) = h \cdot (\text{id} + f \cdot \text{[gd]}) \quad \{47, \text{Funbr } M_0\}$$

$$\Rightarrow f \cdot g \cdot (\text{id} + \text{[gd]}) = h \cdot (\text{id} \cdot \text{id} + f \cdot \text{[gd]}) \quad \{1\}$$

$$\Rightarrow f \cdot g \cdot (\text{id} + \text{[gd]}) = h \cdot (\text{id} + f) \cdot (\text{id} + \text{[gd]}) \quad \{14\}$$

$$\Leftarrow f \cdot g = h \cdot (\text{id} + f) \quad \{5\}$$

5.) $\text{sumprod } a = (a^*) \cdot \text{sum}$

$$\Rightarrow \text{[[zero, add} \cdot ((a^*) \times \text{id})] D} = (a^*) \cdot \text{[[zero, add] D} \quad \{66, \text{def sum}\}$$

$$\Leftarrow (a^*) \cdot \text{[zero, add]} = \text{[zero, add} \cdot ((a^*) \times \text{id})] \cdot (\text{id} + \text{id} \times (a^*)) \quad \{49\}$$

$$\Rightarrow \begin{cases} (a^*) \cdot \text{zero} = \text{zero} \\ (a^*) \cdot \text{add} = \text{add} \cdot ((a^*) \times \text{id}) \cdot (\text{id} \times (a^*)) \end{cases} \quad \{20, 22, 1, 27\}$$

$$\Rightarrow \begin{cases} (a^*) \cdot \text{zero} = \text{zero} \\ (a^*) \cdot \text{add} = \text{add} \cdot ((a^*) \times (a^*)) \end{cases} \quad \{14, 1(x2)\}$$

$$\Rightarrow \begin{cases} a \times 0 = 0 \\ a \times (x + y) = (a \times x) + (a \times y) \end{cases} \quad \begin{matrix} \{72(x2), 73(x3), 75(x2), \\ 78\} \end{matrix}$$

$$\Rightarrow \text{TWE} = \begin{matrix} \{0 \text{ elemento absorvente da multiplicação,} \\ \text{Propriedade distributiva de multiplicação} \end{matrix}$$

$$(6) f \cdot (\text{for } f \ i) = \text{for } f \ (f \ i)$$

$$\Rightarrow f \cdot ([i, f]) = ([f \ i, f])$$

$$\Leftrightarrow f \cdot [i, f] = [f \ i, f] \cdot f$$

$$\Rightarrow [f \cdot i, f \cdot f] = [f \ i, f] \cdot (\text{id} + f)$$

$$\Leftrightarrow \begin{cases} f \cdot i = f \ i \\ f \cdot f = f \cdot f \end{cases}$$

$$\Rightarrow \text{True}$$

{20, 20, 20}

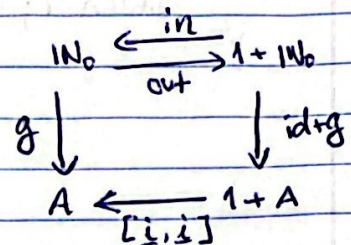
{49}

{20, 20, 20}

{22, 1, 2, 27}

$$(7) f = \text{for id } i = ([i, \text{id}])$$

$$g = \text{for } i \ i = ([i, i])$$



$$i = ([i, \text{id}])$$

$$\Rightarrow i \cdot \text{in} = [i, \text{id}] \cdot (\text{id} + i) \quad \{46\}$$

$$\Rightarrow i = [i, i] \quad \{3, 22, 1(x2)\}$$

$$\Rightarrow i \cdot i_1 = i \wedge i \cdot i_2 = i \quad \{17\}$$

$$\Rightarrow i = i \wedge i = i \quad \{3(x2)\}$$

$$\Rightarrow \text{True}$$

$$i = ([i, i])$$

$$\Rightarrow i \cdot \text{in} = [i, i] \cdot (\text{id} + i) \quad \{46\}$$

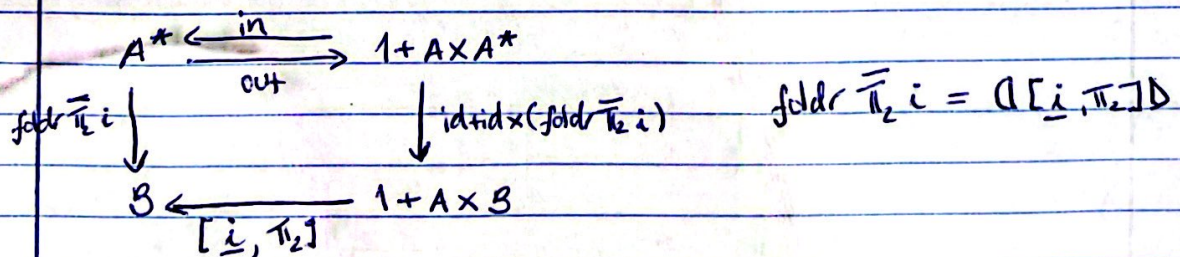
$$\Rightarrow i = [i, i] \quad \{3, 22, 1, 3\}$$

$$\Rightarrow i \cdot i_1 = i \wedge i \cdot i_2 = i \quad \{17\}$$

$$\Rightarrow i = i \wedge i = i \quad \{3(x2)\}$$

$$\Rightarrow \text{True}$$

$$(8) \text{foldr} :: \text{Foldable } t \Rightarrow (a \rightarrow b \rightarrow b) \rightarrow b \rightarrow t a \rightarrow b$$



$$i = ([i, \pi_2]) \Rightarrow i \cdot \text{in} = [i, \pi_2] \cdot (\text{id} + \text{id} \times i) \quad \{46\}$$

$$\Rightarrow i = [i, \pi_2 \cdot (\text{id} \times i)] \quad \{3, 22, 1\}$$

$$\Rightarrow i \cdot i_1 = i \wedge i \cdot i_2 = i \cdot \pi_2 \quad \{17, 13\}$$

$$\Rightarrow \text{True} \wedge i = i \quad \{3(x3)\}$$

$$\Rightarrow \text{True}$$

9. $(a^*) = \text{for } (a^+) 0$

```
int K(int n) {  
    int r = 0;  
    int j;  
    for (j=1; j<n+1; j++) { r = a + r; }  
    return r;  
}
```