Ex2_Precipitation

April 7, 2020

1 Precipitation exercises

1.1 Exercise 2 - Double-mass curve

The table *2MassCurve* in the file *RainfallData.xlsx* provides annual precipitation measured over a 17-year period at five gages in a region. Gage C was moved at the end of 1977. Carry out a double-mass curve analysis to check for consistency in the record of that gage C, and make appropriate adjustments to correct for any inconsistencies.

[1]:
```python
import numpy as np

import pandas as pd

import matplotlib.pyplot as plt
import seaborn as sns
sns.set()
sns.set_context('notebook')
```

1.2 Introduction

A **double mass curve** is a plot of the cumulative data of one variable against the cumulative data of another variable (or against the average cumulative values of the same variable in different locations) during the same period.

Double-mass curve of precipitation data. *(Double-Mass Curves. USGS, 1960)*.

If no change occurred during the period, the plot must be a straight line in which slope is the constant of proportionality between series. A break in the slope means that a change in the constant of proportionality.

The double-mass curve, when applied to precipitation, adopts the form $Y = m \cdot X$, where b is the slope. This form implies that the line should not have an intercept.

1.2.1 Import data

[2]:
```
# import data from sheet '2MassCurve' in file 'RainfallData.xlxs'
data2 = pd.read_excel('..\data\RainfallData.xlsx', sheet_name='2MassCurve',
                      skiprows=4,
             index_col=0, usecols=range(6))
data2.describe()
```

[2]:

	A	B	C	D	E
count	17.000000	17.000000	17.000000	17.000000	17.000000
mean	1094.588235	1059.058824	974.529412	972.117647	1093.058824
std	154.527125	161.344922	214.418142	154.720669	185.289460
min	801.000000	751.000000	710.000000	683.000000	771.000000
25%	1010.000000	978.000000	825.000000	875.000000	967.000000
50%	1140.000000	1056.000000	933.000000	981.000000	1135.000000
75%	1170.000000	1161.000000	1058.000000	1056.000000	1190.000000
max	1411.000000	1353.000000	1584.000000	1286.000000	1483.000000

1.2.2 The double-mass curve

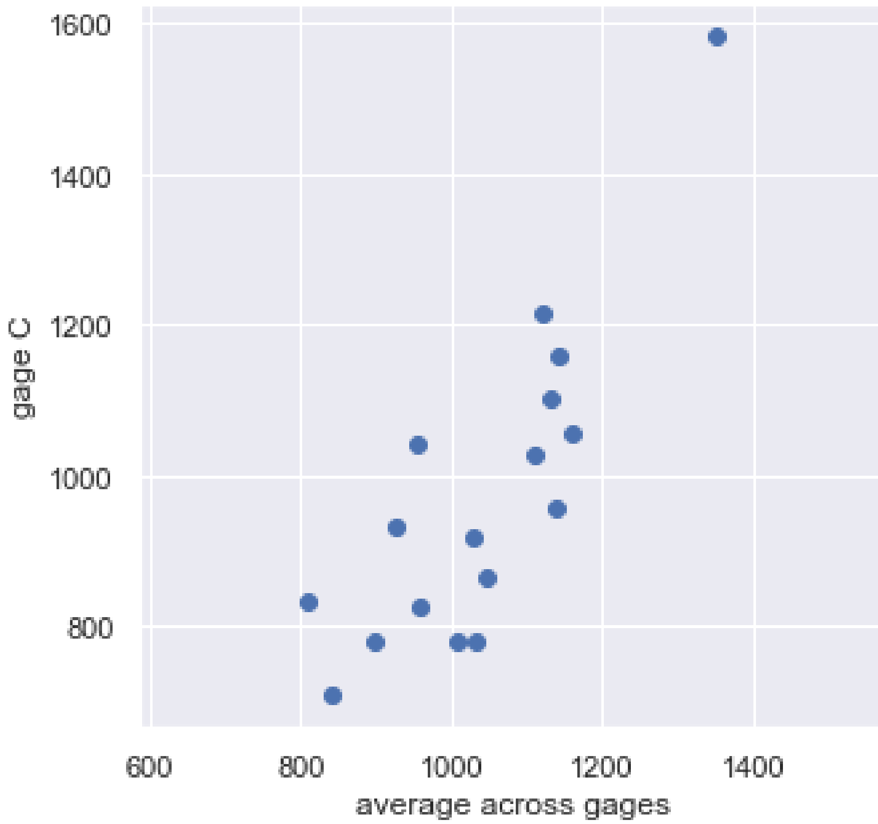
[3]:
```
# compute annual average across all gages
data2['AVG'] = data2.mean(axis=1)
data2.head()
```

[3]:

	A	B	C	D	E	AVG
Year						
1970	1010	1161	780	949	1135	1007.0
1971	1005	978	1041	784	970	955.6
1972	1067	1226	1027	1067	1158	1109.0
1973	1051	880	825	1014	1022	958.4
1974	801	1146	933	923	821	924.8

Visualize the data We will create first a scatter plot comparing the annual series of precipitation in gage C against the average across gages.

[4]:
```
# scatter plot of annual precipitation
plt.figure(figsize=(5, 5))
plt.axis('equal')
plt.scatter(data2.AVG, data2.C)
plt.xlabel('average across gages')
plt.ylabel('gage C');
```



This type of plot has a large dispersion, so it isn't convenient to spot anomalies. We can neither see trends nor the year with possible errors in the data set.

Instead, we will plot a **double mass curve**. This plot is created from the series of **accumulated precipitation**. This way, the plot must have always a positive and continues trend, allowing us to identity anomalies in the precipitation records.

The function `cumsum` in `NumPy` calculates the accumulated series from a series of data. For instance:

[5]:
```
# annual series of accumulated precipitation
data2.cumsum()
```

[5]:

Year	A	B	C	D	E	AVG
1970	1010.0	1161.0	780.0	949.0	1135.0	1007.0
1971	2015.0	2139.0	1821.0	1733.0	2105.0	1962.6
1972	3082.0	3365.0	2848.0	2800.0	3263.0	3071.6
1973	4133.0	4245.0	3673.0	3814.0	4285.0	4030.0
1974	4934.0	5391.0	4606.0	4737.0	5106.0	4954.8
1975	6345.0	6744.0	6190.0	5667.0	6589.0	6307.0
1976	7567.0	7762.0	7405.0	6648.0	7763.0	7429.0
1977	8579.0	8513.0	8237.0	7331.0	8534.0	8238.8

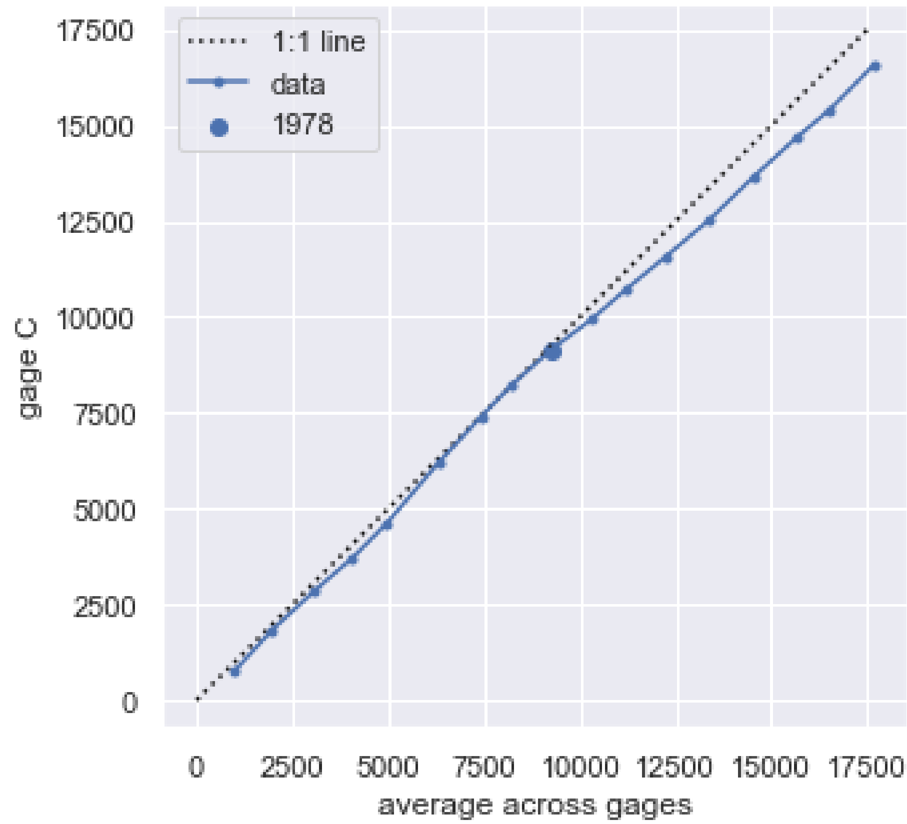
```
1978   9732.0   9572.0   9155.0   8155.0   9722.0   9267.2
1979  10872.0  10795.0   9936.0   9211.0  10689.0  10300.6
1980  11701.0  11798.0  10718.0  10007.0  11777.0  11200.2
1981  12866.0  12918.0  11583.0  11128.0  12740.0  12247.0
1982  14036.0  13907.0  12539.0  12414.0  14027.0  13384.6
1983  15300.0  14963.0  13641.0  13458.0  15217.0  14515.8
1984  16500.0  16224.0  14699.0  14449.0  16500.0  15674.4
1985  17442.0  17035.0  15409.0  15324.0  17373.0  16516.6
1986  18608.0  18004.0  16567.0  16526.0  18582.0  17657.4
```

To avoid duplication of data, we will not save the previous accumulated series, but we will be using the function `cumsum` very often in the following.

Let's plot the double-mass curve for station C.

[6]:
```python
plt.figure(figsize=(5, 5))
# line of slope 1
plt.plot((0, 17500), (0, 17500), ':k', label='1:1 line')
# double-mass curve
plt.plot(data2.AVG.cumsum(), data2.C.cumsum(), '.-', label='data')
# year 1978
plt.scatter(data2.AVG.cumsum().loc[1978], data2.C.cumsum().loc[1978],␣
 ↪label='1978')

# configuration
plt.axis('equal')
plt.xlabel('average across gages')
plt.ylabel('gage C')
plt.legend();
```



We can clearly observe a break in the line in year from year 1918 onwards, exactly matching the year in which the station C was moved.

1.2.3 Correct series

To correct the series we must decide whether we trust the data before or after the break in the double-mass curve. For that, we would need further information about the location and instruments used. In this exercise, we will assume that the data up till 1978 is the one we trust the most, and we will fix the series from 1979 on.

The steps are the following: 1. Calculate the slope of the first part of the double-mass curve (m_1). 2. Calculate the slope of the second part of the double-mass curve (m_2). 3. Correct the series. Assuming that the correct slope is m_1, the corrected precipitation P_c for the observed data P_o is:

$$P_c = \frac{m_1}{m_2} \cdot P_o$$

Therefore, we need to learn how to fit the slope of a linear regression of the form $y = m \cdot x$. We will first define a function that represents that form of the linear regression and later on we will use the function `scipy.optimize.curve_fit` to fit the slope m.

[7]:
```
def linear_reg(x, m):
    """Linear regression with intercept 0:
            y = m · x

    Input:
    ------
    x:          float. Independet value
    m:          float. Slope of the linear regression

    Output:
    -------
    y:          float. Regressed value"""

    y = m * x

    return y
```

[8]:
```
# import function scipy.optimize.curve_fit
from scipy.optimize import curve_fit
```

Fit the regression for the first part This first time we will do it step by step.

[9]:
```
# define x and y in the linear regression
x = data2.loc[:1978, 'AVG']
y = data2.loc[:1978, 'C']
```

[10]:
```
# compute cumulative series
x = x.cumsum()
y = y.cumsum()
```

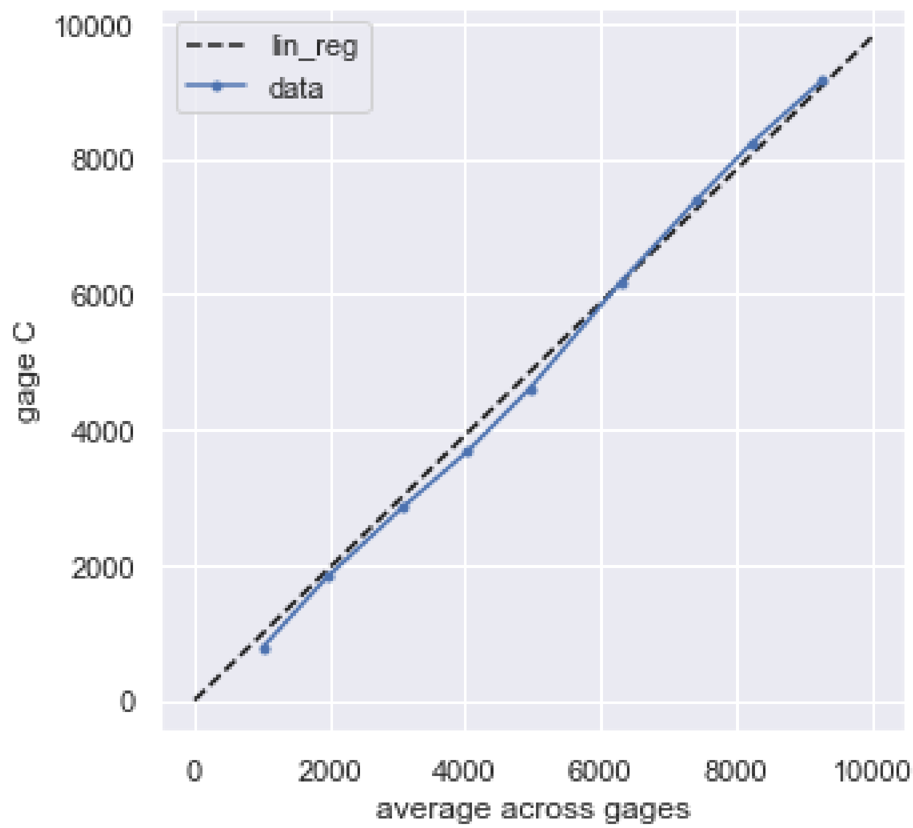
[11]:
```
# fit the regression up till 1978
m1 = curve_fit(linear_reg, x, y)[0][0]
print('m1 = {0:.3f}'.format(m1))
```

```
m1 = 0.979
```

[12]:
```
plt.figure(figsize=(5, 5))
# linear regression
x_ = np.array([0, 10000])
y_ = m1 * x_
plt.plot(x_, y_, linestyle='--', color='k', label='lin_reg')
# double-mass curve
plt.plot(x, y, '.-', label='data')

# configuration
plt.axis('equal')
```

```
plt.xlabel('average across gages')
plt.ylabel('gage C')
plt.legend();
```



Fit the regression for the second part

[13]:
```
# fit the regression from 1978 onwards
m2 = curve_fit(linear_reg, data2.loc[1978:, 'AVG'].cumsum(), data2.loc[1978:,␣
↪'C'].cumsum())[0][0]
print('m1 = {0:.3f}'.format(m2))
```

```
m1 = 0.866
```

Correct the data

[14]:
```
# correction factor
factor = m1 / m2
print('correction factor = {0:.3f}'.format(factor))
```

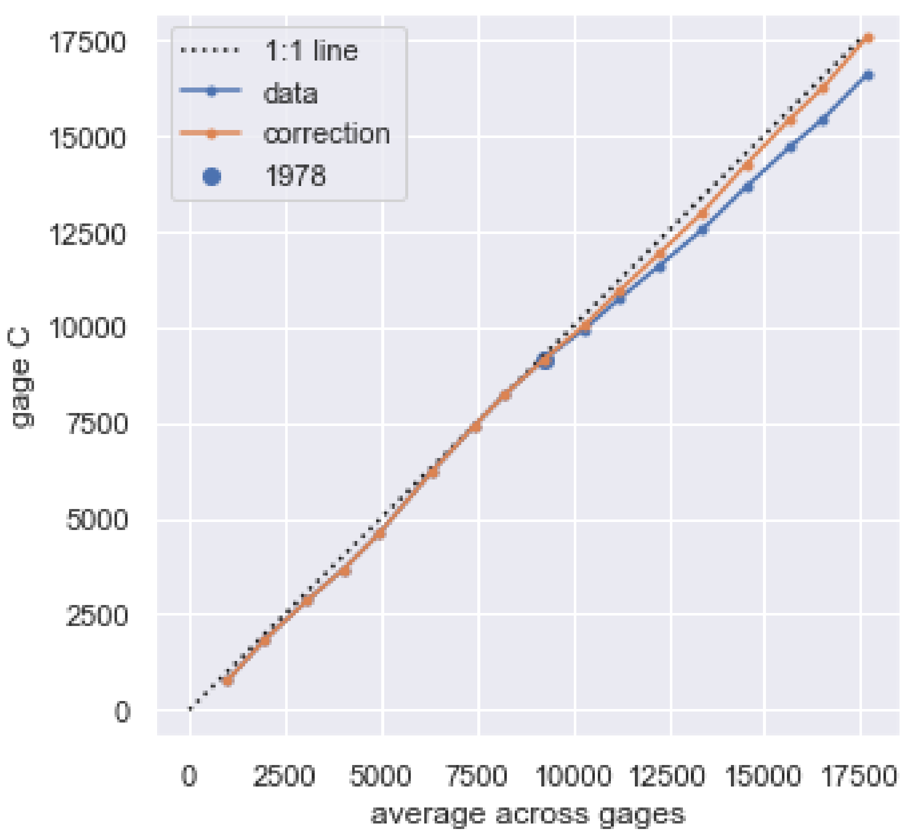
```
correction factor = 1.131
```

[15]:
```
# copy the original data in a new column
data2['C_'] = data2['C']
#multiply the second period by the correction factor
data2.loc[1979:, 'C_'] *= factor
```

[16]:
```
plt.figure(figsize=(5, 5))
# line of slope 1
plt.plot((0, 17500), (0, 17500), ':k', label='1:1 line')
# double-mass curve
plt.plot(data2.AVG.cumsum(), data2.C.cumsum(), '.-', label='data')
plt.plot(data2.AVG.cumsum(), data2.C_.cumsum(), '.-', label='correction')
# year 1978
plt.scatter(data2.AVG.cumsum().loc[1978], data2.C.cumsum().loc[1978],␣
 ↪label='1978')

# configuration
plt.axis('equal')
plt.xlabel('average across gages')
plt.ylabel('gage C')
plt.legend();

# save figure
plt.savefig('../output/Ex2_double-mass curve.png', dpi=300)
```



[17]:
```
data2_ = data2[['A', 'B', 'C_', 'D', 'E']]
data2_.columns = ['A', 'B', 'C', 'D', 'E']
data2_ = data2_.astype(float)
```

[18]:
```
data2_.to_csv('../output/Ex2_corrected series.csv', float_format='%.0f')
```

1.2.4 Useful links:

USGS report on double-mass curves `SciPy.optimize.curve_fit` help