Reference ¶

Coursera Deep learning series by Andrew NG

General

- Different initializations lead to different results
- Random initialization is used to break symmetry and make sure different hidden units can learn different things
- Don't initialize to values that are too large
- He initialization works well for networks with **ReLU activations**.

Zero initialization

In general, initializing all the weights to zero results in the network failing to break symmetry. This means that every neuron in each layer will learn the same thing, and you might as well be training a neural network with $n^{[l]} = 1$ for every layer, and the network is no more powerful than a linear classifier such as logistic regression.

- The weights $W^{[l]}$ should be initialized randomly to break symmetry.
- It is however okay to initialize the biases $b^{[l]}$ to zeros. Symmetry is still broken so long as $W^{[l]}$ is initialized randomly.

Random initialization

- Initializing weights to very large random values does not work well. **The cost might start very high**. This is because with large random-valued weights, the last activation (sigmoid) outputs results that are very close to 0 or 1 for some examples, and when it gets that example wrong it incurs a very high loss for that example. Indeed, when $\log(a^{[3]}) = \log(0)$, the loss goes to infinity. **A more numerically sophisticated implementation would fix this.**
- Initializing with small random values does better. The important question is: **how small should be these random values be? Lets find out in the next part!**

He initialization

Finally, try "He Initialization"; this is named for the first author of He et al., 2015. (If you have heard of "Xavier initialization", this is similar except Xavier initialization uses a scaling factor for the weights $W^{[l]}$ of `sqrt(1./layers_dims[l-1])` where He initialization would use `sqrt(2./layers_dims[l-1])` .)

This function is similar to the previous `initialize_parameters_random(...)` . The only difference is that instead of multiplying `np.random.randn(..,..)` by 10, you will multiply it by $\sqrt{\frac{2}{\text{dimension of the previous layer}}}$, which is what He initialization recommends for layers with a ReLU activation.

In [2]:
```
import numpy as np
import matplotlib.pyplot as plt
import sklearn
import sklearn.datasets

%matplotlib inline
plt.rcParams['figure.figsize'] = (7.0, 4.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'
```

1 - Neural Network model

In [1]:
```python
def model(X, Y, learning_rate=0.01, num_iterations=15000, print_cost=True, initialization="he"):

    grads = {}
    costs = [] # to keep track of the loss
    m = X.shape[1] # number of examples
    layers_dims = [X.shape[0], 10, 5, 1]

    # Initialize parameters dictionary.
    if initialization == "zeros":
        parameters = initialize_parameters_zeros(layers_dims)
    elif initialization == "random":
        parameters = initialize_parameters_random(layers_dims)
    elif initialization == "he":
        parameters = initialize_parameters_he(layers_dims)

    # Loop (gradient descent)

    for i in range(0, num_iterations):

        # Forward propagation: LINEAR -> RELU -> LINEAR -> RELU -> LINEAR -> SIGMOID.
        a3, cache = forward_propagation(X, parameters)

        # Loss
        cost = compute_loss(a3, Y)

        # Backward propagation.
        grads = backward_propagation(X, Y, cache)

        # Update parameters.
        parameters = update_parameters(parameters, grads, learning_rate)

        # Print the loss every 1000 iterations
        if print_cost and i % 1000 == 0:
            print("Cost after iteration {}: {}".format(i, cost))
            costs.append(cost)

    # plot the loss
    plt.plot(costs)
    plt.ylabel('cost')
    plt.xlabel('iterations (per hundreds)')
    plt.title("Learning rate =" + str(learning_rate))
    plt.show()
```

```python
    return parameters
```

2 - Zero initialization

In [3]:
```python
def initialize_parameters_zeros(layers_dims):
    parameters = {}
    L = len(layers_dims)            # number of layers in the network

    for l in range(1, L):
        parameters['W' + str(l)] = np.zeros((layers_dims[l], layers_dims[l - 1]))
        parameters['b' + str(l)] = np.zeros((layers_dims[l], 1))
    return parameters
```

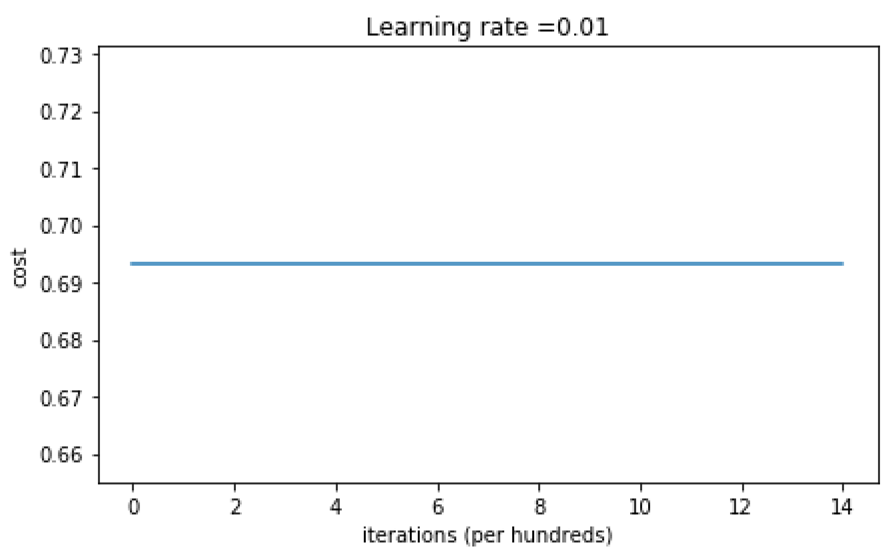
In [4]:
```python
parameters = initialize_parameters_zeros([3,2,1])
print("W1 = " + str(parameters["W1"]))
print("b1 = " + str(parameters["b1"]))
print("W2 = " + str(parameters["W2"]))
print("b2 = " + str(parameters["b2"]))
```

```
W1 = [[ 0.  0.  0.]
 [ 0.  0.  0.]]
b1 = [[ 0.]
 [ 0.]]
W2 = [[ 0.  0.]]
b2 = [[ 0.]]
```

Run the following code to train your model on 15,000 iterations using zeros initialization.

In [5]:
```python
parameters = model(train_X, train_Y, initialization = "zeros")
print ("On the train set:")
predictions_train = predict(train_X, train_Y, parameters)
print ("On the test set:")
predictions_test = predict(test_X, test_Y, parameters)
```

```
Cost after iteration 0: 0.6931471805599453
Cost after iteration 1000: 0.6931471805599453
Cost after iteration 2000: 0.6931471805599453
Cost after iteration 3000: 0.6931471805599453
Cost after iteration 4000: 0.6931471805599453
Cost after iteration 5000: 0.6931471805599453
Cost after iteration 6000: 0.6931471805599453
Cost after iteration 7000: 0.6931471805599453
Cost after iteration 8000: 0.6931471805599453
Cost after iteration 9000: 0.6931471805599453
Cost after iteration 10000: 0.6931471805599455
Cost after iteration 11000: 0.6931471805599453
Cost after iteration 12000: 0.6931471805599453
Cost after iteration 13000: 0.6931471805599453
Cost after iteration 14000: 0.6931471805599453
```




```
On the train set:
Accuracy: 0.5
```

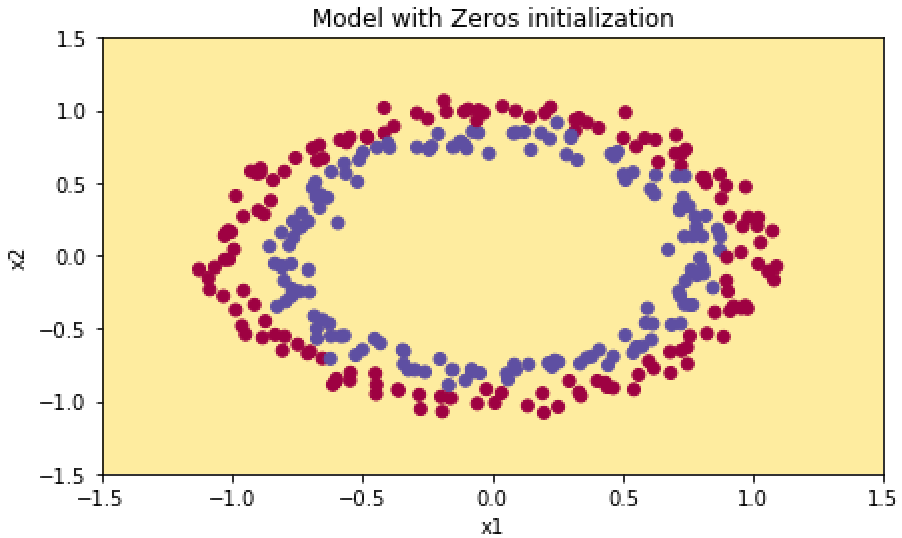
```
On the test set:
Accuracy: 0.5
```

The performance is really bad, and the cost does not really decrease, and the algorithm performs no better than random guessing. Why? Lets look at the details of the predictions and the decision boundary:

In [6]:
```python
print("predictions_train = " + str(predictions_train))
print("predictions_test = " + str(predictions_test))
```

```
predictions_train = [[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0]]
predictions_test = [[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]]
```

In [7]:
```
plt.title("Model with Zeros initialization")
axes = plt.gca()
axes.set_xlim([-1.5, 1.5])
axes.set_ylim([-1.5, 1.5])
plot_decision_boundary(lambda x: predict_dec(parameters, x.T), train_X, train_Y)
```



3 - Random initialization

To break symmetry, lets intialize the weights randomly. Following random initialization, each neuron can then proceed to learn a different function of its inputs. In this exercise, you will see what happens if the weights are intialized randomly, but to very large values.

Exercise: Implement the following function to initialize your weights to large random values (scaled by *10) and your biases to zeros. Use `np.random.randn(..,..) * 10` for weights and `np.zeros((.., ..))` for biases. We are using a fixed `np.random.seed(..)` to make sure your "random" weights match ours, so don't worry if running several times your code gives you always the same initial values for the parameters.

In [8]:
```python
def initialize_parameters_random(layers_dims):
    """
    Arguments:
    layer_dims -- python array (list) containing the size of each layer.
    
    Returns:
    parameters -- python dictionary containing your parameters "W1", "b1", ..., "WL", "bL":
                    W1 -- weight matrix of shape (layers_dims[1], layers_dims[0])
                    b1 -- bias vector of shape (layers_dims[1], 1)
                    ...
                    WL -- weight matrix of shape (layers_dims[L], layers_dims[L-1])
                    bL -- bias vector of shape (layers_dims[L], 1)
    """
    
    np.random.seed(3)               # This seed makes sure your "random" numbers will be the as ours
    parameters = {}
    L = len(layers_dims)            # integer representing the number of layers
    
    for l in range(1, L):
        parameters['W' + str(l)] = np.random.randn(layers_dims[l], layers_dims[l - 1]) * 10
        #If without the multiplication of 10, it might be better.
        parameters['b' + str(l)] = np.zeros((layers_dims[l], 1))

    return parameters
```

In [9]:
```python
parameters = initialize_parameters_random([3, 2, 1])
print("W1 = " + str(parameters["W1"]))
print("b1 = " + str(parameters["b1"]))
print("W2 = " + str(parameters["W2"]))
print("b2 = " + str(parameters["b2"]))
```

```
W1 = [[ 17.88628473   4.36509851   0.96497468]
 [-18.63492703  -2.77388203  -3.54758979]]
b1 = [[ 0.]
 [ 0.]]
W2 = [[-0.82741481 -6.27000677]]
b2 = [[ 0.]]
```

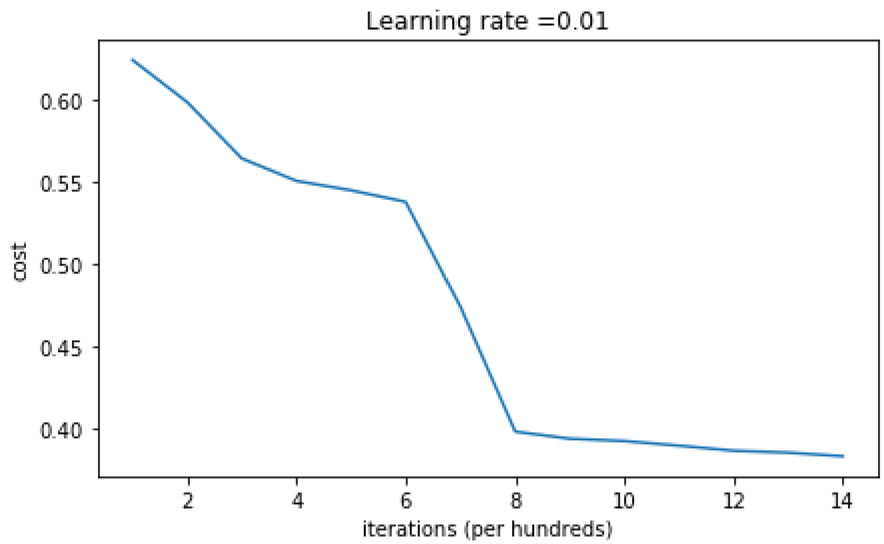
Run the following code to train your model on 15,000 iterations using random initialization.

In [10]:
```python
parameters = model(train_X, train_Y, initialization = "random")
print("On the train set:")
predictions_train = predict(train_X, train_Y, parameters)
print("On the test set:")
predictions_test = predict(test_X, test_Y, parameters)
```

```
Cost after iteration 0: inf

/home/jovyan/work/week5/Initialization/init_utils.py:145: RuntimeWarning: divide by zero encountered in log
  logprobs = np.multiply(-np.log(a3),Y) + np.multiply(-np.log(1 - a3), 1 - Y)
/home/jovyan/work/week5/Initialization/init_utils.py:145: RuntimeWarning: invalid value encountered in multiply
  logprobs = np.multiply(-np.log(a3),Y) + np.multiply(-np.log(1 - a3), 1 - Y)

Cost after iteration 1000: 0.6237287551108738
Cost after iteration 2000: 0.5981106708339466
Cost after iteration 3000: 0.5638353726276827
Cost after iteration 4000: 0.550152614449184
Cost after iteration 5000: 0.5444235275228304
Cost after iteration 6000: 0.5374184054630083
Cost after iteration 7000: 0.47357131493578297
Cost after iteration 8000: 0.39775634899580387
Cost after iteration 9000: 0.3934632865981078
Cost after iteration 10000: 0.39202525076484457
Cost after iteration 11000: 0.38921493051297673
Cost after iteration 12000: 0.38614221789840486
Cost after iteration 13000: 0.38497849983013926
Cost after iteration 14000: 0.38278397192120406
```



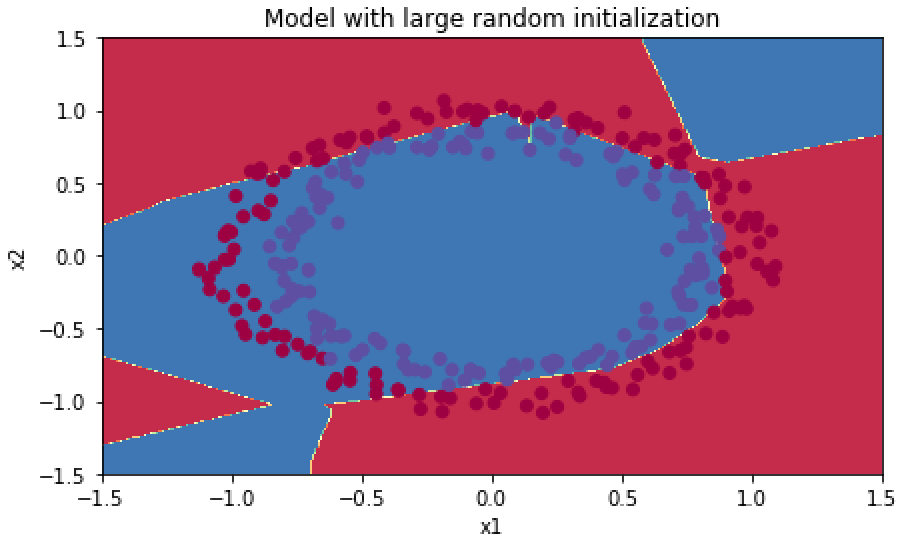

```
On the train set:
Accuracy: 0.83
On the test set:
Accuracy: 0.86
```

In [11]:

```python
print(predictions_train)
print(predictions_test)
```

```
[[1 0 1 1 0 0 1 1 1 1 1 0 1 0 0 1 0 1 1 0 0 0 1 0 1 1 1 1 1 1 0 1 1 0 0 1 1
  1 1 1 1 1 1 0 1 1 1 1 0 1 0 1 1 1 1 0 0 1 1 1 1 0 1 1 0 1 0 1 1 1 1 0 0 0
  0 0 1 0 1 0 1 1 1 0 0 1 1 1 1 1 1 0 0 1 1 1 0 1 1 0 1 0 1 1 0 1 1 0 1 0 1
  1 0 0 1 0 0 1 1 0 1 1 1 0 1 0 0 1 0 1 1 1 1 1 1 1 0 1 1 0 0 1 1 0 0 0 1 0
  1 0 1 0 1 1 1 0 0 1 1 1 1 0 1 1 0 1 0 1 1 0 1 0 1 1 1 1 0 1 1 1 1 0 1 0 1
  0 1 1 1 1 0 1 1 0 1 1 0 1 1 0 1 0 1 1 1 0 1 1 1 0 1 0 1 0 0 1 0 1 1 0 1 1
  0 1 1 0 1 1 1 0 1 1 1 1 0 1 0 0 1 1 0 1 1 1 0 0 0 1 1 0 1 1 1 1 0 1 1 0 1
  1 1 0 0 1 0 0 0 1 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1 0 0 1 1 1 1 1 1 1 0 0 0 1
  1 1 1 0]]
[[1 1 1 1 0 1 0 1 1 0 1 1 1 0 0 0 0 1 0 1 0 0 1 0 1 0 1 1 1 1 1 0 0 0 0 1 0
  1 1 0 0 1 1 1 1 1 0 1 1 1 0 1 0 1 1 0 1 0 1 0 1 1 1 1 1 1 1 1 1 0 1 0 1 1
  1 1 1 0 1 0 0 1 0 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0]]
```

In [12]:
```python
plt.title("Model with large random initialization")
axes = plt.gca()
axes.set_xlim([-1.5, 1.5])
axes.set_ylim([-1.5, 1.5])
plot_decision_boundary(lambda x: predict_dec(parameters, x.T), train_X, train_Y)
```



4 - He initialization

In [13]:
```python
def initialize_parameters_he(layers_dims):

    np.random.seed(3)
    parameters = {}
    L = len(layers_dims) - 1 # integer representing the number of layers

    for l in range(1, L + 1):
        parameters['W' + str(l)] = np.random.randn(layers_dims[l], layers_dims[l - 1]) * np.sqrt(2 / layers_dims
        parameters['b' + str(l)] = np.zeros((layers_dims[l], 1))

    return parameters
```

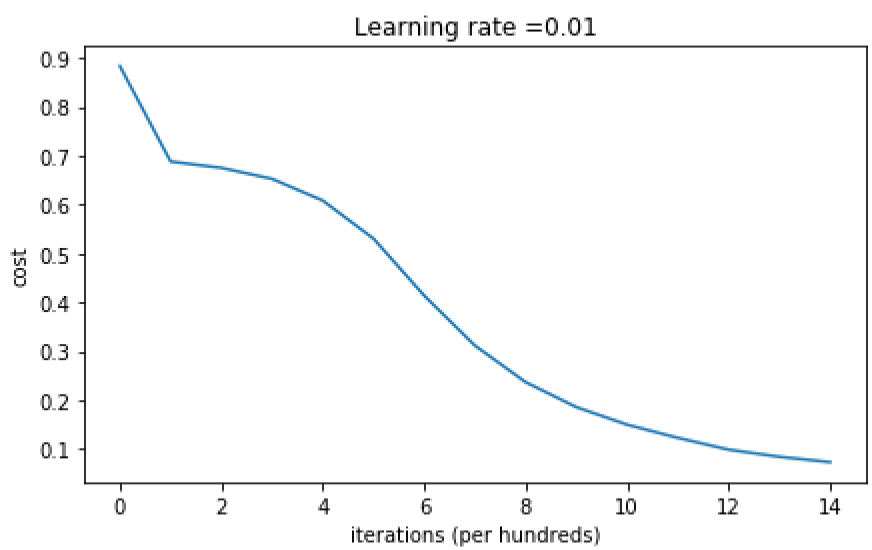
In [14]:
```python
parameters = initialize_parameters_he([2, 4, 1])
print("W1 = " + str(parameters["W1"]))
print("b1 = " + str(parameters["b1"]))
print("W2 = " + str(parameters["W2"]))
print("b2 = " + str(parameters["b2"]))
```

```
W1 = [[ 1.78862847  0.43650985]
 [ 0.09649747 -1.8634927 ]
 [-0.2773882  -0.35475898]
 [-0.08274148 -0.62700068]]
b1 = [[ 0.]
 [ 0.]
 [ 0.]
 [ 0.]]
W2 = [[-0.03098412 -0.33744411 -0.92904268  0.62552248]]
b2 = [[ 0.]]
```

Run the following code to train your model on 15,000 iterations using He initialization.

In [15]:
```
parameters = model(train_X, train_Y, initialization = "he")
print("On the train set:")
predictions_train = predict(train_X, train_Y, parameters)
print("On the test set:")
predictions_test = predict(test_X, test_Y, parameters)
```

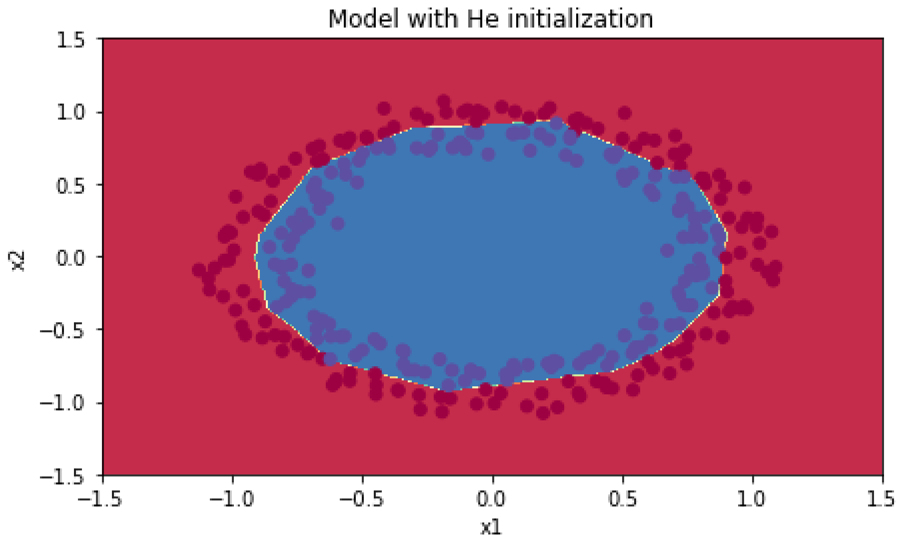
```
Cost after iteration 0: 0.8830537463419761
Cost after iteration 1000: 0.6879825919728063
Cost after iteration 2000: 0.6751286264523371
Cost after iteration 3000: 0.6526117768893807
Cost after iteration 4000: 0.6082958970572938
Cost after iteration 5000: 0.5304944491717495
Cost after iteration 6000: 0.4138645817071794
Cost after iteration 7000: 0.3117803464844441
Cost after iteration 8000: 0.23696215330322562
Cost after iteration 9000: 0.18597287209206836
Cost after iteration 10000: 0.1501555628037182
Cost after iteration 11000: 0.12325079292273548
Cost after iteration 12000: 0.09917746546525937
Cost after iteration 13000: 0.0845705595402428
Cost after iteration 14000: 0.07357895962677366
```




```
On the train set:
Accuracy: 0.993333333333
```

```
On the test set:
Accuracy: 0.96
```

In [16]:
```
plt.title("Model with He initialization")
axes = plt.gca()
axes.set_xlim([-1.5, 1.5])
axes.set_ylim([-1.5, 1.5])
plot_decision_boundary(lambda x: predict_dec(parameters, x.T), train_X, train_Y)
```



5 - Conclusions

You have seen three different types of initializations. For the same number of iterations and same hyperparameters the comparison is:

Model	Train accuracy	Problem/Comment
3-layer NN with zeros initialization	50%	fails to break symmetry
3-layer NN with large random initialization	83%	too large weights
3-layer NN with He initialization	99%	recommended method