Reference

Coursera Deep learning series by Andrew NG

Introduction

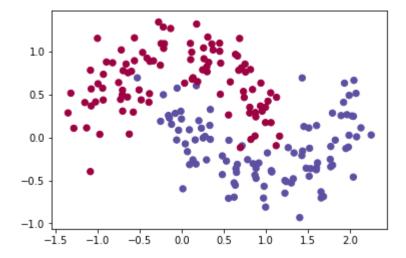
Welcome to the second assignment of this week. Deep Learning models have so much flexibility and capacity that overfitting can be a serious problem.

```
In [1]: # import packages
    import numpy as np
    import matplotlib.pyplot as plt
    from reg_utils import sigmoid, relu, plot_decision_boundary, initialize_parameters, load_2D_dataset, predict_dec
    from reg_utils import compute_cost, predict, forward_propagation, backward_propagation, update_parameters
    import sklearn
    import sklearn.datasets
    import scipy.io
    from testCases_reg import *

    %matplotlib inline
    plt.rcParams['figure.figsize'] = (7.0, 4.0) # set default size of plots
    plt.rcParams['image.interpolation'] = 'nearest'
    plt.rcParams['image.cmap'] = 'gray'
```

(2, 200) (1, 200)

Out[3]: <matplotlib.collections.PathCollection at 0x21d1cc4d4a8>



```
In [4]: import numpy as np
    from sklearn.model_selection import train_test_split
    X=X.T #To be consistent with train_test_split
    Y=Y.T
    print(X.shape,Y.shape)
    train_X, test_X, train_Y, test_Y = train_test_split(X, Y, test_size=0.33, random_state=42)
    print(train_X.shape, test_X.shape, train_Y.shape, test_Y.shape)

    train_X = train_X.T #This is to be consistent to the model used below.
    test_X = test_X.T
    train_Y = train_Y.T
    test_Y = test_Y.T

(200, 2) (200, 1)
```

```
(134, 2) (66, 2) (134, 1) (66, 1)
```

1 - Non-regularized model

You will use the following neural network (already implemented for you below). This model can be used:

- in *regularization mode* -- by setting the lambd input to a non-zero value. We use "lambd" instead of "lambda" because "lambda" is a reserved keyword in Python.
- in dropout mode -- by setting the keep_prob to a value less than one

You will first try the model without any regularization. Then, you will implement:

- L2 regularization -- functions: "compute_cost_with_regularization()" and "backward_propagation_with_regularization()"
- Dropout -- functions: "forward_propagation_with_dropout() "and "backward_propagation_with_dropout() "

In each part, you will run this model with the correct inputs so that it calls the functions you've implemented. Take a look at the code below to familiarize yourself with the model.

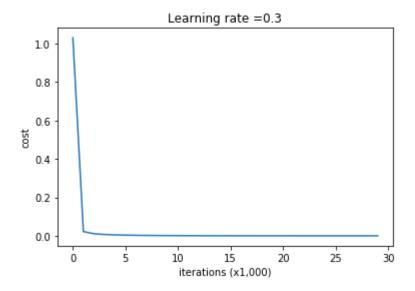
```
In [5]: def model(X, Y, learning rate = 0.3, num iterations = 30000, print cost = True, lambd = 0, keep prob = 1):
            Implements a three-layer neural network: LINEAR->RELU->LINEAR->RELU->LINEAR->SIGMOID.
            Arguments:
            X -- input data, of shape (input size, number of examples)
            Y -- true "label" vector (1 for blue dot / 0 for red dot), of shape (output size, number of examples)
            learning rate -- learning rate of the optimization
            num iterations -- number of iterations of the optimization loop
            print cost -- If True, print the cost every 10000 iterations
            lambd -- regularization hyperparameter, scalar
            keep prob - probability of keeping a neuron active during drop-out, scalar.
            Returns:
            parameters -- parameters learned by the model. They can then be used to predict.
            grads = \{\}
            costs = []
                                                   # to keep track of the cost
            m = X.shape[1]
                                                   # number of examples
            layers dims = [X.shape[0], 20, 3, 1]
            # Initialize parameters dictionary.
            parameters = initialize parameters(layers dims)
            # Loop (gradient descent)
            for i in range(0, num iterations):
                # Forward propagation: LINEAR -> RELU -> LINEAR -> RELU -> LINEAR -> SIGMOID.
                if keep prob == 1:
                    a3, cache = forward propagation(X, parameters)
                elif keep prob < 1:</pre>
                    a3, cache = forward propagation with dropout(X, parameters, keep prob)
                # Cost function
                if lambd == 0:
                    cost = compute cost(a3, Y)
                else:
                    cost = compute cost with regularization(a3, Y, parameters, lambd)
                # Backward propagation.
```

```
assert(lambd == 0 or keep prob == 1)
                                          # it is possible to use both L2 regularization and dropout,
                                            # but this assignment will only explore one at a time
    if lambd == 0 and keep prob == 1:
        grads = backward propagation(X, Y, cache)
    elif lambd != 0:
        grads = backward propagation with regularization(X, Y, cache, lambd)
    elif keep prob < 1:</pre>
        grads = backward propagation with dropout(X, Y, cache, keep prob)
    # Update parameters.
    parameters = update parameters(parameters, grads, learning rate)
    # Print the loss every 10000 iterations
    if print cost and i % 10000 == 0:
        print("Cost after iteration {}: {}".format(i, cost))
    if print cost and i % 1000 == 0:
        costs.append(cost)
# plot the cost
plt.plot(costs)
plt.ylabel('cost')
plt.xlabel('iterations (x1,000)')
plt.title("Learning rate =" + str(learning rate))
plt.show()
return parameters
```

Let's train the model without any regularization, and observe the accuracy on the train/test sets.

```
In [6]: parameters = model(train_X, train_Y)
    print("On the training set:")
    predictions_train = predict(train_X, train_Y, parameters)
    print("On the test set:")
    predictions_test = predict(test_X, test_Y, parameters)
```

Cost after iteration 0: 1.030668564098119 Cost after iteration 10000: 0.0013096475336283563 Cost after iteration 20000: 0.00045300271498517185



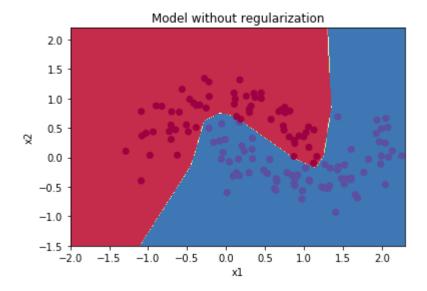
On the training set:

Accuracy: 1.0 On the test set:

Accuracy: 0.9242424242424242

```
In [7]: plt.title("Model without regularization")
    axes = plt.gca()
    axes.set_xlim([-2.0, 2.3])
    axes.set_ylim([-1.5, 2.2])
    plot_decision_boundary(lambda x: predict_dec(parameters, x.T), train_X, train_Y)
```

(1, 134)
(134,)



2 - L2 Regularization

The standard way to avoid overfitting is called **L2 regularization**. It consists of appropriately modifying your cost function, from:

$$J = -\frac{1}{m} \sum_{i=1}^{m} \left(y^{(i)} \log \left(a^{[L](i)} \right) + (1 - y^{(i)}) \log \left(1 - a^{[L](i)} \right) \right) \tag{1}$$

To:

$$J_{regularized} = -\frac{1}{m} \sum_{i=1}^{m} \left(y^{(i)} \log \left(a^{[L](i)} \right) + (1 - y^{(i)}) \log \left(1 - a^{[L](i)} \right) \right) + \underbrace{\frac{1}{m} \frac{\lambda}{2} \sum_{l} \sum_{k} \sum_{j} W_{k,j}^{[l]2}}_{L2 \text{ regularization cost}}$$
(2)

Let's modify your cost and observe the consequences.

Exercise: Implement compute_cost_with_regularization() which computes the cost given by formula (2). To calculate $\sum_{k} \sum_{j} W_{k,j}^{[l]2}$, use:

np.sum(np.square(Wl))

Note that you have to do this for $W^{[1]}$, $W^{[2]}$ and $W^{[3]}$, then sum the three terms and multiply by $\frac{1}{m}\frac{\lambda}{2}$.

Extend this to the L-layer model.

```
In [8]: def compute cost with regularization(A3, Y, parameters, lambd):
            Implement the cost function with L2 regularization. See formula (2) above.
            Arguments:
            A3 -- post-activation, output of forward propagation, of shape (output size, number of examples)
            Y -- "true" labels vector, of shape (output size, number of examples)
            parameters -- python dictionary containing parameters of the model
            Returns:
            cost - value of the regularized loss function (formula (2))
            m = Y.shape[1]
            W1 = parameters["W1"]
            W2 = parameters["W2"]
            W3 = parameters["W3"]
            cross entropy cost = compute cost(A3, Y) # This gives you the cross-entropy part of the cost
            L2 regularization cost = lambd * (np.sum(np.square(W1)) + np.sum(np.square(W2)) + np.sum(np.square(W3))) / (2
            cost = cross entropy cost + L2 regularization cost
            return cost
```

```
In [9]: A3, Y_assess, parameters = compute_cost_with_regularization_test_case()
    print("cost = " + str(compute_cost_with_regularization(A3, Y_assess, parameters, lambd = 0.1)))
```

Because you changed the cost, you have to change backward propagation as well! All the gradients have to be computed with respect to this new cost.

Exercise: Implement the changes needed in backward propagation to take into account regularization. The changes only concern dW1, dW2 and dW3. For each, you have to add the regularization term's gradient $(\frac{d}{dW}(\frac{1}{2}\frac{\lambda}{m}W^2) = \frac{\lambda}{m}W)$.

```
In [10]: def backward propagation with regularization(X, Y, cache, lambd):
             m = X.shape[1]
             (Z1, A1, W1, b1, Z2, A2, W2, b2, Z3, A3, W3, b3) = cache
             dZ3 = A3 - Y
             dW3 = 1. / m * np.dot(dZ3, A2.T) + (lambd * W3) / m
             db3 = 1. / m * np.sum(dZ3, axis=1, keepdims=True)
             dA2 = np.dot(W3.T, dZ3)
             dZ2 = np.multiply(dA2, np.int64(A2 > 0))
             dW2 = 1. / m * np.dot(dZ2, A1.T) + (lambd * W2) / m
             db2 = 1. / m * np.sum(dZ2, axis=1, keepdims=True)
             dA1 = np.dot(W2.T, dZ2)
             dZ1 = np.multiply(dA1, np.int64(A1 > 0))
             dW1 = 1. / m * np.dot(dZ1, X.T) + (lambd * W1) / m
             db1 = 1. / m * np.sum(dZ1, axis=1, keepdims=True)
             gradients = {"dZ3": dZ3, "dW3": dW3, "db3": db3, "dA2": dA2,
                           "dZ2": dZ2, "dW2": dW2, "db2": db2, "dA1": dA1,
                           "dZ1": dZ1, "dW1": dW1, "db1": db1}
             return gradients
```

```
In [11]: X_assess, Y_assess, cache = backward_propagation_with_regularization_test_case()

grads = backward_propagation_with_regularization(X_assess, Y_assess, cache, lambd=0.7)
print ("dW1 = " + str(grads["dW1"]))
print ("dW2 = " + str(grads["dW2"]))
print ("dW3 = " + str(grads["dW3"]))

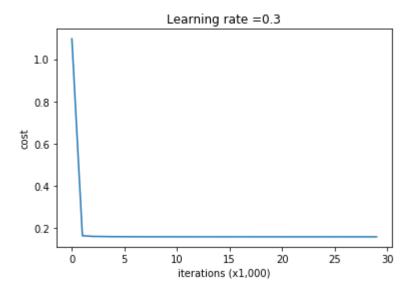
dW1 = [[-0.25604646   0.12298827 -0.28297129]
        [-0.17706303   0.34536094 -0.4410571 ]]
dW2 = [[  0.79276486   0.85133918]
        [-0.0957219   -0.01720463]
        [-0.13100772   -0.03750433]]
dW3 = [[-1.77691347   -0.11832879   -0.09397446]]
```

Let's now run the model with L2 regularization ($\lambda = 0.7$). The model() function will call:

- compute_cost_with_regularization instead of compute_cost
- backward_propagation_with_regularization instead of backward_propagation

```
In [12]: parameters = model(train_X, train_Y, lambd=0.7)
    print("On the train set:")
    predictions_train = predict(train_X, train_Y, parameters)
    print("On the test set:")
    predictions_test = predict(test_X, test_Y, parameters)
```

Cost after iteration 0: 1.0963418369311082 Cost after iteration 10000: 0.15812776244188406 Cost after iteration 20000: 0.15796526408237627



On the train set:

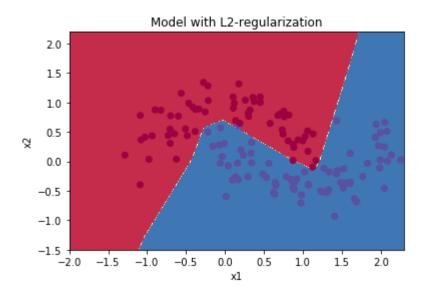
Accuracy: 0.9925373134328358

On the test set:

Accuracy: 0.93939393939394

```
In [13]: plt.title("Model with L2-regularization")
    axes = plt.gca()
    axes.set_xlim([-2.0, 2.3])
    axes.set_ylim([-1.5, 2.2])
    plot_decision_boundary(lambda x: predict_dec(parameters, x.T), train_X, train_Y)

    (1, 134)
    (134,)
```



Observations:

- The value of λ is a hyperparameter that you can tune using a dev set.
- L2 regularization makes your decision boundary smoother. If λ is too large, it is also possible to "oversmooth", resulting in a model with high bias.

3 - Dropout

Finally, **dropout** is a widely used regularization technique that is specific to deep learning. **It randomly shuts down some neurons in each iteration.** Watch these two videos to see what this means!



 $\underline{\text{Figure 2}} : \textbf{Drop-out on the second hidden layer}.$

At each iteration, you shut down (= set to zero) each neuron of a layer with probability $1 - keep_prob$ or keep it with probability $keep_prob$ (50% here). The dropped neurons don't contribute to the training in both the forward and backward propagations of the iteration.

0:00 / 0:08

Figure 3: Drop-out on the first and third hidden layers.

 1^{st} layer: we shut down on average 40% of the neurons. 3^{rd} layer: we shut down on average 20% of the neurons.

When you shut some neurons down, you actually modify your model. The idea behind drop-out is that at each iteration, you train a different model that uses only a subset of your neurons. With dropout, your neurons thus become less sensitive to the activation of one other specific neuron, because that other neuron might be shut down at any time.

3.1 - Forward propagation with dropout

Exercise: Implement the forward propagation with dropout. You are using a 3 layer neural network, and will add dropout to the first and second hidden layers. We will not apply dropout to the input layer or output layer.

Instructions: You would like to shut down some neurons in the first and second layers. To do that, you are going to carry out 4 Steps:

- 1. In lecture, we discussed creating a variable $d^{[1]}$ with the same shape as $a^{[1]}$ using <code>np.random.rand()</code> to randomly get numbers between 0 and 1. Here, you will use a vectorized implementation, so create a random matrix $D^{[1]} = [d^{1}d^{[1](2)}\dots d^{[1](m)}]$ of the same dimension as $A^{[1]}$.
- 2. Set each entry of $D^{[1]}$ to be 0 with probability (1-keep_prob) or 1 with probability (keep_prob), by thresholding values in $D^{[1]}$ appropriately. Hint: to set all the entries of a matrix X to 0 (if entry is less than 0.5) or 1 (if entry is more than 0.5) you would do: X = (X < 0.5). Note that 0 and 1 are respectively equivalent to False and True.
- 3. Set $A^{[1]}$ to $A^{[1]} * D^{[1]}$. (You are shutting down some neurons). You can think of $D^{[1]}$ as a mask, so that when it is multiplied with another matrix, it shuts down some of the values.
- 4. Divide $A^{[1]}$ by keep_prob . By doing this you are assuring that the result of the cost will still have the same expected value as without drop-out. (This technique is also called inverted dropout.)

```
In [14]:
         def forward propagation with dropout(X, parameters, keep prob=0.5):
             Implements the forward propagation: LINEAR -> RELU + DROPOUT -> LINEAR -> RELU + DROPOUT -> LINEAR -> SIGMOID
             Arguments:
             X -- input dataset, of shape (2, number of examples)
             parameters -- python dictionary containing your parameters "W1", "b1", "W2", "b2", "W3", "b3":
                              W1 -- weight matrix of shape (20, 2)
                              b1 -- bias vector of shape (20, 1)
                              W2 -- weight matrix of shape (3, 20)
                              b2 -- bias vector of shape (3, 1)
                              W3 -- weight matrix of shape (1, 3)
                              b3 -- bias vector of shape (1, 1)
             keep prob - probability of keeping a neuron active during drop-out, scalar
             Returns:
             A3 -- last activation value, output of the forward propagation, of shape (1,1)
             cache -- tuple, information stored for computing the backward propagation
             np.random.seed(1)
             # retrieve parameters
             W1 = parameters["W1"]
             b1 = parameters["b1"]
             W2 = parameters["W2"]
             b2 = parameters["b2"]
             W3 = parameters["W3"]
             b3 = parameters["b3"]
             # LINEAR -> RELU -> LINEAR -> RELU -> LINEAR -> SIGMOID
             Z1 = np.dot(W1, X) + b1
             A1 = relu(Z1)
             ### START CODE HERE ### (approx. 4 lines)
                                                                # Steps 1-4 below correspond to the Steps 1-4 described abo
             D1 = np.random.rand(A1.shape[0], A1.shape[1])
                                                                # Step 1: initialize matrix D1 = np.random.rand(..., ...)
             D1 = D1 < keep prob
                                                             # Step 2: convert entries of D1 to 0 or 1 (using keep prob as
             A1 = A1 * D1
                                                                # Step 3: shut down some neurons of A1
             A1 = A1 / keep prob
                                                                # Step 4: scale the value of neurons that haven't been shut
             ### END CODE HERE ###
             Z2 = np.dot(W2, A1) + b2
             A2 = relu(Z2)
```

```
D2 = np.random.rand(A2.shape[0], A2.shape[1])
                                                                # Step 1: initialize matrix D2 = np.random.rand(..., ...)
             D2 = D2 < keep prob
                                                            # Step 2: convert entries of D2 to 0 or 1 (using keep prob as
             A2 = A2 * D2
                                                                # Step 3: shut down some neurons of A2
             A2 = A2 / keep prob
                                                                # Step 4: scale the value of neurons that haven't been shut
             ### END CODE HERE ###
             Z3 = np.dot(W3, A2) + b3
             A3 = sigmoid(Z3)
             cache = (Z1, D1, A1, W1, b1, Z2, D2, A2, W2, b2, Z3, A3, W3, b3)
             return A3, cache
In [15]: X assess, parameters = forward propagation with dropout test case()
         A3, cache = forward propagation with dropout(X assess, parameters, keep prob=0.7)
```

```
print ("A3 = " + str(A3))
```

 $A3 = [[0.36974721 \ 0.00305176 \ 0.04565099 \ 0.49683389 \ 0.36974721]]$

Expected Output:

A3 [[0.36974721 0.00305176 0.04565099 0.49683389 0.36974721]]

3.2 - Backward propagation with dropout

START CODE HERE ### (approx. 4 lines)

Exercise: Implement the backward propagation with dropout. As before, you are training a 3 layer network. Add dropout to the first and second hidden layers, using the masks $D^{[1]}$ and $D^{[2]}$ stored in the cache.

Instruction: Backpropagation with dropout is actually quite easy. You will have to carry out 2 Steps:

- 1. You had previously shut down some neurons during forward propagation, by applying a mask $D^{[1]}$ to A1. In backpropagation, you will have to shut down the same neurons, by reapplying the same mask $D^{[1]}$ to dA1.
- 2. During forward propagation, you had divided A1 by keep prob. In backpropagation, you'll therefore have to divide dA1 by keep prob again (the calculus interpretation is that if $A^{[1]}$ is scaled by keep prob, then its derivative $dA^{[1]}$ is also scaled by the same keep prob).

```
In [16]: def backward propagation with dropout(X, Y, cache, keep prob):
             Implements the backward propagation of our baseline model to which we added dropout.
             Arguments:
             X -- input dataset, of shape (2, number of examples)
             Y -- "true" labels vector, of shape (output size, number of examples)
             cache -- cache output from forward propagation with dropout()
             keep prob - probability of keeping a neuron active during drop-out, scalar
             Returns:
             gradients -- A dictionary with the gradients with respect to each parameter, activation and pre-activation va
             m = X.shape[1]
             (Z1, D1, A1, W1, b1, Z2, D2, A2, W2, b2, Z3, A3, W3, b3) = cache
             dZ3 = A3 - Y
             dW3 = 1. / m * np.dot(dZ3, A2.T)
             db3 = 1. / m * np.sum(dZ3, axis=1, keepdims=True)
             dA2 = np.dot(W3.T, dZ3)
             ### START CODE HERE ### (≈ 2 lines of code)
             dA2 = dA2 * D2
                                         # Step 1: Apply mask D2 to shut down the same neurons as during the forward prope
             dA2 = dA2 / keep_prob
                                                 # Step 2: Scale the value of neurons that haven't been shut down
             ### END CODE HERE ###
             dZ2 = np.multiply(dA2, np.int64(A2 > 0))
             dW2 = 1. / m * np.dot(dZ2, A1.T)
             db2 = 1. / m * np.sum(dZ2, axis=1, keepdims=True)
             dA1 = np.dot(W2.T, dZ2)
             ### START CODE HERE ### (≈ 2 lines of code)
             dA1 = dA1 * D1
                                         # Step 1: Apply mask D1 to shut down the same neurons as during the forward prope
             dA1 = dA1 / keep_prob
                                                 # Step 2: Scale the value of neurons that haven't been shut down
             ### END CODE HERE ###
             dZ1 = np.multiply(dA1, np.int64(A1 > 0))
             dW1 = 1. / m * np.dot(dZ1, X.T)
             db1 = 1. / m * np.sum(dZ1, axis=1, keepdims=True)
             gradients = {"dZ3": dZ3, "dW3": dW3, "db3": db3, "dA2": dA2,
                           "dZ2": dZ2, "dW2": dW2, "db2": db2, "dA1": dA1,
                           "dZ1": dZ1, "dW1": dW1, "db1": db1}
```

```
In [17]: X_assess, Y_assess, cache = backward_propagation_with_dropout_test_case()
         gradients = backward propagation with dropout(X assess, Y assess, cache, keep prob=0.8)
         print ("dA1 = " + str(gradients["dA1"]))
         print ("dA2 = " + str(gradients["dA2"]))
         dA1 = [ [ 0.36544439   0. ]
                                         -0.00188233 0.
                                                                 -0.17408748]
          [ 0.65515713 0.
                                   -0.00337459 0.
                                                           -0.
         dA2 = [ [ 0.58180856   0.
                                         -0.00299679 0.
                                                                 -0.27715731]
          [ 0.
                        0.53159854 -0.
                                                0.53159854 -0.34089673]
          [ 0.
                        0.
                                   -0.00292733 0.
                                                           -0.
                                                                      ]]
```

Expected Output:

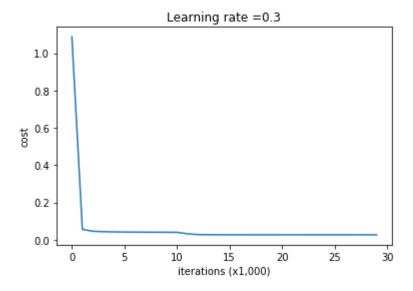
```
dA1 [[ 0.365444439 0. -0.00188233 0. -0.17408748] [ 0.65515713 0. -0.00337459 0. -0. ]]
dA2 [[ 0.58180856 0. -0.00299679 0. -0.27715731] [ 0. 0.53159854 -0. 0.53159854 -0.34089673] [ 0. 0. -0.00292733 0. -0. ]]
```

Let's now run the model with dropout (keep_prob = 0.86). It means at every iteration you shut down each neurons of layer 1 and 2 with 24% probability. The function model() will now call:

- forward_propagation_with_dropout instead of forward_propagation.
- backward_propagation_with_dropout instead of backward_propagation.

```
In [18]: parameters = model(train_X, train_Y, keep_prob=0.86, learning_rate=0.3)
    print("On the train set:")
    predictions_train = predict(train_X, train_Y, parameters)
    print("On the test set:")
    predictions_test = predict(test_X, test_Y, parameters)
```

Cost after iteration 0: 1.087298398064148 Cost after iteration 10000: 0.039881393279895795 Cost after iteration 20000: 0.02647010624682434



On the train set:

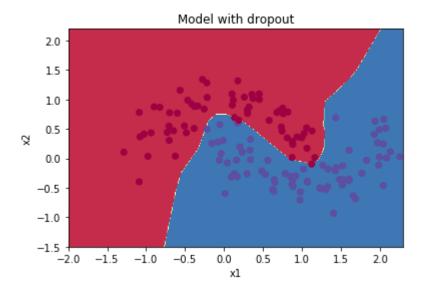
Accuracy: 0.9925373134328358

On the test set:

Accuracy: 0.9090909090909091

```
In [19]: plt.title("Model with dropout")
   axes = plt.gca()
   axes.set_xlim([-2.0, 2.3])
   axes.set_ylim([-1.5, 2.2])
   plot_decision_boundary(lambda x: predict_dec(parameters, x.T), train_X, train_Y)
(1, 134)
```

(1, 134) (134,)



Note:

- A **common mistake** when using dropout is to use it both in training and testing. You should use dropout (randomly eliminate nodes) only in training.
- Deep learning frameworks like <u>tensorflow (https://www.tensorflow.org/api_docs/python/tf/nn/dropout)</u>, <u>PaddlePaddle (http://doc.paddlepaddle.org/release_doc/0.9.0/doc/ui/api/trainer_config_helpers/attrs.html)</u>, <u>keras (https://keras.io/layers/core/#dropout)</u> or <u>caffe (http://caffe.berkeleyvision.org/tutorial/layers/dropout.html)</u> come with a dropout layer implementation. Don't stress you will soon learn some of these frameworks.

What you should remember about dropout:

- Dropout is a regularization technique.
- You only use dropout during training. Don't use dropout (randomly eliminate nodes) during test time.
- Apply dropout both during forward and backward propagation.

• During training time, divide each dropout layer by keep_prob to keep the same expected value for the activations. For example, if keep_prob is 0.5, then we will on average shut down half the nodes, so the output will be scaled by 0.5 since only the remaining half are contributing to the solution. Dividing by 0.5 is equivalent to multiplying by 2. Hence, the output now has the same expected value. You can check that this works even when keep_prob is other values than 0.5.

4 - Conclusions

Note that regularization hurts training set performance! This is because it limits the ability of the network to overfit to the training set. But since it ultimately gives better test accuracy, it is helping your system.

Congratulations for finishing this assignment! And also for revolutionizing French football. :-)

What we want you to remember from this notebook:

- · Regularization will help you reduce overfitting.
- Regularization will drive your weights to lower values.
- L2 regularization and Dropout are two very effective regularization techniques.