Reference

Coursera deep learning series by Andrew NG.

Building your Recurrent Neural Network - Step by Step

Recurrent Neural Networks (RNN) are very effective for Natural Language Processing and other **sequence tasks** because they have "memory". They can read inputs $x^{\langle t \rangle}$ (such as words) one at a time, and remember some information/context through the hidden layer activations that get passed from one time-step to the next. This allows a uni-directional RNN to take information from the past to process later inputs. A bidirection RNN can take context from both the past and the future.

Notation:

- Superscript $[l]$ denotes an object associated with the l^{th} layer.
  - Example: $a^{[4]}$ is the 4^{th} layer activation. $W^{[5]}$ and $b^{[5]}$ are the 5^{th} layer parameters.
- Superscript (i) denotes an object associated with the i^{th} example.
  - Example: $x^{(i)}$ is the i^{th} training example input.
- Superscript $\langle t \rangle$ denotes an object at the t^{th} time-step.
  - Example: $x^{\langle t \rangle}$ is the input x at the t^{th} time-step. $x^{(i)\langle t \rangle}$ is the input at the t^{th} timestep of example i.
- Lowerscript i denotes the i^{th} entry of a vector.
  - Example: $a^{[l]}_i$ denotes the i^{th} entry of the activations in layer l.

In [1]:
```
import numpy as np
from rnn_utils import *
```

1 - Forward propagation for the basic Recurrent Neural Network

Later this week, you will generate music using an RNN. The basic RNN that you will implement has the structure below. **In this example,** $T_x = T_y$.

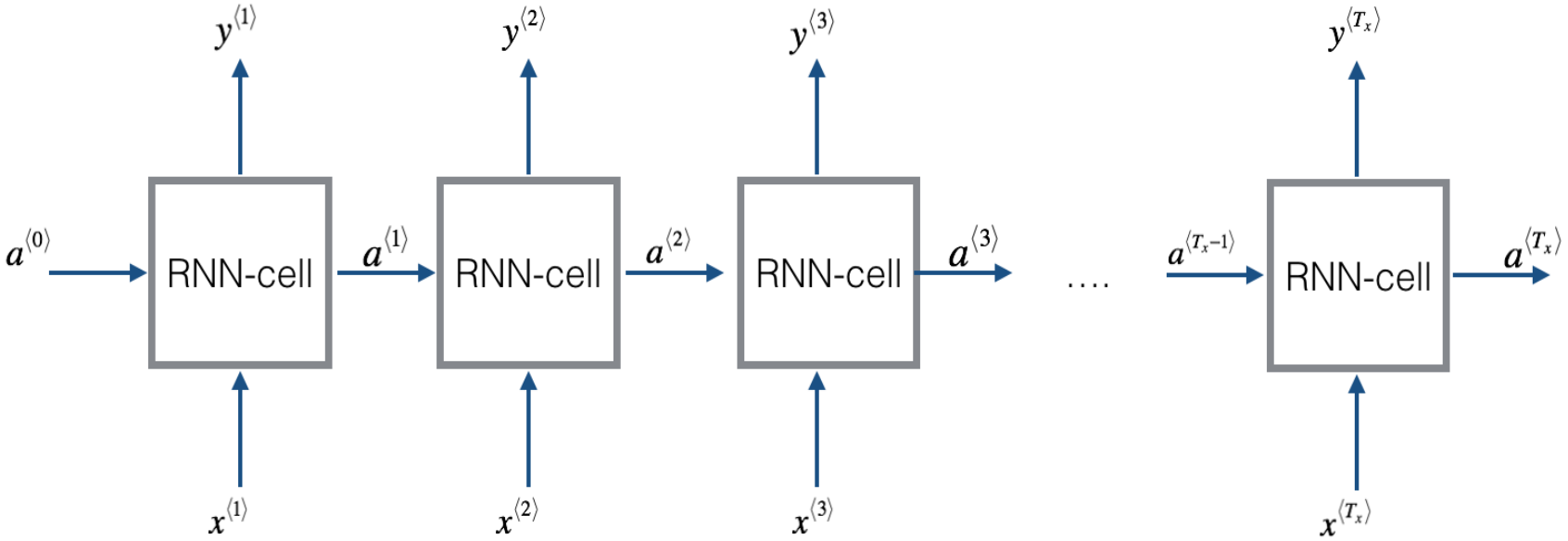


Figure 1: Basic RNN model

Steps:

1. Implement the calculations needed for one time-step of the RNN.
2. Implement a loop over T_x time-steps in order to process all the inputs, one at a time.

1.1 - RNN cell

A Recurrent neural network can be seen as the repetition of a single cell. You are first going to implement the computations for a single time-step. The following figure describes the operations for a single time-step of an RNN cell.

Comments: In the classical neural network, $y^{\langle t \rangle}$ and $a^{\langle t \rangle}$ are same. However, in RNN, they are different as below.

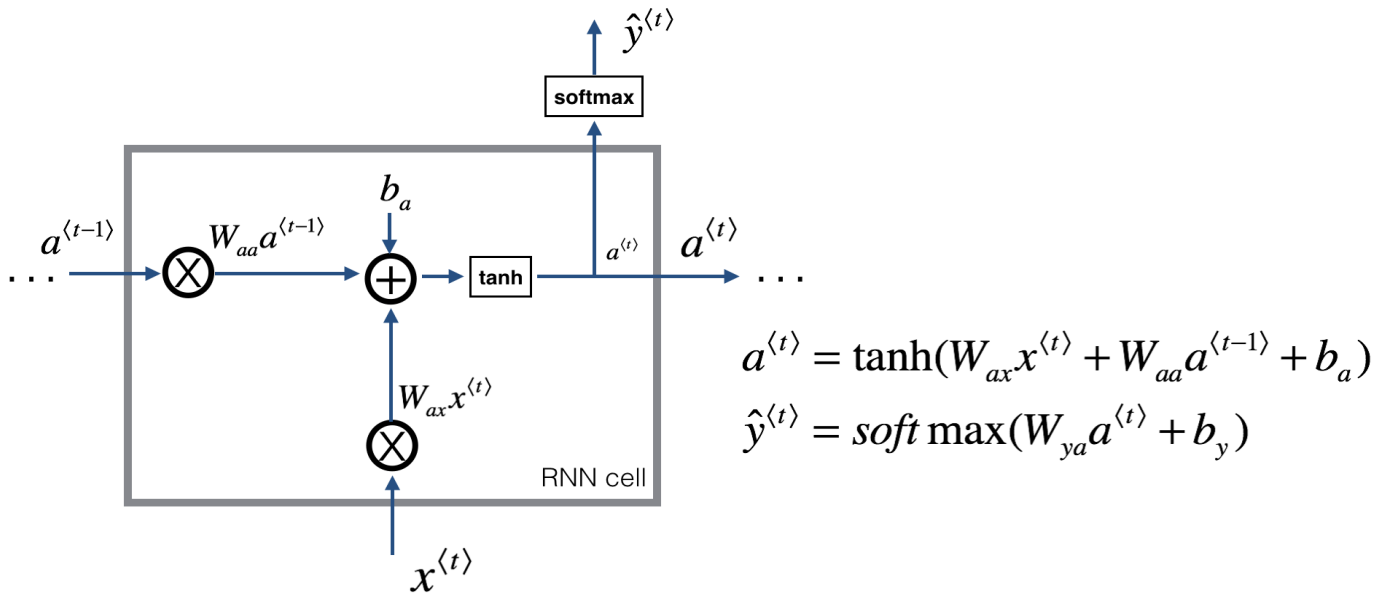


Figure 2: Basic RNN cell. Takes as input $x^{\langle t \rangle}$ (current input) and $a^{\langle t-1 \rangle}$ (previous hidden state containing information from the past), and outputs $a^{\langle t \rangle}$ which is given to the next RNN cell and also used to predict $y^{\langle t \rangle}$

Exercise: Implement the RNN-cell described in Figure (2).

Instructions:

1. Compute the hidden state with tanh activation: $a^{\langle t \rangle} = \tanh(W_{aa}a^{\langle t-1 \rangle} + W_{ax}x^{\langle t \rangle} + b_a)$.
2. Using your new hidden state $a^{\langle t \rangle}$, compute the prediction $\hat{y}^{\langle t \rangle} = softmax(W_{ya}a^{\langle t \rangle} + b_y)$. We provided you a function: `softmax`.
3. Store $(a^{\langle t \rangle}, a^{\langle t-1 \rangle}, x^{\langle t \rangle}, parameters)$ in cache
4. Return $a^{\langle t \rangle}$, $y^{\langle t \rangle}$ and cache

We will vectorize over m examples. Thus, $x^{\langle t \rangle}$ will have dimension (n_x, m), and $a^{\langle t \rangle}$ will have dimension (n_a, m).

About softmax implementation. https://stackoverflow.com/questions/42599498/numercially-stable-softmax/42606665#42606665 (https://stackoverflow.com/questions/42599498/numercially-stable-softmax/42606665#42606665)
The following has three versions for softmax implementation. All versions will give same results. However, this same result will not same as the 'expected output'. Check it out the possible reasons in the future. Note: only part of results are inconsistent.

In [2]:
```
def rnn_cell_forward(xt, a_prev, parameters):

    Wax = parameters["Wax"]
    Waa = parameters["Waa"]
    Wya = parameters["Wya"]
    ba = parameters["ba"]
    by = parameters["by"]

    a_next = np.tanh(np.dot(Wax, xt) + np.dot(Waa, a_prev) + ba)

    yt_pred = softmax(np.dot(Wya, a_next) + by)

    cache = (a_next, a_prev, xt, parameters)

    return a_next, yt_pred, cache
```

In [3]:
```
np.random.seed(1)
xt = np.random.randn(3,10)
a_prev = np.random.randn(5,10)
Waa = np.random.randn(5,5)
Wax = np.random.randn(5,3)
Wya = np.random.randn(2,5)
ba = np.random.randn(5,1)
by = np.random.randn(2,1)
parameters = {"Waa": Waa, "Wax": Wax, "Wya": Wya, "ba": ba, "by": by}

a_next, yt_pred, cache = rnn_cell_forward(xt, a_prev, parameters)
print("a_next[4] = ", a_next[4])
print("a_next.shape = ", a_next.shape)
print("yt_pred[1] =", yt_pred[1])
print("yt_pred.shape = ", yt_pred.shape)
```

```
a_next[4] =  [ 0.59584544  0.18141802  0.61311866  0.99808218  0.85016201  0.99980978
 -0.18887155  0.99815551  0.6531151   0.82872037]
a_next.shape =  (5, 10)
yt_pred[1] = [0.9888161  0.01682021 0.21140899 0.36817467 0.98988387 0.88945212
 0.36920224 0.9966312  0.9982559  0.17746526]
yt_pred.shape =  (2, 10)
```

Expected Output:

a_next[4]: [0.59584544 0.18141802 0.61311866 0.99808218 0.85016201 0.99980978 -0.18887155 0.99815551 0.6531151 0.82872037]

a_next.shape: (5, 10)

yt[1]: [0.9888161 0.01682021 0.21140899 0.36817467 0.98988387 0.88945212 0.36920224 0.9966312 0.9982559 0.17746526]

yt.shape: (2, 10)

1.2 - RNN forward pass

You can see an RNN as the repetition of the cell you've just built. If your input sequence of data is carried over 10 time steps, then you will copy the RNN cell 10 times. Each cell takes as input the hidden state from the previous cell ($a^{\langle t-1 \rangle}$) and the current time-step's input data ($x^{\langle t \rangle}$). It outputs a hidden state ($a^{\langle t \rangle}$) and a prediction ($y^{\langle t \rangle}$) for this time-step.

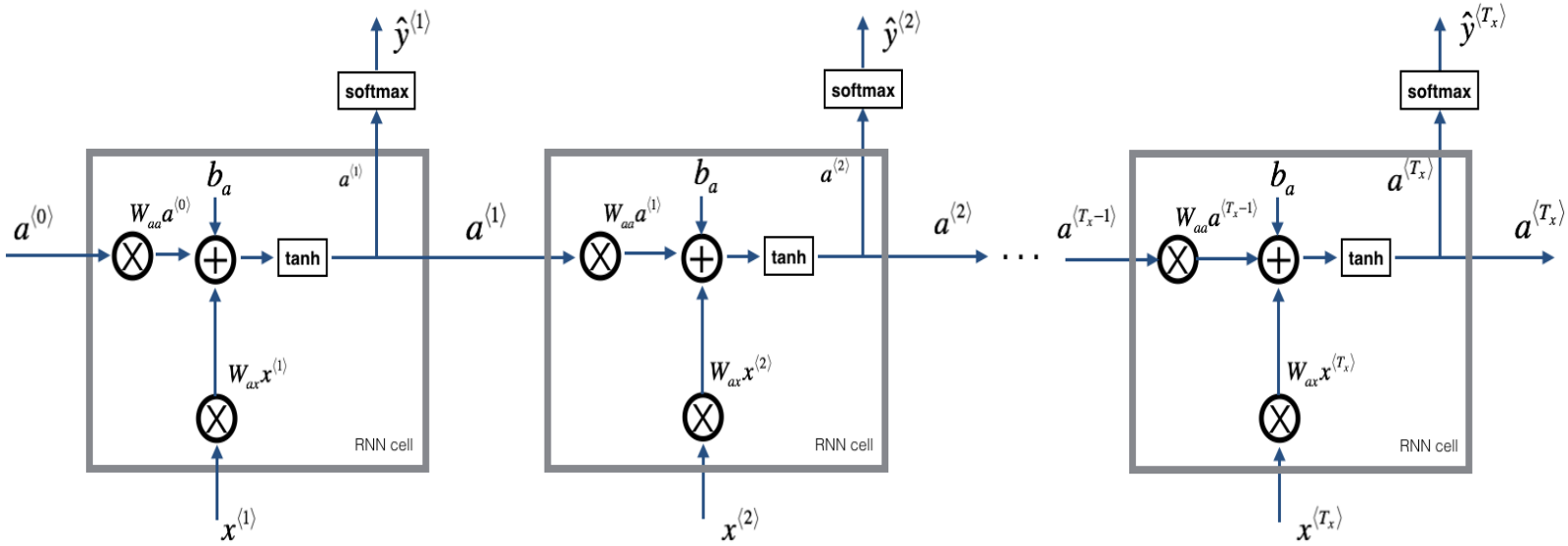


Figure 3: Basic RNN. The input sequence $x = (x^{\langle 1 \rangle}, x^{\langle 2 \rangle}, \ldots, x^{\langle T_x \rangle})$ is carried over T_x time steps. The network outputs $y = (y^{\langle 1 \rangle}, y^{\langle 2 \rangle}, \ldots, y^{\langle T_x \rangle})$.

Exercise: Code the forward propagation of the RNN described in Figure (3).

Instructions:

1. Create a vector of zeros (a) that will store all the hidden states computed by the RNN.
2. Initialize the "next" hidden state as a_0 (initial hidden state).
3. Start looping over each time step, your incremental index is t :

- Update the "next" hidden state and the cache by running `rnn_cell_forward`
- Store the "next" hidden state in a (t^{th} position)
- Store the prediction in y
- Add the cache to the list of caches

4. Return a, y and caches

In [4]:
```
def rnn_forward(x, a0, parameters):
    """
    Implement the forward propagation of the recurrent neural network described in Figure (3).

    Arguments:
    x -- Input data for every time-step, of shape (n_x, m, T_x).
    a0 -- Initial hidden state, of shape (n_a, m)
    parameters -- python dictionary containing:
                        Waa -- Weight matrix multiplying the hidden state, numpy array of shape (n_a, n_a)
                        Wax -- Weight matrix multiplying the input, numpy array of shape (n_a, n_x)
                        Wya -- Weight matrix relating the hidden-state to the output, numpy array of shape (n_y,
                        ba --  Bias numpy array of shape (n_a, 1)
                        by -- Bias relating the hidden-state to the output, numpy array of shape (n_y, 1)

    Returns:
    a -- Hidden states for every time-step, numpy array of shape (n_a, m, T_x)
    y_pred -- Predictions for every time-step, numpy array of shape (n_y, m, T_x)
    caches -- tuple of values needed for the backward pass, contains (list of caches, x)
    """

    caches = []

    n_x, m, T_x = x.shape
    n_y, n_a = parameters["Wya"].shape

    a = np.zeros((n_a, m, T_x))
    y_pred = np.zeros((n_y, m, T_x))

    a_next = a0

    for t in range(T_x):
        # Update next hidden state, compute the prediction, get the cache (≈1 line)
        a_next, yt_pred, cache = rnn_cell_forward(x[:,:,t], a_next, parameters)
        # Save the value of the new "next" hidden state in a (≈1 line)
        a[:,:,t] = a_next
        # Save the value of the prediction in y (≈1 line)
        y_pred[:,:,t] = yt_pred
        # Append "cache" to "caches" (≈1 line)
        caches.append(cache)

    # store values needed for backward propagation in cache
    caches = (caches, x)
```

```
    return a, y_pred, caches
```

In [5]:
```
np.random.seed(1)
x = np.random.randn(3,10,4)
a0 = np.random.randn(5,10)
Waa = np.random.randn(5,5)
Wax = np.random.randn(5,3)
Wya = np.random.randn(2,5)
ba = np.random.randn(5,1)
by = np.random.randn(2,1)
parameters = {"Waa": Waa, "Wax": Wax, "Wya": Wya, "ba": ba, "by": by}

a, y_pred, caches = rnn_forward(x, a0, parameters)
print("a[4][1] = ", a[4][1])
print("a.shape = ", a.shape)
print("y_pred[1][3] =", y_pred[1][3])
print("y_pred.shape = ", y_pred.shape)
print("caches[1][1][3] =", caches[1][1][3])
print("len(caches) = ", len(caches))
```

```
a[4][1] =  [-0.99999375  0.77911235 -0.99861469 -0.99833267]
a.shape =  (5, 10, 4)
y_pred[1][3] = [0.79560373 0.86224861 0.11118257 0.81515947]
y_pred.shape =  (2, 10, 4)
caches[1][1][3] = [-1.1425182  -0.34934272 -0.20889423  0.58662319]
len(caches) =  2
```

You've successfully built the forward propagation of a recurrent neural network from scratch. This will work well enough for some applications, **but it suffers from vanishing gradient problems**. So it works best when each output $y^{\langle t \rangle}$ can be estimated using mainly "local" context (meaning information from inputs $x^{\langle t' \rangle}$ where t' is not too far from t).

In the next part, you will build a more complex LSTM model, which is better at addressing vanishing gradients. The LSTM will be better able to remember a piece of information and keep it saved for many timesteps.

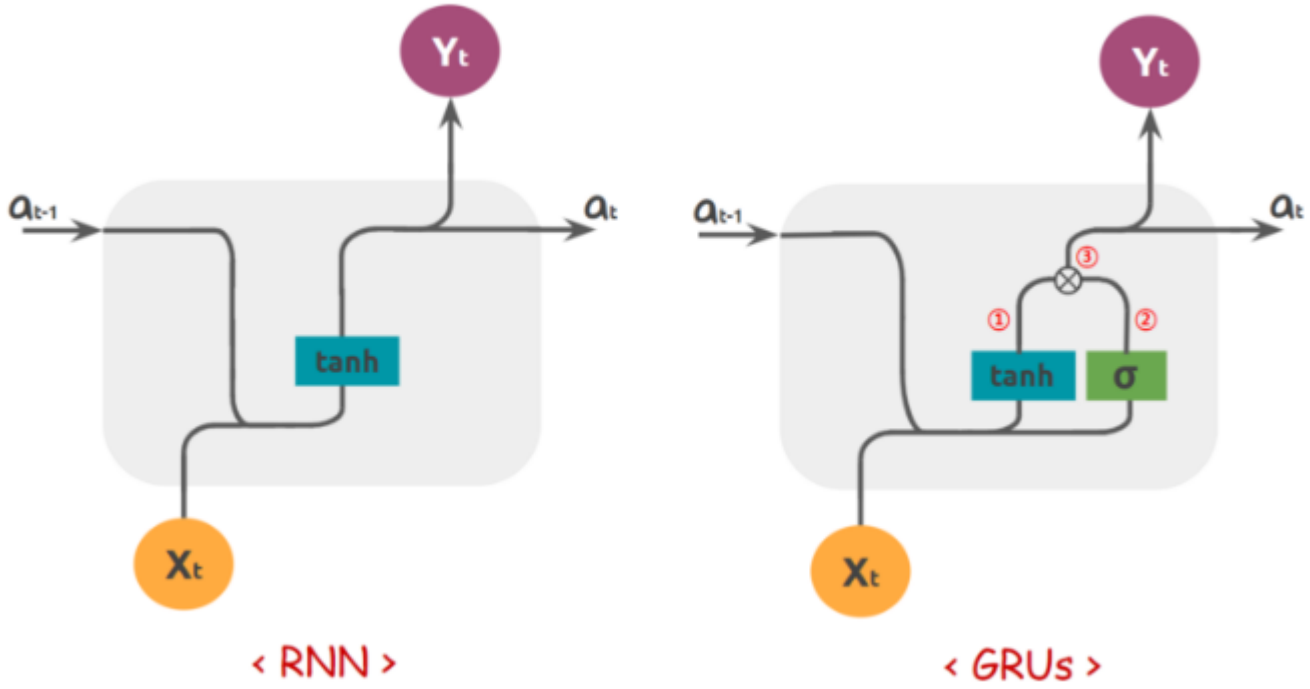
2 - Long Short-Term Memory (LSTM) network

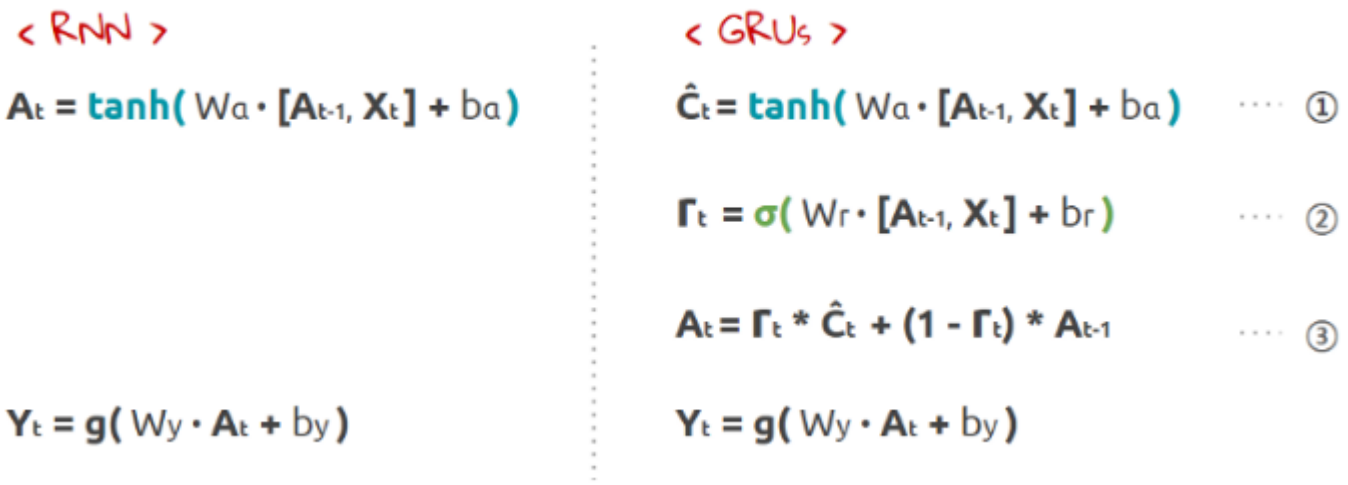
In LSTM, typically there are three gates determine whether or not to let new input in (input gate), delete the information because it isn't important (forget gate) or to let it impact the output at the current time step (output gate).

Basics of gated recurrent unit (GRU)

https://towardsdatascience.com/the-most-intuitive-and-easiest-guide-for-recurrent-neural-network-873c29da73c7
(https://towardsdatascience.com/the-most-intuitive-and-easiest-guide-for-recurrent-neural-network-873c29da73c7)

Below is a simple illustration comparing GRU and LTSM from the link above. Schematic digram and the corresponding equations are shown for each type of cell unit. The notation here is different from other places in this note.





Comments

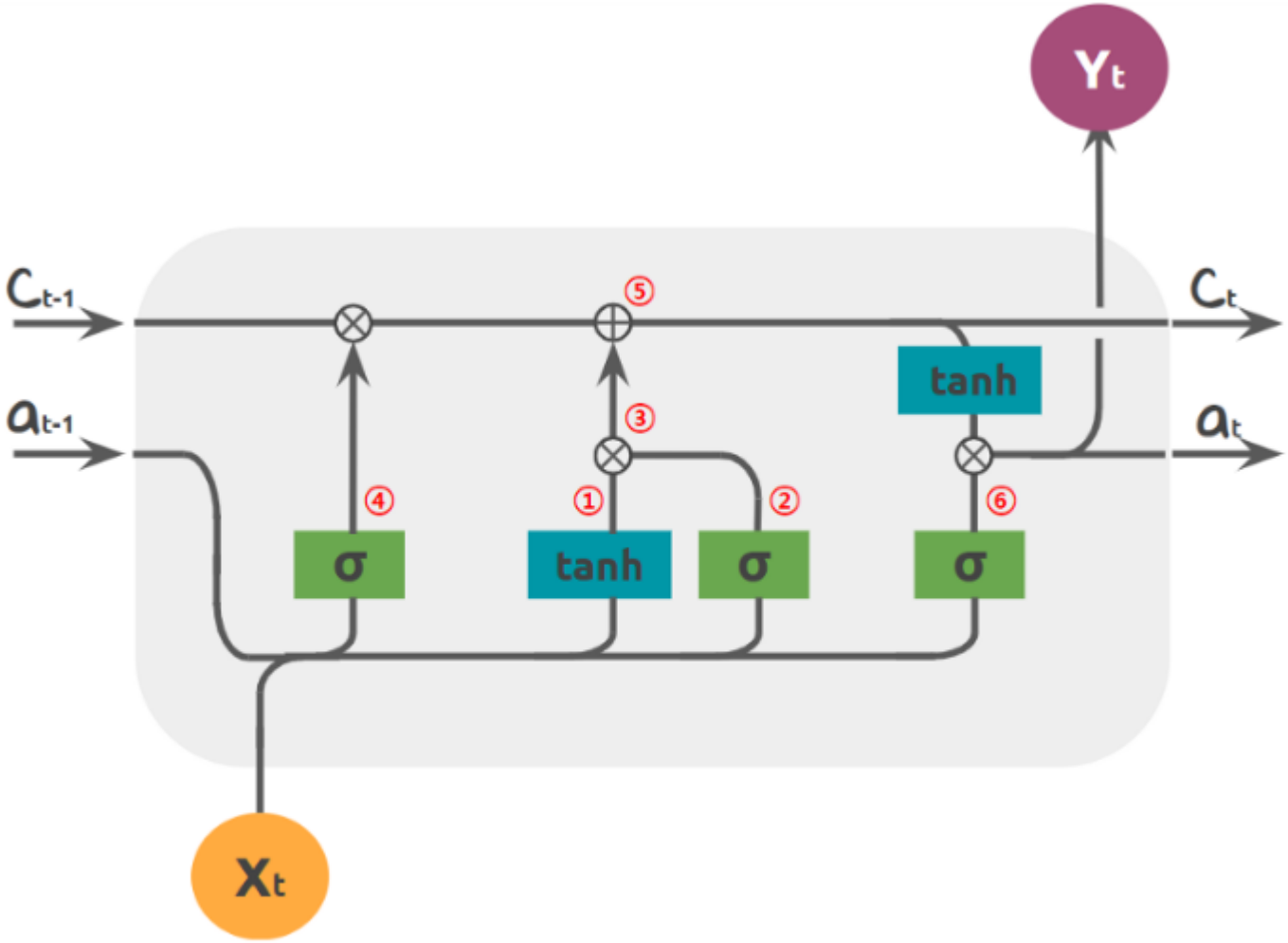
- The following is some understanding of GRU with slightly different notation.
- The c below indicates memory cell. In GRU $c^{\langle t \rangle} = a^{\langle t \rangle}$.
- The unit is called a long short-term memory block because the program is using a structure founded on short-term memory processes to create longer-term memory. The idea is like this (see p.32 of slides C5M1): When setting $c^{\langle t \rangle} = 1$ and $\Gamma_u = 1$ at a time step. Later, if the information for next many time steps is not so strongly dependent, then Γ_u for those time steps will be almost zero. If this is the case, then $c^{\langle t \rangle}$ at those steps will be kept as 1 without gradually going down. The key here is to keep the almost the whole information if it is above some probability. This is because in most cases Γ_u will be either very close to 0 or close to 1. This is implemented by the following logic, where if Γ_u is or close to 0, then $c^{\langle t \rangle}$ will keep the same value (memorize) as $c^{\langle t-1 \rangle}$.

$$\tilde{c}^{\langle t \rangle} = \tanh\left(W_c[c^{\langle t-1 \rangle}, x^{\langle t \rangle}] + b_c\right)$$

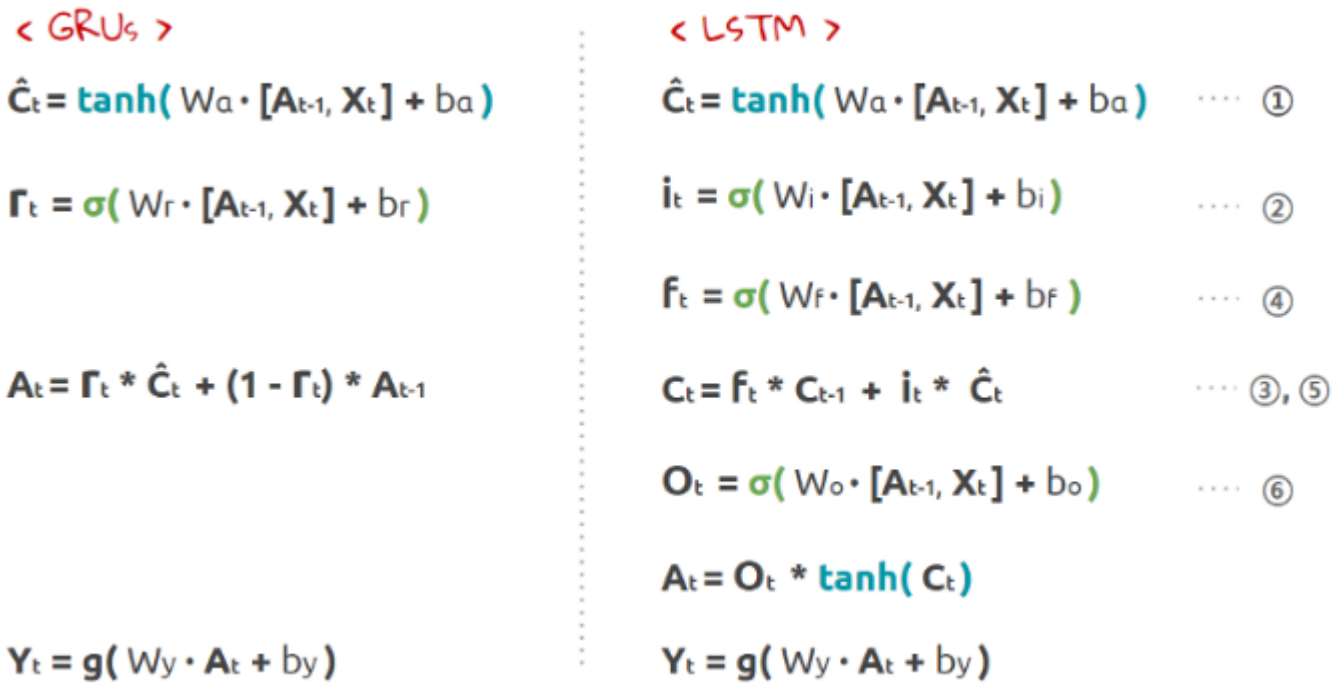
$$c^{\langle t-1 \rangle} = \Gamma_u * \tilde{c}^{\langle t \rangle} + (1 - \Gamma_u) * c^{\langle t-1 \rangle}$$

- In LSTM, the second part will be separated into a forget gate.
- Watch the videos on GRU (RNN W1L09) to help understand LSTM. Historically LSTM appeared much earlier than GRU. But simpler GRU may be useful in constructing big models.

Basics of LSTM



‹ LSTM ›



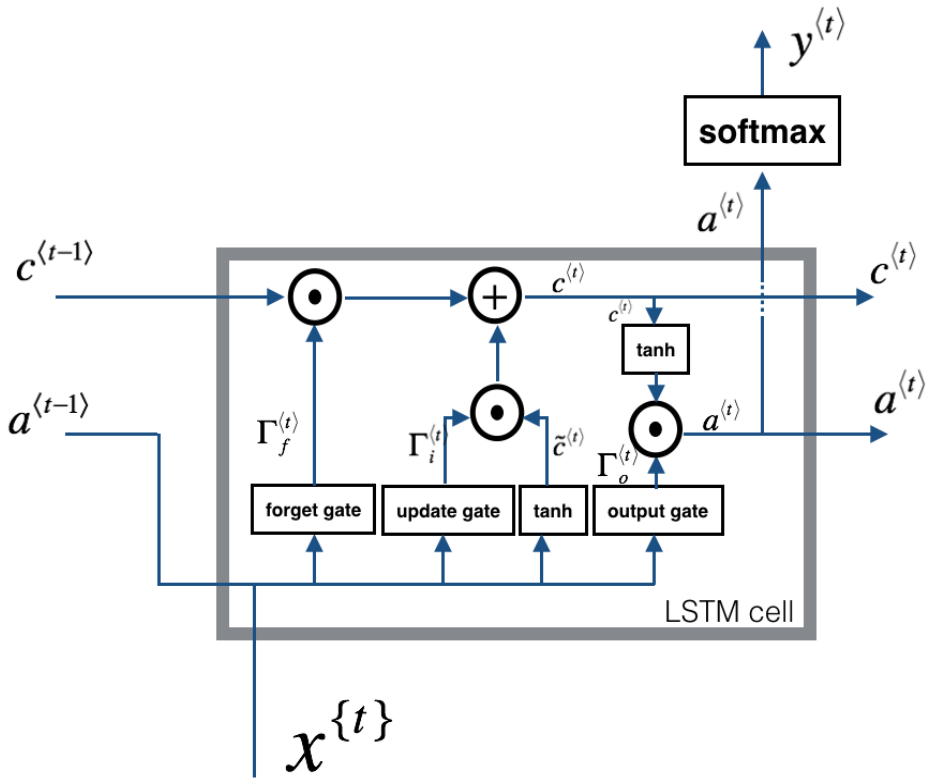
Comments:

- Except the parameters W and b, three gates i_t, f_t and O_t have same component $[A_{t-1}, X_t]$.
- In LSTM, f_t has a similar role as the $1 - \Gamma_t$ in GRU.
- Forget gate is to forget information of earlier cell, so we have $f_t * C_{t-1}$. Input gate is to decide whether to keep the information of current cell, so we have $i_t * \hat{C}_t$. Similar output gate is also for current cell, and there fore we have $A_t = O_t * \tanh(C_t)$.
- Map the equations and the diagrams as much as possible, so as to be familiar with both.
- The 'x' and '+' signs in the diagrams indicate multiplication and addition respectively. In the diagram below, multiplication is indicated by a dot.
- The middle portion of the figure below for LSTM cell is actually the GRU cell.

More details about LSTM

This following figure shows the operations of an LSTM-cell. **Note the $\Gamma_u^{\langle t \rangle}$ in the equation should be changed to $\Gamma_i^{\langle t \rangle}$ in order to be**

consistent with the diagram on the left.



$$\Gamma_f^{\langle t \rangle} = \sigma(W_f[a^{\langle t-1 \rangle}, x^{\langle t \rangle}] + b_f)$$
$$\Gamma_u^{\langle t \rangle} = \sigma(W_u[a^{\langle t-1 \rangle}, x^{\langle t \rangle}] + b_u)$$
$$\tilde{c}^{\{t\}} = \tanh(W_C[a^{\langle t-1 \rangle}, x^{\langle t \rangle}] + b_C)$$
$$c^{\langle t \rangle} = \Gamma_f^{\langle t \rangle} \circ c^{\langle t-1 \rangle} + \Gamma_u^{\langle t \rangle} \circ \tilde{c}^{\langle t \rangle}$$
$$\Gamma_o^{\langle t \rangle} = \sigma(W_o[a^{\langle t-1 \rangle}, x^{\langle t \rangle}] + b_o)$$
$$a^{\langle t \rangle} = \Gamma_o^{\langle t \rangle} \circ \tanh(c^{\langle t \rangle})$$

Figure 4: LSTM-cell. This tracks and updates a "cell state" or memory variable $c^{\langle t \rangle}$ at every time-step, which can be different from $a^{\langle t \rangle}$.

Similar to the RNN example above, you will start by implementing the LSTM cell for a single time-step. Then you can iteratively call it from inside a for-loop to have it process an input with T_x time-steps.

About the gates

- Forget gate

For the sake of this illustration, lets assume we are reading words in a piece of text, and want use an LSTM to keep track of grammatical structures, such as whether the subject is singular or plural. If the subject changes from a singular word to a plural word, we need to find a way to get rid of our previously stored memory value of the singular/plural state. In an LSTM, the forget gate lets us do this:

$$\Gamma_f^{\langle t \rangle} = \sigma(W_f[a^{\langle t-1 \rangle}, x^{\langle t \rangle}] + b_f) \tag{1}$$

Here, W_f are weights that govern the forget gate's behavior. We concatenate $[a^{\langle t-1 \rangle}, x^{\langle t \rangle}]$ and multiply by W_f. The equation above results in a vector $\Gamma_f^{\langle t \rangle}$ with values between 0 and 1. This forget gate vector will be multiplied element-wise by the previous cell state $c^{\langle t-1 \rangle}$. So if one of the values of $\Gamma_f^{\langle t \rangle}$ is 0 (or close to 0) then it means that the LSTM should remove that piece of information (e.g. the singular subject) in the corresponding component of $c^{\langle t-1 \rangle}$. If one of the values is 1, then it will keep the information.

- Update gate

Once we forget that the subject being discussed is singular, we need to find a way to update it to reflect that the new subject is now plural. Here is the formulat for the update gate:

$$\Gamma_u^{\langle t \rangle} = \sigma(W_u[a^{\langle t-1 \rangle}, x^{\{t\}}] + b_u) \tag{2}$$

Similar to the forget gate, here $\Gamma_u^{\langle t \rangle}$ is again a vector of values between 0 and 1. This will be multiplied element-wise with $\tilde{c}^{\langle t \rangle}$, in order to compute $c^{\langle t \rangle}$.

- Updating the cell

To update the new subject we need to create a new vector of numbers that we can add to our previous cell state. The equation we use is:

$$\tilde{c}^{\langle t \rangle} = \tanh(W_c[a^{\langle t-1 \rangle}, x^{\langle t \rangle}] + b_c) \tag{3}$$

Finally, the new cell state is:

$$c^{\langle t \rangle} = \Gamma_f^{\langle t \rangle} * c^{\langle t-1 \rangle} + \Gamma_u^{\langle t \rangle} * \tilde{c}^{\langle t \rangle} \tag{4}$$

- Output gate

To decide which outputs we will use, we will use the following two formulas:

$$\Gamma_o^{\langle t \rangle} = \sigma(W_o[a^{\langle t-1 \rangle}, x^{\langle t \rangle}] + b_o) \tag{5}$$
$$a^{\langle t \rangle} = \Gamma_o^{\langle t \rangle} * \tanh(c^{\langle t \rangle}) \tag{6}$$

Where in equation 5 you decide what to output using a sigmoid function and in equation 6 you multiply that by the $\tanh$ of the previous state.

2.1 - LSTM cell

Exercise: Implement the LSTM cell described in the Figure (3).

Instructions:

1. Concatenate $a^{\langle t-1 \rangle}$ and $x^{\langle t \rangle}$ in a single matrix: $concat = \begin{bmatrix} a^{\langle t-1 \rangle} \\ x^{\langle t \rangle} \end{bmatrix}$
2. Compute all the formulas 1-6. You can use `sigmoid()` (provided) and `np.tanh()`.
3. Compute the prediction $y^{\langle t \rangle}$. You can use `softmax()` (provided).

In [6]:
```python
def lstm_cell_forward(xt, a_prev, c_prev, parameters):
    """
    Implement a single forward step of the LSTM-cell as described in Figure (4)

    Arguments:
    xt -- your input data at timestep "t", numpy array of shape (n_x, m).
    a_prev -- Hidden state at timestep "t-1", numpy array of shape (n_a, m)
    c_prev -- Memory state at timestep "t-1", numpy array of shape (n_a, m)
    parameters -- python dictionary containing:
                        Wf -- Weight matrix of the forget gate, numpy array of shape (n_a, n_a + n_x)
                        bf -- Bias of the forget gate, numpy array of shape (n_a, 1)
                        Wi -- Weight matrix of the update gate, numpy array of shape (n_a, n_a + n_x)
                        bi -- Bias of the update gate, numpy array of shape (n_a, 1)
                        Wc -- Weight matrix of the first "tanh", numpy array of shape (n_a, n_a + n_x)
                        bc -- Bias of the first "tanh", numpy array of shape (n_a, 1)
                        Wo -- Weight matrix of the output gate, numpy array of shape (n_a, n_a + n_x)
                        bo -- Bias of the output gate, numpy array of shape (n_a, 1)
                        Wy -- Weight matrix relating the hidden-state to the output, numpy array of shape (n_y, n
                        by -- Bias relating the hidden-state to the output, numpy array of shape (n_y, 1)

    Returns:
    a_next -- next hidden state, of shape (n_a, m)
    c_next -- next memory state, of shape (n_a, m)
    yt_pred -- prediction at timestep "t", numpy array of shape (n_y, m)
    cache -- tuple of values needed for the backward pass, contains (a_next, c_next, a_prev, c_prev, xt, paramete

    Note: ft/it/ot stand for the forget/update/output gates, cct stands for the candidate value (c tilde),
          c stands for the memory value
    """

    # Retrieve parameters from "parameters"
    Wf = parameters["Wf"]
    bf = parameters["bf"]
    Wi = parameters["Wi"]
    bi = parameters["bi"]
    Wc = parameters["Wc"]
    bc = parameters["bc"]
    Wo = parameters["Wo"]
    bo = parameters["bo"]
    Wy = parameters["Wy"]
    by = parameters["by"]
```

```
    n_x, m = xt.shape
    n_y, n_a = Wy.shape

    # Concatenate a_prev and xt (≈3 lines)
    concat = np.zeros((n_a + n_x, m))
    concat[: n_a, :] = a_prev
    concat[n_a :, :] = xt

    # Compute values for ft, it, cct, c_next, ot, a_next using the formulas given figure (4)
    ft = sigmoid(np.dot(Wf, concat) + bf)
    it = sigmoid(np.dot(Wi, concat) + bi)
    cct = np.tanh(np.dot(Wc, concat) + bc)
    c_next = ft * c_prev + it * cct
    ot = sigmoid(np.dot(Wo, concat) + bo)
    a_next = ot * np.tanh(c_next)

    # Compute prediction of the LSTM cell (≈1 line)
    yt_pred = softmax(np.dot(Wy, a_next) + by)

    # store values needed for backward propagation in cache
    cache = (a_next, c_next, a_prev, c_prev, ft, it, cct, ot, xt, parameters)

    return a_next, c_next, yt_pred, cache
```

In [7]:
```
np.random.seed(1)
xt = np.random.randn(3,10)
a_prev = np.random.randn(5,10)
c_prev = np.random.randn(5,10)
Wf = np.random.randn(5, 5+3)
bf = np.random.randn(5,1)
Wi = np.random.randn(5, 5+3)
bi = np.random.randn(5,1)
Wo = np.random.randn(5, 5+3)
bo = np.random.randn(5,1)
Wc = np.random.randn(5, 5+3)
bc = np.random.randn(5,1)
Wy = np.random.randn(2,5)
by = np.random.randn(2,1)

parameters = {"Wf": Wf, "Wi": Wi, "Wo": Wo, "Wc": Wc, "Wy": Wy, "bf": bf, "bi": bi, "bo": bo, "bc": bc, "by": by}

a_next, c_next, yt, cache = lstm_cell_forward(xt, a_prev, c_prev, parameters)
print("a_next[4] = ", a_next[4])
print("a_next.shape = ", c_next.shape)
print("c_next[2] = ", c_next[2])
print("c_next.shape = ", c_next.shape)
print("yt[1] =", yt[1])
print("yt.shape = ", yt.shape)
print("cache[1][3] =", cache[1][3])
print("len(cache) = ", len(cache))
```

```
a_next[4] =  [-0.66408471  0.0036921   0.02088357  0.22834167 -0.85575339  0.00138482
  0.76566531  0.34631421 -0.00215674  0.43827275]
a_next.shape =  (5, 10)
c_next[2] =  [ 0.63267805  1.00570849  0.35504474  0.20690913 -1.64566718  0.11832942
  0.76449811 -0.0981561  -0.74348425 -0.26810932]
c_next.shape =  (5, 10)
yt[1] = [0.79913913 0.15986619 0.22412122 0.15606108 0.97057211 0.31146381
 0.00943007 0.12666353 0.39380172 0.07828381]
yt.shape =  (2, 10)
cache[1][3] = [-0.16263996  1.03729328  0.72938082 -0.54101719  0.02752074 -0.30821874
  0.07651101 -1.03752894  1.41219977 -0.37647422]
len(cache) =  10
```

2.2 - Forward pass for LSTM

Now that you have implemented one step of an LSTM, you can now iterate this over this using a for-loop to process a sequence of T_x inputs.

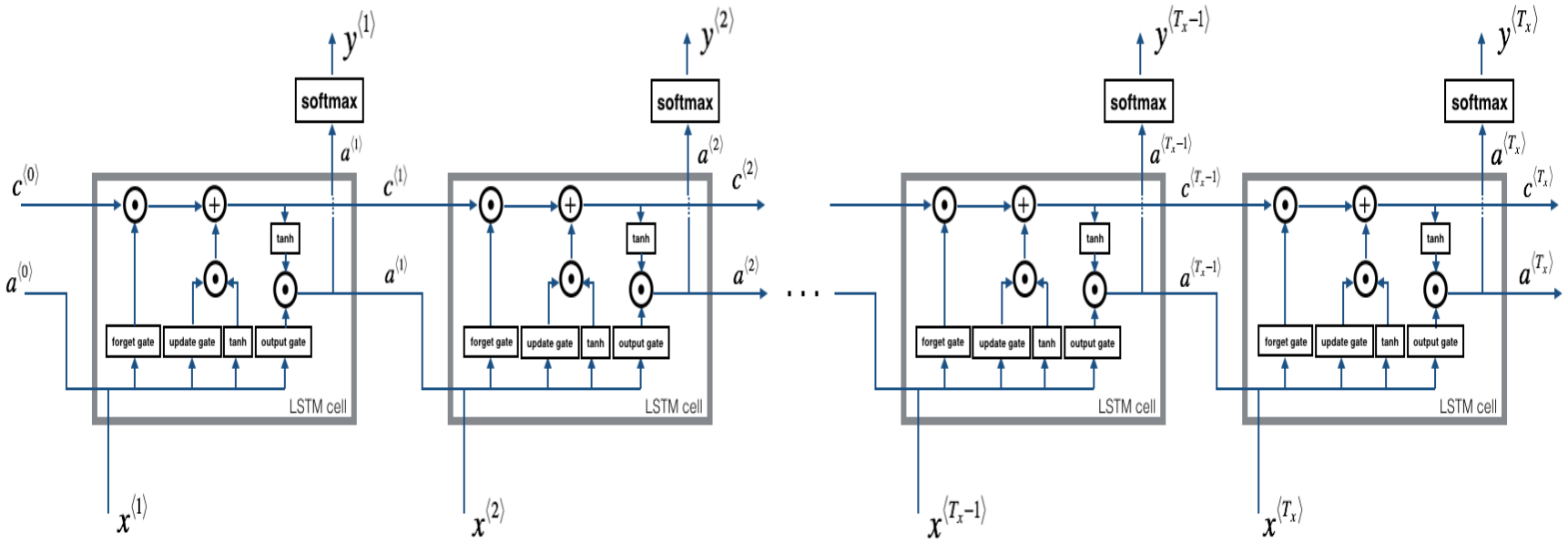


Figure 4: LSTM over multiple time-steps.

Exercise: Implement `lstm_forward()` to run an LSTM over T_x time-steps.

Note: $c^{\langle 0 \rangle}$ is initialized with zeros.

In [8]:
```
def lstm_forward(x, a0, parameters):
    """
    Implement the forward propagation of the recurrent neural network using an LSTM-cell described in Figure (3)

    Arguments:
    x -- Input data for every time-step, of shape (n_x, m, T_x).
    a0 -- Initial hidden state, of shape (n_a, m)
    parameters -- python dictionary containing:
                        Wf -- Weight matrix of the forget gate, numpy array of shape (n_a, n_a + n_x)
                        bf -- Bias of the forget gate, numpy array of shape (n_a, 1)
                        Wi -- Weight matrix of the update gate, numpy array of shape (n_a, n_a + n_x)
                        bi -- Bias of the update gate, numpy array of shape (n_a, 1)
                        Wc -- Weight matrix of the first "tanh", numpy array of shape (n_a, n_a + n_x)
                        bc -- Bias of the first "tanh", numpy array of shape (n_a, 1)
                        Wo -- Weight matrix of the output gate, numpy array of shape (n_a, n_a + n_x)
                        bo -- Bias of the output gate, numpy array of shape (n_a, 1)
                        Wy -- Weight matrix relating the hidden-state to the output, numpy array of shape (n_y, n
                        by -- Bias relating the hidden-state to the output, numpy array of shape (n_y, 1)

    Returns:
    a -- Hidden states for every time-step, numpy array of shape (n_a, m, T_x)
    y -- Predictions for every time-step, numpy array of shape (n_y, m, T_x)
    caches -- tuple of values needed for the backward pass, contains (list of all the caches, x)
    """

    # Initialize "caches", which will track the list of all the caches
    caches = []

    # Retrieve dimensions from shapes of x and Wy (≈2 lines)
    n_x, m, T_x = x.shape
    n_y, n_a = parameters["Wy"].shape

    # initialize "a", "c" and "y" with zeros (≈3 lines)
    a = np.zeros((n_a, m, T_x))
    c = np.zeros((n_a, m, T_x))
    y = np.zeros((n_y, m, T_x))

    # Initialize a_next and c_next (≈2 lines)
    a_next = a0
    c_next = np.zeros(a_next.shape)

    # loop over all time-steps
```

```
    for t in range(T_x):
        # Update next hidden state, next memory state, compute the prediction, get the cache (≈1 line)
        a_next, c_next, yt, cache = lstm_cell_forward(x[:, :, t], a_next, c_next, parameters)
        # Save the value of the new "next" hidden state in a (≈1 line)
        a[:,:,t] = a_next
        # Save the value of the prediction in y (≈1 line)
        y[:,:,t] = yt
        # Save the value of the next cell state (≈1 line)
        c[:,:,t]  = c_next
        # Append the cache into caches (≈1 line)
        caches.append(cache)

    # store values needed for backward propagation in cache
    caches = (caches, x)

    return a, y, c, caches
```

In [10]:
```
np.random.seed(1)
x = np.random.randn(3,10,7)
a0 = np.random.randn(5,10)
Wf = np.random.randn(5, 5+3)
bf = np.random.randn(5,1)
Wi = np.random.randn(5, 5+3)
bi = np.random.randn(5,1)
Wo = np.random.randn(5, 5+3)
bo = np.random.randn(5,1)
Wc = np.random.randn(5, 5+3)
bc = np.random.randn(5,1)
Wy = np.random.randn(2,5)
by = np.random.randn(2,1)

parameters = {"Wf": Wf, "Wi": Wi, "Wo": Wo, "Wc": Wc, "Wy": Wy, "bf": bf, "bi": bi, "bo": bo, "bc": bc, "by": by}

a, y, c, caches = lstm_forward(x, a0, parameters)
print("a[4][3][6] = ", a[4][3][6])
print("a.shape = ", a.shape)
print("y[1][4][3] =", y[1][4][3])
print("y.shape = ", y.shape)
print("caches[1][1[1]] =", caches[1][1][1])
print("c[1][2][1]", c[1][2][1])
print("len(caches) = ", len(caches))
```

```
a[4][3][6] =  0.17211776753291672
a.shape =  (5, 10, 7)
y[1][4][3] = 0.9508734618501101
y.shape =  (2, 10, 7)
caches[1][1[1]] = [ 0.82797464  0.23009474  0.76201118 -0.22232814 -0.20075807  0.18656139
  0.41005165]
c[1][2][1] -0.8555449167181981
len(caches) =  2
```

You have now implemented the forward passes for the basic RNN and the LSTM. When using a deep learning framework, **implementing the forward pass is sufficient to build systems that achieve great performance**.

The rest of this notebook is optional.

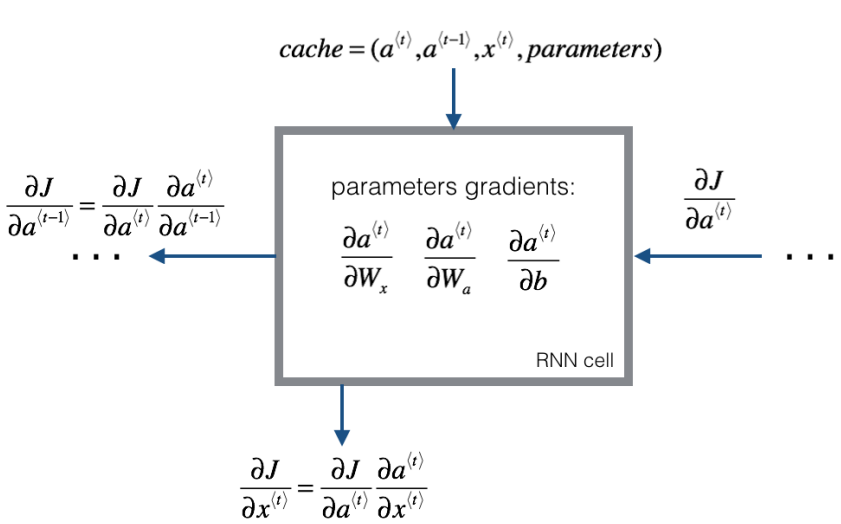
3 - Backpropagation in recurrent neural networks (OPTIONAL / UNGRADED)

In modern deep learning frameworks, you only have to implement the forward pass, and the framework takes care of the backward pass, so most deep learning engineers do not need to bother with the details of the backward pass. If however you are an expert in calculus and want to see the details of backprop in RNNs, you can work through this optional portion of the notebook.

When in an earlier course you implemented a simple (fully connected) neural network, you used backpropagation to compute the derivatives with respect to the cost to update the parameters. Similarly, in recurrent neural networks you can to calculate the derivatives with respect to the cost in order to update the parameters. The backprop equations are quite complicated and we did not derive them in lecture. However, we will briefly present them below.

3.1 - Basic RNN backward pass

We will start by computing the backward pass for the basic RNN-cell.



$$a^{\langle t \rangle} = \tanh(W_{ax} x^{\langle t \rangle} + W_{aa} a^{\langle t-1 \rangle} + b)$$

$$\frac{\partial \tanh(x)}{\partial x} = 1 - \tanh(x)^2$$

$$\frac{\partial a^{\langle t \rangle}}{\partial W_{ax}} = (1 - \tanh(W_{ax} x^{\langle t \rangle} + W_{aa} a^{\langle t-1 \rangle} + b)^2) . x^{\langle t \rangle T}$$

$$\frac{\partial a^{\langle t \rangle}}{\partial W_{aa}} = (1 - \tanh(W_{ax} x^{\langle t \rangle} + W_{aa} a^{\langle t-1 \rangle} + b)^2) . a^{\langle t-1 \rangle T}$$

$$\frac{\partial a^{\langle t \rangle}}{\partial b} = \sum_{batch} (1 - \tanh(W_{ax} x^{\langle t \rangle} + W_{aa} a^{\langle t-1 \rangle} + b)^2)$$

$$\frac{\partial a^{\langle t \rangle}}{\partial x^{\langle t \rangle}} = W_{ax}{}^T . (1 - \tanh(W_{ax} x^{\langle t \rangle} + W_{aa} a^{\langle t-1 \rangle} + b)^2)$$

$$\frac{\partial a^{\langle t \rangle}}{\partial a^{\langle t-1 \rangle}} = W_{aa}{}^T . (1 - \tanh(W_{ax} x^{\langle t-1 \rangle} + W_{aa} a^{\langle t-1 \rangle} + b)^2)$$

Figure 5: RNN-cell's backward pass. Just like in a fully-connected neural network, the derivative of the cost function J backpropagates through the RNN by following the chain-rule from calculas. The chain-rule is also used to calculate $(\frac{\partial J}{\partial W_{ax}}, \frac{\partial J}{\partial W_{aa}}, \frac{\partial J}{\partial b})$ to update the parameters (W_{ax}, W_{aa}, b_a).

Deriving the one step backward functions:

To compute the `rnn_cell_backward` you need to compute the following equations. It is a good exercise to derive them by hand.

The derivative of $\tanh$ is $1 - \tanh(x)^2$. You can find the complete proof here (https://www.wyzant.com/resources/lessons/math/calculus/derivative_proofs/tanx). Note that: $\sec(x)^2 = 1 - \tanh(x)^2$

Similarly for $\frac{\partial a^{\langle t \rangle}}{\partial W_{ax}}$, $\frac{\partial a^{\langle t \rangle}}{\partial W_{aa}}$, $\frac{\partial a^{\langle t \rangle}}{\partial b}$, the derivative of $\tanh(u)$ is $(1 - \tanh(u)^2)du$.

The final two equations also follow same rule and are derived using the $\tanh$ derivative. Note that the arrangement is done in a way to get the same dimensions to match.

In [11]:
```python
def rnn_cell_backward(da_next, cache):
    """
    Implements the backward pass for the RNN-cell (single time-step).

    Arguments:
    da_next -- Gradient of loss with respect to next hidden state
    cache -- python dictionary containing useful values (output of rnn_cell_forward())

    Returns:
    gradients -- python dictionary containing:
                        dx -- Gradients of input data, of shape (n_x, m)
                        da_prev -- Gradients of previous hidden state, of shape (n_a, m)
                        dWax -- Gradients of input-to-hidden weights, of shape (n_a, n_x)
                        dWaa -- Gradients of hidden-to-hidden weights, of shape (n_a, n_a)
                        dba -- Gradients of bias vector, of shape (n_a, 1)
    """

    # Retrieve values from cache
    (a_next, a_prev, xt, parameters) = cache

    # Retrieve values from parameters
    Wax = parameters["Wax"]
    Waa = parameters["Waa"]
    Wya = parameters["Wya"]
    ba = parameters["ba"]
    by = parameters["by"]

    # compute the gradient of tanh with respect to a_next (≈1 line)
    dtanh = (1 - a_next ** 2) * da_next

    # compute the gradient of the loss with respect to Wax (≈2 lines)
    dxt = np.dot(Wax.T, dtanh)
    dWax = np.dot(dtanh, xt.T)

    # compute the gradient with respect to Waa (≈2 lines)
    da_prev = np.dot(Waa.T, dtanh)
    dWaa = np.dot(dtanh, a_prev.T)

    # compute the gradient with respect to b (≈1 line)
    dba = np.sum(dtanh, axis = 1,keepdims=1)

    # Store the gradients in a python dictionary
```

```
    gradients = {"dxt": dxt, "da_prev": da_prev, "dWax": dWax, "dWaa": dWaa, "dba": dba}

    return gradients
```

In [12]:
```
np.random.seed(1)
xt = np.random.randn(3,10)
a_prev = np.random.randn(5,10)
Wax = np.random.randn(5,3)
Waa = np.random.randn(5,5)
Wya = np.random.randn(2,5)
b = np.random.randn(5,1)
by = np.random.randn(2,1)
parameters = {"Wax": Wax, "Waa": Waa, "Wya": Wya, "ba": ba, "by": by}

a_next, yt, cache = rnn_cell_forward(xt, a_prev, parameters)

da_next = np.random.randn(5,10)
gradients = rnn_cell_backward(da_next, cache)
print("gradients[\"dxt\"][1][2] =", gradients["dxt"][1][2])
print("gradients[\"dxt\"].shape =", gradients["dxt"].shape)
print("gradients[\"da_prev\"][2][3] =", gradients["da_prev"][2][3])
print("gradients[\"da_prev\"].shape =", gradients["da_prev"].shape)
print("gradients[\"dWax\"][3][1] =", gradients["dWax"][3][1])
print("gradients[\"dWax\"].shape =", gradients["dWax"].shape)
print("gradients[\"dWaa\"][1][2] =", gradients["dWaa"][1][2])
print("gradients[\"dWaa\"].shape =", gradients["dWaa"].shape)
print("gradients[\"dba\"][4] =", gradients["dba"][4])
print("gradients[\"dba\"].shape =", gradients["dba"].shape)
```

```
gradients["dxt"][1][2] = -0.4605641030588796
gradients["dxt"].shape = (3, 10)
gradients["da_prev"][2][3] = 0.08429686538067718
gradients["da_prev"].shape = (5, 10)
gradients["dWax"][3][1] = 0.3930818739219303
gradients["dWax"].shape = (5, 3)
gradients["dWaa"][1][2] = -0.2848395578696067
gradients["dWaa"].shape = (5, 5)
gradients["dba"][4] = [0.80517166]
gradients["dba"].shape = (5, 1)
```

Backward pass through the RNN

Computing the gradients of the cost with respect to $a^{\langle t \rangle}$ at every time-step t is useful because it is what helps the gradient backpropagate to the previous RNN-cell. To do so, you need to iterate through all the time steps starting at the end, and at each step, you increment the overall db_a, dW_{aa}, dW_{ax} and you store dx.

Instructions:

Implement the `rnn_backward` function. Initialize the return variables with zeros first and then loop through all the time steps while calling the `rnn_cell_backward` at each time timestep, update the other variables accordingly.

In [13]:
```
def rnn_backward(da, caches):
    """
    Implement the backward pass for a RNN over an entire sequence of input data.

    Arguments:
    da -- Upstream gradients of all hidden states, of shape (n_a, m, T_x)
    caches -- tuple containing information from the forward pass (rnn_forward)

    Returns:
    gradients -- python dictionary containing:
                         dx -- Gradient w.r.t. the input data, numpy-array of shape (n_x, m, T_x)
                         da0 -- Gradient w.r.t the initial hidden state, numpy-array of shape (n_a, m)
                         dWax -- Gradient w.r.t the input's weight matrix, numpy-array of shape (n_a, n_x)
                         dWaa -- Gradient w.r.t the hidden state's weight matrix, numpy-arrayof shape (n_a, n_a)
                         dba -- Gradient w.r.t the bias, of shape (n_a, 1)
    """

    # Retrieve values from the first cache (t=1) of caches (≈2 lines)
    (caches, x) = caches
    (a1, a0, x1, parameters) = caches[0]

    # Retrieve dimensions from da's and x1's shapes (≈2 lines)
    n_a, m, T_x = da.shape
    n_x, m = x1.shape

    # initialize the gradients with the right sizes (≈6 lines)
    dx = np.zeros((n_x, m, T_x))
    dWax = np.zeros((n_a, n_x))
    dWaa = np.zeros((n_a, n_a))
    dba = np.zeros((n_a, 1))
    da0 = np.zeros((n_a, m))
    da_prevt = np.zeros((n_a, m))


    # Loop through all the time steps
    for t in reversed(range(T_x)):
        # Compute gradients at time step t. Choose wisely the "da_next" and the "cache" to use in the backward pr
        gradients = rnn_cell_backward(da[:,:,t] + da_prevt, caches[t])
        # Retrieve derivatives from gradients (≈ 1 line)
        dxt, da_prevt, dWaxt, dWaat, dbat = gradients["dxt"], gradients["da_prev"], gradients["dWax"], gradients
        # Increment global derivatives w.r.t parameters by adding their derivative at time-step t (≈4 lines)
        dx[:, :, t] = dxt
```

```
        dWax += dWaxt
        dWaa += dWaat
        dba += dbat

    # Set da0 to the gradient of a which has been backpropagated through all time-steps (≈1 line)
    da0 = da_prevt

    # Store the gradients in a python dictionary
    gradients = {"dx": dx, "da0": da0, "dWax": dWax, "dWaa": dWaa,"dba": dba}

    return gradients
```

In [14]:
```
np.random.seed(1)
x = np.random.randn(3,10,4)
a0 = np.random.randn(5,10)
Wax = np.random.randn(5,3)
Waa = np.random.randn(5,5)
Wya = np.random.randn(2,5)
ba = np.random.randn(5,1)
by = np.random.randn(2,1)
parameters = {"Wax": Wax, "Waa": Waa, "Wya": Wya, "ba": ba, "by": by}
a, y, caches = rnn_forward(x, a0, parameters)
da = np.random.randn(5, 10, 4)
gradients = rnn_backward(da, caches)

print("gradients[\"dx\"][1][2] =", gradients["dx"][1][2])
print("gradients[\"dx\"].shape =", gradients["dx"].shape)
print("gradients[\"da0\"][2][3] =", gradients["da0"][2][3])
print("gradients[\"da0\"].shape =", gradients["da0"].shape)
print("gradients[\"dWax\"][3][1] =", gradients["dWax"][3][1])
print("gradients[\"dWax\"].shape =", gradients["dWax"].shape)
print("gradients[\"dWaa\"][1][2] =", gradients["dWaa"][1][2])
print("gradients[\"dWaa\"].shape =", gradients["dWaa"].shape)
print("gradients[\"dba\"][4] =", gradients["dba"][4])
print("gradients[\"dba\"].shape =", gradients["dba"].shape)
```

```
gradients["dx"][1][2] = [-2.07101689 -0.59255627  0.02466855  0.01483317]
gradients["dx"].shape = (3, 10, 4)
gradients["da0"][2][3] = -0.31494237512664996
gradients["da0"].shape = (5, 10)
gradients["dWax"][3][1] = 11.264104496527777
gradients["dWax"].shape = (5, 3)
gradients["dWaa"][1][2] = 2.303333126579893
gradients["dWaa"].shape = (5, 5)
gradients["dba"][4] = [-0.74747722]
gradients["dba"].shape = (5, 1)
```

3.2 - LSTM backward pass

3.2.1 One Step backward

The LSTM backward pass is slighltly more complicated than the forward one. We have provided you with all the equations for the LSTM backward pass below. (If you enjoy calculus exercises feel free to try deriving these from scratch yourself.)

3.2.2 gate derivatives

$$d\Gamma_o^{\langle t \rangle} = da_{next} * \tanh(c_{next}) * \Gamma_o^{\langle t \rangle} * (1 - \Gamma_o^{\langle t \rangle}) \tag{7}$$

$$d\tilde{c}^{\langle t \rangle} = dc_{next} * \Gamma_i^{\langle t \rangle} + \Gamma_o^{\langle t \rangle}(1 - \tanh(c_{next})^2) * i_t * da_{next} * \tilde{c}^{\langle t \rangle} * (1 - \tanh(\tilde{c})^2) \tag{8}$$

$$d\Gamma_u^{\langle t \rangle} = dc_{next} * \tilde{c}^{\langle t \rangle} + \Gamma_o^{\langle t \rangle}(1 - \tanh(c_{next})^2) * \tilde{c}^{\langle t \rangle} * da_{next} * \Gamma_u^{\langle t \rangle} * (1 - \Gamma_u^{\langle t \rangle}) \tag{9}$$

$$d\Gamma_f^{\langle t \rangle} = dc_{next} * \tilde{c}_{prev} + \Gamma_o^{\langle t \rangle}(1 - \tanh(c_{next})^2) * c_{prev} * da_{next} * \Gamma_f^{\langle t \rangle} * (1 - \Gamma_f^{\langle t \rangle}) \tag{10}$$

3.2.3 parameter derivatives

$$dW_f = d\Gamma_f^{\langle t \rangle} * \begin{pmatrix} a_{prev} \\ x_t \end{pmatrix}^T \tag{11}$$

$$dW_u = d\Gamma_u^{\langle t \rangle} * \begin{pmatrix} a_{prev} \\ x_t \end{pmatrix}^T \tag{12}$$

$$dW_c = d\tilde{c}^{\langle t \rangle} * \begin{pmatrix} a_{prev} \\ x_t \end{pmatrix}^T \tag{13}$$

$$dW_o = d\Gamma_o^{\langle t \rangle} * \begin{pmatrix} a_{prev} \\ x_t \end{pmatrix}^T \tag{14}$$

To calculate db_f, db_u, db_c, db_o you just need to sum across the horizontal (axis= 1) axis on $d\Gamma_f^{\langle t \rangle}, d\Gamma_u^{\langle t \rangle}, d\tilde{c}^{\langle t \rangle}, d\Gamma_o^{\langle t \rangle}$ respectively. Note that you should have the `keep_dims = True` option.

Finally, you will compute the derivative with respect to the previous hidden state, previous memory state, and input.

$$da_{prev} = W_f^T * d\Gamma_f^{\langle t \rangle} + W_u^T * d\Gamma_u^{\langle t \rangle} + W_c^T * d\tilde{c}^{\langle t \rangle} + W_o^T * d\Gamma_o^{\langle t \rangle} \tag{15}$$

Here, the weights for equations 13 are the first n_a, (i.e. $W_f = W_f[:n_a, :]$ etc...)

$$dc_{prev} = dc_{next}\Gamma_f^{\langle t \rangle} + \Gamma_o^{\langle t \rangle} * (1 - \tanh(c_{next})^2) * \Gamma_f^{\langle t \rangle} * da_{next} \tag{16}$$

$$dx^{\langle t \rangle} = W_f^T * d\Gamma_f^{\langle t \rangle} + W_u^T * d\Gamma_u^{\langle t \rangle} + W_c^T * d\tilde{c}_t + W_o^T * d\Gamma_o^{\langle t \rangle} \tag{17}$$

where the weights for equation 15 are from n_a to the end, (i.e. $W_f = W_f[n_a :, :]$ etc...)

Exercise: Implement `lstm_cell_backward` by implementing equations $7 - 17$ below. Good luck! :)

In [15]:
```
def lstm_cell_backward(da_next, dc_next, cache):
    """
    Implement the backward pass for the LSTM-cell (single time-step).

    Arguments:
    da_next -- Gradients of next hidden state, of shape (n_a, m)
    dc_next -- Gradients of next cell state, of shape (n_a, m)
    cache -- cache storing information from the forward pass

    Returns:
    gradients -- python dictionary containing:
                         dxt -- Gradient of input data at time-step t, of shape (n_x, m)
                         da_prev -- Gradient w.r.t. the previous hidden state, numpy array of shape (n_a, m)
                         dc_prev -- Gradient w.r.t. the previous memory state, of shape (n_a, m, T_x)
                         dWf -- Gradient w.r.t. the weight matrix of the forget gate, numpy array of shape (n_a, n
                         dWi -- Gradient w.r.t. the weight matrix of the update gate, numpy array of shape (n_a, n
                         dWc -- Gradient w.r.t. the weight matrix of the memory gate, numpy array of shape (n_a, n
                         dWo -- Gradient w.r.t. the weight matrix of the output gate, numpy array of shape (n_a, n
                         dbf -- Gradient w.r.t. biases of the forget gate, of shape (n_a, 1)
                         dbi -- Gradient w.r.t. biases of the update gate, of shape (n_a, 1)
                         dbc -- Gradient w.r.t. biases of the memory gate, of shape (n_a, 1)
                         dbo -- Gradient w.r.t. biases of the output gate, of shape (n_a, 1)
    """

    # Retrieve information from "cache"
    (a_next, c_next, a_prev, c_prev, ft, it, cct, ot, xt, parameters) = cache

    # Retrieve dimensions from xt's and a_next's shape (≈2 lines)
    n_x, m = xt.shape
    n_a, m = a_next.shape

    # Compute gates related derivatives, you can find their values can be found by looking carefully at equations
    dot = da_next * np.tanh(c_next) * ot * (1 - ot)
    dcct = (da_next * ot * (1 - np.tanh(c_next) ** 2) + dc_next) * it * (1 - cct ** 2)
    dit = (da_next * ot * (1 - np.tanh(c_next) ** 2) + dc_next) * cct * (1 - it) * it
    dft = (da_next * ot * (1 - np.tanh(c_next) ** 2) + dc_next) * c_prev * ft * (1 - ft)

    # Compute parameters related derivatives. Use equations (11)-(14) (≈8 lines)

    dWf = np.dot(dft, np.hstack([a_prev.T, xt.T]))
    dWi = np.dot(dit, np.hstack([a_prev.T, xt.T]))
    dWc = np.dot(dcct, np.hstack([a_prev.T, xt.T]))
```

```
    dWo = np.dot(dot, np.hstack([a_prev.T, xt.T]))
    dbf = np.sum(dft, axis=1, keepdims=True)
    dbi = np.sum(dit, axis=1, keepdims=True)
    dbc = np.sum(dcct, axis=1, keepdims=True)
    dbo = np.sum(dot, axis=1, keepdims=True)

    # Compute derivatives w.r.t previous hidden state, previous memory state and input. Use equations (15)-(17).
    da_prev = np.dot(Wf[:, :n_a].T, dft) + np.dot(Wc[:, :n_a].T, dcct) + np.dot(Wi[:, :n_a].T, dit) + np.dot(Wo[
    dc_prev = (da_next * ot * (1 - np.tanh(c_next) ** 2) + dc_next) * ft
    dxt = np.dot(Wf[:, n_a:].T, dft) + np.dot(Wc[:, n_a:].T, dcct) + np.dot(Wi[:, n_a:].T, dit) + np.dot(Wo[:, n_

    # Save gradients in dictionary
    gradients = {"dxt": dxt, "da_prev": da_prev, "dc_prev": dc_prev, "dWf": dWf,"dbf": dbf, "dWi": dWi,"dbi": dbi
                 "dWc": dWc,"dbc": dbc, "dWo": dWo,"dbo": dbo}

    return gradients
```

In [16]:
```
np.random.seed(1)
xt = np.random.randn(3,10)
a_prev = np.random.randn(5,10)
c_prev = np.random.randn(5,10)
Wf = np.random.randn(5, 5+3)
bf = np.random.randn(5,1)
Wi = np.random.randn(5, 5+3)
bi = np.random.randn(5,1)
Wo = np.random.randn(5, 5+3)
bo = np.random.randn(5,1)
Wc = np.random.randn(5, 5+3)
bc = np.random.randn(5,1)
Wy = np.random.randn(2,5)
by = np.random.randn(2,1)

parameters = {"Wf": Wf, "Wi": Wi, "Wo": Wo, "Wc": Wc, "Wy": Wy, "bf": bf, "bi": bi, "bo": bo, "bc": bc, "by": by}

a_next, c_next, yt, cache = lstm_cell_forward(xt, a_prev, c_prev, parameters)

da_next = np.random.randn(5,10)
dc_next = np.random.randn(5,10)
gradients = lstm_cell_backward(da_next, dc_next, cache)
print("gradients[\"dxt\"][1][2] =", gradients["dxt"][1][2])
print("gradients[\"dxt\"].shape =", gradients["dxt"].shape)
print("gradients[\"da_prev\"][2][3] =", gradients["da_prev"][2][3])
print("gradients[\"da_prev\"].shape =", gradients["da_prev"].shape)
print("gradients[\"dc_prev\"][2][3] =", gradients["dc_prev"][2][3])
print("gradients[\"dc_prev\"].shape =", gradients["dc_prev"].shape)
print("gradients[\"dWf\"][3][1] =", gradients["dWf"][3][1])
print("gradients[\"dWf\"].shape =", gradients["dWf"].shape)
print("gradients[\"dWi\"][1][2] =", gradients["dWi"][1][2])
print("gradients[\"dWi\"].shape =", gradients["dWi"].shape)
print("gradients[\"dWc\"][3][1] =", gradients["dWc"][3][1])
print("gradients[\"dWc\"].shape =", gradients["dWc"].shape)
print("gradients[\"dWo\"][1][2] =", gradients["dWo"][1][2])
print("gradients[\"dWo\"].shape =", gradients["dWo"].shape)
print("gradients[\"dbf\"][4] =", gradients["dbf"][4])
print("gradients[\"dbf\"].shape =", gradients["dbf"].shape)
print("gradients[\"dbi\"][4] =", gradients["dbi"][4])
print("gradients[\"dbi\"].shape =", gradients["dbi"].shape)
print("gradients[\"dbc\"][4] =", gradients["dbc"][4])
print("gradients[\"dbc\"].shape =", gradients["dbc"].shape)
```

```
print("gradients[\"dbo\"][4] =", gradients["dbo"][4])
print("gradients[\"dbo\"].shape =", gradients["dbo"].shape)
```

```
gradients["dxt"][1][2] = 3.2305591151091884
gradients["dxt"].shape = (3, 10)
gradients["da_prev"][2][3] = -0.06396214197109239
gradients["da_prev"].shape = (5, 10)
gradients["dc_prev"][2][3] = 0.7975220387970015
gradients["dc_prev"].shape = (5, 10)
gradients["dWf"][3][1] = -0.1479548381644968
gradients["dWf"].shape = (5, 8)
gradients["dWi"][1][2] = 1.0574980552259903
gradients["dWi"].shape = (5, 8)
gradients["dWc"][3][1] = 2.304562163687667
gradients["dWc"].shape = (5, 8)
gradients["dWo"][1][2] = 0.3313115952892109
gradients["dWo"].shape = (5, 8)
gradients["dbf"][4] = [0.18864637]
gradients["dbf"].shape = (5, 1)
gradients["dbi"][4] = [-0.40142491]
gradients["dbi"].shape = (5, 1)
gradients["dbc"][4] = [0.25587763]
gradients["dbc"].shape = (5, 1)
gradients["dbo"][4] = [0.13893342]
gradients["dbo"].shape = (5, 1)
```

3.3 Backward pass through the LSTM RNN

This part is very similar to the `rnn_backward` function you implemented above. You will first create variables of the same dimension as your return variables. You will then iterate over all the time steps starting from the end and call the one step function you implemented for LSTM at each iteration. You will then update the parameters by summing them individually. Finally return a dictionary with the new gradients.

Instructions: Implement the `lstm_backward` function. Create a for loop starting from T_x and going backward. For each step call `lstm_cell_backward` and update the your old gradients by adding the new gradients to them. Note that `dxt` is not updated but is stored.

In [17]:
```
def lstm_backward(da, caches):

    """
    Implement the backward pass for the RNN with LSTM-cell (over a whole sequence).

    Arguments:
    da -- Gradients w.r.t the hidden states, numpy-array of shape (n_a, m, T_x)
    dc -- Gradients w.r.t the memory states, numpy-array of shape (n_a, m, T_x)
    caches -- cache storing information from the forward pass (lstm_forward)

    Returns:
    gradients -- python dictionary containing:
                        dx -- Gradient of inputs, of shape (n_x, m, T_x)
                        da0 -- Gradient w.r.t. the previous hidden state, numpy array of shape (n_a, m)
                        dWf -- Gradient w.r.t. the weight matrix of the forget gate, numpy array of shape (n_a, r
                        dWi -- Gradient w.r.t. the weight matrix of the update gate, numpy array of shape (n_a, r
                        dWc -- Gradient w.r.t. the weight matrix of the memory gate, numpy array of shape (n_a, r
                        dWo -- Gradient w.r.t. the weight matrix of the save gate, numpy array of shape (n_a, n_a
                        dbf -- Gradient w.r.t. biases of the forget gate, of shape (n_a, 1)
                        dbi -- Gradient w.r.t. biases of the update gate, of shape (n_a, 1)
                        dbc -- Gradient w.r.t. biases of the memory gate, of shape (n_a, 1)
                        dbo -- Gradient w.r.t. biases of the save gate, of shape (n_a, 1)
    """

    # Retrieve values from the first cache (t=1) of caches.
    (caches, x) = caches
    (a1, c1, a0, c0, f1, i1, cc1, o1, x1, parameters) = caches[0]

    # Retrieve dimensions from da's and x1's shapes (≈2 lines)
    n_a, m, T_x = da.shape
    n_x, m = x1.shape

    # initialize the gradients with the right sizes (≈12 lines)
    dx = np.zeros((n_x, m, T_x))
    da0 = np.zeros((n_a, m))
    da_prevt = np.zeros((n_a, m))
    dc_prevt = np.zeros((n_a, m))
    dWf = np.zeros((n_a, n_a + n_x))
    dWi = np.zeros((n_a, n_a + n_x))
    dWc = np.zeros((n_a, n_a + n_x))
    dWo = np.zeros((n_a, n_a + n_x))
    dbf = np.zeros((n_a, 1))
```

```
    dbi = np.zeros((n_a, 1))
    dbc = np.zeros((n_a, 1))
    dbo = np.zeros((n_a, 1))

    # loop back over the whole sequence
    for t in reversed(range(T_x)):
        # Compute all gradients using lstm_cell_backward
        gradients = lstm_cell_backward(da[:,:,t] + da_prevt, dc_prevt, caches[t])
        # Store or add the gradient to the parameters' previous step's gradient
        dx[:,:,t] = gradients["dxt"]
        dWf += gradients["dWf"]
        dWi += gradients["dWi"]
        dWc += gradients["dWc"]
        dWo += gradients["dWo"]
        dbf += gradients["dbf"]
        dbi += gradients["dbi"]
        dbc += gradients["dbc"]
        dbo += gradients["dbo"]
    # Set the first activation's gradient to the backpropagated gradient da_prev.
    da0 = gradients["da_prev"]

    # Store the gradients in a python dictionary
    gradients = {"dx": dx, "da0": da0, "dWf": dWf,"dbf": dbf, "dWi": dWi,"dbi": dbi,
                 "dWc": dWc,"dbc": dbc, "dWo": dWo,"dbo": dbo}

    return gradients
```

In [18]:
```
np.random.seed(1)
x = np.random.randn(3,10,7)
a0 = np.random.randn(5,10)
Wf = np.random.randn(5, 5+3)
bf = np.random.randn(5,1)
Wi = np.random.randn(5, 5+3)
bi = np.random.randn(5,1)
Wo = np.random.randn(5, 5+3)
bo = np.random.randn(5,1)
Wc = np.random.randn(5, 5+3)
bc = np.random.randn(5,1)

parameters = {"Wf": Wf, "Wi": Wi, "Wo": Wo, "Wc": Wc, "Wy": Wy, "bf": bf, "bi": bi, "bo": bo, "bc": bc, "by": by}

a, y, c, caches = lstm_forward(x, a0, parameters)

da = np.random.randn(5, 10, 4)
gradients = lstm_backward(da, caches)

print("gradients[\"dx\"][1][2] =", gradients["dx"][1][2])
print("gradients[\"dx\"].shape =", gradients["dx"].shape)
print("gradients[\"da0\"][2][3] =", gradients["da0"][2][3])
print("gradients[\"da0\"].shape =", gradients["da0"].shape)
print("gradients[\"dWf\"][3][1] =", gradients["dWf"][3][1])
print("gradients[\"dWf\"].shape =", gradients["dWf"].shape)
print("gradients[\"dWi\"][1][2] =", gradients["dWi"][1][2])
print("gradients[\"dWi\"].shape =", gradients["dWi"].shape)
print("gradients[\"dWc\"][3][1] =", gradients["dWc"][3][1])
print("gradients[\"dWc\"].shape =", gradients["dWc"].shape)
print("gradients[\"dWo\"][1][2] =", gradients["dWo"][1][2])
print("gradients[\"dWo\"].shape =", gradients["dWo"].shape)
print("gradients[\"dbf\"][4] =", gradients["dbf"][4])
print("gradients[\"dbf\"].shape =", gradients["dbf"].shape)
print("gradients[\"dbi\"][4] =", gradients["dbi"][4])
print("gradients[\"dbi\"].shape =", gradients["dbi"].shape)
print("gradients[\"dbc\"][4] =", gradients["dbc"][4])
print("gradients[\"dbc\"].shape =", gradients["dbc"].shape)
print("gradients[\"dbo\"][4] =", gradients["dbo"][4])
print("gradients[\"dbo\"].shape =", gradients["dbo"].shape)
```

```
gradients["dx"][1][2] = [-0.00173313  0.08287442 -0.30545663 -0.43281115]
gradients["dx"].shape = (3, 10, 4)
```

```
gradients["da0"][2][3] = -0.09591150195400468
gradients["da0"].shape = (5, 10)
gradients["dWf"][3][1] = -0.06981985612744009
gradients["dWf"].shape = (5, 8)
gradients["dWi"][1][2] = 0.10237182024854774
gradients["dWi"].shape = (5, 8)
gradients["dWc"][3][1] = -0.06249837949274524
gradients["dWc"].shape = (5, 8)
gradients["dWo"][1][2] = 0.04843891314443013
gradients["dWo"].shape = (5, 8)
gradients["dbf"][4] = [-0.0565788]
gradients["dbf"].shape = (5, 1)
gradients["dbi"][4] = [-0.15399065]
gradients["dbi"].shape = (5, 1)
gradients["dbc"][4] = [-0.29691142]
gradients["dbc"].shape = (5, 1)
gradients["dbo"][4] = [-0.29798344]
gradients["dbo"].shape = (5, 1)
```

Expected Output:

gradients["dx"][1][2] =	[-0.00173313 0.08287442 -0.30545663 -0.43281115]
gradients["dx"].shape =	(3, 10, 4)
gradients["da0"][2][3] =	-0.095911501954
gradients["da0"].shape =	(5, 10)
gradients["dWf"][3][1] =	-0.0698198561274
gradients["dWf"].shape =	(5, 8)
gradients["dWi"][1][2] =	0.102371820249
gradients["dWi"].shape =	(5, 8)
gradients["dWc"][3][1] =	-0.0624983794927
gradients["dWc"].shape =	(5, 8)
gradients["dWo"][1][2] =	0.0484389131444
gradients["dWo"].shape =	(5, 8)
gradients["dbf"][4] =	[-0.0565788]
gradients["dbf"].shape =	(5, 1)

gradients["dbi"][4] =	[-0.06997391]
gradients["dbi"].shape =	(5, 1)
gradients["dbc"][4] =	[-0.27441821]
gradients["dbc"].shape =	(5, 1)
gradients["dbo"][4] =	[0.16532821]
gradients["dbo"].shape =	(5, 1)

A few results in the end is not consistent with the real output.