## Counting or Probability Theory

Counting/Probability

Addition Rule: OR

Multiplication Rule: AND or STEPS

B:

~Mutual

Exclusive

A:

Mutual

Exclusive

D:

Independent

C1:

Dependent

C2:

Permutation &

Combination

**Try to understand or connect all the following problems with the two main branches, and four small branches above. For the additional rule, we have normally a few problems involving Venn diagram. Most of other problems involve multi-steps, and thus we need multiplication rule. In multiplication rule, we need make sure the sum of numbers in each step equal to the total number to be arranged**.

### Permutation and combination

This is taken out because many (not all) probability problems involve permutation and combination. The following examples show almost all possible problems for nCr and nPr.

* How many ways to order 1, 2, 3, 4, 5 in five positions? 5P5;
* How many ways to order 1, 2, 3, 4, 5 in five positions if 3rd position is 4? 4P4\*1P1;
* How many ways to order 1, 2, 3, 4, 5 in five positions if 2nd and 4th positions are 3 and 2? 3P3\*1P1\*1P1.
* How many ways to obtain ordered three number from 1, 2, 3, 4, 5? 5P3;
* How many ways to obtain ordered three number from 1, 2, 3, 4, 5 if 3rd position is 4? 4P2\*1P1;
* How many ways to obtain ordered three number from 1, 2, 3, 4, 5 if 2nd and 3rd positions are 3 and 2? 3P1\*1P1\*1P1;
* How many three-number groups from 1, 2, 3, 4, 5? 5C3;
* How many three-number groups from 1, 2, 3, 4, 5 if one of group member is 3? 4C2\*1C1;
* How many three-number groups from 1, 2, 3, 4, 5 if 2nd and 3rd positions are 3 and 2? 3C1\*1C1;

### Common Problems

* At the tire store, 5 out of every 50 tires are defective. If you purchase 4 tires and they are randomly selected from a newly shipped 50 tires. [1] What is the probability of two tires from the four are defective (no replacement in the process)? [2] How about there are replacements in the choosing process for the two cases above.

Answer:

[1] 5C2\*45C2/50C4. Note here I cannot use the method of probability multiplication described in next section.

* At the tire store, 5 out of every 50 tires are defective. If you purchase 4 tires and they are randomly selected from a newly shipped 50 tires. [1] What is the probability of all four tires are defective (no replacement in the process)? [2] What is the probability of none of them are defective (no replacement in the process)? [3] How about there are replacements in the choosing process for the two cases above.

Answer:

[1] 5C4/50C4. Note this is the most general method. Sometimes we can also calculate with (5/50)\*(4/49)\*(3/48)\*(2/47). However, better not use this method because a lot of times you can get wrong results.

[2] 45C4/50C4.

* Arrange 10 persons A, B, C, …. J into 10 offices. What is probability when three persons B, E, G can only take offices 2, 5, 8?

Answer: Step1: Arranging three persons in offices 2, 5 and 8 has 3P3 ways. Step2: Arranging other 7 persons has 7P7 ways. So the probability is 3P3\*7P7/10P10.

* Arrange 10 persons A, B, C, …. J into 10 offices. What is probability and B, E, G can take only the offices 1, 7, 9 respectively.

Answer: Step1, 2, 3: Arranging B has 1P1 way. Same thing for E and G. Step 4: Arranging other 7 persons has 7P7 ways. Thus the probability is 1P1\*1P1\*1P1\*7P7/10P10.

* Arrange 10 persons A, B, C, …. J into 10 offices. What is probability and three persons B, E, G are arranged in three adjacent offices?

Answer: Step1: When BEG are together, we have 10-3+1 = 8 ways of arrange them within 10 offices. Step2: To arrange B,E,G within three offices we have 3P3 ways. Step3: After the BEG is fixed, we have 7P7 of ways arrange other 7 person into the 7 other positions. Therefore, the probability is 3\*3P3\*7P7/10P10.

* After tossing three coins, [1] what is the probability that two of them are head and one of them is tail; [2] What is the probability for the first coin is H, second is T and final coin is H?

Answer:

[1] 3C2\*1C1/(2\*2\*2). Note that the total events are not permutation because each toss is independent.

[2] 1C1\*1C1\*1C1/(2\*2\*2). This is the one we need consider order.

* menuX contains 5 main dishes names A, B, C, D, E. menu Y contains 7 dishes named M, N, O, P, Q, R, S. How many ways one customer can choose a main dish?

Answer: Addition rule with mutually exclusive condition. So the total is 5+7 = 12.

* menuX contains 5 main dishes names A, B, C, D, E. menu Y contains 7 dishes named A, B, O, P, Q, R, S. How many ways one customer can choose a main dish?

Answer: Addition rule with mutually non-exclusive condition. The total is 5+7-2 = 10.

* menuX contains 5 main dishes. menu Y contains 7 dishes. From the two menus, one customer can only choose a maximum of 10 dishes. How many dishes overlap in manuX and menuY?

Answer: