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Redshift evolution of the SN stretch distribution

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ABSTRACT

Context. Type Ia supernovae (SNe Ia) allow for the construction of the Hubble diagram, giving us information about the Universe's expansion and its fondamental components, one of which is dark energy. But systematic uncertainties are now starting to be limiting in our ability to measure those parameters. In particular, the physics of SNe Ia is still mostly unknown, and is thought not to change in time/with the redshift.
Aims. In an attempt to reduce those uncertainties, we try to find an empirical law describing SNe Ia's length of explosion (stretch) evolution with the redshift.
Methods. We started by getting a complete sample representing all of the stretch distribution that Nature can give us, before using LsSFR measurments, an age tracer which evolution with redshift is known, that has been shown to have a strong correlation with the stretch. We compare their AICc, an estimator of the relative quality of statistical models that includes the number of free parameters, to determine which ones describe besto the data.
Results. Models with an evolution of the stretch with the redshift have a better AICc than the ones without.
Conclusions. We find that implementing these models allows us to fit the data better than models without stretch evolution.

Key words. Cosmology – Type Ia Supernova – Systematic uncertainties

1. Introduction

Type Ia supernovae (SNe Ia) are now well-known for their capacity to determine cosmological parameters: their study led to the discovery of the accelerated expansion of the Universe (Riess 98, Perlmutter 99) through the name of "dark energy", and they have been used continuously for better measurments since then (Betoule 2014). They are acquired through their lightcurves, giving the evolution of their luminosity from the time of explosion, in different wavelength. 3 parameters are used to describe those: an amplitude, a width (named "stretch") and a color (magnitude difference in the B and V bands).

The simple use of those is not enough for our aim, as SNe Ia have an intrinsec dispersion of their luminosity of $\approx 0.4\ mag$ that gives a huge uncertainty on the determination of their distance modulus. Henceforth they are standardized using the "brigther-slower" and "brighter-bluer" relations (Philipps 93, Riess 96, Tripp 98) in the SALT2 algorithm (Guy 2007, 2010) that fits the distance modulus which is expressed as

$$\mu = m_b + \alpha x_1 - \beta c - M \tag{1}$$

with m_B the logarithm of their flux, x_1 their stretch, c their color and M their intrinsec magnitude. This relation lowered their magnitude dispersion to $\approx 0.15\ mag$, allowing for previously mentioned accelerated expansion to be discovered.

This Tripp estimator lies on the idea that this standardization doesn't change with the redshift. However, Rigault 2015 showed that SNe Ia depend on their environments, and these environments' properties evolve with the redshift. In this Letter, we try to determine whether a stretch evolution with the redshift allows for a better description of the collected data.

2. Sample

We use data from 5 different surveys: Hubble Space Telescope (HST, REF), Supernova Nearby Factory (SNf, REF), Supernova Legacy Survey (SNLS, Aster 2006, Betoule 2014), Sloan DSS (SDSS, REF) and Panstarr-1 (PS1, REF). The last 3 had selection effects that come from the "brighter-slower" and "brighter-bluer" relations: the first SNe Ia that are missed by the instruments are the redder and faster ones. That implies that taking all of these surveys' data would not represent the stretch distribution that Nature could give us.

As such, we determined the maximum redshift at which we would cut these samples to get complete ones. We used a statistical approach in lack of precise data concerning the instruments' capacity to acquire fluxes. We proceeded by making histograms of the surveys, and defining a function of the expected SNe Ia rate:

$$N_{\text{SNe Ia}} = a \times V(z) \tag{2}$$

with a the density of SNe Ia and $V(z)$ the volume of Universe in each bin. Before selection effects, the histograms are supposed to grow proportionnaly to $V(z)$ assuming an homogeneous repartition of SNe Ia. When we start to miss some supernovae, this growth is not followed anymore.

To determine this limit, we first took the observed counts in each bin of the histograms. We then fitted a on these counts for different numbers of bins used to fit: the quality of the fit diminishes when we use bins that include selection effects. To quantize this, we computed the expected counts from the fitted a in the same bins we used for the observed counts. That allowed us to compute the poissonian probability in each bin, given by

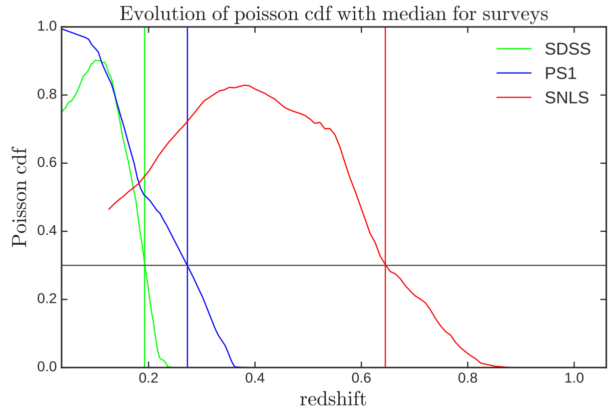
$$p(k) = \frac{\lambda^k}{k!} e^{-\lambda} \tag{3}$$

with k the expected counts, λ the observed counts. This work is done 100 times with a random number of total bins, a being fitted 10 times on a different and random number of bins used to fit each time. We interpolate these 1000 poissonian evolutions to get one final result that is as close as possible to accurately represent how the probability to follow the expected rate evolves.

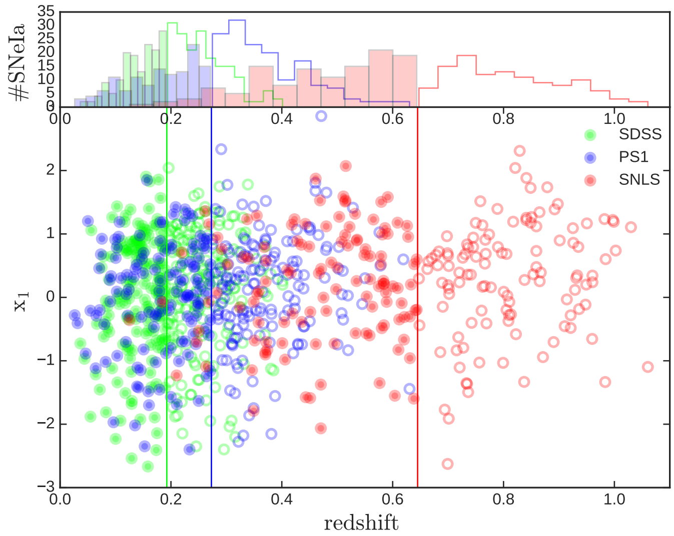
To get the maximum redshift after which we stop following the expected rate, we took the redshift for which we are at a 0.3 on the interpolated cumulative distribution function. We chose that value to have a systematic analytical result that matches what we expect from the histograms; its impact on the work after is to be studied.

Table 1. Number of SNe Ia effectively used for each survey, totalling 689 Sne Ia. TO CORRECT FOR NEW ZMAX

Survey	Number of SNe Ia
SNf	141
PS1	178
SDSS	206
SNLS	138
HST	26



(a) Method



(b) Result

Fig. 1. (a) $z_{\rm max}$ determination from the interpolated poissonian evolutions; (b) SDSS, SNLS and PS1 samples cut at $z_{\rm max}$ as explained in section 2. The data we use is in plain markers.

3. Method

We used the LsSFR and stretch measurments from the SNf sample. The LsSFR has an evolution with the redshift that is analytically known: calling $\delta(z)$ the fraction of young stars and $\psi(z)$ the fraction of old ones, Rigault 2018 et + find

$$\frac{\delta(z)}{\psi(z)} \equiv \mathrm{LsSFR}(z) = K \times (1+z)^{\varphi} \tag{4}$$

with $\varphi = 2.8$, and knowing $\delta(z) + \psi(z) = 1$:

$$\delta(z) = \left(K^{-1} \times (1+z)^{-\varphi} + 1\right)^{-1} \tag{5}$$

$$\psi(z) = \left(K \times (1+z)^{+\varphi} + 1\right)^{-1} \tag{6}$$

The goal is to find how the stretch depends on the LsSFR, distinguishing old and young SNe which fractions evolve with the redshift, in order to have an analytical law for the mean redshift evolution of the stretch. The measured data on which we based our first model is plotted in figure 2; based of the shape of the x_1 vs LsSFR cloud, we implemented the following model:

- **young**: a gaussian of mean μ_1 and standard deviation σ_1, namely $\mathcal{N}_1 \equiv \mathcal{N}(\mu_1, \sigma_1)$;
- **old**: a linear combination between $\mathcal{N}_1$ and another gaussian $\mathcal{N}_2 \equiv \mathcal{N}(\mu_2, \sigma_2)$

As such, the probability to observe a young SNe Ia labeled "i" with a stretch x_1^i and error $\mathrm{d}x_1^i$ is

$$p\left(x_1^i, \mathrm{d}{x_1}^i | \mu_1, \sigma_1\right) = \mathcal{N}\left(\mu_1, \sqrt{{\sigma_1}^2 + {\mathrm{d}x_1^i}^2}\right)(x_1^i) \tag{7}$$

and for an old one:

$$\begin{aligned} p(x_1^i, \mathrm{d}x_1^i | \mu_{1,2}, \sigma_{1,2}, a) = {} & a \times \mathcal{N}\left(\mu_1, \sqrt{{\sigma_1}^2 + {\mathrm{d}x_1^i}^2}\right)(x_1^i) + \\ & (1-a) \times \mathcal{N}\left(\mu_2, \sqrt{{\sigma_2}^2 + {\mathrm{d}x_1^i}^2}\right)(x_1^i), \end{aligned} \tag{8}$$

where a is the relative amplitude between the two gaussians. Finally, the normalized stretch distribution at a given z $\Delta(x_1|z)$ is the weigthed sum of both young and old stretch distributions given their relative fraction $\delta(z)$:

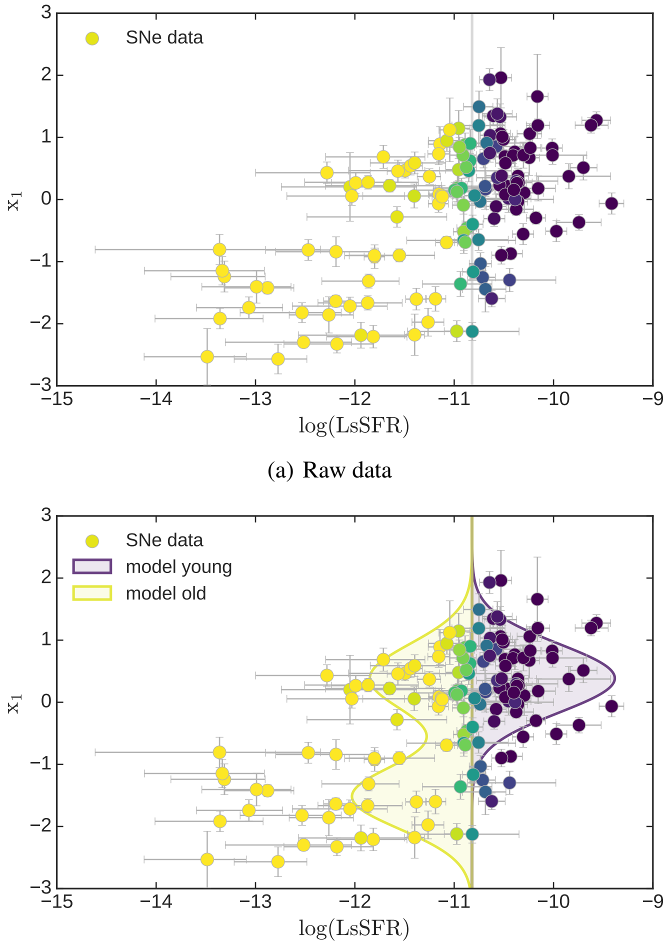
$$\Delta(x_1|z) = \delta(z) \times \mathcal{N}_1 + (1 - \delta(z)) \times (a\mathcal{N}_1 + (1-a)\mathcal{N}_2) \tag{9}$$

We fitted it on SNf data, giving the results table 2. For clarity with the next models, we named it 3G2M2S$_{\rm SNf}$ for it has a total of 3 gaussians but with only 2 means and 2 standard deviations, and has been fitted on SNf data only.

We implemented and compared 10 models in total, 4 of which have an evolution with the redshift from $\delta(z)$, and 6 don't ($\delta(z) = f = \mathrm{cst}$). The ones with and evolution are:

- 3G2M2S, the one we described (but fitted on all the data);
- 3G2M1S, where this time $\sigma_1 \equiv \sigma_2$;
- 2G2M2S, model taken from HOWELL 2009 where we added $\delta(z)$;
- 3G3M3S, with three independent gaussians.

The ones without a stretch evolution are the same ones but with an "F" implying we set $\delta(z) = f = \mathrm{cst}$, and two others:



(a) Raw data

(b) First implemented model

Fig. 2. Stretch des supernovae étudiées par la collaboration SNF en fonction de log(LsSFR). La couleur représente la probabilité pour une supernova d'être issue d'un jeune progéniteur.

- 1G1M1S, where there is no distinction between old and young SNe;
- 1G1M2S, taken from KESLLER 2017 and used in recent cosmological analysis SCOLNIC 2018. It's an asymetric model where

$$p(x_1^i,\, \mathrm{d}x_1^i|\mu,\sigma_-,\sigma_+) = \begin{cases} \mathcal{N}\left(\mu,\ \sqrt{{\sigma_-}^2 + {\mathrm{d}x_1^i}^2}\right)(x_1^i) & \text{if } x_1^i \geq \mu \\ \mathcal{N}\left(\mu,\ \sqrt{{\sigma_+}^2 + {\mathrm{d}x_1^i}^2}\right)(x_1^i), & \text{else} \end{cases} \tag{10}$$

The fitted parameters are showed table 2.

4. Results

To compare all these models, we use the Akaike Information Criterion corrected for sample size (BURNHAM 2002), that penalises the increase of free parameters in order to discourage overfitting:

$$\mathrm{AICc} = \mathrm{AIC} + \left(\frac{2k\,(k+1)}{n-k-1}\right) \tag{11}$$

Table 2. Valeurs des paramètres pour différents modèles. En rouge les données aberrantes.

Modèle	a	f	μ_1	σ_1	μ_2	σ_2
3G2M2Ssnf	0.48± 0.07	none	0.39± 0.07	0.56± 0.05	−1.5± 0.1	0.52± 0.09
3G2M2S	0.48± 0.17	none	0.36± 0.08	0.61± 0.05	−1.3± 0.2	0.60± 0.12
3G2M2SF	0.1 ± 0.6	0.2 ± 0.6	−0.9± 0.7	0.7 ± 0.3	0.5 ± 0.2	0.6 ± 0.1
3G2M1S	0.47± 0.07	none	0.35± 0.04	0.61± 0.03	−1.25± 0.10	σ_1
3G2M1SF	0.2 ± 0.9	0.7 ± 0.3	0.36± 0.04	0.60± 0.03	−1.23± 0.10	σ_1
2G2M2S	none	none	0.49± 0.04	0.54± 0.03	−0.72± 0.08	0.83± 0.07
2G2M2SF	none	0.3 ± 0.2	−0.9± 0.6	0.7 ± 0.2	0.5 ± 0.2	0.56± 0.09

Modèle	μ	σ
1G1M1S	0.01 ± 0.04	0.90 ± 0.03

Modèle	μ	σ_-	σ_+
1G1M2S	0.16617 ± 0.00004	1.07 ± 0.04	0.69 ± 0.03

Modèle	a	f	μ_1	σ_1	μ_2	σ_2	m_3	σ_3
3G3M3S	0.14± 0.08	none	0.51± 0.06	0.54± 0.04	−1.9± 0.2	0.29± 0.11	−0.55± 0.12	0.67± 0.15
3G3M3SF	0.2± 0.2	0.10± 0.04	−1.7± 0.2	0.4± 0.1	0.9± 0.1	0.3± 0.2	0.0± 0.2	0.7± 0.1

with AIC = $2k - 2\ln(\mathcal{L})$, where k is the number of free parameters and $\mathcal{L}$ the likelihood. The probability for a model to be as representative as the "best" one is given by:

$$p(\text{other} > \text{best}) = \exp(\Delta\mathrm{AICc}/2) \tag{12}$$

As such, we obtain the results table 3.

We find that every model lacking an evolution of the stretch with the redshift is systematically worse than those that implement it.

5. Conclusion

Stretch evolution with the redshift is a thing. Need to see if it has an impact on the cosmology though.

Table 3. Comparaison des modèles. NR représente les modèles implémentés durant ce stage. (F) indique les modèles pour lesquels il n'y a pas d'évolution de la fraction de SNe Ia jeunes et vieilles en fonction du redshift.

Name	Descrip	Free param	$\ln \mathcal{L}$	$AICc$	$\Delta AICc$	Proba
3G2M1S	NR 1S	4	1815	1823	0.0	1.0
3G2M2S	NR 2S	5	1815	1825	-2.0	3.6×10^{-1}
2G2M2S	Howell	4	1818	1826	-3.4	1.8×10^{-1}
3G3M3S	NR 3S	7	1812	1826	-3.6	1.6×10^{-1}
3G3M3SF	NR 3S (F)	8	1813	1829	-6.3	4.3×10^{-2}
2G2M2SF	Howell (F)	5	1823	1833	-9.9	7.0×10^{-3}
3G2M1SF	NR 1S (F)	5	1823	1833	-10.4	5.5×10^{-3}
3G2M2SF	NR 2S (F)	6	1823	1835	-12.0	2.5×10^{-3}
1G1M2S	Kessler (F)	3	1837	1843	-20.2	4.1×10^{-5}
1G1M1S	1 gauss. (F)	2	1872	1876	-53.5	2.4×10^{-12}