Redshift Evolution of the Underlying Type Ia Supernova Stretch Distribution

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ABSTRACT

The true nature of type Ia supernovae (SNe Ia) remains largely unknown, and as survey statistics increase, the question of astrophysical systematic uncertainties rises, notably that of the SN Ia population redshift drift. In this paper, we study the redshift evolution of SN Ia `SALT2.4` lightcurve stretch, a purely intrinsic SN property, to probe this drift. The SN stretch has been shown to strongly correlate with the SN's environment, notably with stellar age tracers. Following the evolution of fraction of young and old SNe Ia as predicted by Rigault et al. (2018), and assuming constant underlying stretch distribution for each population as a function of redshift, model the expected SN Ia stretch distribution as a function of redshift. We test our prediction against data from the literature chosen to have negligible selection effects to ensure that any observed drift is indeed astrophysical and not observational. We clearly demonstrate that the underlying SNe Ia stretch distribution is evolving as a function of redshift, and that the young/old drifting model is a much better description of the data than any non-drifting model, including the sample-based asymmetric distributions used by the Beams with Bias Correction algorithm to correct Malmquist bias. Our favored underlying stretch model is bimodal: a high-stretch mode shared by both young and old environments, and a low-stretch mode exclusive to old environments. The precise impact of the redshift evolution of the underlying SNe Ia population on cosmology remains to be studied. Yet, the astrophysical drift of the SN stretch distribution does affect current Malmquist bias corrections and thereby distances derived from SN affected by selection effects. We highlight that such a bias will increase with surveys covering larger redshift ranges, which is particularly important for LSST.

Key words. Cosmology – Type Ia Supernova – Systematic uncertainties

1. Introduction

Type Ia supernovae (SNe Ia) are powerful cosmological distance indicators that have enabled the discovery of the acceleration of the Universe's expansion (Riess et al. 1998; Perlmutter et al. 1999). They remain today a key cosmological probe to understand the properties of dark energy (DE) as it is the only tool able to precisely map the recent expansion rate ($z < 0.5$), when DE is driving it (e.g. Scolnic et al. 2019). They also are key to directly measure the Hubble Constant (H_0), provided one can calibrate their absolute magnitude (Riess et al. 2016; Freedman et al. 2019). Interestingly, the value of H_0 derived when the SNe Ia are anchored on Cepheids (the SH0ES project, Riess et al. 2009, 2016) is ∼ 5σ higher than what is predicted from cosmic microwave background (CMB) data measured by Planck assuming the standard ΛCDM (Planck Collaboration et al. 2018; Riess et al. 2019; Reid et al. 2019), or when the SN luminosity is anchored at intermediate redshift by the baryon acoustic oscillation (BAO) scale (Feeney et al. 2019). While using the tip of the red giant branch technique in place of the Cepheids seem to favor an intermediate value of H_0 (Freedman et al. 2019, 2020), time delay measurements from strong lensing seem to also favor high H_0 values (Wong et al. 2019).

The H_0 tension has received a lot of attention, as it could be a sign of new fundamental physics. Yet, no simple solution is able to accommodate this H_0 tension when accounting for all other probes (Knox & Millea 2019), but see e.g. Poulin et al. (2019). Alternatively, systematic effects affecting one or several of the aforementioned analyses could also explain the tension. Rigault et al. (2015) suggested that SNe Ia from the Cepheid calibrator sample differ by construction from the Hubble flow sample ones as the former strongly favors young stellar populations, while the latter not. This selection effect would impact the derivation of H_0 if SNe Ia from young and older environments differ in average standardized magnitudes.

For the last decades, numerous analyses have studied the relation between SNe Ia and host properties, finding first that the standardized SNe Ia magnitude significantly depends on the host stellar mass, SNe Ia from high-mass host being brighter on average (e.g. Kelly et al. 2010; Sullivan et al. 2010; Childress et al. 2013; Betoule et al. 2014; Rigault et al. 2018; Kim et al. 2019). This mass-step correction is currently used in cosmological analyses (e.g. Betoule et al. 2014; Scolnic et al. 2018), including for deriving H_0 (Riess et al. 2016, 2019). Yet, the underlying connection between the SNe and their host remains unclear when using global properties such as the host stellar mass, which raises the question of the accuracy of the correction. More recently, studies have used the local SN environment to probe more direct

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connections between the SN and its environments (Rigault et al. 2013), showing that local age tracers such as the Local specific Star Formation Rate (LsSFR) or the local color are more strongly correlated with the standardized SN magnitude (Rigault et al. 2018; Roman et al. 2018; Kim et al. 2018), suggesting age as the driving parameter underlying the mass-step. If true, this would have a significant impact for cosmology, since the environmental correction to apply to SN standardization could strongly vary with redshift. (Rigault et al. 2013; Childress et al. 2014; Scolnic et al. 2018). Yet, the importance of local SN environmental studies remains highly debated (e.g. Jones et al. 2015, 2019) and especially the impact of such an astrophysical bias has on the derivation of H$_0$ (Jones et al. 2015; Riess et al. 2016, 2018; Rose et al. 2019).

The concept of the SNe Ia age dichotomy arose with the study of the SN Ia rate. AK: This grammatical error appears a lot in this article. In compound nouns the first noun is typically singular, so this should be SN Ia age dichotomy. Mannucci et al. (2005); Scannapieco & Bildsten (2005); Sullivan et al. (2006); Aubourg et al. (2008) have shown that the relative SNe Ia rate in galaxies could be explained if two populations existed, one young, following the host star formation activity, and one old following the host stellar mass (the so called "prompt and delayed" or "A+B" model). In Rigault et al. (2018) we used the LsSFR to classify which are the younger (those with a high LsSFR) and which are the older (those with low LsSFR). Furthermore, since the first SNe Ia host analyses, the SN stretch has been known to be strongly correlated with the SN host properties (Hamuy et al. 1996, 2000), correlation that has been extensively confirmed since (e.g. Neill et al. 2009; Sullivan et al. 2010; Lampeitl et al. 2010; Kelly et al. 2010; Gupta et al. 2011; D'Andrea et al. 2011; Childress et al. 2013; Rigault et al. 2013; Pan et al. 2014; Kim et al. 2019). Following the "A+B" model and the connection between SN stretch and host properties, Howell et al. (2007) first discussed the potential redshift drift of the SN stretch distribution. In this paper we revisit this analysis with the most recent SNe Ia dataset.

In this paper, we take a step aside to probe the validity of our modeling of the SN population, which we claim to be constituted of two age-populations (Rigault et al. 2013; Rigault et al. 2015, 2018): one old and one younger, the former having on average lower lightcurve stretches and being brighter after standardization. We use the correlation between the SN age, as probed by the LsSFR, and the SN stretch to model the expected evolution of the underlying SN stretch distribution as a function of redshift. This modeling relies on three assumptions: (1) there are two distinct populations of SNe Ia; (2) the relative fraction of each of these populations as a function of redshift follows the model presented in Rigault et al. (2018) and (3) the underlying distribution of stretch for each age sample is constant. This paper aims at testing this specific model with datasets from the literature. Note that the progenitor age as traced by the LsSFR seems to capture physical feature intrinsic to the progenitor and/or explosion mechanism that the stretch alone is not capturing (Nordin et al. 2018).

We present in Section 2 the sample we are using for this analysis, derived from the Pantheon catalog (Scolnic et al. 2018). We discuss the importance of obtaining a "complete" sample, i.e. representative of the true underlying SNe Ia distribution, and how we build one from the Pantheon sample. We then present in Section 3 our modeling of the distribution of stretch and our results are presented in Section 4. In this section we test whether the SN stretch distribution evolves as a function of redshift and if the aforementioned age model is in good agreement with this

evolution. We briefly discuss these results in the context of SN cosmology in Section 5 and we conclude in Section 6.

2. Complete Sample Construction

We base our analysis on the most recent comprehensive SNe Ia compilation, the Pantheon catalog from Scolnic et al. (2018). A naive approach to test the SN stretch redshift drift would be to simply compare the observed SN stretch distributions in a few bins of redshift. In practice, however, differential selection effects are affecting the observed SN stretch distributions. Indeed, because the observed SN Ia magnitude correlates with the lightcurve stretch (and color), the first SNe Ia that a magnitude-limited survey will miss are the lowest-stretch (and reddest) ones. Consequently, if magnitude-related selection effects are not accounted for, one might confuse true population drift with survey properties, and conversely.

Assuming sufficient (and unbiased) spectroscopic follow-up for acquiring typing and host redshift, the selection effects of magnitude-limited surveys should be negligible below a given redshift at which even the faintest SNe Ia can be observed. In contrast, targeted surveys have highly complex selection functions and will be discarded from our analysis. Fortunately, modern SN cosmology samples such as the Pantheon one are now dominated by magnitude-limited surveys.

We present in Fig. 1 the lightcurve stretch and color of SNe Ia from the following surveys: PanStarrs (PS1 Rest et al. 2014), the Sloan Digital Sky Survey (SDSS Frieman et al. 2008) and the SuperNovae Legacy Survey (SNLS Astier et al. 2006). An ellipse in the SALT2.4 (x_1, c) plane with $x_1 = \pm 3$ and $c = \pm 0.3$ encapsulates the full distribution (Guy et al. 2007; Betoule et al. 2014); see also Bazin et al. (2011) and Campbell et al. (2013) for similar contours, the second using a more conservative $|c| \leq 0.2$ cut. Assuming the SN absolute magnitude with $x_1 = 0$ and $c = 0$ is $M_0 = -19.36$ (Kessler et al. 2009; Scolnic et al. 2014), we can derive the absolute standardized magnitude at maximum of light $M = M_0 - \alpha x_1 + \beta c$ along the aforementioned ellipse given the standardization coefficient $\alpha = 0.156$ and $\beta = 3.14$ from Scolnic et al. (2018): the faintest SN Ia is that with ($x_1 = -1.65, c = 0.25$) and an absolute standardized magnitude at peak in Bessel B band of $M_{min}^{t_0} = -18.31$ mag. Since one ought to detect this object typically 5 days before and a week after peak to build a suitable lightcurve, the effective limiting standardized absolute magnitude is approximately $M_{lim} = -18.00$ mag. Hence, given the magnitude limit m_{lim} of a magnitude limited survey, one can derive the maximum redshift z_{lim} above which the faintest SNe Ia will be missed using the relation between apparent magnitude, redshift and absolute magnitude $\mu(z_{lim}) = m_{lim} - M_{lim}$.

SNLS typically acquired SNe Ia in the redshift range $0.4 < z < 0.8$; at these redshifts, the rest-frame Bessel B band roughly corresponds to the SNLS i filter, that has a 5σ depth of 24.8 mag[1]. This converts to a $z_{lim} = 0.60$, in agreement with Neill et al. (2006), Perrett et al. (2010) and Bazin et al. (2011). Fig. 14 of (Perrett et al. 2010, see their Section 5) suggests a lower limit of $z_{lim} = 0.55$; both limits will be considered, as discussed below.

Similarly, PS1 observed SNe Ia in the range $0.2 < z < 0.4$, their g-band 5σ depth is 23.1 mag (Rest et al. 2014), which yields to $z_{lim} = 0.30$ in agreement with, e.g., Fig. 6 of Scolnic et al. (2018). This figure is also suggestive of a more conservative $z_{lim} = 0.27$.
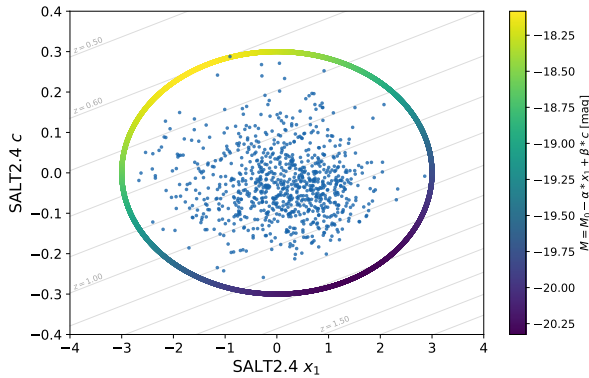
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[1] CFHT final release website.

Fig. 1. `SALT2.4` stretch (x_1) and color (c) lightcurve parameters of SNe Ia from the SDSS, PS1 and SNLS samples of the Pantheon catalog. The individual SNe are shown as blue dots. The ellipse ($x_1 = \pm 3, c = \pm 0.3$) is displayed, colored by the corresponding standardized absolute magnitude using the α and β coefficients from Scolnic et al. (2018). The grey diagonal lines represent the (x_1, c) evolution for $m = m_{lim}$, for z between 0.50 and $z = 1.70$ using SNLS's m_{lim} of 24.8 mag.

Table 1. Composition of the SNe Ia dataset used in this analysis. Conservative cuts are indicated in brackets. AK: parentheses not brackets

Survey	z_{lim}	N_{SN}
SNf	–	114
SDSS	0.20 (0.15)	167 (82)
PS1	0.30 (0.27)	160 (122)
SNLS	0.60 (0.55)	102 (78)
HST	–	26
Total	–	569 (422)

In a similar redshift range, SDSS has a limiting magnitude of 22.5 (Dilday et al. 2008; Sako et al. 2008), which would lead to a $z_{lim} = 0.24$. However, the SDSS surveys were more sensitive to limited spectroscopic resources; (Kessler et al. 2009, see their Section 2) pointed out that during the first year of SDSS, SNe Ia with $r - mag < 20.5$ were favored for spectroscopic follow-up, corresponding to a redshift cut at 0.15. For the rest of the SDSS survey, additional spectroscopic resources were available, and Kessler et al. (2009) and Dilday et al. (2008) show a reasonable completeness up to $z_{lim} = 0.2$. Following these analyses, we will use $z_{lim} = 0.2$ as the baseline SDSS redshift limit.

The sample selection is summarized in Table 1, and the redshift distribution of these three surveys is shown in Fig. 2. As expected, the selected redshift limits roughly correspond to the peak of these histograms. We show in Section 4 that this sample selection indeed provides a subset of SNe Ia with insignificant selection effects when compared to state-of-the-art Malmquist correction techniques.

In addition, we use the SNe Ia from the Nearby Supernova Factory (SNfactory, Aldering et al. 2002) published in Rigault et al. (2018) and that have been discovered from non-targeted searches (114 SNe Ia, see their sections 3 and 4.2.2; SNe Ia time series are published in **?** *Greg: for now you may need to cite SNEMO and SUGAR to remove this "?"*, see also Aldering in prep. and Ponder in prep.). The spectroscopic follow-up was done over a redshift range of $0.02 < z < 0.09$ (as in Rigault et al. 2018), while the search was much deeper. These 114 SNfactory SNeIa thus are part of the volume limited part of the survey (Aldering in prep.). As such, these SNe Ia are assumed to be a random sampling of the underlying SN population. The SNfac-
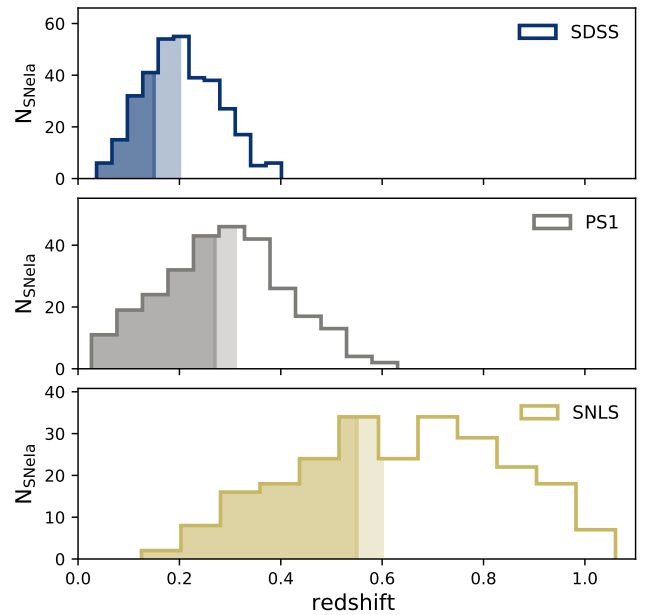

Fig. 2. *From top to bottom*: Redshift histograms of SNe Ia from the SDSS, PS1 and SNLS dataset respectively (data from Pantheon, Scolnic et al. 2018). The colored parts represent the distribution of SNe Ia kept in our analysis for they are supposedly free from selection bias (see Section 2). The darker (resp. lighter) color responds to the conservative (resp. fiducial) selection cut.

tory sample is particularly useful for studying SN property drift, as it enables us to have a large SN Ia sample at $z < 0.1$.

Finally, we include the HST sample from Pantheon, that similarly have a search deeper than the follow-up and that we therefore kept entirely (Strolger et al. 2004).

We present the stretch distribution and redshift histogram of these five surveys up to their respective z_{lim} in Fig. 3. The evolution of the mean stretch is also shown in Fig. 5, where the data are split in redshift bins of regular sample size. We see that SNe Ia at higher redshift have on average larger stretch (0.34 ± 0.10 at $z \sim 0.65$) than those at lower redshift (-0.17 ± 0.10 at $z \sim 0.05$), suggesting that the underlying stretch distribution is drifting. AK: how can this be asserted when you just showed that none of the surveys are complete at $z \sim 0.65$!? MR: Alex, There is HST too, see top of Fig. 3. | in the latest bin 68 SNLS, 26 HST

3. Modeling the redshift drift

Rigault et al. (2018) presented a model for the evolution of the fraction of younger and older SNe Ia as a function of redshift following former work on rates and delay time distributions (e.g., Mannucci et al. 2005; Scannapieco & Bildsten 2005; Sullivan et al. 2006; Aubourg et al. 2008; Childress et al. 2014; Maoz et al. 2014). In short, it was assumed that the number of "young" SNe Ia follows the star formation rate (SFR) in the Universe, while the number of "old" SNe Ia follows the number of Gyr-old stars in the Universe, i.e. the stellar mass (M^*). Hence, if we denote $\delta(z)$ (resp. $\psi(z) = 1 - \delta(z)$) the fraction of young (resp. old) SNe Ia in the Universe as a function of redshift, then the ratio δ/ψ is expected to follow the evolution of the specific star formation rate (SFR/M^*), which goes as $(1+z)^{2.8}$ until $z \sim 2$ (e.g., Tasca et al. 2015). Since $\delta(0.05) \sim \psi(0.05)$ (Rigault et al. 2013; Rigault et al. 2018; Wiseman et al. 2020), in agreement with rate
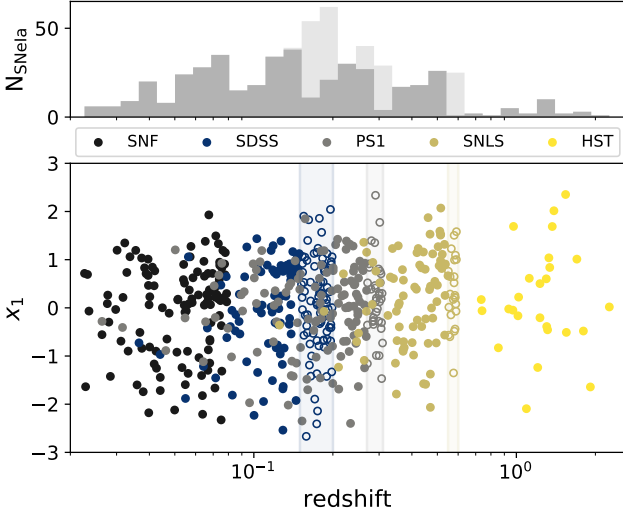
Fig. 3. *Bottom:* `SALT2.4` lightcurve stretch as a function of redshift for each survey considered in this analysis (see legend). Solid (resp. open) markers correspond to the conservative (resp. fiducial) redshift cuts. *Top:* combined redshift histogram in dark (resp. light) gray for the conservative (resp. fiducial) redshift cuts.

expectations ([Mannucci et al. 2006](); [Rodney et al. 2014](), [Rigault et al. (2018)]() concluded that

$$\delta(z) = \left(K^{-1} \times (1+z)^{-2.8} + 1 \right)^{-1} \quad (1)$$

with $K = 0.87$. This model is comparable to the evolution predicted by [Childress et al. (2014)]() based on SN rates in galaxies depending on their quenching time as a function of their stellar mass.

3.1. "Base" underlying stretch distribution

To model the evolution of the full SN stretch distribution as a function of redshift, given our aforementioned model of the evolution of the fraction of younger and older SNe Ia with cosmic time, we need to model the SN stretch distribution for each age subsample.

[Rigault et al. (2018)]() presented the relation between SN stretch and LsSFR measurement, a progenitor age tracer, using the SNfactory sample. This relation is shown in Fig. 4 for the SNfactory SNe used in the current analysis. Given the structure of the stretch-LsSFR scatter plot, our model of the underlying SN Ia stretch distribution is defined as follows:

– for the younger population (i.e., log(LsSFR) ≥ −10.82), the stretch distribution is modeled as a single normal distribution $\mathcal{N}(\mu_1, \sigma_1{}^2)$;
– the older population (i.e., log(LsSFR) < −10.82) stretch distribution is modeled as a bimodal Gaussian mixture $a \times \mathcal{N}(\mu_1, \sigma_1{}^2) + (1-a) \times \mathcal{N}(\mu_2, \sigma_2{}^2)$, where one mode is the same as for the young population, a representing the relative influence of the two modes.

The stretch probability distribution function (pdf) of a given SN will be the linear combination of the stretch distributions of these two population weighted by its probability y^i to be young (see Section 3.2). But generally, the fraction of young SNe Ia as a function of redshift is given by $\delta(z)$ AK: Convention is that letters that are variables are italicized. z, H_0. (see Eq. 1) and therefore, our redshift drift model of the underlying distribution of

SNe Ia as a function of redshift $X_1(z)$ is given by:

$$X_1(z) = \delta(z) \times \mathcal{N}(\mu_1, \sigma_1{}^2) +$$
$$(1 - \delta(z)) \times \left[a \times \mathcal{N}(\mu_1, \sigma_1{}^2) + (1-a) \times \mathcal{N}(\mu_2, \sigma_2{}^2) \right]. \quad (2)$$

3.2. Base model applied to data

Given the probability y^i that a given SN is young, and assuming our Base (see Section 3.1), the probability to measure a `SALT2.4` stretch x_1^i with an error dx_1^i is given by:

$$\mathcal{P}\left(x_1^i \mid \boldsymbol{\theta}; dx_1^i, y^i \right) = y^i \times \mathcal{N}\left(x_1^i \mid \mu_1, \sigma_1{}^2 + dx_1^{i\,2} \right) +$$
$$(1-y^i) \times \left[a \times \mathcal{N}\left(x_1^i \mid \mu_1, \sigma_1{}^2 + dx_1^{i\,2} \right) +$$
$$(1-a) \times \mathcal{N}\left(x_1^i \mid \mu_2, \sigma_2{}^2 + dx_1^{i\,2} \right) \right] \quad (3)$$

The maximum-likelihood estimate of the 5 free parameters $\boldsymbol{\theta} \equiv (\mu_1, \mu_2, \sigma_1, \sigma_2, a)$ of the model is obtained by minimizing the following:

$$-2\ln(L) = -2\sum_i \ln \mathcal{P}\left(x_1^i \mid \boldsymbol{\theta}; dx_1^i, y^i \right). \quad (4)$$

Depending on whether y^i can be estimated directly from LsSFR measurements or not, there are two ways to proceed, which we now discuss.

3.2.1. With LsSFR measurements

For the SNfactory sample, we can readily set $y^i = p_y^i$, the probability to have $\log \mathrm{LsSFR} \geq -10.82$ (see Fig. 4), to minimize Eq. 4 with respect to $\boldsymbol{\theta}$. Results on fitting the SNf SNe with this model are shown Table 2 and illustrated in Fig. 4.

AK: You don't define p_y^i. The language is is sloppy. What is relevant is the opposite, the probability that a galaxy with low mass gets the measured LsSFR. These are in general different. The solution you use is only correct for a model in which the rates for low and high-LsSFR galaxies is the same. The probability in general should include a prior on l.

$$p(x_{o1}^i, l_o^i | \mu_1, \mu_2, \sigma_1, \sigma_2, a) = \int dl^i\, p(x_{o1}^i, l_o^i | l^i, \mu_1, \mu_2, \sigma_1, \sigma_2, a) p(l^i | \mu_1, \mu_2, \sigma$$

$$= \int dl^i\, p(x_{o1}^i | l^i, \mu_1, \mu_2, \sigma_1, \sigma_2, a) p(l_o^i | li) p(l^i | \mu_1, \mu$$

$$= \int_{-\infty}^{-10.82} dl^i\, [a\mathcal{N}(\mu_1, \sigma_1{}^2) + (1-a)\mathcal{N}(\mu_2, \sigma_2{}^2)]l$$

$$+ \int_{-10.82}^{\infty} dl^i\, \mathcal{N}(\mu_1, \sigma_1{}^2) p(l_o^i | l^i) p(l^i | \mu_1, \mu_2, \sigma_1, \sigma_2,$$

Note that you use rates without LsSFR measurements, so you are doing something inconsistent here. There should be a δz term here. However, since the rates are the same at $z = 0.05$, maybe this doesn't matter so much.

3.2.2. Without LsSFR measurements

When lacking direct LsSFR measurements (i.e. p_y^i), we can extend the analysis to non-SNfactory samples by using the redshift-evolution of the fraction $\delta(z)$ of young SNe Ia (Eq. 1) as a proxy for the probability of a SN to be young. This still corresponds to minimizing Eq. 4 with respect to the parameters
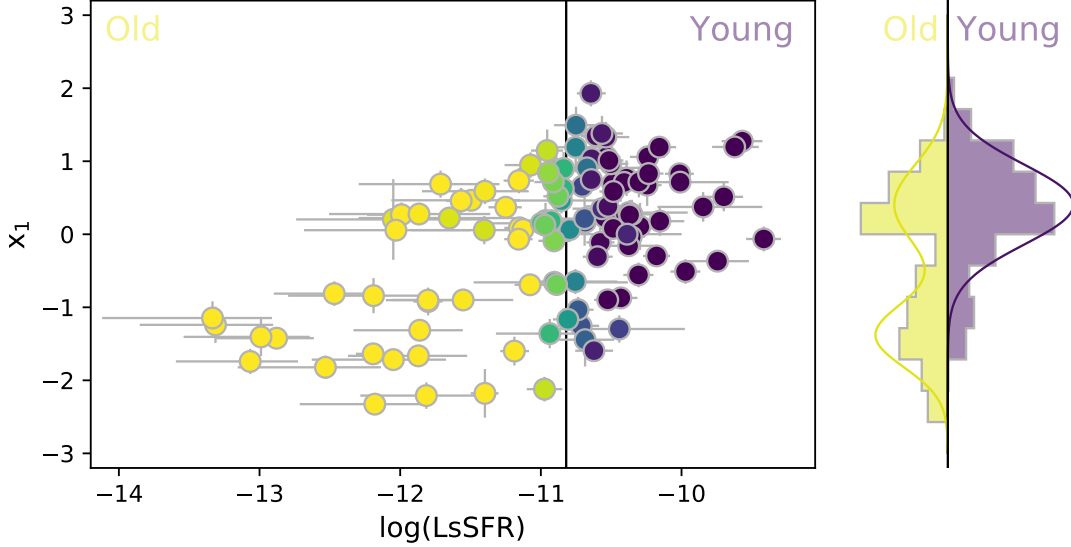
Fig. 4. *Main*: `SALT2.4` lightcurve stretch (x_1) as a function of the local specific star formation rate (LsSFR) for SNfactory SNe used in this analysis. The color corresponds to the probability, p_y, for the SNe Ia to be young, i.e. to have $\log \mathrm{LsSFR} \geq -10.82$ (see Rigault et al. 2018). *Right*: p_y-weighted histogram of the SN stretches, as well as the adjusted Base model; the younger and older population contributions are shown in purple and yellow, respectively.

Table 2. Best fit values of the parameters for the Base stretch distribution model when applied to the SNfactory dataset only (114 SNe Ia), the fiducial 569 SN Ia sample or the conservative one (422).

Sample	μ_1	σ_1	μ_2	σ_2	a
SNfactory	0.41 ± 0.08	0.55 ± 0.06	-1.38 ± 0.10	0.44 ± 0.08	$0.48^{+0.08}_{-0.08}$
Fiducial	0.37 ± 0.05	0.61 ± 0.04	-1.22 ± 0.16	0.56 ± 0.10	$0.51^{+0.09}_{-0.10}$
Conservative	0.38 ± 0.05	0.60 ± 0.04	-1.26 ± 0.13	0.53 ± 0.08	$0.47^{+0.09}_{-0.08}$

$\boldsymbol{\theta} \equiv (\mu_1, \mu_2, \sigma_1, \sigma_2, a)$ of the stretch distribution X_1 (Eq. 2), but this time assuming $y^i = \delta(z^i)$ for any given SN i.

For the rest of the analysis, we will therefore minimize Eq. 4 using p_y^i – the probability for the SN i to be young – when available (i.e. for SNfactory dataset), and $\delta(z^i)$ – the expected fraction of young SNe Ia at the SN redshift z^i – otherwise.

Results of fitting this model to all the 569 (resp. 422) SNe from the fiducial (resp. conservative) sample are given Table 2, and the predicted redshift evolution of mean stretch (mean of Eq. 2 AK: This is not the mean of Eq. 4. It is the expected value of x_1 given the distribution of Eq. 4. Are the uncertainties in the parameters (and their covariances) taken into account in this expected value and the shaded blue band? If not, they be.) illustrated as a blue band in Fig. 5. We see in this figure that the measured mean SN Ia stretch per redshift bins of equal sample size closely follows our redshift drift modeling. This is indeed what is expected if old environments favor low SN stretches (e.g. Howell et al. 2007) *and* if the fraction of old SNe Ia declines as a function of redshift. See Section 4 for a more quantitative discussion.

3.3. Alternative models

In Section 3.1, we have modeled the underlying stretch distribution following Rigault et al. (2018), i.e. as a single Gaussian for the "young" SNe Ia and a mixture of two Gaussians for the "old" SNe Ia population, one being the same as for the young population, plus another one for the fast-declining SNe Ia that seem to only exist in old local environments. This is our so-called "Base" model. However, to test different modeling choices, we have implemented a suite of alternative parametrizations that we also adjust to the data following the procedure described in Section 3.2.2.

Howell et al. (2007) used a simpler unimodal model per age category, assuming a single normal distribution for each of the young and old populations. We thus consider a "Howell+drift" model, with one single Gaussian per age group and the $\delta(z)$ drift from Eq. 1.

Alternatively, since we aim at probing the existence of an evolution with redshift, we also test constant models by restricting the "Base" and "Howell" models to use a supposedly redshift-independent fraction $\delta(z) \equiv f$ of young SNe; these models are hereafter labeled "Base+constant" and "Howell+constant".

We also consider another intrinsically non-drifting model, the one developed for Beams with Bias Correction (BBC, Scolnic & Kessler 2016; Kessler & Scolnic 2017), used in recent SN cosmological analyses (e.g. Scolnic et al. 2018; Abbott et al. 2019; Riess et al. 2016, 2019) to account for Malmquist biases. The BBC formalism assumes sample-based (hence intrinsically non-drifting) asymmetric Gaussian stretch distributions: $\mathcal{N}\left(\mu, \sigma_-^2 \text{ if } x_1 < \mu, \text{ else } \sigma_+^2\right)$. The idea behind this sample-based approach is twofold: (1) Malmquist biases are driven by survey properties and (2) since current surveys cover limited redshift ranges, having a sample-based approach covers some potential redshift evolution information (Scolnic & Kessler 2016; Scolnic et al. 2018). See further discussion concerning BBC in Section 5.
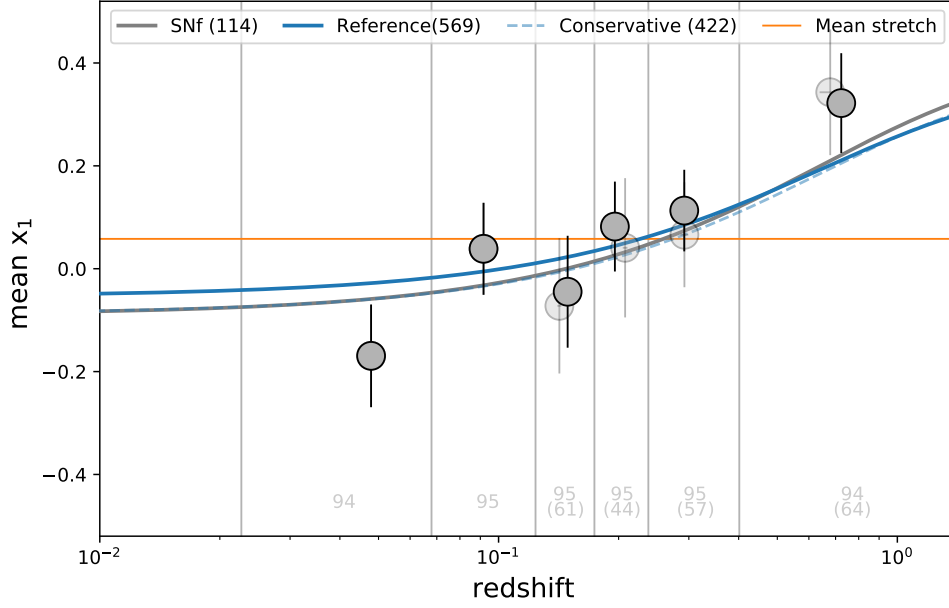
Fig. 5. Evolution of the mean SN `SALT2.4` stretch (x_1) as a function of redshift. Markers show the mean stretch measured in redshift bins of equal sample size, indicated in light gray at the bottom of each redshift bins. Full and light markers are used when considering the fiducial or the conservative samples, respectively. The orange horizontal line represents the mean stretch of the fiducial sample, illustrating the expectation if the SN stretch distribution is not evolving with redshift. Best fits of our Base drifting model are shown as blue, dashed-blue and gray, when fitted on the fiducial sample, the conservative one or the SNfactory dataset only, respectively; all are compatible. The light-blue band illustrates the amplitude of the error of the best fit model when considering the fiducial dataset. AK: Confirming this band includes parameter covariance.

Finally, for the sake of completeness, we also consider simple AK: ? the "Gaussian" and "Asymmetric" Gaussian non-drifting models.

4. Results

We adjusted each of the models described above on both the fiducial and conservative samples (cf. Section 2); results are gathered in Table 3, and illustrated in Fig. 6.

Since the various models have different degrees of freedom, we use the Akaike Information Criterion (AIC, e.g. Burnham & Anderson 2004) to compare their ability to properly describe the observations. This estimator penalizes extra degrees of freedom to avoid over-fitting the data, and is defined as follow:

$$\text{AIC} = -2\ln(L) + 2k \tag{5}$$

where $-2\ln(L)$ is derived by minimizing Eq. (4), and k is the number of free parameters to be adjusted. The reference model is the one with the smallest AIC; in comparison to this model, the models with $\Delta\text{AIC} < 5$ are coined acceptable, the ones with $5 < \Delta\text{AIC} < 20$ are unfavored, and those with $\Delta\text{AIC} > 20$ are deemed excluded. This roughly corresponds to 2, 3 and 5 σ limits for a Gaussian probability distribution.

We first note that the best model (with smallest AIC) is the so-called Base model, implementing the Rigault et al. (2018) description of the stretch distribution and its redshift evolution. This is therefore the reference model, both on the fiducial and conservative samples.

Furthermore, we find that redshift-independent stretch distributions are all excluded as suitable descriptions of the data relative to the Base model. In fact, the best non-drifting model (the Asymmetric one) has a very marginal chance ($p \equiv \exp(\Delta\text{AIC}/2) = 5 \times 10^{-6}$) to describe the data as well as the
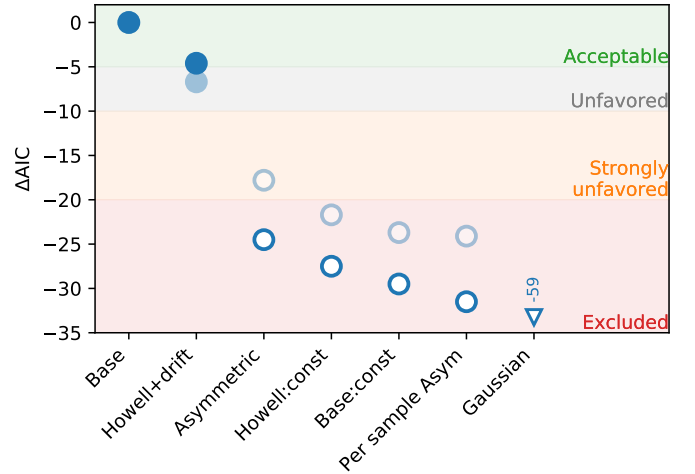

Fig. 6. ΔAIC between "Base" model (reference) and other models (see Table 3). Full and open blue markers correspond to models with and without redshift drift, respectively. Light markers show the results when the analysis is performed on the conservative sample rather than the fiducial one. Color-bands illustrate the validity of the models, from Acceptable (ΔAIC > −5) to Excluded (ΔAIC < −20), see text. According to the AIC, all non-drifting models (open symbols) are excluded to be as good a representation of the data as the Base (drifting) model.

Base model. This result is just a quantitative assessment of qualitative facts clearly visible in Fig. 5: the mean SN stretch per bin of redshift strongly suggests a significant redshift evolution rather than a constant value, and this evolution is well described by Eq. 1.

Surprisingly, the sample-based Gaussian asymmetric modeling used by current implementations of the BBC technique (Scolnic & Kessler 2016; Kessler & Scolnic 2017), is one of

Table 3. Comparison of the relative ability of each model to describe the data. For each considered model, we report if the model is drifting or not, its number of free parameters and, for both the fiducial and the conservative cuts, $-2\ln(L)$ (see Eq. 4), the AIC and the AIC difference (ΔAIC) between this model and the Base model used as reference for it has the lowest AIC.

Name	drift	k	Fiducial sample (569 SNe)			Conservative sample (422 SNe)		
			$-2\ln(L)$	AIC	ΔAIC	$-2\ln(L)$	AIC	Δ AIC
Base	$\delta(z)$	5	1456.7	1466.7	–	1079.5	1089.5	–
Howell+drift	$\delta(z)$	4	1463.3	1471.3	−4.6	1088.2	1096.2	−6.7
Asymmetric	–	3	1485.2	1491.2	−24.5	1101.3	1107.3	−17.8
Howell+const	f	5	1484.2	1494.2	−27.5	1101.2	1111.2	−21.7
Base+const	f	6	1484.2	1496.2	−29.5	1101.2	1113.2	−23.7
Per Sample Asym.	per sample	3×5	1468.2	1498.2	−31.5	1083.6	1113.6	−24.1
Gaussian	–	2	1521.8	1525.8	−59.1	1142.6	1146.6	−57.1

Table 4. Best-fit parameters for our sample-based asymmetric modeling of the underlying stretch distribution.

Asymmetric	σ_-	σ_+	μ_0
SNfactory	1.34 ± 0.13	0.41 ± 0.10	0.68 ± 0.15
SDSS	1.31 ± 0.11	0.42 ± 0.09	0.72 ± 0.13
PS1	1.01 ± 0.11	0.52 ± 0.12	0.38 ± 0.16
SNLS	1.41 ± 0.13	0.15 ± 0.13	1.22 ± 0.15
HST	0.76 ± 0.36	0.79 ± 0.35	0.11 ± 0.44

the worst AK: the word "worst" is inappropriate for the most highly AIC excluded model modelisations of our analysis (see Section 4). While its $-2\ln(L)$ is the smallest of all redshift-independent models (but still −11.5 worse than the reference Base model), it is strongly penalized for requiring 15 free parameters ($\mu_0, \sigma_\pm$ for each of the 5 samples of the analysis). AK: I think it does worse not because of the more parameters but that it has 5 complexity penalties whereas the base model only has 1. AIC doesn't account for the fact that each sample is fit independently. It seems a fairer metric would be the BIC, where you adjust n for each sample. This would recover ~ 16 in the IC for this model. Hence, its ΔAIC < -20, which could be interpreted as a probability $p = 2 \times 10^{-7}$ of being an as good representation of the data as the Base model. We report in Table 4 the samples' μ_0 and $\sigma_\pm$ adjusted on the nominally selection-free samples using our fiducial cuts (see Section 2). We find our results in close agreement with Scolnic & Kessler (2016) for SNLS and SDSS and with Scolnic et al. (2018) for PS1, who derived these model parameters using the full BBC formalism, using numerous simulations to model the selection effects (see details e.g., Section 3 of Kessler & Scolnic 2017). The agreement between our fit of the asymmetric Gaussians on the supposedly selection-free part of the samples and the results derived using the BBC formalism supports our approach to get a sample with negligible selection effects. If we were to use Scolnic & Kessler (2016) and Scolnic et al. (2018) best fit values of the $\mu_0, \sigma_\pm$ asymmetric parameters for SNLS, SDSS and PS1, respectively, the ΔAIC between our Base drifting model and the BBC modeling would go even deeper from −32 to −47. We further discuss the consequence of this result for cosmology in Section 5.

We also performed tests allowing the high-stretch mode of the old population to differ from the young population mode, hence adding two degrees of freedom. The corresponding fit is not significantly better, with a ΔAIC of −0.4. this strengthens the fact that the young and old populations indeed appear to share the same underlying high-stretch mode. Furthermore, one might wonder whether a low-stretch mode might also exist in the young-population, see Fig. 4. We tested that by allowing this population to also be bi-modal, finding the amplitude of this low-

stretch mode to be compatible with 0 in this young population ($< 2\%$). More generally, this raises the question of inaccurate tracing of age by a given environmental tracer (here the LsSFR). A dedicated analysis will be presented in Briday et al. in prep.

Finally, ignoring the LsSFR measurements – available only for the SNfactory dataset, see Section 3 – reduces the significance of the results presented in this section, as expected. Yet, non-drifting models remain strongly disfavored, and for instance, the best fitted sample-based Gaussian asymmetric modeling still is ΔAIC < -10 less representative of the data than our Base drifting modeling.

5. Discussion

To the best of our knowledge, a SN Ia stretch redshift drift modeling has never been explicitly used in cosmological analyses, though bayesian hierarchy formalism such as UNITY (Rubin et al. 2015), BAHAMAS (Shariff et al. 2016) or Steve (Hinton et al. 2019) can easily allow it; see e.g., section 1.3 and 2.5 of Rubin et al. (2015). Not doing so is a second order issue for SN cosmology, as it only affects the way one accounts for Malmquist bias. Indeed, as long as Phillips' relation (Phillips 1993) standardization parameter α is not redshift dependent (a study behind the scope of this paper, but see e.g. Scolnic et al. 2018), the stretch-corrected SNe Ia magnitudes used for cosmology are blind to the underlying stretch distribution for complete samples. However, surveys usually do have significant Malmquist bias at least for the upper half of their SN redshift distribution. As a consequence, an ill-modeling of the underlying stretch distribution will bias the SN magnitude derived from such surveys.

Commonly used Malmquist bias correction techniques, such as the BBC-formalism, assume per Sample Asymmetric Gaussian functions for modeling the underlying stretch and color distributions. Yet, as shown in Section 4, such a sample-based distribution is excluded as being as good as our drifting model. Then, unlike what Scolnic & Kessler (2016, Section 2) and Scolnic et al. (2018, Section 5.4) suggested, i.e. that traditional surveys span limited redshift ranges and that therefore the per-sample approach accounts for implicit redshift drifts, a direct modeling of the redshift drift is more appropriate than a sample-based approach. We stress here that, as measurements of modern surveys try to cover increasingly larger redshift ranges in order to reduce calibration systematic uncertainties, this sample-based approach is becoming less valid, notably for PS1, DES and, soon, LSST.

We illustrate in Fig. 7 the prediction difference in the underlying stretch distribution between the per-sample asymmetric modeling and our Base drifting model for the PS1 sample. Our model is bimodal and the relative amplitude of each mode
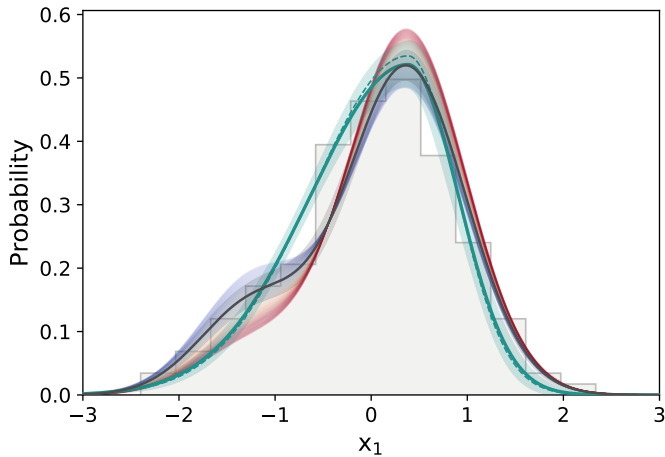
Fig. 7. Distribution of the PS1 SN Ia `SALT2.4` stretch (x_1) after the fiducial redshift limit cut (grey histogram). This distribution is supposed to be a random draw from the underlying stretch distribution. The green line shows the BBC model of this underlying distribution (asymmetric Gaussian). The full line (band) is our best fit (its error); the dashed line shows the Scolnic et al. (2018) result. The black line (band) show our best fitted Base-modeling (its error, see Table 2) that includes redshift drift. For illustration, we show as colored (from blue to red with increasing redshifts) the evolution of the underlying stretch distribution as a function of redshift for the redshift range covered by PS1 data.

depends on the redshift-dependent fraction of old and young SNe Ia in the sample: the higher the fraction of old SNe Ia (at lower redshift), the higher the amplitude of the old-specific low-stretch mode. This redshift dependency is shown as blue to red underlying distributions in Fig. 7 for redshift ranges covered by PS1. The observed x_1 histogram follows our modeling defined as the sum of individual underlying SN-redshift distributions. As expected, the two modeling approaches differ mostly in the negative part of the SN stretch distribution. The asymmetric Gaussian distribution goes through the middle of the bimodal distribution, over-estimating the number of SNe Ia at $x_1 \sim -0.7$ and under-estimating it at $x_1 \sim -1.7$ in comparison to our Base drifting model for typical PS1 SN redshifts. This means that the SN bias-corrected standardized magnitude estimated at redshifts affected by selection effect would be biased by an ill-modeling of the true underlying stretch distribution.

The amplitude of this magnitude bias for cosmology is beyond the scope of this paper given the complexity of the BBC analysis. It would require a full study using our Base model (Eq. 2) in place of the sample-based asymmetric modeling as part of the BBC simulations.

Nonetheless, it is enlightening to study the expected mean stretch difference as a function of redshift when considering that the underlying SN stretch distribution follows either the per-sample Asymmetric Gaussian modeling or our Base drifting model. This is shown Fig. 8, where, for clarity we convert this difference in mean stretch x_1 to a difference in magnitude using $\alpha = 0.156$ (Scolnic et al. 2018). We stress here that this does not correspond to the actual impact on cosmological distances derived from SNe Ia as this simple stretch average comparison does not account for the amplitude of the Malmquist bias correction to be applied as a function of redshift and surveys. AK: The AG modeling contains Malmquist bias. To be fair you should plot Base with the survey-specific Malmquist bias. There is something strange going on at $z < 0.2$, can you add SNfactory to this
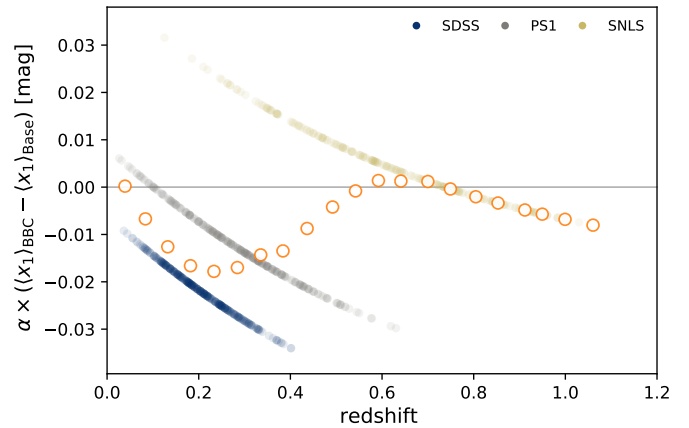
Fig. 8. Difference of mean stretch standardization (in mag, using $\alpha = 0.156$ from Scolnic et al. 2018) between the per-sample Gaussian asymmetric modeling and our Base drifting model, as a function of redshift, considering the SDSS, PS1 and SNLS datasets. The colored markers show the difference for SDSS, PS1 and SNLS samples (see legend) considering the individual SN redshifts (no redshift cut applied). The orange markers represent their mean evolution in 30 regularly-spaced redshift bins. This must be multiplied by the amplitude of the Malmquist bias correction to derive the SN distance modulus bias affecting cosmology, see text. *Apparently, there is some issue with the mean of all surveys combined. See Greg's comments.*

plot? I think you fill find that the systematic difference is driven by differences in the x_1 distribution of SNf.

We see that the amplitude of the modeling difference is a few tens of millimag. In the era of modern cosmology, where we aim at probing w_0 at a sub-percent level and w_a at the ten-percent precision (e.g., Ivezić et al. 2019), we stress that correctly modeling of potential SN redshift drift should be further studied and care should be made when using samples affected by selection effects.

6. Conclusion

We have presented a study of the drift of the underlying SNe Ia stretch distribution as a function of redshift. We built a magnitude-limited SN Ia sample from the Pantheon dataset (Scolnic et al. 2018, SDSS, PS1 and SNLS), to which we added HST and SNfactory data from Rigault et al. (2018) for the high-and low-redshift bins. We only considered the SNe that have been discovered in the redshift range of each survey where selection effects are negligible, so that the observed SNe Ia stretches are random sampling of the true underlying distribution. This resulted in a 569 SN Ia fiducial sample (422 SNe when more conservative cuts were considered).

Following predictions made in Rigault et al. (2018), we introduced a redshift drift model which depends on the expected fraction of "young" and "old" SNe Ia as a function of redshift, each age population having its own underlying stretch distribution.

In addition to this "base" modeling, we have studied various distributions, including redshift independent models; we also studied the prediction from the "Beams with Bias Correction", a Malmquist bias correction algorithm used by several recent SN Ia cosmological analyses, implemented as a per-sample asymmetric Gaussian distribution.

Our conclusions are the following:

1. The underlying SN Ia stretch distribution is redshift dependent, as previously suggested by e.g. Howell et al. (2007). This result is largely independent of details on each age-population modeling.

2. Non-drifting models are indeed excluded as suitable descriptions of the data relative to our Base model. This model assumes that: (1) the younger population has a unimodal Gaussian stretch distribution, while the older population stretch distribution is bimodal, one mode being the same as the young one; (2) the evolution of the relative fraction of younger and older SNe Ia follows the prediction made in Rigault et al. (2018).

3. Using survey-based asymmetric Gaussian distributions, as done, e.g., in the current implementation of the BBC, is excluded to be good descriptions of the data relative to our drifting model. Hence, the sample-based approach does not accurately account for redshift drift and even less so as survey span larger redshift ranges. We warn that, even if extra degrees of freedom might be acceptable given the large number of SNe Ia in cosmological studies, extrapolating the SN property distributions from the volume-limited part of a survey to the magnitude-limited one affected by sample selection would still be inaccurate since the distribution would drift.

4. An inaccurate underlying stretch distribution model is estimated to bias the mean standardized magnitude of SNe Ia affected by selection effects at the order of a few percents. This is comparable in scale to the observational signature of exotic forms of dark energy and is therefore a significant source of systematic that should be carefully studied.

5. Given the current dataset, we suggest the use of the following stretch population model as a function of redshift:

$$X_1(z) = \delta(z) \times \mathcal{N}(\mu_1, \sigma_1^2) +$$
$$(1 - \delta(z)) \times \left[a \times \mathcal{N}(\mu_1, \sigma_1^2) + (1 - a) \times \mathcal{N}(\mu_2, \sigma_2^2) \right] \quad (6)$$

with: $a = 0.51, \mu_1 = 0.37, \mu_2 = -1.22, \sigma_1 = 0.61, \sigma_2 = 0.56$ (see Table 2) and using the age-population drift modelization $\delta(z)$ with $K = 0.87$:

$$\delta(z) = \left(K^{-1} \times (1 + z)^{-2.8} + 1 \right)^{-1}. \quad (7)$$

In this paper, we considered a simple Gaussian mixture modeling, but additional data free from significant Malmquist bias would enable us to refine it. We note that samples at the low- and high-redshift ends of the Hubble diagram would be particularly helpful for this drifting analysis; fortunately this will soon be provided by the Zwicky Transient Facility (Bellm et al. 2019; Graham et al. 2019) and Subaru and SeeChange SNe Ia programs, respectively.

References

Abbott, T. M. C., Allam, S., Andersen, P., et al. 2019, ApJ, 872, L30
Aldering, G., Adam, G., Antilogus, P., et al. 2002, Proc. SPIE, 61
Astier, P., Guy, J., Regnault, N., et al. 2006, A&A, 447, 31
Aubourg, É., Tojeiro, R., Jimenez, R., et al. 2008, A&A, 492, 631
Bazin, G., Ruhlmann-Kleider, V., Palanque-Delabrouille, N., et al. 2011, A&A, 534, A43
Bellm, E. C., Kulkarni, S. R., Graham, M. J., et al. 2019, PASP, 131, 018002
Betoule, M., Kessler, R., Guy, J., et al. 2014, A&A, 568, A22
Brout, D., Scolnic, D., Kessler, R., et al. 2019, ApJ, 874, 150
Burnham, K., Anderson, D., 2004, Sociological Methods & Research, 33, 2
Campbell, H., D'Andrea, C. B., Nichol, R. C., et al. 2013, ApJ, 763, 88
Childress, M., Aldering, G., Antilogus, P., et al. 2013, ApJ, 770, 108
Childress, M. J., Wolf, C., & Zahid, H. J. 2014, MNRAS, 445, 1898
D'Andrea, C. B., Gupta, R. R., Sako, M., et al. 2011, ApJ, 743, 172
Dilday, B., Kessler, R., Frieman, J. A., et al. 2008, ApJ, 682, 262
Feeney, S. M., Peiris, H. V., Williamson, A. R., et al. 2019, Phys. Rev. Lett., 122, 061105
Freedman, W. L., Madore, B. F., Hatt, D., et al. 2019, ApJ, 882, 34
Freedman, W. L., Madore, B. F., Hoyt, T., et al. 2020, arXiv e-prints, arXiv:2002.01550
Frieman, J. A., Bassett, B., Becker, A., et al. 2008, AJ, 135, 338
Graham, M. J., Kulkarni, S. R., Bellm, E. C., et al. 2019, PASP, 131, 078001
Gupta, R. R., D'Andrea, C. B., Sako, M., et al. 2011, ApJ, 740, 92
Guy, J., Astier, P., Baumont, S., et al. 2007, A&A, 466, 11
Hamuy, M., Phillips, M. M., Suntzeff, N. B., et al. 1996, AJ, 112, 2391
Hamuy, M., Trager, S. C., Pinto, P. A., et al. 2000, AJ, 120, 1479
Hinton, S. R., Davis, T. M., Kim, A. G., et al. 2019, ApJ, 876, 15
Howell, D. A., Sullivan, M., Conley, A., et al. 2007, ApJ, 667, L37
Ivezić, Ž., Kahn, S. M., Tyson, J. A., et al. 2019, ApJ, 873, 111
Jones, D. O., Riess, A. G., & Scolnic, D. M. 2015, ApJ, 812, 3 1
Jones, D. O., Riess, A. G., Scolnic, D. M., et al. 2018, ApJ, 867, 108
Jones, D. O., Scolnic, D. M., Riess, A. G., et al. 2018, ApJ, 857, 51
Jones, D. O., Scolnic, D. M., Foley, R. J., et al. 2019, ApJ, 881, 19
Kelly, P. L., Hicken, M., Burke, D. L., et al. 2010, ApJ, 715, 743
Kessler, R., Becker, A. C., Cinabro, D., et al. 2009, ApJS, 185, 32
Kessler, R., & Scolnic, D. 2017, ApJ, 836, 56
Kim, Y.-L., Smith, M., Sullivan, M., et al. 2018, ApJ, 854, 24
Kim, Y.-L., Kang, Y., & Lee, Y.-W. 2019, Journal of Korean Astronomical Society, 52, 181
Knox, L., & Millea, M. 2019, arXiv e-prints, arXiv:1908.03663
Lampeitl, H., Smith, M., Nichol, R. C., et al. 2010, ApJ, 722, 566
Mannucci, F., Della Valle, M., Panagia, N., et al. 2005, A&A, 433, 807
Mannucci, F., Della Valle, M., & Panagia, N. 2006, MNRAS, 370, 773
Maoz, D., Mannucci, F., & Nelemans, G. 2014, ARA&A, 52, 107
Neill, J. D., Sullivan, M., Balam, D., et al. 2006, AJ, 132, 1126
Neill, J. D., Sullivan, M., Howell, D. A., et al. 2009, ApJ, 707, 1449
Nordin, J., Aldering, G., Antilogus, P., et al. 2018, A&A, 614, A71
Pan, Y.-C., Sullivan, M., Maguire, K., et al. 2014, MNRAS, 438, 1391
Perlmutter, S., Aldering, G., Goldhaber, G., et al. 1999, ApJ, 517, 565
Perrett, K., Balam, D., Sullivan, M., et al. 2010, AJ, 140, 518
Phillips, M. M. 1993, ApJ, 413, L105
Planck Collaboration, Aghanim, N., Akrami, Y., et al. 2018, arXiv e-prints, arXiv:1807.06209
Poulin, V., Smith, T. L., Karwal, T., et al. 2019, Phys. Rev. Lett., 122, 221301
Reid, M. J., Pesce, D. W., & Riess, A. G. 2019, arXiv e-prints, arXiv:1908.05625
Rest, A., Scolnic, D., Foley, R. J., et al. 2014, ApJ, 795, 44
Riess, A. G., Filippenko, A. V., Challis, P., et al. 1998, AJ, 116, 1009
Riess, A. G., Macri, L., Casertano, S., et al. 2009, ApJ, 699, 539
Riess, A. G., Macri, L. M., Hoffmann, S. L., et al. 2016, ApJ, 826, 56
Riess, A. G., Casertano, S., Yuan, W., et al. 2018, ApJ, 861, 126
Riess, A. G., Casertano, S., Yuan, W., et al. 2019, ApJ, 876, 85
Rigault, M., Copin, Y., Aldering, G., et al. 2013, A&A, 560, A66
Rigault, M., Aldering, G., Kowalski, M., et al. 2015, ApJ, 802, 20
Rigault, M., Brinnel, V., Aldering, G., et al. 2018, arXiv:1806.03849
Rodney, S. A., Riess, A. G., Strolger, L.-G., et al. 2014, AJ, 148, 13
Roman, M., Hardin, D., Betoule, M., et al. 2018, A&A, 615, A68
Rose, B. M., Garnavich, P. M., & Berg, M. A. 2019, ApJ, 874, 32
Rubin, D., Aldering, G., Barbary, K., et al. 2015, ApJ, 813, 137
Rubin, D., & Hayden, B. 2016, ApJ, 833, L30
Sako, M., Bassett, B., Becker, A., et al. 2008, AJ, 135, 348
Saunders, C., Aldering, G., Antilogus, P., et al. 2020, VizieR Online Data Catalog, J/ApJ/869/167
Scannapieco, E., & Bildsten, L. 2005, ApJ, 629, L85
Scolnic, D., Rest, A., Riess, A., et al. 2014, ApJ, 795, 45
Scolnic, D., & Kessler, R. 2016, ApJ, 822, L35
Scolnic, D. M., Jones, D. O., Rest, A., et al. 2018a, ApJ, 859, 101
Scolnic, D., Perlmutter, S., Aldering, G., et al. 2019, Astro2020: Decadal Survey on Astronomy and Astrophysics, 2020, 270
Shariff, H., Jiao, X., Trotta, R., et al. 2016, ApJ, 827, 1
Strolger, L.-G., Riess, A. G., Dahlen, T., et al. 2004, ApJ, 613, 200
Sullivan, M., Le Borgne, D., Pritchet, C. J., et al. 2006, ApJ, 648, 868
Sullivan, M., Conley, A., Howell, D. A., et al. 2010, MNRAS, 406, 782
Tasca, L. A. M., Le Fèvre, O., Hathi, N. P., et al. 2015, A&A, 581, A54
Wiseman, P., Smith, M., Childress, M., et al. 2020, arXiv e-prints, arXiv:2001.02640
Wong, K. C., Suyu, S. H., Chen, G. C.-F., et al. 2019, arXiv e-prints, arXiv:1907.04869