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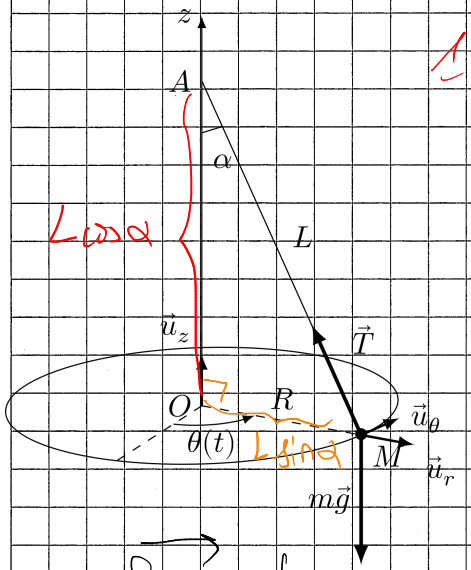

1) ☐ Système : {masse} M(m)

☐ Référentiel galiléen

☐ Repère : cylindrique $(O, \vec{u_x}, \vec{u_\theta}, \vec{u_z})$

☐ Repère Frenet : $\overrightarrow{OM} = R\vec{u_r} = L\sin\alpha\,\vec{u_r}$

$\vec{v} = L\sin\alpha\,\dot{\theta}\,\vec{u_\theta} = L\omega\sin\alpha\,\vec{u_\theta}$

$\vec{a} = -L\omega^2\sin\alpha\,\vec{u_r}$

☐ $\vec{\mathcal{L}}_A(M) = \overrightarrow{AM} \wedge \vec{p}(M) = (\overrightarrow{AO} + \overrightarrow{OM}) \wedge m\vec{v}$

$\Leftrightarrow \vec{\mathcal{L}}_A(M) = (-L\cos\alpha\,\vec{u_z} \wedge L\sin\alpha\,\vec{u_r}) \wedge (mL\omega\sin\alpha\,\vec{u_\theta})$

$\vec{u_z} \wedge \vec{u_r} = \vec{u_\theta}$; $\vec{u_r} \wedge \vec{u_\theta} = \vec{u_z}$

$\Leftrightarrow \vec{\mathcal{L}}_A(M) = -mL^2\omega\sin\alpha\cos\alpha\,(-\vec{u_\theta}) + mL^2\omega\sin^2\alpha\,\vec{u_z}$

$\Leftrightarrow \boxed{\vec{\mathcal{L}}_A(M) = mL^2\omega\left(\sin^2\alpha\,\vec{u_z} + \sin\alpha\cos\alpha\,\vec{u_r}\right)}$

2) BdF : $\vec{P} = m\vec{g} = -mg\,\vec{u_z}$

$\vec{T} = -T\,\vec{u_r}$

BdM : $\mathcal{M}_A(\vec{P}) = \overrightarrow{AM} \wedge \vec{P} = L(\sin\alpha\,\vec{u_r} - \cos\alpha\,\vec{u_z}) \wedge (-mg\,\vec{u_z})$

$\mathcal{M}_A(\vec{T}) = \overrightarrow{AM} \wedge \vec{T} = \vec{0}$ ($\overrightarrow{AM} \parallel \vec{T}$)

Donc $\mathcal{M}_A(\vec{P}) = +mgL\sin\alpha\,\vec{u_\theta}$

TMC : $\dfrac{d\vec{\mathcal{L}}_A(M)}{dt} = \sum \mathcal{M}_A(\vec{F}) = \mathcal{M}_A(\vec{P})$

$\Leftrightarrow mL^2\omega\left(\sin^2\alpha\,\dfrac{d\vec{u_z}}{dt} + \sin\alpha\cos\alpha\,\omega\,\vec{u_\theta}\right) = mgL\sin\alpha\,\vec{u_\theta}$

($\dfrac{d\vec{u_z}}{dt} = \vec{0}$; $\dfrac{d\vec{u_r}}{dt} = \omega\,\vec{u_\theta}$)

Par rapport à $\vec{u_\theta}$:

$$\Leftrightarrow m L^2 \omega^2 \sin\alpha\cos\alpha = mgL\sin\alpha$$

$$\Leftrightarrow \boxed{\cos\alpha = \frac{g}{L\omega^2}}$$

3) TFD : $\frac{d\vec{p}}{dt} = \sum \vec{F}_{ext}$

$$\Leftrightarrow -mL\omega^2 \sin\alpha\, \vec{u_r} = -mg\vec{u_z} + T(\cos\alpha\, \vec{u_z} - \sin\alpha\, \vec{u_r})$$

$$\Leftrightarrow \begin{cases} +mL\omega^2\sin\alpha = +T\sin\alpha \\ 0 = -mg + T\cos\alpha \end{cases} \Leftrightarrow \begin{cases} T = mL\omega^2 \\ T = \frac{mg}{\cos\alpha} \end{cases} \Leftrightarrow \boxed{\cos\alpha = \frac{g}{L\omega^2}}$$