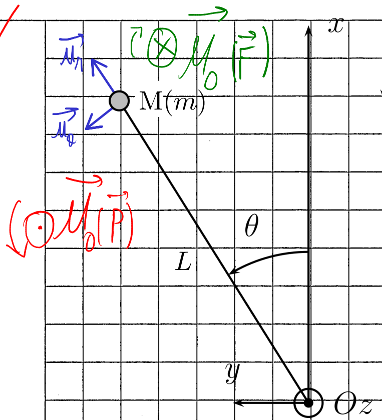
I/



1) ◇ Système = {masse} repérée par M(m)

◇ Repère : cylindrique $(O, \vec{u_r}, \vec{u_\theta}, \vec{u_z})$

◇ Repérage : $\vec{OM} = L\vec{u_r}$

$\vec v = L\dot\theta \vec{u_\theta}$

$\vec a = L\ddot\theta \vec{u_\theta} - L\dot\theta^2 \vec{u_r}$

◇ BDF : $\begin{cases} \vec P = -mg\vec{u_x} = -mg(\cos\theta + \sin\theta \vec{u_\theta}) \\ \vec F \text{ inconnue} \end{cases}$

◇ BdM : $\begin{cases} \vec{\mathcal{M}_0}(\vec P) = \vec{OM} \wedge \vec P = +mgL\vec{u_r} \wedge (\cos\theta \vec{u_r} - \sin\theta \vec{u_\theta}) = +mgL\sin\theta \vec{u_z} \\ \vec{\mathcal{M}_0}(\vec F) = -C\theta \vec{u_z} \end{cases}$

$= \vec 0$, $\vec{u_z}$

et $\vec{\mathcal{L}_0}(M) = \vec{OM} \wedge m\vec v = mL\vec{u_r} \wedge L\dot\theta \vec{u_\theta} = mL^2\dot\theta \vec{u_z}$

$= \vec{u_z}$

D'où sur l'axe $(O, \vec{u_z})$:

$\boxed{\mathcal{M}_z(\vec P) = +mgL\sin\theta}$ $\boxed{\mathcal{M}_z(\vec F) = -C\theta}$ $\boxed{\mathcal{L}_z(M) = mL^2\dot\theta}$

Ainsi avec le TMC, $\dfrac{d\mathcal{L}_z(M)}{dt} = \mathcal{M}_z(\vec P) + \mathcal{M}_z(\vec F)$

$\Leftrightarrow mL^2\ddot\theta = mgL\sin\theta - C\theta$

$\Leftrightarrow \boxed{\ddot\theta + \dfrac{C}{mL^2}\theta - \dfrac{g}{L}\sin\theta = 0}$

2) Autour de $\theta_{eq} = 0$, $\sin\theta \sim \theta$

$\ddot\theta - \left(\dfrac{g}{L} - \dfrac{C}{mL^2}\right)\theta = 0$

D'où $\ddot\theta + \underbrace{\left(\dfrac{C}{mL^2} - \dfrac{g}{L}\right)}_{>0}\theta = 0$ (**)

Stable si

$\Leftrightarrow \boxed{C > mgL}$ car éq. carac : $r^2 + \omega_0^2 = 0 \Rightarrow A\cos + B\sin$

ssi $\omega_0^2 > 0$

3) $\vec F = -\dfrac{C}{L}\theta \vec{u_\theta}$

$\delta W(\vec F) = -dE_{p,el}$ $\quad d\vec{OM} = dr\,\vec{u_r} + r\,d\theta\,\vec{u_\theta}$

$\Leftrightarrow -\frac{C}{L}\theta \vec{u_\theta} \cdot L\, d\theta\, \vec{u_\theta} = -dE_{p,el}$

$\Leftrightarrow \boxed{\frac{dE_{p,el}}{d\theta} = C\theta} \Rightarrow E_{pel} = \frac{C\theta^2}{2} + K$

Or, $E_{pp} = mgl\cos\theta$ donc

$E_{p,tot} = mgl\cos\theta + \frac{C}{2}\theta^2 + K$

$\frac{dE_{p,tot}}{d\theta} = -mgl\sin\theta + C\theta = 0$

$\frac{d^2E_p}{d\theta^2} = -mgl\cos\theta + C > 0$

Soit pour $\theta_{eq} = 0$, $\left.\frac{d^2E_p}{d\theta^2}\right|_{\theta eq} > 0 \Leftrightarrow C > mgl$! Pareil ✓

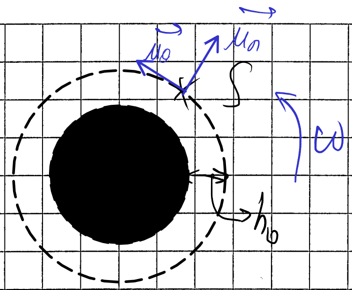
4) $\omega = \sqrt{\frac{C - mgL}{mL^2}} \Leftrightarrow \frac{2\pi}{T} = \ldots \Leftrightarrow \boxed{T = 2\pi\sqrt{\frac{mL^2}{C - mgL}}}$

5) $g_0 = \frac{C}{mL} \Rightarrow T = 2\pi\sqrt{\frac{\cancel{mL} \times L}{\cancel{mL}(g_0 - g)}}$

$\Leftrightarrow T = 2\pi\boxed{\sqrt{\frac{L}{g_0 - g}}}$: $g_0 \simeq g \Rightarrow$ vario° de g se traduisent par fortes vario° de T, mesurables

Remarque : $g_0 < g \Rightarrow$ ~~CH~~, plus d'oscillo°.

I/1) ◇ Système : {satellite}



◇ Référentiel : géocentrique

◇ Repère : polaire, $(T, \vec{u_r}, \vec{u_\theta})$

◇ Repérage :

▷ $\vec{OM} = (R_T + h_0)\vec{u_r} = R\vec{u_r}$

▷ $\vec{v} = \underbrace{\cancel{\dot{R}\vec{u_r}}}_{=0} + R\dot{\theta}\vec{u_\theta} \Rightarrow v = |R\dot{\theta}|$

▷ $\vec{a} = R\ddot{\theta}\vec{u_\theta} - R\dot{\theta}^2\vec{u_r}$

◇ BDF : $\vec{F_g} = -\mathcal{G}\frac{m_T m}{R^2}\vec{u_r}$ $\left(\Rightarrow E_p = -\mathcal{G}\frac{m_T m}{R} = -\mathcal{G}\frac{m_T m}{R_T+h_0}\right)$

◇ PFD : $m\vec{a} = \vec{F_g}$

$$\Leftrightarrow \begin{cases} \cancel{-}\cancel{m}R\dot{\theta}^2 = \cancel{-}\mathcal{G}\frac{m_T \cancel{m}}{R^3} \\ R\ddot{\theta} = 0 \end{cases} \Leftrightarrow \begin{cases} \dot{\theta} = \pm\sqrt{\mathcal{G}\frac{m_T}{R^3}} \\ \dot{\theta} = cte \end{cases}$$

Ainsi, $v = \sqrt{\frac{\mathcal{G}m_T}{R}} \Leftrightarrow \boxed{v = \sqrt{\frac{\mathcal{G}m_T}{R_T+h_0}}}$

Or, $E_m = E_c + E_p \Leftrightarrow E_m = \frac{1}{2}mv^2 - \mathcal{G}\frac{m_T m}{R_T+h_0}$

$$\Leftrightarrow E_m = \frac{\mathcal{G}m m_T}{R_T+h_0}\left(\frac{1}{2} - 1\right)$$

$$\Leftrightarrow \boxed{E_m = -\frac{1}{2}\mathcal{G}\frac{m_T m}{R_T+h_0}}$$

2)a- $\vec{F_f}$ est tjs résistant, car $\vec{F} = -\underbrace{\alpha}_{>0}\vec{v}$ donc $P(\vec{F_f}) = -\alpha v^2 < 0$

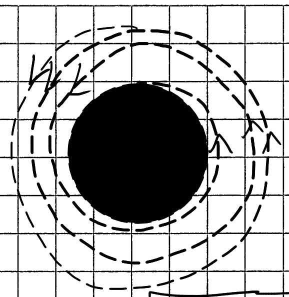
Ainsi, par le TPM,

$$\frac{dE_m}{dt} = \sum P(\vec{F_{NC}}) = P(\vec{F_f}) < 0$$

et l'énergie mécanique décroît.

b- Il diminue forcé^t, puisque $E_m = -\frac{k}{R_T+h(t)}$ ↓ quand h ↓

(attention, $\underline{E_m < 0}$, donc $|E_m|$ ↗ quand h ↓ mais E_m ↓)

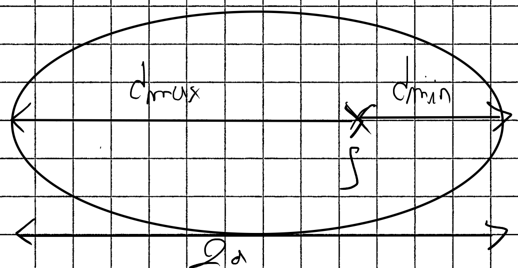


C- $v = \sqrt{\frac{G m m_T}{R_T + h(t)}}$, donc $\boxed{v \nearrow \text{ qd } h \downarrow}$

Peu intuitif a priori car frott^t ralentissent, mais si $\|\vec{F}_g\|$ presque négligeable c'est logique (3ème loi Kepler)

III/ $T_H = 76$ ans, $d_{min} = 0,59$ UA

1)



2) $d_{max} = 2a - d_{min}$

Or, $\frac{T_H^2}{a_H^3} = \frac{4\pi^2}{G m_S}$ $\Leftrightarrow$ $a_H = \sqrt[3]{\frac{G m_S T_H^2}{4\pi^2}}$ avec $G = 6,67.10^{-11}$ SI, $m_S = 2,0.10^{30}$ kg, $T_H = 76$ ans $= 76 \times 365,25 \times 24 \times 3600$ s $= 2,40.10^{40}$ s

Ainsi, $a_H = 2,69.10^{12}$ m

$\boxed{a_H = 17,9 \text{ UA}}$

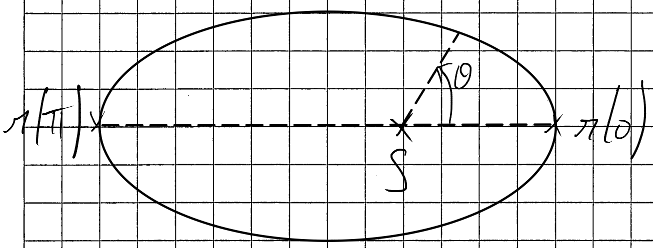
Or $\frac{T_H^2}{a_H^3} = \frac{T_T^2}{a_T^3}$ $\Leftrightarrow$ $a_H = \sqrt[3]{\frac{T_H^2 a_T^3}{T_T^2}}$ $\left\{ T_H = 76 \text{ y}, \ a_T = 1 \text{ UA}, \ T_T = 1 \text{ y} \right.$

$\Leftrightarrow \boxed{\frac{a_H}{1 \text{UA}} = \left(\frac{T_H}{1 \text{y}}\right)^{2/3}} = 17,9$ UA ✓

Ainsi, $d_{max} = 2a - d_{min}$ avec $\left\{ a = 17,9 \text{ UA}, \ d_{min} = 0,59 \text{ UA} \right.$

A.N : $\boxed{d_{max} = 35,3 \text{ UA}}$

3)



$$\begin{cases} d_{min} = r(0) = \dfrac{p}{1-e} \\ d_{max} = r(\pi) = \dfrac{p}{1+e} \end{cases}$$

$$\Leftrightarrow \begin{cases} 1-e = \dfrac{p}{d_{min}} & (1) \\ 1+e = \dfrac{p}{d_{max}} & (2) \end{cases}$$

Ainsi,

$$(1)+(2) \Rightarrow 1 - \cancel{e} + 1 + \cancel{e} = p\left(\frac{1}{d_{min}} + \frac{1}{d_{max}}\right)$$

$$\Leftrightarrow 2 = p\left(\frac{d_{max} + d_{min}}{d_{min}\, d_{max}}\right)$$

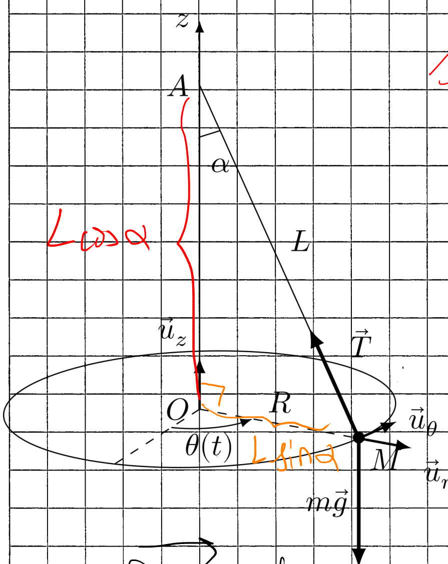
$$\Leftrightarrow \boxed{p = 2\,\frac{d_{min}\, d_{max}}{d_{min} + d_{max}}}$$

$$(2)-(1) \Rightarrow \underbrace{\cancel{1} + e - (\cancel{1} - e)}_{2e} = p\left(\frac{1}{d_{max}} - \frac{1}{d_{min}}\right)$$

$$\Leftrightarrow \cancel{2}e = \cancel{2}\,\frac{d_{min}\,\cancel{d_{max}}}{d_{min} + d_{max}}\left(\frac{d_{min} - d_{max}}{\cancel{d_{min}\, d_{max}}}\right)$$

$$\Leftrightarrow \boxed{e = \frac{d_{min} - d_{max}}{d_{min} + d_{max}}}$$

IV/



1) □ Système : {masse} M(m)

□ Référentiel galiléen

□ Repère : cylindrique $(O, \vec{u_r}, \vec{u_\theta}, \vec{u_z})$

□ Repérage : $\overrightarrow{OM} = R\,\vec{u_r} = L \sin\alpha\, \vec{u_r}$

$\vec{v} = L \sin(\alpha)\,\dot{\theta}\,\vec{u_\theta} = L\omega \sin\alpha\, \vec{u_\theta}$

$\vec{a} = -L\omega^2 \sin\alpha\, \vec{u_r}$

□ $\overrightarrow{\mathcal{L}_A}(M) = \overrightarrow{AM} \wedge \vec{p}(M) = (\overrightarrow{AO} + \overrightarrow{OM}) \wedge m\vec{v}$

$\Leftrightarrow \overrightarrow{\mathcal{L}_A}(M) = (-L\cos\alpha\, \vec{u_z} + L\sin\alpha\, \vec{u_r}) \wedge (m L\omega \sin\alpha\, \vec{u_\theta})$

$\vec{u_z}$

$-\vec{u_r}$

$\Leftrightarrow \vec{L}_A(M) = -mL^2\omega\sin\alpha\cos\alpha\,(-\vec{u_r}) + mL^2\omega\sin^2\alpha\,\vec{u_z}$

$$\Leftrightarrow \boxed{\vec{L}_A(M) = mL^2\omega\left(\sin^2\alpha\,\vec{u_z} + \sin\alpha\cos\alpha\,\vec{u_r}\right)}$$

2) BdF : $\begin{cases} \vec{P} = m\vec{g} = -mg\,\vec{u_z} \\ \vec{T} = -T\,\vec{u_n} \end{cases}$

BdM : $\begin{cases} \vec{M}_A(\vec{P}) = \vec{AM}\wedge\vec{P} = L(\sin\alpha\,\vec{u_r} - \cos\alpha\,\vec{u_z})\wedge(-mg\,\vec{u_z}) \\ \vec{M}_A(\vec{T}) = \underbrace{\vec{AM}\wedge\vec{T}}_{\vec{AM}\,/\!/\,\vec{T}\ =\ \vec{0}} = \vec{0} \end{cases}$

$\vec{u_z}\wedge\vec{u_z} = \vec{0}$; $-\vec{u_\theta}$

D'où $\vec{M}_A(\vec{P}) = +mgL\sin\alpha\,\vec{u_\theta}$

TMC : $\dfrac{d\vec{L}_A(M)}{dt} = \sum \vec{M}_A(\vec{F}) = \vec{M}_A(\vec{P})$

$\Leftrightarrow mL^2\omega\left(\sin^2\alpha\,\cancel{\dfrac{d\vec{u_z}}{dt}} + \sin\alpha\cos\alpha\,\omega\,\vec{u_\theta}\right) = mgL\sin\alpha\,\vec{u_\theta}$

$\dfrac{d\vec{u_z}}{dt} = \vec{0}$; $\dfrac{d\vec{u_r}}{dt}$

Par projec° sur $\vec{u_\theta}$:

$\Leftrightarrow \cancel{m}L^{\cancel{2}}\omega^2\cancel{\sin\alpha}\cos\alpha = \cancel{m}g\cancel{L}\cancel{\sin\alpha}$

$$\Leftrightarrow \boxed{\cos\alpha = \frac{g}{L\omega^2}}$$

3) PFD : $\dfrac{d\vec{p}}{dt} = \sum \vec{F}_{ext}$

$\Leftrightarrow -mL\omega^2\sin\alpha\,\vec{u_r} = -mg\,\vec{u_z} + T(\cos\alpha\,\vec{u_z} - \sin\alpha\,\vec{u_r})$

$\Leftrightarrow \begin{cases} +mL\omega^2\cancel{\sin\alpha} = +T\cancel{\sin\alpha} \\ 0 = -mg + T\cos\alpha \end{cases} \Leftrightarrow \begin{cases} T = mL\omega^2 \\ T = \dfrac{mg}{\cos\alpha} \end{cases} \Leftrightarrow \boxed{\cos\alpha = \dfrac{g}{L\omega^2}}$

IV/1) $v_1 = r_1 \dot\theta_1$

$$\begin{cases} + \cancel{m} \cancel{r_1} \dot\theta_1^2 = + \mathcal{G}\dfrac{\cancel{m} M}{r_1^{\cancel{2}\,3}} \\ + m r_1 \ddot\theta_1 = 0 \end{cases} \Leftrightarrow \begin{cases} \dot\theta_1 = \sqrt{\dfrac{\mathcal{G}M}{r_1^3}} \\ \dot\theta_1 = cte \end{cases}$$

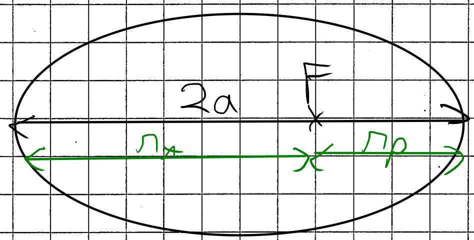
$$v_1 = \sqrt{\frac{\mathcal{G}M}{r_1}} = 7800\ m.s^{-1}$$

2) $$\mathcal{E}_m = \frac{1}{2} m v^2 - \mathcal{G}\frac{Mm}{r_1} = \frac{1}{2}\frac{m\mathcal{G}M}{r_1} - \mathcal{G}\frac{Mm}{r_1}$$

$$\Leftrightarrow \boxed{\mathcal{E}_{m,1} = -\mathcal{G}\frac{mM}{2r_1}} \qquad \boxed{\mathcal{E}_{m,2} = -\mathcal{G}\frac{mM}{2r_2}}$$

Orbite elliptique : $2a = r_p + r_A = r_1 + r_2$

et $$\boxed{\mathcal{E}_{m,e} = -\mathcal{G}\frac{mM}{r_1 + r_2}}$$



3) Changement de vitesse instantané

$\Leftrightarrow$ position fixe

$\Leftrightarrow$ $\mathcal{E}_p = cte$.

Donc $$\Delta_{1,e}\mathcal{E}_m = \mathcal{E}_{m,e} - \mathcal{E}_{m,1} = \frac{1}{2} m (v_{e_1}^2 - v_1^2)$$

$$\Leftrightarrow -\mathcal{G}Mm\left(\frac{1}{r_1 + r_2} - \frac{1}{2r_1}\right) = \frac{1}{2} m v_{e_1}^2 - \frac{1}{2} m v_1^2$$

$$\Leftrightarrow \frac{1}{2} m v_{e_1}^2 = -\mathcal{G}mM\left(\frac{\cancel{2}r_1 - \cancel{r_1} - r_2}{2r_1(r_1 + r_2)}\right) + \mathcal{G}\frac{mM}{2r_1}\frac{(r_1 + r_2)}{(r_1 + r_2)}$$

$$\Leftrightarrow \frac{1}{2}\cancel{m} v_{e_1}^2 = \frac{\mathcal{G}\cancel{m}M}{\cancel{2}r_1(r_1 + r_2)}\Big(\underbrace{\cancel{r_1} + r_2 - (\cancel{r_1} - r_2)}_{\cancel{2}\,r_2}\Big)$$

Soit $\boxed{v_{e1} = \sqrt{\dfrac{2 G_s M r_2}{r_1(r_1+r_2)}}} = 9520 \ m.s^{-1}$

Ainsi, $\Delta v_p = \dots\dots = 2220 \ m.s^{-1}$

TEM : $\Delta_{1,2} \mathcal{E}_m = \Delta_{1,2} W = W_p$

$\Leftrightarrow \boxed{W_p = \dfrac{G_s m M (r_2 - r_1)}{2 r_1 (r_1 + r_2)}} = 1{,}7.10^{10} \ J = 17 GJ.$

4) En P et A, $\vec{OM} \perp \vec{v}$; soit $\vec{v} \parallel \vec{u_\theta}$ ($\dot{r} = 0$)

$\vec{L_0(P)} = \vec{OP} \wedge m\vec{v} = r_1 \vec{u_r} \wedge m v_{e1} \vec{u_\theta} = m r_1 v_{e1} \vec{e_z}$

et $\vec{L_0(A)} = \dots\dots = m r_2 v_{e2} \vec{e_z}$

Or, $\dfrac{d\vec{L_0}}{dt} = \vec{0}$, soit $m r_1 v_{e1} \cancel{\vec{e_z}} = m r_2 v_{e2} \cancel{\vec{e_z}}$

$\Leftrightarrow r_1 v_{e1} = r_2 v_{e2}$

$\Leftrightarrow \boxed{v_{e2} = v_{e1} \dfrac{r_1}{r_2} = \sqrt{\dfrac{2 G_s M r_1}{r_2(r_1+r_2)}}}$

A.N. : $v_{e2} = 1690 \ m.s^{-1}$

5) $v_2 = \sqrt{\dfrac{G_s M}{r_2}}$, $\Delta v_A = \sqrt{\dfrac{G_s M}{r_2}} - \sqrt{\dfrac{2 G_s M r_1}{r_2(r_1+r_2)}} = 1390 \ m.s^{-1}$

$\hookrightarrow = 3080 \ m.s^{-1}$

$W_A = \mathcal{E}_{m2} - \mathcal{E}_{me} = -\dfrac{G_s m M}{2 r_2} + \dfrac{G_s m M}{r_1 + r_2} = \boxed{\dfrac{G_s m M (r_2 - r_1)}{2 r_2 (r_1 + r_2)}}$

A.N. : $W_A = 3{,}0 GJ$.

$\hookrightarrow v \downarrow$ qd. $r \nearrow$, donc $W \downarrow$ de P à A ✓.