

# Électrocinétique en RSF : oscillateurs et filtrage

/10 [1] À partir de  $U(x) = \frac{E_0}{\sqrt{(1-x^2)^2 + (\frac{x}{Q})^2}}$ , démontrer la condition de résonance ainsi que l'amplitude à la résonance.

## Condition de résonance

$$\begin{aligned}
 U(x_r) &\stackrel{\textcircled{1}}{=} U_{\max} \\
 \Leftrightarrow f(X_r) &\stackrel{\textcircled{1}}{=} (1 - X_r)^2 + \frac{X_r}{Q^2} \quad \min. \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} X = x^2 \\
 \Leftrightarrow f'(X_r) &= 0 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{On dérive} \\
 \Leftrightarrow -2(1 - X_r) + \frac{1}{Q^2} &\stackrel{\textcircled{1}}{=} 0
 \end{aligned}$$

$$\begin{aligned}
 \Leftrightarrow X_r - 1 &= -\frac{1}{2Q^2} \Leftrightarrow X_r = 1 - \frac{1}{2Q^2} \\
 \Leftrightarrow x_r &= \sqrt{1 - \frac{1}{2Q^2}} \Leftrightarrow x_r \stackrel{\textcircled{1}}{=} \frac{1}{Q} \sqrt{Q^2 - \frac{1}{2}} \\
 \Rightarrow x_r > 0 &\Leftrightarrow Q > \frac{1}{\sqrt{2}}
 \end{aligned}$$

$\left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} X_r = x^2 \\ x_r \in \mathbb{R} \end{array}$

## Amplitude de résonance

$$\begin{aligned}
 f(X_r) &\stackrel{\textcircled{1}}{=} \left(1 - \left(1 - \frac{1}{2Q^2}\right)\right)^2 + \frac{1}{Q^2} \left(1 - \frac{1}{2Q^2}\right) \\
 \Leftrightarrow f(X_r) &= \left(\frac{1}{2Q^2}\right)^2 + \frac{1}{Q^2} - \frac{1}{2Q^4} \\
 \Leftrightarrow f(X_r) &\stackrel{\textcircled{1}}{=} \frac{1}{4Q^4} - \frac{2}{4Q^4} + \frac{1}{Q^2} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{Même dénom.} \\ \text{On calcule} \end{array} \\
 \Leftrightarrow f(X_r) &= \frac{1}{Q^2} - \frac{1}{4Q^4} \\
 \Leftrightarrow f(X_r) &\stackrel{\textcircled{1}}{=} \frac{1}{Q^2} \left(1 - \frac{1}{4Q^2}\right) \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{On factorise}
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow U(x_r) &\stackrel{\textcircled{1}}{=} \frac{E_0}{\sqrt{f(X_r)}} \\
 \Leftrightarrow U(x_r) &= \frac{E_0}{\frac{1}{Q} \sqrt{1 - \frac{1}{4Q^2}}} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \sqrt{\cdot} \\ \text{Simplifie} \end{array} \\
 \Leftrightarrow U(x_r) &\stackrel{\textcircled{1}}{=} \frac{QE_0}{\sqrt{1 - \frac{1}{4Q^2}}}
 \end{aligned}$$

/5 [2] Montrer que diviser l'amplitude par 10 revient à réduire le gain en décibel de 20 dB. Montrer ensuite qu'on trouve la bande passante d'un filtre en trouvant les  $\omega$  tels que  $G_{\text{dB}}(\omega) \geq G_{\text{dB,max}} - 3 \text{ dB}$ .

$$\begin{aligned}
 |\underline{H}(\omega_2)| &\stackrel{\textcircled{1}}{=} \frac{|\underline{H}(\omega_1)|}{10} \\
 \Leftrightarrow 20 \log(|\underline{H}(\omega_2)|) &= 20 \log\left(\frac{|\underline{H}(\omega_1)|}{10}\right) \\
 \Leftrightarrow 20 \log(|\underline{H}(\omega_2)|) &= 20 \log(|\underline{H}(\omega_1)|) - 20 \log(10) \\
 \Leftrightarrow G_{\text{dB}}(\omega_2) &\stackrel{\textcircled{1}}{=} G_{\text{dB}}(\omega_1) - 20 \text{ dB} \quad \blacksquare
 \end{aligned}$$

$$\begin{aligned}
 |\underline{H}(\omega)| &\stackrel{\textcircled{1}}{\geq} \frac{|\underline{H}|_{\max}}{\sqrt{2}} \\
 \Leftrightarrow 20 \log(|\underline{H}(\omega)|) &\geq 20 \log\left(\frac{|\underline{H}|_{\max}}{\sqrt{2}}\right) \\
 \Leftrightarrow G_{\text{dB}}(\omega) &\stackrel{\textcircled{1}}{\geq} \underbrace{20 \log(|\underline{H}|_{\max})}_{= G_{\text{dB, max}}} - \underbrace{20 \log(\sqrt{2})}_{\approx 3 \text{ dB}} \quad \blacksquare
 \end{aligned}$$

/5 [3] À partir de  $\underline{H}(x) = \frac{1}{1+jx}$ , déterminer les asymptotes de  $G_{\text{dB}}(x)$  et  $\varphi(x)$ .

$$\begin{aligned}
 \underline{H}(x) &\stackrel{\textcircled{1}}{\underset{x \rightarrow 0}{\sim}} \frac{1}{1+0} = 1 \quad \text{et} \quad \underline{H}(x) \stackrel{\textcircled{1}}{\underset{x \rightarrow \infty}{\sim}} \frac{1}{jx} \\
 \Rightarrow G_{\text{dB}}(x) &\stackrel{\textcircled{1}}{\underset{x \rightarrow 0}{\sim}} 20 \log(1) = 0 \quad \text{et} \quad G_{\text{dB}}(x) \stackrel{\textcircled{1}}{\underset{x \rightarrow \infty}{\sim}} 20 \log\left|\frac{1}{jx}\right| = -20 \log x \\
 \Rightarrow \varphi(x) &\underset{x \rightarrow 0}{\sim} \arg(1) = 0 \quad \text{et} \quad \varphi(x) \underset{x \rightarrow \infty}{\sim} \arg\left(\frac{1}{jx}\right) = -\frac{\pi}{2}
 \end{aligned}$$

$$\begin{aligned}
 G_{\text{dB}}(x) &= 20 \log |\underline{H}(x)| \\
 \varphi(x) &= \arg(\underline{H}(x))
 \end{aligned}$$