

TD C6 Exercise 7

$$\arg\left(R_1 + \frac{R_2 + jL\omega}{1 + jR_2C\omega - L\omega^2}\right)$$

$$= \arg\left(\frac{R_1 + jR_1R_2C\omega - L(R_1\omega^2 + R_2 + jL\omega)}{1 + jR_2C\omega - L\omega^2}\right)$$

$$= \arg\left(\frac{(R_1 + R_2 - L(R_1\omega^2)) + j(R_1R_2C\omega + L\omega)}{1 + jR_2C\omega - L\omega^2}\right)$$

$$\tan(\arg) = \frac{R_1R_2C\omega + L\omega}{R_1 + R_2 - L(R_1\omega^2)} - \frac{R_2C\omega}{1 - L\omega^2} = 0$$

$$\Leftrightarrow \frac{R_1 + L\omega}{R_2C\omega} = \frac{1}{1 - L\omega^2}$$

$$\Leftrightarrow (R_1 + L\omega)(1 - L\omega^2) = R_2C\omega$$

$$\Leftrightarrow \cancel{R_1} - L\cancel{R_1}\omega^2 + \cancel{L\omega} - \cancel{L^2}\omega^3 = \cancel{R_2C\omega}$$

$$\Leftrightarrow \frac{L\omega}{R_2C\omega} = R_2 + \frac{L^2\omega^2}{R_2}$$

$$\Leftrightarrow \omega^2 L^2 C = L - R_2^2 C$$

$$\Leftrightarrow L^2 C \omega^2 - L = -R_2^2 C$$

$$\Leftrightarrow \omega^2 = -\frac{R_2^2 C}{L^2} + \frac{1}{L^2 C}$$

$$\Leftrightarrow \omega^2 = \frac{1}{LC} - \frac{R_2^2}{L^2}$$

$$R_2^2 C = L(1 - LC\omega^2)$$

$$\Leftrightarrow \omega^2 = \frac{1}{LC} \left(1 - \frac{R_2^2 C}{L} \right)$$

$$0 = \arg \left(\frac{(R_1 + R_2 - L(R_1 \omega^2)) + j(R_1 R_2 C \omega + L \omega)}{1 + j R_2 C \omega - L C \omega^2} \right)$$

$$\text{As } \arg(\text{top}) = \arg(\text{bot})$$

$$\Rightarrow \frac{R_1 R_2 C \omega + L \omega}{R_1 + R_2 - L(R_1 \omega^2)} = \frac{R_2 C \omega}{1 - L C \omega^2}$$