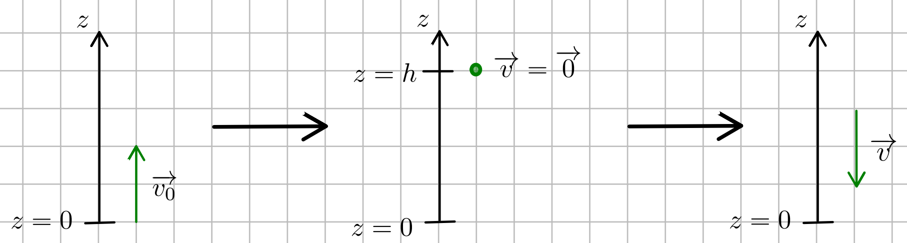
TD application : approche énergétique

I/1)



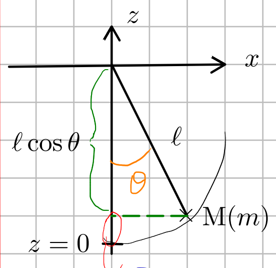
À $t=0$, la balle est en $z=0$ avec $v(0)=v_0$.
Elle va monter en altitude en perdant de l'énergie cinétiq, et en gagnant de l'énergie potentielle.
Le système étant conservatif (BdF $\Rightarrow \vec{P}$), l'énergie mécanique se conserve. Ainsi, avec le TEM :

$$\mathcal{E}_m(0) = \mathcal{E}_m(t_{max})$$

$$\Leftrightarrow \frac{1}{2}\cancel{m}v_0^2 + \underbrace{\cancel{m}gz_0}_{=0} = \frac{1}{2}\cancel{m}\underbrace{v(t_{max})^2}_{=0} + \cancel{m}gh$$

$$\Leftrightarrow \boxed{h = \sqrt{v_0^2/2g}}$$

2)



$z(\theta) = \ell(1-\cos\theta)$

◇ Système conservatif car $\left|\begin{array}{l}\vec{P} \text{ conservatif} \\ W_{AB}(\vec{T}) = 0\end{array}\right.$

◇ $\Delta_{AB}\mathcal{E}_m = 0$

$$\Leftrightarrow \frac{1}{2}\cancel{m}v_0^2 + \underbrace{\cancel{m}gz_0}_{=0} = \frac{1}{2}m\underbrace{v_{max}^2}_{=0} + \cancel{m}gz_{max}$$

$$\Leftrightarrow \ell(1-\cos\theta_{max}) = \frac{v_0^2}{2g}$$

$$\Leftrightarrow \boxed{\cos\theta_{max} = 1 - \frac{v_0^2}{2g\ell}}$$

Valable si $\frac{v_0^2}{2g\ell} < 2$; sinon $\cos\theta_{max} < -1$
$\Rightarrow$ pendule ne fait pas un tour

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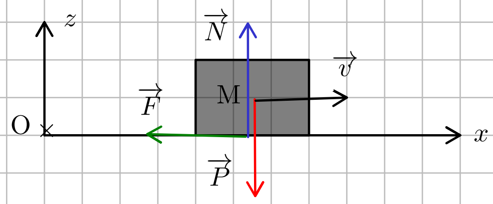
II/1) $\mathcal{E}_{c,I} = \frac{1}{2} m v_0^2$; $\mathcal{E}_{c,F} = 0$



2)
- Syst. = {pierre}
- $\mathcal{R}_{piste}$, galiléen
- $(O, \vec{u_x}, \vec{u_z})$
- $\vec{OM} = x\,\vec{u_x}$, $\vec{OM_0} = D\,\vec{u_x}$
- $\vec{v} = \dot{x}\,\vec{u_x}$
- <u>BDF</u> : $\vec{P} = -mg\,\vec{u_z}$
  $\vec{N} = N\,\vec{u_z}$
  $\vec{F} = -F_0\,\vec{u_x}$
- <u>BdW</u> : $W_{OM_0}(\vec{P}) = \vec{P}\cdot\vec{OM_0} = -mgD\,.\underbrace{(\vec{u_z}\cdot\vec{u_x})}_{=0} = 0$
  $W_{OM_0}(\vec{N}) = \dots = 0$
  $\boxed{W_{OM_0}(\vec{F}) = -F_0 D}$

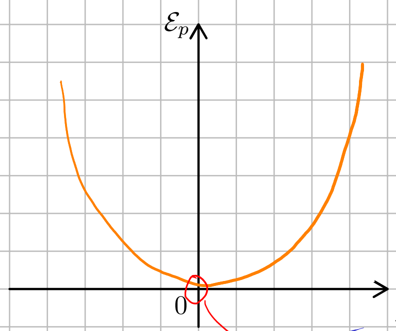
3) <u>TEC</u> : $\Delta_{AB}\,\mathcal{E}_c = \sum_i W_{AB}(\vec{F_i})$

Ici, $\Delta_{OM_0}\,\mathcal{E}_c = \sum_i W_{OM_0}(\vec{F_i})$

$\Leftrightarrow 0 - \frac{1}{2} m v_0^2 = -F_0 D$

$\Leftrightarrow \boxed{v_0 = \sqrt{\frac{2F_0 D}{m}}}$

III/1)



équilibre en $z_{éq} = 0$

car $\left.\frac{d\mathcal{E}_p}{dz}\right|_{z_{éq}} = \frac{eV_0}{d^2} z_{éq} = 0 \Leftrightarrow \boxed{z_{éq} = 0}$

On voit que l'équilibre est stable, et en effet $\boxed{\left.\frac{d^2\mathcal{E}_p}{dz^2}\right|_{z_{éq}} = \frac{eV_0}{d^2} > 0}$

III/2) Autour de z_{eq},

$$E_p(z) \simeq \underbrace{E_p(z_{eq})}_{=0} + (z - z_{eq}) \underbrace{\frac{dE_p}{dz}\Big|_{z_{eq}}}_{=0} + \frac{(z - \overbrace{z_{eq}}^{=0})^2}{2} \underbrace{\frac{d^2E_p}{dz^2}\Big|_{z_{eq}}}_{\frac{eV_0}{d^2}}$$

Et $E_c = \frac{1}{2} m \dot{z}^2$

Système conservatif $\Rightarrow \frac{dE_m}{dt} = 0$

$$\Leftrightarrow \frac{d}{dt}\left(\frac{1}{2} m \dot{z}^2 + \frac{z^2}{2}\frac{eV_0}{d^2}\right) = 0$$

$$\Leftrightarrow m\cancel{\dot{z}}\ddot{z} + \cancel{\dot{z}}\, z \frac{eV_0}{d^2} = 0$$

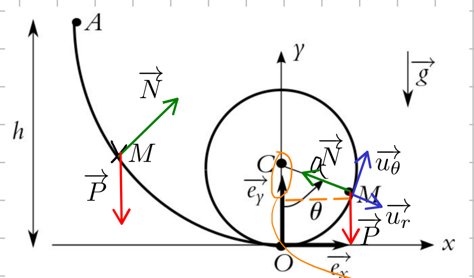
$$\Leftrightarrow \boxed{\ddot{z} + \frac{eV_0}{md^2} z = 0}$$ O.H. avec $\omega_0^2 = \frac{eV_0}{md^2} \Leftrightarrow \boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{eV_0}{md^2}}}$

$f_0 = 25$ MHz

3) $\vec{F} = -\vec{grad}\, E_p(z) = -\left(0.\vec{u_x} + 0\,\vec{u_y} + \frac{eV_0}{d^2} z\, \vec{u_z}\right)$

$$\boxed{\vec{F} = -\frac{eV_0}{d^2} z\, \vec{u_z}}$$

IV/1)



□ $v_A = 0$; $z_A = h$

□ $v_O = v_0$; $z_O = 0$ Référence ∀ exo

□ $W_{AO}(\vec{N}) = 0$ $(\vec{N} \perp d\vec{OM})$

□ $\Delta_{AO} E_m = 0 \Leftrightarrow mgh = \frac{1}{2} m v_0^2$

Soit $\boxed{v_0 = \sqrt{2gh}}$

Dans le tonneau, $z(\theta) = a(1 - \cos\theta)$

□ $\vec{CM} = a\vec{u_r}$

□ $\vec{v_M} = a\dot{\theta}\,\vec{u_\theta}$

□ $\Delta_{AM} E_m = 0 \Leftrightarrow \underbrace{\frac{1}{2}\cancel{m} v_M^2}_{E_c(M)} + \underbrace{\cancel{m} g a(1 - \cos\theta)}_{E_p(M)} = \underbrace{\frac{1}{2}\cancel{m} v_0^2}_{E_c(0)} + \underbrace{mg \overbrace{z_0}^{=0}}_{E_p(0)}$

$\Leftrightarrow v_M = \sqrt{v_0^2 + 2ga(\cos\theta - 1)} = \boxed{\sqrt{2g}\sqrt{h + a(\cos\theta - 1)}} = a\dot{\theta}$

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IV/2)

- $\vec{N} = -N\,\vec{u_r}$
- $\vec{P} = mg\left(\cos\theta\,\vec{u_r} - \sin\theta\,\vec{u_\theta}\right)$
- $\vec{a_M} = a\ddot{\theta}\,\vec{u_\theta} - a\dot{\theta}^2\,\vec{u_r}$

PFD : $\begin{cases} -ma\dot{\theta}^2 = mg\cos\theta - N \\ ma\ddot{\theta} = -mg\sin\theta \end{cases}$

$\hookrightarrow N = mg\cos\theta + ma\dot{\theta}^2$

Or, $v = a\dot{\theta}$ donc $v_M^2 = a^2\dot{\theta}^2$ soit $a\dot{\theta}^2 = \frac{v_M^2}{a}$

Donc

$$N = m\left(g\cos\theta + \frac{2g}{\cancel{a}}\left(h + \cancel{a}(\cos\theta - 1)\right)\right)$$

$$\Leftrightarrow N = m\left(g\cos\theta + 2g\cos\theta - 2g + 2g\frac{h}{a}\right)$$

$$\Leftrightarrow \boxed{N = mg\left(3\cos\theta - 2 + 2\frac{h}{a}\right)}$$

3)

Bille ne tombe pas $\Leftrightarrow N > 0$

Or, N minimal pour $\theta = \pi$

Soit $N(\pi) > 0$

$$\Leftrightarrow \cancel{mg}\left(-3 - 2 + \frac{2h}{a}\right) > 0$$

$$\Leftrightarrow \frac{2h}{a} > 5$$

$$\Leftrightarrow \boxed{h > \frac{5}{2}a} = h_{min}.$$