Robot Arm

6DOF

Table 1: The DH parameters of the DFROBOT

Joint	Link	a_{i-1} min	α_{i-1} degree	d_i mm	θi degree
0-1	1	0	0	45	θ_1
1-2	2	0	90	0	θ_2
2-3	3	90	0	0	θ_3
3-4	4	90	0-90	0	θ_4
4-5	5	0	-90	30	θ_5
5-6	6	0	0	0	gripper

Kinematics Analysis and Modeling of 6 Degree of Freedom Robotic Arm from DEROBOT on Labview Paper.

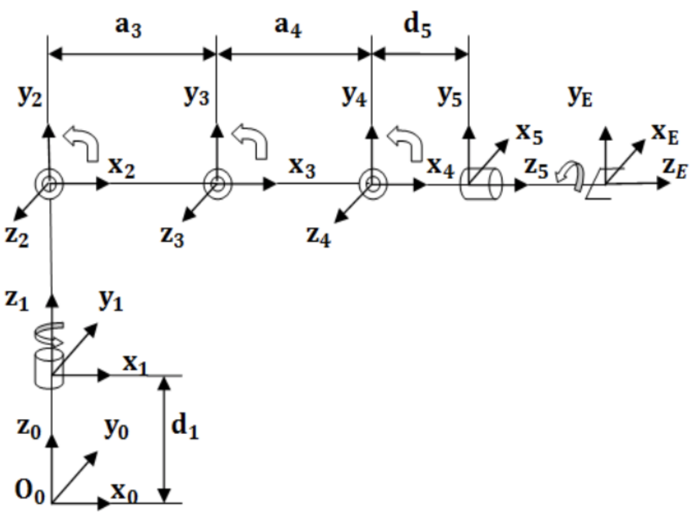

Fig. 3: The robot coordinate frame

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- ## Forward Kinematics:

  To find the position (p_6^0) and rotation or the orientation (R_6^0) for the end effector.

  1- Use the homogenous transformation matrix: to find the matrix of the link from joint i to i+1.

$$A_i = \begin{bmatrix} C_i\theta_i & -S_i\theta_i C_i\alpha_i & S_i\theta_i S_i\alpha_i & a_i C_i\theta_i \\ S_i\theta_i & C_i\theta_i C_i\alpha_i & -C_i\theta_i S_i\alpha_i & a_i S_i\theta_i \\ 0 & S_i\alpha_i & C_i\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hint:

$C_i = \cos\theta_i$, $S_i = \sin\theta_i$

$C_{i,i+1} = \cos(\theta_i + \theta_{i+1})$, $S_{i,i+1} = \sin(\theta_i + \theta_{i+1})$

2- Sub. in A_i from Table1:

$$A_1^0 = A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^1 = A_2 = \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & a_3 C_3 \\ S_3 & C_3 & 0 & a_3 S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4^3 = A_4 = \begin{bmatrix} C_4 & -S_4 & 0 & a_4 C_4 \\ S_4 & C_4 & 0 & a_4 S_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5^4 = A_5 = \begin{bmatrix} C_5 & 0 & -S_5 & 0 \\ S_5 & 0 & C_5 & 0 \\ 0 & -1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6^5 = A_6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3- Transformation from 6th joint to the base frame 0th joint.

Using matrix multiplication:

$$A_6^0 = A_1.A_2.A_3.A_4.A_5.A_6 = \begin{bmatrix} R_6^0 & P_6^0 \\ 0 & 1 \end{bmatrix}$$

$$R_6^0 = \begin{bmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{bmatrix}_{3\times 3, rotation\ for\ the\ end\ effector}$$

$$R_6^0 = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}_{3\times 1, position\ for\ the\ end\ effector}$$

Thus,

$$A_6^0 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4- Find the position (p_6^0) and rotation or the orientation (R_6^0) for the end effector in term of joint angle:

Orientation (R_6^0) for the end effector:

$$n_x = \cos(\theta_6)\cos(\theta_1+\theta_2)\cos(\theta_3+\theta_4+\theta_5) - sin(\theta_6)\sin(\theta_1+\theta_2)$$

$$n_y = \cos(\theta_6)\sin(\theta_1+\theta_2)\cos(\theta_3+\theta_4+\theta_5) + sin(\theta_6)\cos(\theta_1+\theta_2)$$

$$n_z = \cos(\theta_6)\sin(\theta_3+\theta_4+\theta_5)$$

$$o_x = \sin(\theta_6)\cos(\theta_1+\theta_2)\cos(\theta_3+\theta_4+\theta_5) - cos(\theta_6)\sin(\theta_1+\theta_2)$$

$$o_y = -\sin(\theta_6)\sin(\theta_1+\theta_2)\cos(\theta_3+\theta_4+\theta_5) + cos(\theta_6)\cos(\theta_1+\theta_2)$$

$$o_z = -\sin(\theta_6)\sin(\theta_3+\theta_4+\theta_5)$$

$$a_x = -\cos(\theta_1+\theta_2)\sin(\theta_3+\theta_4+\theta_5)$$

$$a_y = -\sin(\theta_1+\theta_2)\sin(\theta_3+\theta_4+\theta_5)$$

$$a_z = \cos(\theta_3+\theta_4+\theta_5)$$

Position (p_6^0) for the end effector:

$$p_x = a_4\cos(\theta_1+\theta_2)\cos(\theta_3)\cos(\theta_4) - a_4\cos(\theta_1+\theta_2)\sin(\theta_3)\sin(\theta_4) + \sin(\theta_1+\theta_2)\,d_5 + a_3\cos(\theta_1+\theta_2)\cos(\theta_3)$$

$$p_y = a_4\sin(\theta_1+\theta_2)\cos(\theta_3)\cos(\theta_4) - a_4\sin(\theta_1+\theta_2)\sin(\theta_3)\sin(\theta_4) - \cos(\theta_1+\theta_2)\,d_5 + a_3\sin(\theta_1+\theta_2)\cos(\theta_3)$$

$$p_z = a_4\sin(\theta_3)\cos(\theta_4) + a_4\cos(\theta_3)\sin(\theta_4) + a_3\sin(\theta_3) + d_1$$

- ## Inverse Kinematics:

To find the joint angle (θ_i).

1- Use the algebra solution:

$$A_6^0 = A_1.A_2.A_3.A_4.A_5.A_6$$

Multiply the both side of the above equation by $A_n^{-1}, n = 1,2,3,4,5,6$

Hint: The inverse the transformation matrix.

$$A_1^{-1} = \begin{bmatrix} C_1 & S_1 & 0 & 0 \\ -S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & -d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^{-1} = \begin{bmatrix} C_1 & S_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ S_2 & -C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^{-1} = \begin{bmatrix} C_3 & S_3 & 0 & -a_3 \\ -S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4^{-1} = \begin{bmatrix} C_4 & S_4 & 0 & -a_4 \\ -S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5^{-1} = \begin{bmatrix} C_5 & S_5 & 0 & 0 \\ 0 & 0 & -1 & d_5 \\ -S_5 & C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6^{-1} = \begin{bmatrix} C_6 & S_6 & 0 & 0 \\ -S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2- Find the joint angles $(\theta_1), (\theta_2)\ and\ (\theta_6)$:

$$A_2^{-1} A_1^{-1}. A_6^0 = A_1. A_2. A_3. A_4. A_5. A_6. A_1^{-1}. A_2^{-1}$$

$$A_2^{-1}. A_1^{-1}. \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_3. A_4. A_5. A_6$$

$$\begin{bmatrix} . & . & . & C_1C_2p_x + C_1S_2p_y + S_1p_z \\ . & . & . & -S_1C_2p_x - S_1S_2p_y - C_1p_z \\ S_2n_x - C_2n_y & S_2o_x - C_2o_y & . & S_2p_x - C_2p_y - d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_6C_{345} & -S_6C_{345} & -S_{345} & a_4C_{34} + a_3C_3 \\ C_6S_{345} & -S_6S_{345} & C_{345} & a_4S_{34} + a_3S_3 \\ -S_6 & -C_6 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find (θ_2):

$$S_2p_x - C_2p_y - d_1 = d_5$$

$$p_x \sin(\theta_2) - p_y \cos(\theta_2) = d_5 + d_1$$

Using Pythagoras and trigonometric relationships:

$$\sin(\theta + A) = \frac{(d_5 + d_1)\sqrt{{p_x}^2 + {p_y}^2}}{{p_x}^2 + {p_y}^2}$$

$$A = \tan^{-1}\left(\frac{-p_y}{p_x}\right)$$

$$\theta_2 = \sin^{-1}\left(\frac{(d_5 + d_1)\sqrt{{p_x}^2 + {p_y}^2}}{{p_x}^2 + {p_y}^2}\right) - \tan^{-1}\left(\frac{-p_y}{p_x}\right)$$

Find (θ_1):

$$a_x = -\cos(\theta_1 + \theta_2)\sin(\theta_3 + \theta_4 + \theta_5)$$
$$a_y = -\sin(\theta_1 + \theta_2)\sin(\theta_3 + \theta_4 + \theta_5)$$

Dividing the two equations:

$$\theta_{12} = \tan^{-1}\left(\frac{a_y}{a_x}\right)$$
$$\theta_1 = \theta_{12} - \theta_2$$

Find (θ_6):

$$-S_6 = S_2 n_x - C_2 n_y$$
$$-C_6 = S_2 o_x - C_2 o_y$$

Dividing the two equations:

$$\tan(\theta_6) = \frac{-C_2 n_y + S_2 n_x}{-C_2 o_y + S_2 o_x} \rightarrow \theta_6 = \tan^{-1}\left(\frac{-C_2 n_y + S_2 n_x}{-C_2 o_y + S_2 o_x}\right)$$

3- Find the joint angles (θ_3):

$$A_3^{-1}.A_2^{-1}.A_1^{-1}.\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_4.A_5.A_6$$
$$S_{23}p_x - C_{23}p_y - a_3 S_2 - d_1 = d_5$$

$$S_{23}p_x - C_{23}p_y = d_5 + a_3S_2 + d_1$$

Using trigonometric relationships:

$$\theta_{23} = atan2(px, -py) \mp atan2(\sqrt{p_x{}^2 + p_y{}^2 - (a_3S_2 + d_1 + d_5)^2}, (a_3S_2 + d_1 + d_5)$$

$$\theta_3 = \theta_{23} - \theta_2$$

4- Find the joint angles (θ_4):

$$A_1^{-1}.\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_1.A_2.A_3.A_4.A_5.A_6.A_1^{-1}$$

$$\begin{bmatrix} . & . & C_1a_x + S_1a_y & C_1p_x + S_1p_y \\ . & . & -S_1a_x + C_1a_y & -S_1p_x + C_1p_y \\ . & . & a_z & p_z - d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} . & . & -C_2S_{345} & a_4C_2C_{34} + a_3C_2C_3 + S_2d_5 \\ . & . & -S_2S_{345} & a_4S_2C_{34} + a_3S_2C_3 - C_2d_5 \\ . & . & C_{345} & a_4S_{34} + a_3S_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find (θ_4):

$$p_z - d_1 = a_4S_{34} + a_3S_3$$

$$S_{34} = \frac{-a_3S_3 + p_z - d_1}{a_4}$$

$$\theta_{34} = \sin^{-1}\left(\frac{-a_3S_3 + p_z - d_1}{a_4}\right)$$

$$\theta_4 = \theta_{34} - \theta_3$$

5- Find the joint angles (θ_5):

Find (θ_5):

$$a_z = \cos(\theta_3 + \theta_4 + \theta_5)$$

$$\theta_{345} = \cos^{-1}(a_z)$$

$$\theta_5 = \theta_{345} - \theta_3 - \theta_4$$