Homework 1

Tianjian Shi

Due date: Thursday, September 20

1. **Textbook problem 1.3** The investigator of a large clinical trial would like to assess factors that might be associated with drop-out over the course of the trial. Describe what would be the event and which observations would be considered censored for such a study.

   Event:drop-out; Censored:patients remained at the end of the study.

2. Let T be a positive continuous random variable, show $E(T) = \int_0^\infty S(t)\,dt$.

$$E(T) = \int_0^\infty t\;dF(t) = t \cdot F(t)|_0^\infty - \int_0^\infty F(t)\,dt = t - \int_0^\infty F(t)\,dt = \int_0^\infty (1 - F(t))\,dt = \int_0^\infty S(t)\;dt$$

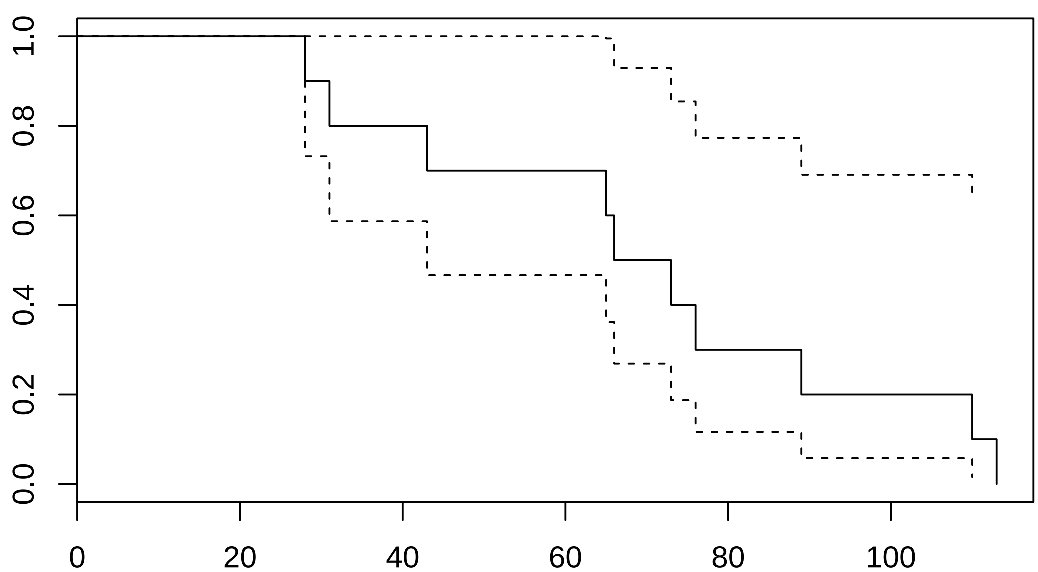
3. Question 2 suggests that the area under the survival curve can be interpreted as the expected survival time. Consider the following hypothetical data set with 10 death times.

```
> library(survival)
```

```
Warning: package 'survival' was built under R version 3.4.4
```

```
> dat <- c(43, 110, 113, 28, 73, 31, 89, 65, 66, 76)
> surv <- survfit(Surv(dat) ~ 1)
```

```
> plot(surv)
```



b. Find the expected survival time for the hypothetical data set.

```
> summary(dat)
```

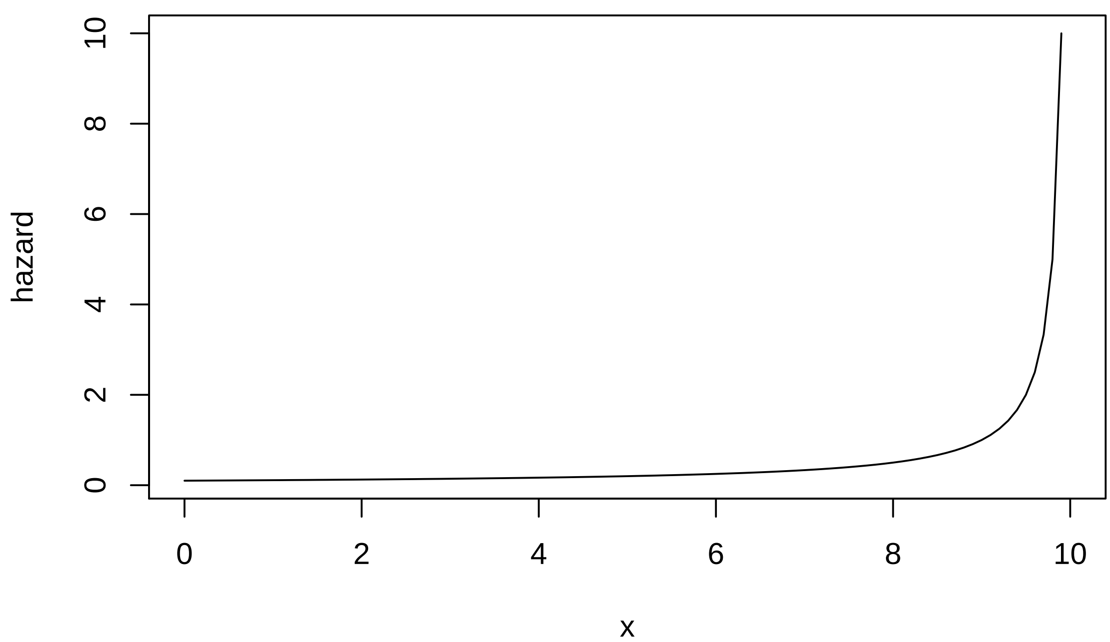
```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
  28.00   48.50   69.50   69.40   85.75  113.00 
```

Sample mean is 69.40 which is exactly the expected survival time.

4. Consider a survival time random variable with hazard $\lambda(t) = \frac{1}{10-x}$ in $[0, 10)$.

a. Plot the hazard function.

```
> hazard= function(x){1/(10-x)}
> plot(hazard, 0,10)
```

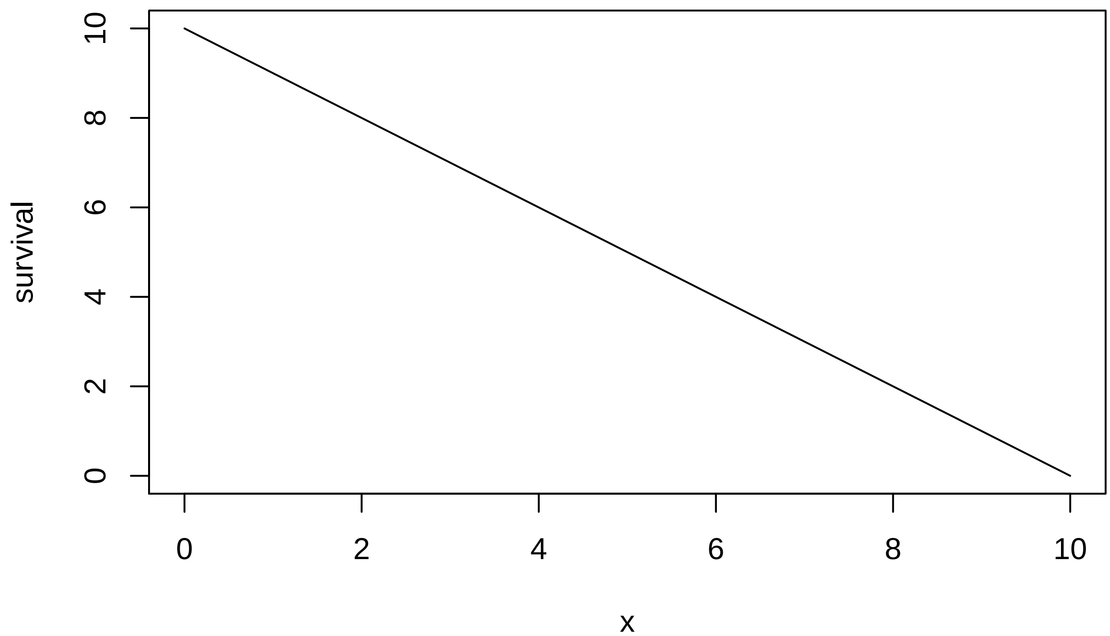


b. Plot the survival function.

$$H(t) = \int h(t)dt = \int \frac{1}{10-t}dt = -\log(10-t)$$

$$S(t) = e^{-H(t)} = e^{log(10-t)} = 10 - t$$

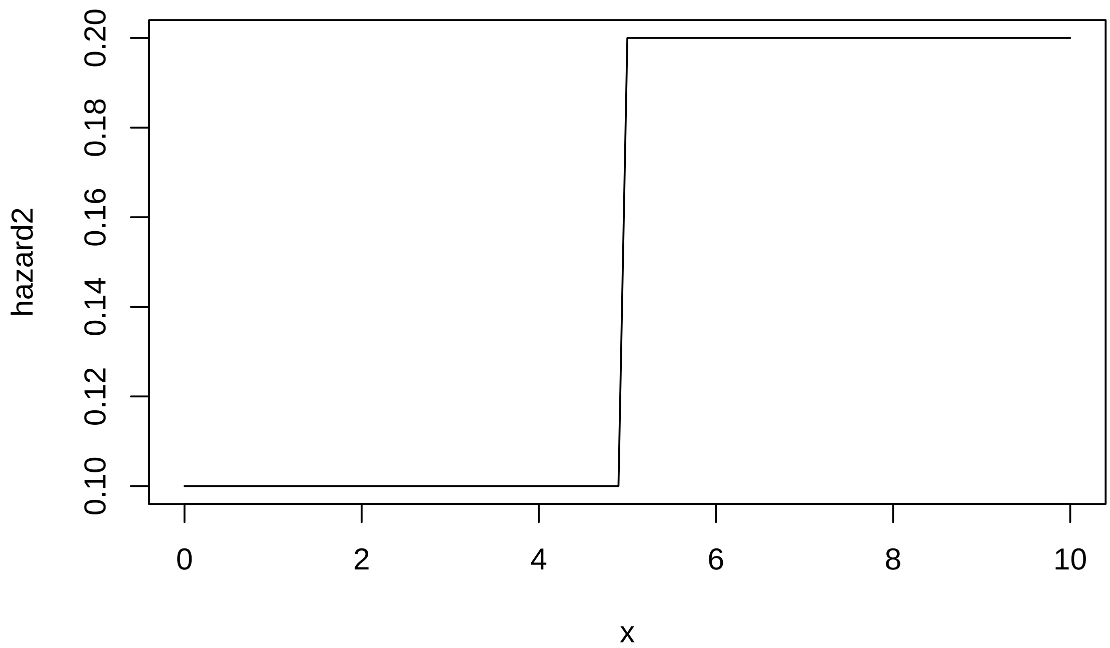
```
> survival=function(x){10-x}
> plot(survival,0,10)
```



5. Consider a survival time random variable with constant hazard $\lambda = 0.1$ in $[0, 5)$, and $\lambda = 0.2$ in $[5, \infty)$. This is known as a piece-wise constant hazard.

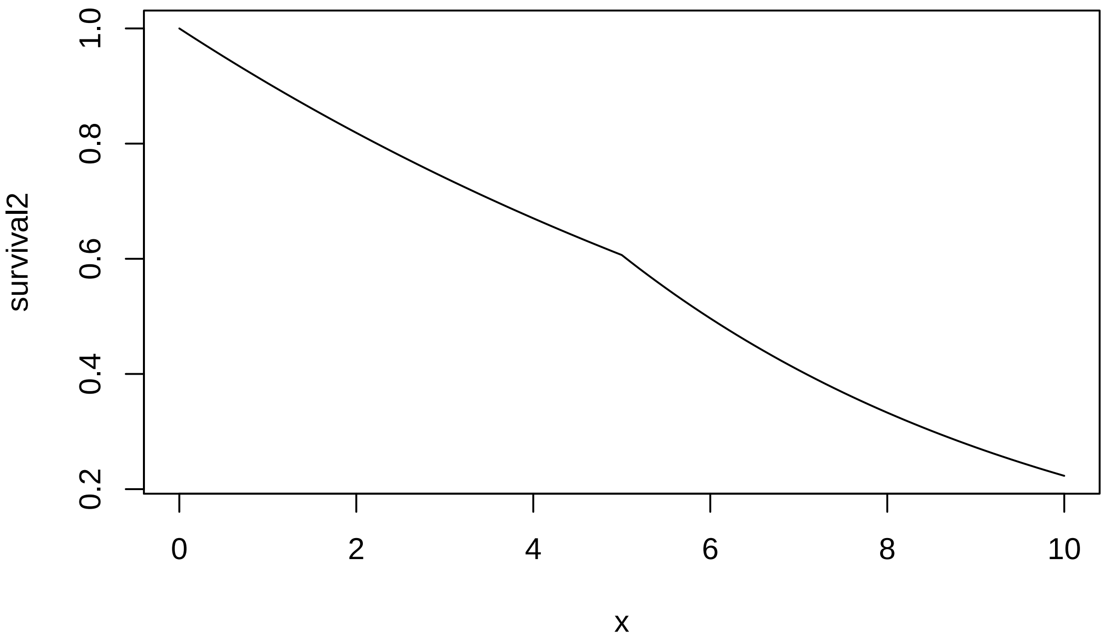
   a. Plot the hazard function.

```
> hazard2=function(x)(x<5)*0.1+(x>=5)*0.2
> plot(hazard2, 0,10)
```



b. Plot the survival function.

```
> survival2=function(x)(x<5)*exp(-0.1*x)+(x>=5)*exp(0.5-0.2*x)
> plot(survival2,0,10)
```



$$H(t)|t \in [0,5) = \int_0^5 h(t)dt = \int_0^5 0.1dt = 0.1t|_0^5$$

$$H(t)|t \in [5,\infty) = \int_5^\infty h(t)dt + c = \int_5^\infty 0.2dt + c = -0.5 + 0.2t|_5^\infty$$

$$S(t)|t \in [0,5) = \exp(-H(t)|t \in [0,5)) = \exp(-0.1t)|_0^5$$

$$S(t)|t \in [5,\infty) = \exp(-H(t)|t \in [5,\infty)) = \exp(0.5 - 0.2t)|_5^\infty$$