Basic Principle of Cyclotron

V. S. Pandit
vspandit@gmail.com
(Outstanding Scientist (Retired), VECC, Kolkata)

E.O. Lawrence originated the concept of the cyclotron accelerator in 1929. It is based on a combination of RF acceleration and bending of charged particles in a magnetic field. This way the same electrode is used over and over again to give acceleration to the particles. Lawrence built the first cyclotron in 1931 and it produced protons of 1.25 MeV.

The basic operating principle of a cyclotron is shown in Figure 1. Particles are accelerated in spiral paths inside two semicircular, flat metallic electrodes called dees (D's). The dees are connected to an RF generator and are placed in a nearly uniform magnetic field. Charged particles are produced by an ion source located at the centre of the cyclotron between the dees and extracted by a puller electrode at the same potential as the dee. The magnetic field causes the particles to move in a plane called median plane in approximately circular orbits inside the dee and across the gap between them. At each gap, particles are accelerated and, therefore, they follow a spiral path as they gain energy. This is because the radius of orbit, being a function of the particle velocity, increases with time. At the edge of the magnet, full energy particle beam is pulled out (extracted) as an external beam by an electrostatic deflector. The dees, deflector, ion source etc are enclosed in a flat vacuum chamber where a low pressure of the order of ~10^{-6} torr is maintained. High vacuum is essential to generate high voltages on dee and deflector as well as to avoid the beam loss by scattering during acceleration.

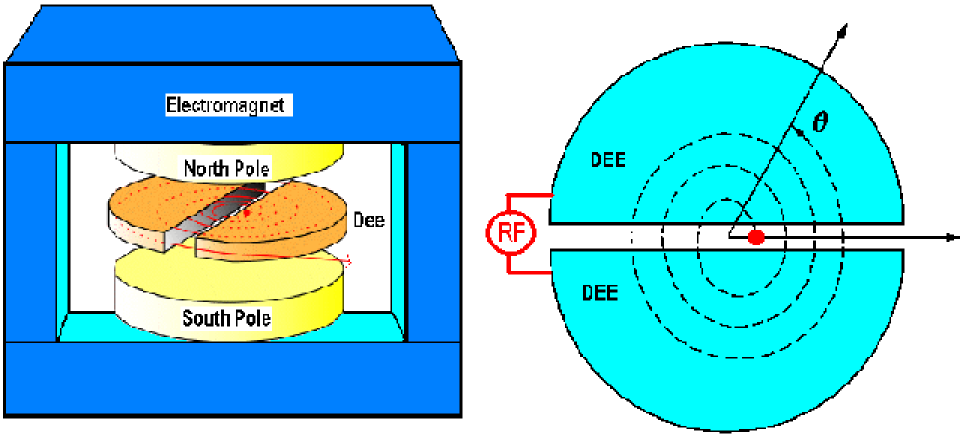

Figure 1: Schematic layout of a cyclotron, showing the two dees (hollow semi-circular electrodes for acceleration), driven by an RF power source. The charged particles move on a spiral orbit from their point of creation in an ion source located in the centre.

If r be the radius of the circular path of a particle of mass m and charge q moving with velocity v in a magnetic field B, then at every instant the force due to magnetic field supplies the centripetal force required for a circular path. The basic equations are:

$$\frac{mv^2}{r} = qvB, \qquad p = mv = qBr, \qquad \omega = \frac{qB}{m} \tag{1}$$

The orbital frequency ω depends only on B and is independent of the energy of the particle as long as m is constant. For a given B, orbit radius r is proportional to momentum p of the particle. If the voltage on one dee is $V = V_D \cos(\omega_{rf} t)$ and on the other dee it is 180^0 out of phase and if $\omega_{rf} = \omega$, then a particle that starts out at right time (at the peak of RF) will experience an increase in the kinetic energy $\Delta T = 2qV_D$ each time it crosses the gap. The particle does not experience the electric field except while crossing the gaps

Resonance condition: For repeated acceleration, we need resonance between the particle revolution frequency and the accelerating RF frequency i.e.

$$\omega_{rf} = \omega \quad or \quad \omega_{rf} = h\omega \tag{2}$$

Here, h=1,2,3,…. is known as harmonic number. Cyclotrons are operated in harmonic mode h>1 mostly for heavy ion acceleration where the revolution frequency is, generally, low. Resonance condition must be fulfilled throughout the acceleration process. Any discrepancy between particle revolution frequency ω and RF frequency ω_{rf} results in a phase shift ϕ between the beam and the driving RF given by:

$$d(\sin\phi) = 2\pi hn \frac{\omega_{rf} - \omega}{\omega} \tag{3}$$

where, n is the number of turns. In this case particles will receive an increase in the kinetic energy $\Delta T = 2qV_D \cos(\phi)$ each time it crosses the gap. The limit of acceleration occurs when ϕ reaches the value $\pm 90^0$. Particles are decelerated and lost if the phase is beyond $+90^0$ or -90^0.

Final kinetic energy of the particle: As long as the particles do not reach relativistic energies the maximum achievable kinetic energy depends on the type of the particle, the magnetic field B, and the maximum orbit radius R (extraction radius) possible in the cyclotron and is given by:

$$T = \frac{1}{2}mv^2 = \frac{q^2 B^2 R^2}{2m} = \frac{e^2 B^2 R}{2m_p} \cdot \frac{Q^2}{A} = K \cdot \frac{Q^2}{A} \tag{4}$$

where, e is electronic charge, m_p is the mass of proton, A and Q are the mass and charge numbers of the ion, respectively. K is, generally, called the K-value or K-bending of a cyclotron. It is measure of the maximum rigidity that a particle can achieve in the cyclotron. The final kinetic energy can also be, roughly, expressed in terms of dee voltage V_D as:

$$T = \text{number of turns} \times \text{energy gain /turn} = n \times 2qV_D \tag{5}$$

Classical Cyclotrons

In a truly uniform magnetic field particle orbits have no stability in the axial direction. A particle moving with even a small component of velocity in the axial direction will soon strike the inside of a dee and be lost. Moreover, in a nominal beam current (μA), there are trillions of particles that repel each other due to coulomb repulsion. It is, therefore, necessary to provide some additional field components so that particles, which have an upward or downward velocity component, must feel a stabilizing force back towards the median plane. In the earlier cyclotrons (before 1955) designers used to provide axial (vertical) focusing to the beam by decreasing the average magnetic field with radius. In this kind of magnetic field, particles away from the designed orbit or equilibrium orbit execute simple harmonic motion about the equilibrium orbit in axial and radial directions and remain confined. These oscillations are known as betatron oscillations as discussed earlier. Focusing forces in axial and radial directions in a cyclotron are characterized by two dimensionless parameters, ν_z and ν_r, known as betatron tunes, respectively. They represent the number of oscillations per revolution and are given by:

$$\nu_z = \frac{\omega_z}{\omega} = \sqrt{n} \qquad \nu_r = \frac{\omega_r}{\omega} = \sqrt{1-n} \tag{6}$$

where, $n = -\frac{r}{B}\frac{dB}{dr}$, is called the field index. Stability in both the directions corresponds to real values of ν_z and ν_r and thus field index must satisfy the inequality $0 \leq n \leq 1$. This corresponds to a magnetic field decreasing with radius.

Limitations of classical cyclotrons: The resonance condition requires that $\omega_{rf} = \omega = qB/m$ should be constant throughout the acceleration. This is in conflict with the axial focusing requirement (B should decrease as radius r increases), as discussed above. The situation is in fact worse than this because of the relativistic variation of mass with velocity. As the energy of the particle increases, so do the mass and the radius of the orbit r. As a consequence, ω does not remain constant at higher radii. If particle revolution frequency is different from RF frequency there is a shift of phase and particle will not reach the accelerating gaps at the proper time. As a consequence of phase shift a limit will be reached where there will be no acceleration. The maximum kinetic energy of ions in a classical cyclotron is limited to ~2% of the rest mass energy of the particle.

It is clear from the resonance condition that either the RF frequency or the magnetic field or both must be modified to compensate for the increasing mass in order to have beam particles arrive at the gap in phase. The former is done in a synchrocylotron. These machines are not built now a day due to complexity and limitation on beam intensity.

Azimuthally Varying Field (AVF) Cyclotron

The conventional cyclotron cannot accelerate ions to very high energy due to the loss of synchronization caused by the relativistic increase in mass of the ion. The synchronization between ω and ω_{rf} can be preserved only if the average magnetic field increases with radius. This leads to the axial defocusing. In order to provide vertical stability an extra source of axial force is required. One way is to introduce azimuthal variation in the magnet field (first proposed by L.H. Thomas in 1938). In an AVF cyclotron the orbital frequency is maintained constant i.e. $\omega = \omega_{rf}$ by increasing the average magnetic field according to:

$$\langle B(r) \rangle = \gamma B_0 = (1 + \frac{T}{E_0})B_0 = \frac{B_0}{\sqrt{1-(r/r_0)^2}} \tag{7}$$

where, γ is the well known relativistic factor ($m=\gamma m_0$), B_0 is the magnetic field at the center of the cyclotron and $r_0 = c/\omega$. The magnetic field with this feature is called isochronous field. Such a field profile gives an axial defocusing given by:

$$\Delta \nu_z^2 = -\frac{r}{\langle B \rangle}\frac{d\langle B \rangle}{dr} = -\beta^2\gamma^2 = -(\gamma^2 - 1) \tag{8}$$

In order to achieve the axial stability, magnet pole faces are contoured to provide an azimuthal variation in the field strength. In practice this can be achieved by dividing the poles into N symmetrical straight sectors (see Figure 2) each consisting of a hill with small gap and high field B_H and a valley with large gap and low field B_V. The different radii of curvature in hill and valley lead to a scalloped closed orbit oscillating around a perfect circle. This implies a radial velocity component v_r, strongest at the hill valley boundary where the hill-fringing field provides B_θ component. The combined effect is an axial focusing force at every hill valley boundary. This is called Thomas focusing and its contribution to the axial focusing is:

$$\Delta \nu_{zThomas}^2 = F^2 = \frac{(B_H - \langle B \rangle)(\langle B \rangle - B_V)}{\langle B \rangle^2} \tag{9}$$

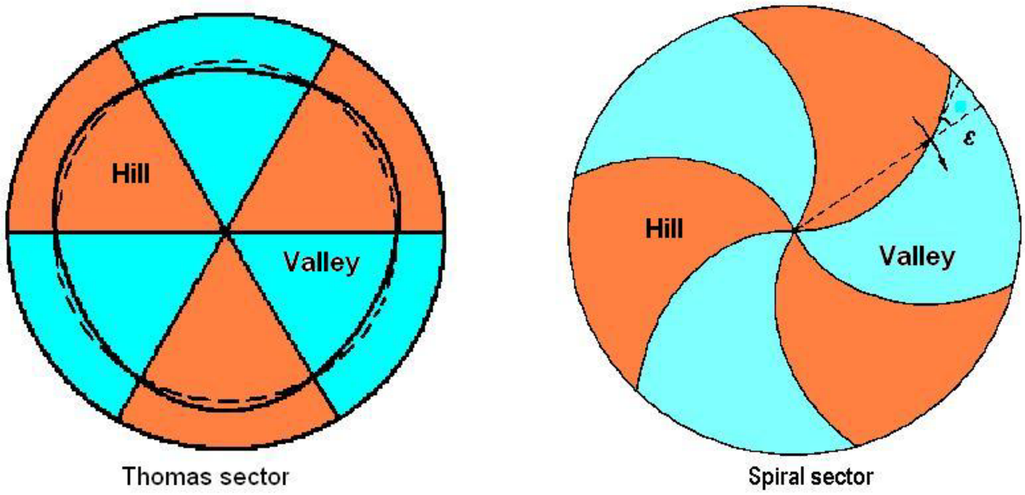

Figure 2: Geometry of the magnet pole faces of a straight sector and spiral sector AVF cyclotrons. Solid line shows the scalloped particle orbit oscillating around a perfect circle (dashed line).

An additional axial focusing force can also be obtained from the field pattern produced by spiral shaped sectors. In this case at every hill-valley boundary the fringing field has a radial component B_r with a gradient that points in at one edge and out at the other. While passing through these radial fields, a particle with azimuthal velocity v_θ, experiences a focusing force at one edge and defocusing force at the other. The net effect is focusing due to the alternating gradient principle. The contribution of spiral effect to the axial focusing is:

$$\Delta \nu_{zspiral}^2 = 2F^2 \tan^2 \varepsilon \tag{10}$$

where, ε is known as the spiral angle, which at any point is defined as the angle between the radius vector and tangent to the spiral at that point. Adding all the contributions we have

$$\nu_z^2 = -(\gamma^2 - 1) + F^2(1 + 2\tan^2 \varepsilon) \tag{11}$$

From equation (11) it is clear that axial focusing will be achieved as long as the magnitude of the second term will be greater than that of the first term. When the second term becomes equal to the first term we have a limiting value of the kinetic energy above which there will be no axial stability. This energy is known as the focusing limit of the cyclotron. It reaches quickly for the particles with small rest mass energy E_0 and that is why we cannot accelerate electrons in a cyclotron. The radial betatron tune is not affected (in the first order) by the sectors and the spirals and is given by:

$$\nu_r^2 = 1 + \frac{r}{\langle B \rangle}\frac{d\langle B \rangle}{dr} + ... = \gamma^2 + ... \tag{12}$$

In an AVF cyclotron, isochronous average magnetic field $<B(r)>$ increases with radius as:

$$\langle B(r) \rangle = \langle B_0 \rangle \cdot \gamma(r)$$

where, $<B_0>$ is the average magnetic field at the centre. The required shape of the magnetic field can be obtained by following different methods:

(a) By using circular trim coils placed at the pole faces with careful tuning. This facility permits the acceleration of different types of ions to different final kinetic energy.

(b) By increasing the azimuthal width of the hill at the expense of azimuthal width of the valley as radius r increases. This technique severely limits the flexibility in acceleration.

(c) By decreasing the gap between the poles with radius. This method is not suitable when high beam intensity is required.

The focusing properties of cyclotrons are determined by betatron tunes ν_z and ν_r which depend on hill field B_H, valley field B_V, average field $<B>$, number of sectors N and the spiral angle ε. Magnetic fields must be carefully designed to avoid harmful resonances during the acceleration process. Cyclotrons with N=2 sectors will be unstable from the centre and, therefore, they do not exist. In the medium energy range, say up to 100 MeV protons, either N=3 or N=4 sectors cyclotron is most appropriate. However, for high-energy protons more number of sectors is needed.

In earlier cyclotrons there were only two gap crossings per turn, which corresponds to two dees with each having an angle 180^0. But many AVF cyclotrons have now been constructed with four, six or even eight gap crossings per turn. Dee's are no more in the shape of "D". They have now wide variety of structures (even spiral) with much lower angles. The energy gained by ions per turn is given by;

$$\Delta E = 2NqV_D \sin\left(\frac{hD}{2}\right)\cos\phi \tag{13}$$

where, N is the number of dees, q is the charge of ion, V_D is the dee voltage, D is the dee angle, h is harmonic number and ϕ is the phase of the RF voltage when ion is in the middle of the gap. Clearly, more number of dees produces more energy gain per turn.

The Variable Energy Cyclotron (VEC) at Kolkata is a three spiral sector compact AVF cyclotron with K=130 MeV. It can accelerate protons from 6-60 MeV, deuterons 12-65 MeV and heavy ions 130 Q^2/A MeV. It has only one dee (vacuum chamber or the RF liner acts as another dee) for acceleration and an electrostatic deflector for beam extraction. A single dee facilitates installation and operation of the injection and extraction elements. The VEC provides light ions using internal Penning Ion Gauge (PIG) ion source and heavy ions using external Electron Cyclotron Resonance (ECR) ion source with injection line.

Separated sector cyclotron (SSC): SSC is also an AVF isochronous cyclotron where magnet sectors are separated by empty (zero field) valleys. The powerful RF accelerating structures are situated in the valleys between the magnets. This feature helps to make the magnet gaps quite small and thus improves the vertical focusing properties. It also enables to use high injection energy. The injection is usually from another accelerator. The radial separation between the orbits is sufficiently large which leads to almost loss less extraction. These unique features enable SSC to accelerate high beam current. Example: Ring Cyclotron at PSI, Switzerland, which delivers highest output beam power (2mA, 590 MeV) and is used for the production of pions and spallation neutrons.

Superconducing cyclotrons: Superconducting cyclotrons are compact AVF cyclotrons where the magnetic force is supplied by superconducting coils, which consume little power. Superconducting coils (made of Nb-Ti and cooled with liquid helium) are placed around the magnet pole in a cryostat. Magnetic field of ~5-6 Tesla is produced for bending the particles. Since the maximum energy is proportional to the square of the magnetic field, the size of the magnet and, hence, the weight of the iron is significantly reduced (by a factor of ~20), compared to the same energy room-temperature cyclotron. The superconducting cyclotron K-500 at MSU, USA is a compact AVF cyclotron having three-spiral sectors with three-spiral dees (six accelerating gaps). It is capable of accelerating variety of light as well as heavy ions. An external ECR ion source produces the ions to be accelerated. A similar cyclotron is in an advanced stage of commissioning at VECC, Kolkata.

Injection into Cyclotrons

The acceleration of heavy ions or polarized ions requires ion sources of larger dimensions that could not be confined in the central region of the cyclotron. This calls for external injection systems injecting ions either radially or axially. It is also needed when a cyclotron works as a booster accelerator for another cyclotron or a linear accelerator. Radial injection is used mostly in separated sector cyclotrons where there is plenty of space available to house large inflector and/or stripper. Most compact cyclotrons utilize versatile external ion source together with an external injection system injecting the ions axially. A typical layout of an external injection system (e.g. for VEC) is shown in figure 31.

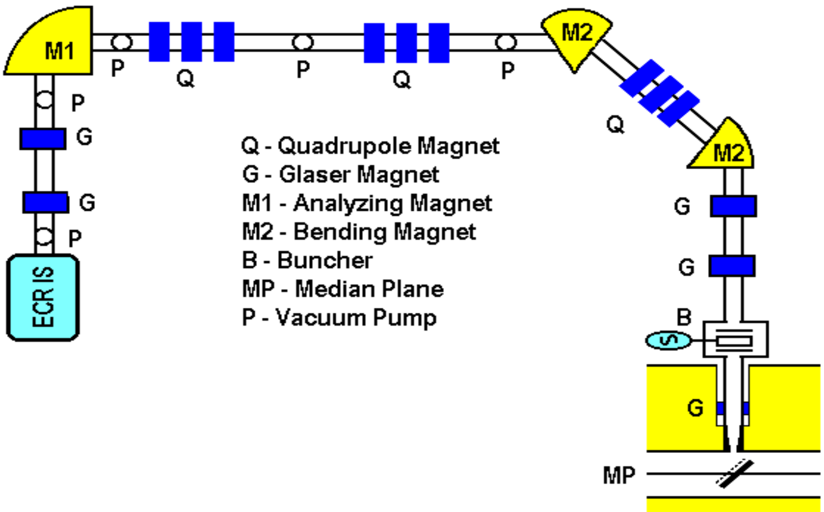

Figure 3: A typical external injection system used to axially inject heavy ion beam into a cyclotron using an external ion source.

The beam from ECR ion source is extracted using a high voltage electrode (usually 10–20 keV). The extracted heavy ion beam from the source is transported to an analyzing magnet where the required charge state of the ion is selected. The analyzed beam is then transported to the buncher, which converts the dc beam from ECR ion source into bunched beam for proper RF matching. This bunched beam is guided to the inflector in the central region, which bends the beam by 90^0 into the median plane and places the beam on the orbit for acceleration.

Extraction from the Cyclotron

After acceleration to maximum energy, particles must somehow be pulled out of their orbits to form an external beam for utilization by experimentalists. The device used for this purpose is known as deflector. It consists of a channel formed by two electrodes across which an electrostatic field directed outwards (for positive ions) is maintained. As the particle advances in the channel, magnetic field gets progressively weaker and thus the channel width increases. The basic equation that holds at each point in the channel for central trajectory is:

$$qvB - qE = \frac{mv^2}{\rho} \tag{6.14}$$

where, E is the electric field in the channel and ρ is the radius of curvature of the channel at the point of consideration. In addition to an electrostatic deflector, magnetic shielding i.e. a magnetic channel is also used to reduce the fringe field. A properly designed magnetic channel helps in the extraction process and also provides radial focusing to the beam. The combination of electrostatic deflector and magnetic channel gives high extraction efficiency particularly at high beam energy.

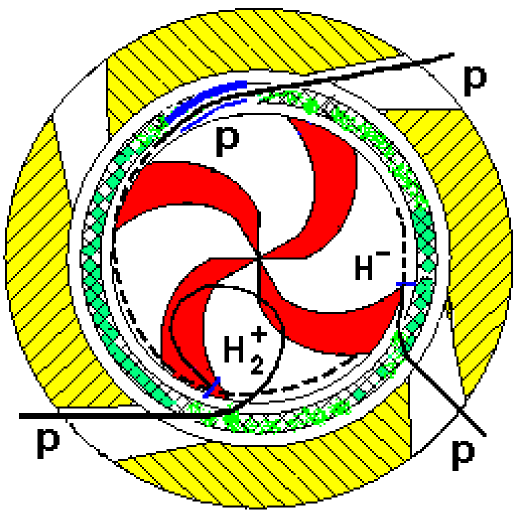

Figure 4: Beam extraction processes used in cyclotrons: electrostatic deflector for proton (or positive ions), stripping for negative hydrogen and molecular hydrogen.

In many cyclotrons, designed to accelerate only protons, negative hydrogen ions (H^-) are accelerated. At a chosen radius in the acceleration chamber a thin foil (usually carbon) is inserted into the circulating beam. As a result, virtually all the H^- ions are converted into protons by stripping. The radius of curvature of the orbit is, therefore, reversed and all the protons bend out of the cyclotron magnet. By changing the radial position of the stripper foil one can also easily change the extraction energy. The extraction efficiency is ~99% or more. Unfortunately, it is not possible to simply increase the magnetic field in the cyclotron to achieve any desired extraction energy due to electromagnetic stripping forces, which tend to remove the outer electron bound only by .775eV and leaves a neutral hydrogen atom. For energies in the range of 20 MeV to 50 MeV, the compact H^- cyclotron is a far more economical choice for high intensity beam than a proton cyclotron.

The compact AVF cyclotrons are widely utilized for medical applications e.g. cancer therapy, isotope production etc. Modern isotope production facilities employ compact H^- ion cyclotrons in the energy rang 10-30 MeV with extracted beam current ~500 μA. For cancer therapy, cyclotrons in the energy range 70-250 MeV with low beam current are utilized.

In order to obtain high intensity proton beams, another way is to accelerate molecular hydrogen H_2^+ beam. The extraction of the beam can be accomplished by a stripper foil, which produces two protons by breaking the molecule. The binding energy of hydrogen molecule is 16.3 eV and it allows utilization of a high magnetic field. Ion sources that produce highly intense beams of H_2^+ should be available.

Beams from AVF Cyclotron

The beam from a cyclotron consists of short pulses arriving at the frequency of the RF oscillator. The time duration and energy width of each pulse are largely dependent on the phase width accepted from the ion source into the acceleration process. The quality of the beam is generally defined by its two transverse emittances (relating position and divergence) and one longitudinal emittance (relating energy resolution and time structure).

Transverse emittance: It is a measure of the correlation between the position and divergence of the particles in the beam. Typical values of horizontal and vertical emittance of the extracted cyclotron beam are ~20 to 80 mm-mrad and vary from cyclotron to cyclotron. The brightness of the central part is intense and thus the emittance can be reduced using a collimator. The emittance of the beam emerging from an accelerator is usually limited by the space available for axial and radial oscillations during acceleration and sometimes by extraction system rather than the characteristics at the injection.

Energy resolution: In an AVF cyclotron under normal operating conditions the beam has energy spread $\Delta E/E \sim 0.5\%$ (FWHM). The major contributing factor to this energy spread comes from the multi-turn extraction nature as a result of overlapping of orbits at the extraction radii. The energy resolution can be improved by using slits at the central region or an external analyzing magnet and slits in the beam line.

Time structure: In a cyclotron, out of the full RF cycle (360^0), acceleration takes place only for a small phase width (~ 40^0 - 60^0). Thus, AVF cyclotrons normally produce a continuous train of nano-second (~5-15 ns) pulses separated in time equal to the period of the RF (~50-200 ns) depending on the RF frequency. The pulse width (FWHM) is generally called as the time structure and is influenced by the accuracy of the isochronous magnetic field and the stability of the dee voltage. For slow experiments, cyclotron appears to yield a continuous beam (100% macro duty cycle) whereas to fast experiment it appears to yield a sharply pulsed beam (1-3% micro duty cycle).

References

[1] J. J. Livingood; **Principles of Cyclic Particle Accelerator**,
D. Van. Nostrand Company, New Jersey (1961).

[2] J. R. Richardson; **Sector Focusing Cyclotron**,
Prog. Nucl. Techniques and Instrumentation (1965)

[3] Martin Reiser; **Theory and Design of Charged particle Beams**,

John Wiley and sons, Inc. New York (1994)

[4] E. J. N. Wilson; **An Introduction to particle accelerators**
Oxford University Press Inc. New York (2001)

[5] Humphries, S., Jr., **Principles of Charged Particle Acceleration,**
John Wiley and Sons, New York (1986) (Freely available on internet)

[6] Proceedings of **CERN Accelerator schools** (Freely available on internet)

[7] S.Y. Lee, **Accelerator Physics**, World Scientific Publishing, Singapore, (1999)