Sekventy 9.6

CVIČENIE 9.6. Pomocou sekventov odvoďte:

a) $\vdash (\neg A \Rightarrow B) \Rightarrow ((A \Rightarrow B) \Rightarrow B)$

b) $\vdash A \Rightarrow (B \Rightarrow (A \wedge B))$

c) $\vdash B \Rightarrow (A \vee B)$

d) $\vdash A \Rightarrow (A \vee B)$

e) $\vdash (A \wedge B) \Rightarrow B$

f) $\vdash (A \wedge B) \Rightarrow A$

g) $\vdash (A \wedge B) \Rightarrow (A \vee B)$

h) $\vdash (A \wedge B) \Rightarrow (B \wedge A)$

i) $\vdash A \Rightarrow (A \wedge A)$

j) $\vdash \neg(A \wedge B) \Rightarrow (\neg A \vee \neg B)$

k) $\vdash (A \wedge (A \vee B)) \Rightarrow A$

l) $\vdash A \Rightarrow (A \wedge (A \vee B))$

m) $\vdash ((A \Rightarrow A) \Rightarrow A) \Rightarrow A$

n) $\vdash (\neg A \Rightarrow A) \Rightarrow A$

o) $\vdash (\neg A \vee \neg B) \Rightarrow \neg(A \wedge B)$

p) $\vdash (A \Rightarrow B) \vee (A \wedge \neg B)$

q) $\vdash (\ldots((\underbrace{A \Rightarrow A) \Rightarrow A) \Rightarrow \ldots) \Rightarrow A}_{2k \text{ } A\text{-čiek}}$

r) $\vdash (\ldots((\neg A \Rightarrow \underbrace{A) \Rightarrow A) \Rightarrow \ldots) \Rightarrow A}_{2k \text{ } A\text{-čiek}}$

s) $\vdash ((A \Rightarrow B) \Rightarrow A) \Rightarrow A$

t) $A \Rightarrow B \vdash (C \vee A) \Rightarrow (C \vee B)$

u) $A \Rightarrow B \vdash (C \wedge A) \Rightarrow (C \wedge B)$

v) $A \Rightarrow B,\ B \Rightarrow C \vdash \neg(A \wedge \neg C)$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}(I)}{\vdash \neg A, A}(\neg L) \quad \dfrac{}{B \vdash B}(I)}{\neg A \Rightarrow B \vdash A, B}(\Rightarrow L) \quad \dfrac{}{B \vdash B}(I)}{\neg A \Rightarrow B, A \Rightarrow B \vdash B, B}(\Rightarrow L)}{\neg A \Rightarrow B, A \Rightarrow B \vdash B}(CR)}{(\neg A \Rightarrow B) \vdash (A \Rightarrow B) \Rightarrow B}(\Rightarrow R)$$

$$\dfrac{(\neg A \Rightarrow B) \vdash (A \Rightarrow B) \Rightarrow B}{\text{a)} \vdash (\neg A \Rightarrow B) \Rightarrow ((A \Rightarrow B) \Rightarrow B)}(\Rightarrow R)$$

$$\dfrac{\dfrac{\dfrac{}{A \vdash A}\,I \qquad \dfrac{}{A \vdash B}\,I}{A, B \vdash A \wedge B}\,(\wedge R)}{A \vdash B \Rightarrow (A \wedge B)}\,(\Rightarrow R)}{\vdash A \Rightarrow (B \Rightarrow (A \wedge B))}\,(\Rightarrow R)$$

b) $\vdash A \Rightarrow (B \Rightarrow (A \wedge B))$

$$\dfrac{\dfrac{\dfrac{}{B \vdash B}\,(I)}{B \vdash A \vee B}\,(\vee R)}{\vdash B \Rightarrow (A \vee B)}\,(\Rightarrow R)$$

c) $\vdash B \Rightarrow (A \vee B)$

$$\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I)}{A \vdash A \vee B}\,(\vee R)}{\vdash A \Rightarrow (A \vee B)}\,(\Rightarrow R)$$

d) $\vdash A \Rightarrow (A \vee B)$

$$\dfrac{\dfrac{\dfrac{}{B \vdash B}\,(I)}{A \wedge B \vdash B}\,(\wedge L_2)}{\vdash (A \wedge B) \Rightarrow B}\,(\Rightarrow R)$$

e) $\vdash (A \wedge B) \Rightarrow B$

$$\dfrac{\dfrac{}{A \vdash A}\,(\mathrm{I})}{A \wedge B \vdash A}\,(\wedge L_1)$$

$$\Rightarrow 2$$

f) $\vdash (A \wedge B) \Rightarrow A$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I)}{A \vdash A \vee B}\,(\vee R_1)}{A \wedge B \vdash A \vee B}\,(\wedge L_1)}{\text{g)}\ \vdash (A \wedge B) \Rightarrow (A \vee B)}\,(\Rightarrow R)$$

$$\dfrac{\dfrac{\dfrac{}{B \vdash B}\,(I)}{A \wedge B \vdash B}\,(\wedge L_2) \quad \dfrac{\dfrac{}{A \vdash A}\,(I)}{A \wedge B \vdash A}\,(\wedge L_1)}{A \wedge B,\ A \wedge B \vdash B \wedge A}\,(\wedge R)$$

$$\dfrac{A \wedge B,\ A \wedge B \vdash B \wedge A}{A \wedge B \vdash B \wedge A}\,(cL)$$

$$\dfrac{A \wedge B \vdash B \wedge A}{\vdash (A \wedge B) \Rightarrow (B \wedge A)}\,(\Rightarrow R)$$

h) $\vdash (A \wedge B) \Rightarrow (B \wedge A)$

$$\dfrac{\dfrac{\dfrac{}{A \vdash A}(I) \qquad \dfrac{}{A \vdash A}(I)}{A, A \vdash A \wedge A}(\wedge R)}{A \vdash A \wedge A}\,CL}{\vdash A \Rightarrow (A \wedge A)}(\Rightarrow R)$$

i) $\vdash A \Rightarrow (A \wedge A)$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I)}{\vdash \neg A, A}\,(\neg R)}{\vdash \neg A \vee \neg B, A}\,(\vee R_1)}{\vdash A, \neg A \vee \neg B}\,PR \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{B \vdash B}\,(I)}{\vdash \neg B, B}\,(\neg R)}{\vdash \neg A \vee \neg B, B}\,(\vee R_2)}{\vdash B, \neg A \vee \neg B}\,PR}{\vdash A \wedge B, \neg A \vee \neg B, \neg A \vee \neg B}\,(\wedge R)}{\vdash A \wedge B, \neg A \vee \neg B}\,(PR, CR, PR)}{\neg(A \wedge B) \vdash \neg A \vee \neg B}\,(\neg L)}{\vdash \neg(A \wedge B) \Rightarrow (\neg A \vee \neg B)}\,(\Rightarrow R)$$

j) $\vdash \neg(A \wedge B) \Rightarrow (\neg A \vee \neg B)$

$$\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I)}{A \wedge (A \vee B) \vdash A}\,(\wedge L_1)}{\mathrm{k})\ \vdash (A \wedge (A \vee B)) \Rightarrow A}\,(\Rightarrow R)$$

$$\dfrac{\dfrac{\dfrac{}{A \vdash A}(I) \qquad \dfrac{\dfrac{}{A \vdash A}(I)}{A \vdash A \vee B}(\vee R_1)}{A, A \vdash A \wedge (A \vee B)}(\wedge R)}{A \vdash A \wedge (A \vee B)}\,CL$$

$$\dfrac{}{\vdash A \Rightarrow (A \wedge (A \vee B))}(\Rightarrow R)$$

1) $\vdash A \Rightarrow (A \wedge (A \vee B))$

$$\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I)}{\vdash A \Rightarrow A}\,(\Rightarrow R) \qquad \dfrac{}{A \vdash A}\,(I)}{\dfrac{}{(A \Rightarrow A) \Rightarrow A \vdash A}\,\Rightarrow L}{\text{m)} \vdash ((A \Rightarrow A) \Rightarrow A) \Rightarrow A}\,\Rightarrow R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I)}{\vdash \neg A, A}\,(\neg R) \qquad \dfrac{}{A \vdash A}\,(I)}{\neg A \Rightarrow A \vdash A, A}\,(\Rightarrow L)}{\neg A \Rightarrow A \vdash A}\,(\text{CR})}{\text{n)}\ \vdash (\neg A \Rightarrow A) \Rightarrow A}\,(\Rightarrow R)$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}(I)}{A \wedge B \vdash A}(\wedge L_1)}{A \wedge B, \neg A \vdash}(\neg L) \qquad \dfrac{\dfrac{\dfrac{}{B \vdash B}(I)}{A \wedge B \vdash B}(\wedge L_2)}{A \wedge B, \neg B \vdash}(\neg L)}{A \wedge B, A \wedge B, \neg A \vee \neg B \vdash}(\vee L)}{\neg A \vee \neg B, A \wedge B, A \wedge B \vdash}PL$$

$$\dfrac{\dfrac{\neg A \vee \neg B, A \wedge B \vdash}{\neg A \vee \neg B \vdash \neg(A \wedge B)}CL}{\vdash (\neg A \vee \neg B) \Rightarrow \neg(A \wedge B)}(\neg R)(\Rightarrow R)$$

o) $\vdash (\neg A \vee \neg B) \Rightarrow \neg(A \wedge B)$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}(I)}{A \vdash B, A}WR}{\vdash A \Rightarrow B, A}\Rightarrow R}{\vdash A, A \Rightarrow B}PR \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{B \vdash B}(I)}{A, B \vdash B}PL, WL}{A \vdash B, \neg B}PR, \neg R}{\vdash A \Rightarrow B, \neg B}\Rightarrow R}{\vdash \neg B, A \Rightarrow B}PR}{\vdash A \wedge \neg B, A \Rightarrow B, A \Rightarrow B}(\wedge R)$$

$$\dfrac{\vdash A \Rightarrow B, A \Rightarrow B, A \wedge \neg B}{\vdash A \Rightarrow B, A \Rightarrow B}PR$$

$$\dfrac{\vdash A \Rightarrow B, A \wedge \neg B}{\vdash A \Rightarrow B, A \wedge \neg B}CR$$

$$\dfrac{\vdash (A \wedge \neg B), (A \Rightarrow B) \vee (A \wedge \neg B)}{\vdash (A \Rightarrow B) \vee (A \wedge \neg B), (A \Rightarrow B) \vee (A \wedge \neg B)}(vR_2)$$

(PR)(vR)

p) $\vdash (A \Rightarrow B) \vee (A \wedge \neg B)$

zzz

$$\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I)}{A \vdash B, A}\,WR}{\vdash A \Rightarrow B, A}\,\Rightarrow R \qquad \dfrac{}{A \vdash A}\,(I)$$

$$\dfrac{\dfrac{\dfrac{}{(A \Rightarrow B) \Rightarrow A \vdash A, A}\,\Rightarrow L}{(A \Rightarrow B) \Rightarrow A \vdash A}\,CR}{\text{s)}\ \vdash ((A \Rightarrow B) \Rightarrow A) \Rightarrow A}\,\Rightarrow R$$

$$\dfrac{\dfrac{}{A \vdash A}(I) \qquad \dfrac{}{B \vdash B}(I)}{\dfrac{A,\ A \Rightarrow B \vdash B}{A \Rightarrow B,\ A \vdash B}(\Rightarrow L)\ (PL)} \qquad \dfrac{}{C \vdash C}(I)$$

$$\dfrac{}{A \Rightarrow B,\ C \vee A \vdash B,\ C}(\vee L)$$

$$\dfrac{}{A \Rightarrow B,\ C \vee A \vdash C \vee B}(CR)\,(\vee R_1)\,(PR)\,(\vee R_2)\,(\Rightarrow R)$$

t) $A \Rightarrow B \vdash (C \vee A) \Rightarrow (C \vee B)$

$$\dfrac{\dfrac{}{A \vdash A}\,(I) \qquad \dfrac{}{B \vdash B}\,(I)}{A,\ A \Rightarrow B \vdash B}\,\Rightarrow L$$

$$\dfrac{\dfrac{}{C \vdash C}\,(I) \qquad A,\ A \Rightarrow B \vdash B}{A \Rightarrow B,\ A \vdash B}\,PL$$

$$\dfrac{}{A \Rightarrow B,\ C,\ A \vdash C \wedge B}\,(\wedge R)$$

$$\dfrac{}{A \Rightarrow B,\ C \wedge A \vdash C \wedge B}\,(CL)(\wedge L_1)(PL)\ (\wedge L_2)$$

$$\dfrac{}{A \Rightarrow B \vdash (C \wedge A) \Rightarrow (C \wedge B)}\,\Rightarrow R$$

u) $A \Rightarrow B \vdash (C \wedge A) \Rightarrow (C \wedge B)$

$$\frac{}{A \vdash A}\,(I) \qquad \frac{}{B \vdash B}\,(I)$$

$$\frac{}{A, A \Rightarrow B \vdash B}\,\Rightarrow L$$

$$\frac{}{A \Rightarrow B, A \vdash B}\,(PL) \qquad \frac{}{C \vdash C}\,(I)$$

$$\frac{}{A \Rightarrow B, A, B \Rightarrow C \vdash C}\,\Rightarrow L$$

$$\frac{}{A \Rightarrow B, A, B \Rightarrow C, \neg C \vdash}\,\neg L$$

$$\frac{}{A \Rightarrow B, B \Rightarrow C, A, \neg C \vdash}\,PL$$

$$\frac{}{A \Rightarrow B, B \Rightarrow C, A \wedge \neg C, A \wedge \neg C \vdash}\,(\wedge L_1)(PL)(\wedge L_2)$$

$$\frac{}{A \Rightarrow B, B \Rightarrow C, A \wedge \neg C \vdash}\,(CL)$$

$$\frac{}{A \Rightarrow B, B \Rightarrow C \vdash \neg(A \wedge \neg C)}\,(\neg R)$$

v) $A \Rightarrow B,\ B \Rightarrow C \vdash \neg(A \wedge \neg C)$