

9.6 a)

(9 kapitola)

$$\begin{array}{l}
 (I) \text{ ---} \\
 (7R) \frac{A \vdash A}{\vdash A \vdash A} \quad \text{---} (I) \\
 \frac{\vdash A \vdash A \quad B \vdash B}{\vdash A \vdash A \quad B \vdash B} (\Rightarrow L) \\
 (CR) \frac{\frac{\frac{A \Rightarrow B \vdash A, A, B}{A \Rightarrow B \vdash A, B} \quad \text{---} (I)}{A \Rightarrow B \vdash A, B} \quad B \vdash B}{A \Rightarrow B, A \Rightarrow B \vdash B, B} (\Rightarrow L) \\
 \frac{A \Rightarrow B, A \Rightarrow B \vdash B, B}{A \Rightarrow B, (A \Rightarrow B) \vdash B} (CR) \\
 \frac{A \Rightarrow B, (A \Rightarrow B) \vdash B}{A \Rightarrow B \vdash ((A \Rightarrow B) \Rightarrow B)} (\Rightarrow R) \\
 \frac{A \Rightarrow B \vdash ((A \Rightarrow B) \Rightarrow B)}{(\vdash A \Rightarrow B) \Rightarrow ((A \Rightarrow B) \Rightarrow B)} (\Rightarrow R)
 \end{array}$$

9.6 b)

9.6 e)

$$\begin{array}{l}
 \text{---} (I) \quad \text{---} (I) \\
 \frac{A \vdash A \quad B \vdash B}{A \vdash A \quad B \vdash B} (\wedge R) \\
 \frac{A, B \vdash A \wedge B}{A \vdash B \Rightarrow A \wedge B} \\
 \frac{A \vdash B \Rightarrow A \wedge B}{A \Rightarrow (B \Rightarrow (A \wedge B))}
 \end{array}$$

$$\begin{array}{l}
 \text{---} (I) \\
 \frac{B \vdash B}{A \wedge B \vdash B} (\wedge L2) \\
 \frac{A \wedge B \vdash B}{\vdash (A \wedge B) \Rightarrow B} (\Rightarrow R)
 \end{array}$$

9.6 c)

9.6 d)

$$\begin{array}{l}
 \text{---} (I) \\
 \frac{B \vdash B}{B \vdash A \vee B} (\vee R2) \\
 \frac{B \vdash A \vee B}{B \Rightarrow (A \vee B)} (\Rightarrow R)
 \end{array}$$

$$\begin{array}{l}
 \frac{A \vdash A}{A \vdash A \vee B} (\vee R1) \\
 \frac{A \vdash A \vee B}{\vdash A \Rightarrow (A \vee B)} (\Rightarrow R)
 \end{array}$$

b)

$$\frac{\frac{\frac{}{A \vdash A}}{A \wedge B \vdash A} (\wedge L_1)}{\vdash (A \wedge B) \Rightarrow A} (\Rightarrow e)$$

g)

$$\begin{array}{r} \text{--- (I)} \\ A \vdash A \quad (VR) \\ \text{---} \\ A \wedge B \quad A \vee B \\ \text{--- (AL)} \\ A \wedge B \vdash A \vee B \\ \text{---} \\ \vdash (A \wedge B) \Rightarrow (A \vee B) \quad (\Rightarrow R) \end{array}$$

h)

~~$$\vdash A \vee B \vdash B \quad (AR)$$

$$A \vee B \vdash B \wedge A, B \wedge A \quad (CR)$$

$$A \vee B \vdash B \wedge A \quad (\Rightarrow R)$$

$$\vdash (A \vee B) \Rightarrow (B \wedge A)$$

zle opisanie~~

$$\begin{array}{c}
 (I) \quad \underline{\quad \quad \quad} \quad \quad \quad \underline{\quad \quad \quad} \quad (I) \\
 (1L2) \quad \underline{B \vdash B} \quad \quad \quad \underline{A \vdash A} \quad (1L1) \\
 \underline{A \wedge B \vdash B} \quad \quad \quad \underline{A \wedge B \vdash A} \quad (1R) \\
 \hline
 A \wedge B \vdash \underline{B \wedge A} \quad (=R) \\
 \vdash (A \wedge B) \Rightarrow (B \wedge A) \quad (=CL)
 \end{array}$$

i)

$$\begin{array}{l} (I) \quad \underline{\quad \quad} \quad \quad (I) \\ A \vdash A \quad A \vdash A \quad (1R) \\ \hline A \vdash (A \wedge A) \\ \hline \vdash A \Rightarrow (A \wedge A) \quad (\Rightarrow R) \end{array}$$

८

$$\begin{array}{l}
 (I) \quad \underline{\quad} \\
 \quad \underline{A \vdash A} \\
 (\neg R) \quad \underline{\vdash \neg A, A} \\
 (\vee R_1) \quad \underline{\vdash \neg A \vee B, A} \\
 (PR) \quad \underline{\vdash A, (\neg A \vee B)} \\
 \quad \underline{\vdash A \wedge B, \neg A \vee B, \neg A \vee B} \quad (PR) \\
 \quad \underline{\vdash \neg A \vee B, \neg A \vee B, A \wedge B} \quad CR \\
 \quad \underline{\vdash \neg A \vee B, A \wedge B} \quad PR \\
 \quad \underline{\vdash (A \wedge B) \Rightarrow \neg A \vee B} \quad (\neg I) \\
 \quad \neg(A \wedge B) \vdash (\neg A \vee B) \\
 \underline{\vdash \neg(A \wedge B) \Rightarrow (\neg A \vee B)} \Rightarrow R
 \end{array}$$

LA

k)

$$\frac{\frac{}{A \vdash A} (I)}{A \vdash A} (\wedge L_1)$$

$$\frac{[A \wedge (A \vee B)] \vdash A}{\vdash (A \wedge (A \vee B)) \Rightarrow A} (\Rightarrow R)$$

l)

$$\frac{\frac{}{A \vdash A} (I)}{A \vdash A} (\vee R_1)$$

$$\frac{A \vdash A \quad A \vdash A \vee B}{A, A \vdash (A \wedge (A \vee B))} (\wedge R)$$

$$\frac{A, A \vdash (A \wedge (A \vee B))}{A \vdash (A \wedge (A \vee B))} (CL)$$

$$\frac{A \vdash (A \wedge (A \vee B))}{\vdash A \Rightarrow (A \wedge (A \vee B))} (\Rightarrow R)$$

m)

$$\frac{\frac{}{A \vdash A} (I)}{\vdash A \Rightarrow A, A} (\Rightarrow R)$$

$$\frac{\vdash A \Rightarrow A, A \quad A \vdash A}{\vdash A \Rightarrow A} (\Rightarrow L)$$

$$\frac{\vdash A \Rightarrow A \quad \vdash A, A}{\vdash A \Rightarrow A} (CR)$$

$$\frac{\vdash A \Rightarrow A \quad \vdash A}{\vdash ((A \Rightarrow A) \Rightarrow A) \Rightarrow A}$$

n)

$$\frac{}{A \vdash A} (I)$$

$$\frac{A \vdash A}{\vdash ?A, A} (?R)$$

$$\frac{\vdash ?A, A \quad A \vdash A}{\vdash ?A \Rightarrow A} (\Rightarrow L)$$

$$\frac{\vdash ?A \Rightarrow A \quad \vdash A, A}{\vdash ?A \Rightarrow A} (CR)$$

$$\frac{\vdash ?A \Rightarrow A \quad \vdash A}{\vdash (?A \Rightarrow A) \Rightarrow A} (\Rightarrow R)$$

$$\sigma) \frac{}{A \vdash A} (I)$$

$$\frac{}{B \vdash B} (I)$$

$$\frac{A \vdash A}{\vdash ?A, A} (?L)$$

$$\frac{B \vdash B}{\vdash ?B, B} (?L)$$

$$\frac{\vdash ?A, A \quad \vdash ?B, B}{\vdash ?A, A \wedge B} (\wedge L_2)$$

$$\frac{\vdash ?A, A \wedge B}{\vdash ?A, A \wedge B} (?R)$$

$$\frac{\vdash ?A, ?(A \wedge B) \quad \vdash ?B, ?(A \wedge B)}{\vdash ?A \vee ?B, ?(A \wedge B)} (\vee L)$$

$$\frac{\vdash ?A \vee ?B, ?(A \wedge B)}{\vdash ?A \vee ?B, ?(A \wedge B)} (CR)$$

$$\frac{\vdash ?A \vee ?B \quad \vdash ?(A \wedge B)}{\vdash (?A \vee ?B) \Rightarrow ?(A \wedge B)} (\Rightarrow R)$$

p)

$$\frac{}{B \vdash B} (I)$$

$$\frac{B \vdash B}{\vdash B, ?B} (?R)$$

$$\frac{\vdash B, ?B}{\vdash A \vdash B, ?B} (WL)$$

$$\frac{\vdash A \vdash B, ?B}{\vdash A \Rightarrow B, ?B} (\Rightarrow R)$$

$$\frac{\vdash A \Rightarrow B, ?B}{\vdash (A \Rightarrow B) \vee (A \wedge ?B), ?B} (\vee R)$$

$$\frac{\vdash (A \Rightarrow B) \vee (A \wedge ?B), ?B}{\vdash A, (A \Rightarrow B) \vee (A \wedge ?B)} (PR)$$

$$\frac{\vdash A, (A \Rightarrow B) \vee (A \wedge ?B)}{\vdash A \wedge ?B, (A \Rightarrow B) \vee (A \wedge ?B)} (\wedge R)$$

$$\frac{\vdash A \wedge ?B, (A \Rightarrow B) \vee (A \wedge ?B)}{\vdash (A \Rightarrow B) \vee (A \wedge ?B), (A \Rightarrow B) \vee (A \wedge ?B)} (\vee R_2)$$

$$\frac{\vdash (A \Rightarrow B) \vee (A \wedge ?B), (A \Rightarrow B) \vee (A \wedge ?B)}{\vdash (A \Rightarrow B) \vee (A \wedge ?B)} (CR)$$

3)

$$\begin{array}{c}
 \frac{B \vdash B}{B, A \vdash B, A} \text{ (CR)(CL)} \\
 \text{(\Rightarrow R)} \frac{A, B, A \vdash B, A}{A \vdash B, A \Rightarrow B, A} \\
 \text{(\Rightarrow R)} \frac{A \vdash B, A \Rightarrow B, A}{\vdash A \Rightarrow B, A \Rightarrow B, A} \\
 \text{(CR)} \frac{\vdash A \Rightarrow B, A \Rightarrow B, A}{\vdash (A \Rightarrow B), A} \text{ (I)} \\
 \frac{\vdash (A \Rightarrow B), A \quad A \vdash A}{(A \Rightarrow B) \Rightarrow A \vdash A, A} \text{ (\Rightarrow L)} \\
 \frac{(A \Rightarrow B) \Rightarrow A \vdash A, A}{(A \Rightarrow B) \Rightarrow A \vdash A} \text{ (CR)} \\
 \vdash ((A \Rightarrow B) \Rightarrow A) \Rightarrow A \text{ (\Rightarrow R)}
 \end{array}$$

4)

$$\begin{array}{c}
 \text{(I)} \frac{}{A \vdash A} \text{ (I)} \frac{}{B \vdash B} \\
 \text{(I)} \frac{}{A, A \Rightarrow B \vdash B} \text{ (PR)} \\
 \text{(VR}_2\text{)} \frac{C \vdash C}{C \vdash C \vee B} \text{ (I)} \frac{A \Rightarrow B, A \vdash B}{A \Rightarrow B, A \vdash C \vee B} \text{ (VR}_1\text{)} \\
 \frac{C \vdash C \vee B \quad A \Rightarrow B, A \vdash C \vee B}{A \Rightarrow B, C \vee A \vdash C \vee B, C \vee B} \text{ (CR)} \\
 \frac{A \Rightarrow B, C \vee A \vdash C \vee B}{A \Rightarrow B, C \vee A \vdash C \vee B} \text{ (\Rightarrow R)} \\
 A \Rightarrow B \vdash (C \vee A) \Rightarrow (C \vee B)
 \end{array}$$

5)

$$\begin{array}{c}
 \text{(I)} \frac{}{A \vdash A} \text{ (I)} \frac{}{C \vdash C} \text{ (I)} \frac{}{B \vdash B} \\
 \text{(AL)} \frac{A \vdash A}{C \wedge A \vdash A} \text{ (AL)} \frac{C \vdash C}{C \wedge A \vdash C} \text{ (PR)} \\
 \frac{C \wedge A \vdash A \quad C \wedge A \vdash C}{C \wedge A, C \wedge A \vdash C \wedge B} \text{ (\Rightarrow L)} \\
 \frac{C \wedge A, C \wedge A \vdash C \wedge B}{A \Rightarrow B, C \wedge A, C \wedge A \vdash C \wedge B} \text{ (PR)(PR)} \\
 \frac{A \Rightarrow B, C \wedge A, C \wedge A \vdash C \wedge B}{A \Rightarrow B, C \wedge A \vdash C \wedge B} \text{ (CL)} \\
 \frac{A \Rightarrow B, C \wedge A \vdash C \wedge B}{A \Rightarrow B \vdash (C \wedge A) \Rightarrow (C \wedge B)} \text{ (\Rightarrow R)}
 \end{array}$$

6)

$$\begin{array}{c}
 \text{(I)} \frac{}{A \vdash A} \text{ (AL)} \frac{}{A \wedge C \vdash A} \\
 \text{(PR)} \frac{A \wedge C \vdash A}{\vdash ?(A \wedge C), A} \\
 \text{(\Rightarrow L)} \frac{\vdash A, ?(A \wedge C) \quad B \vdash B}{A \Rightarrow B \vdash ?(A \wedge C), B} \text{ (I)} \frac{}{C \vdash C} \text{ (I)} \frac{}{C, C \vdash C} \text{ (I)} \frac{}{C, C \vdash C} \text{ (I)} \\
 \text{PR} \frac{A \Rightarrow B \vdash ?(A \wedge C), B}{A \Rightarrow B \vdash B, ?(A \wedge C)} \text{ (PR)} \frac{C, C \vdash C}{C, A \wedge C \vdash C} \text{ (PR)} \\
 \frac{A \Rightarrow B \vdash B, ?(A \wedge C) \quad C, A \wedge C \vdash C}{A \Rightarrow B, B \Rightarrow C \vdash ?(A \wedge C), ?(A \wedge C)} \text{ (\Rightarrow L)} \\
 A \Rightarrow B, B \Rightarrow C \vdash ?(A \wedge C) \text{ CR}
 \end{array}$$