9.6 a)

(9 kapitolas)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}(I)}{\neg A \vdash A}(\neg R) \quad \dfrac{}{B \vdash B}(I)}{\neg A \Rightarrow B \vdash A, B}(\Rightarrow L) \quad \dfrac{}{B \vdash B}(I)}{\neg A \Rightarrow B \vdash A, A, B}(CR)}{\neg A \Rightarrow B \vdash A, B \quad B \vdash B}(\Rightarrow L)}{\neg A \Rightarrow B, A \Rightarrow B \vdash B, B}(CR)}{\neg A \Rightarrow B, (A \Rightarrow B) \vdash B}(\Rightarrow R)}{\neg A \Rightarrow B \vdash ((A \Rightarrow B) \Rightarrow B)}(\Rightarrow R)}{(\neg A \Rightarrow B) \Rightarrow ((A \Rightarrow B) \Rightarrow B)}$$

9.6 b)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}(I) \quad \dfrac{}{B \vdash B}(I)}{A, B \vdash A \wedge B}(\wedge R)}{A \vdash B \Rightarrow A \wedge B}}{A \Rightarrow (B \Rightarrow (A \wedge B))}$$

9.6 c)

$$\dfrac{\dfrac{\dfrac{}{B \vdash B}(I)}{B \vdash A \vee B}(\vee R_2)}{B \Rightarrow (A \vee B)}(\Rightarrow R)$$

9.6 d)

$$\dfrac{\dfrac{\dfrac{}{A \vdash A}(I)}{A \vdash A \vee B}(\vee R_1)}{\vdash A \Rightarrow (A \vee B)}(\Rightarrow R)$$

9.6 e)

$$\dfrac{\dfrac{\dfrac{}{B \vdash B}(I)}{A \wedge B \vdash B}(\wedge L_2)}{\vdash (A \wedge B) \Rightarrow B}(\Rightarrow R)$$

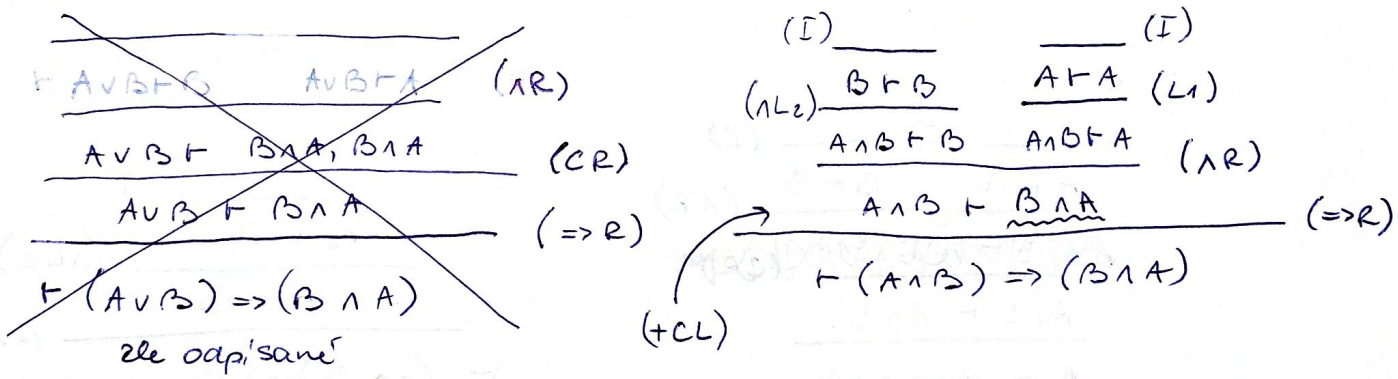
f)

$$\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I)}{A \wedge B \vdash A}\,(\wedge L_1)}{\vdash (A \wedge B) \Rightarrow A}\,(\Rightarrow R)$$

g)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I)}{A \vdash A \vee B}\,(\vee R)}{A \wedge B \vdash A \vee B}\,(\wedge L)}{\vdash (A \wedge B) \Rightarrow (A \vee B)}\,(\Rightarrow R)$$

h)



i)

$$\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I) \quad \dfrac{}{A \vdash A}\,(I)}{A \vdash (A \wedge A)}\,(\wedge R)}{\vdash A \Rightarrow (A \wedge A)}\,(\Rightarrow R)$$

j)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I)}{\vdash \neg A, A}\,(\neg R)}{\vdash \neg A \vee \neg B, A}\,(\vee R_1)}{\vdash A, (\neg A \vee \neg B)}\,(PR) \quad \dfrac{\dfrac{\dfrac{}{B \vdash B}\,(I)}{\vdash \neg B, B}\,(\neg R)}{\vdash \neg A \vee \neg B, B}\,(\vee R_2)}{\vdash B, (\neg A \vee \neg B)}\,(PR)}{\vdash A \wedge B, \neg A \vee \neg B, \neg A \vee \neg B}\,(\wedge R)}{\vdash \neg A \vee \neg B, \neg A \vee \neg B, A \wedge B}\,(PR)}{\vdash \neg A \vee \neg B, A \wedge B}\,CR}{\vdash (A \wedge B), \neg A \vee \neg B}\,PR}{\neg (A \wedge B) \vdash (\neg A \vee \neg B)}\,(\neg L)}{\vdash \neg (A \wedge B) \Rightarrow (\neg A \vee \neg B)}\,(\Rightarrow R)$$

k)

$$\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I)}{[A \wedge (A \vee B)] \vdash A}\,(\wedge L_1)}{\vdash (A \wedge (A \vee B)) \Rightarrow A}\,(\Rightarrow R)$$

l)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I) \quad \dfrac{\dfrac{}{A \vdash A}\,(I)}{A \vdash A \vee B}\,(\vee R_1)}{A, A \vdash (A \wedge (A \vee B))}\,(\wedge R)}{A \vdash (A \wedge (A \vee B))}\,(CL)}{\vdash A \Rightarrow (A \wedge (A \vee B))}\,(\Rightarrow R)$$

m)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I)}{\vdash A \Rightarrow A, A}\,(\Rightarrow R) \quad \dfrac{}{A \vdash A}\,(I)}{(A \Rightarrow A) \Rightarrow A \vdash A, A}\,(\Rightarrow L)}{(A \Rightarrow A) \Rightarrow A \vdash A}\,CR}{((A \Rightarrow A) \Rightarrow A) \Rightarrow A}$$

m)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I)}{\vdash \neg A, A}\,(\neg R) \quad \dfrac{}{A \vdash A}\,I}{\neg A \Rightarrow A \vdash A, A}\,(\Rightarrow L)}{\neg A \Rightarrow A \vdash A}\,(CR)}{\vdash (\neg A \Rightarrow A) \Rightarrow A}\,(\Rightarrow R)$$

o)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I)}{\neg A, A \vdash}\,(\neg L)}{\neg A, A \wedge B \vdash}\,(\wedge L_1)}{\neg A \vdash \neg (A \wedge B)}\,(\neg R) \quad \dfrac{\dfrac{\dfrac{}{B \vdash B}\,(I)}{\neg B, B \vdash}\,(\neg L)}{\neg B, A \wedge B \vdash}\,(\wedge L_2)}{\neg B \vdash \neg (A \wedge B)}\,(\neg R)}{\neg A \vee \neg B \vdash \neg (A \wedge B), \neg (A \wedge B)}\,(\vee L)}{\neg A \vee \neg B \vdash \neg (A \wedge B)}\,(CR)}{\vdash (\neg A \vee \neg B) \Rightarrow \neg (A \wedge B)}\,(\Rightarrow R)$$

p)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I)}{\vdash A, A}\,(WR)}{\vdash A \vdash B, A}\,(\Leftrightarrow R)}{\vdash A \Rightarrow B, A}\,(\Rightarrow R)}{\vdash (A \Rightarrow B) \vee (A \wedge \neg B), A}\,(\vee R_1) \quad \dfrac{\dfrac{\dfrac{\dfrac{}{B \vdash B}\,(I)}{\vdash B, \neg B}\,(\neg R)}{\vdash A \vdash B, \neg B}\,(WL)}{\vdash A \Rightarrow B, \neg B}\,(\Rightarrow R)}{\vdash (A \Rightarrow B) \vee (A \wedge \neg B), \neg B}\,(\vee R)}{\vdash A, (A \Rightarrow B) \vee (A \wedge \neg B)}\,(PR) \quad \vdash \neg B, (A \Rightarrow B) \vee (A \wedge \neg B)}{\vdash A \wedge \neg B, (A \Rightarrow B) \vee (A \wedge \neg B)}\,(\wedge R)}{\vdash (A \Rightarrow B) \vee (A \wedge \neg B), (A \Rightarrow B) \vee (A \vee \neg B)}\,(\vee R_2)}{\vdash (A \Rightarrow B) \vee (A \wedge \neg B)}\,(CR)}{\vdash (A \Rightarrow B) \vee (A \wedge \neg B)}\,(PR)$$

3)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{B \vdash B}{B, A \vdash B, A}\,(CR)(CL)}{A, B, A \vdash B, A}\,(\Rightarrow R)}{A \vdash B, A \Rightarrow B, A}\,(\Rightarrow R)}{\vdash A \Rightarrow B, A \Rightarrow B, A}\,(CR) \quad \dfrac{}{A \vdash A}\,(I)}{\vdash (A \Rightarrow B), A}\,(\Rightarrow L)}{(A \Rightarrow B) \Rightarrow A \vdash A, A}\,(CR)}{(A \Rightarrow B) \Rightarrow A \vdash A}\,(\Rightarrow R)}{\vdash ((A \Rightarrow B) \Rightarrow A) \Rightarrow A}$$

4)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I) \quad \dfrac{}{B \vdash B}\,(I)}{A, A \Rightarrow B \vdash B}\,(PR)}{A \Rightarrow B, A \vdash B}\,(vR_1)}{A \Rightarrow B, A \vdash C \vee B}\,(vL) \quad \dfrac{\dfrac{}{C \vdash C}\,(I)}{C \vdash C \vee B}\,(vR_2)}{A \Rightarrow B, C \vee A \vdash C \vee B, C \vee B}\,(CR)}{A \Rightarrow B, C \vee A \vdash C \vee B}\,(CR)}{A \Rightarrow B \vdash (C \vee A) \Rightarrow (C \vee B)}\,(\Rightarrow R)$$

w)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I)}{C \wedge A \vdash A}\,(\wedge L_1) \quad \dfrac{\dfrac{}{C \vdash C}\,(I)}{C \wedge A \vdash C}\,(\wedge L_2) \quad \dfrac{}{B \vdash B}\,(I)}{C \wedge A, B \vdash C \wedge B}\,(\wedge R)}{C \wedge A, C \wedge A, A \Rightarrow B \vdash C \wedge B}\,(\Rightarrow L)}{A \Rightarrow B, C \wedge A, C \wedge A \vdash C \wedge B}\,(PR)(PR)}{A \Rightarrow B, C \wedge A \vdash C \wedge B}\,(CL)}{A \Rightarrow B \vdash (C \wedge A) \Rightarrow (C \wedge B)}\,(\Rightarrow R)$$

N)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,(I)}{A \wedge \neg C \vdash A}\,(\wedge L)}{\vdash \neg(A \wedge \neg C), A}\,(\neg R) \quad \dfrac{\dfrac{}{B \vdash B}\,(I)}{\vdash A, \neg(A \wedge \neg C), B}\,(PR)}{A \Rightarrow B \vdash \neg(A \wedge \neg C), B}\,(\Rightarrow L) \quad \dfrac{\dfrac{\dfrac{}{C \vdash C}\,(I)}{C, \neg C \vdash}\,(\neg L)}{C, A \wedge \neg C \vdash}\,(\wedge L)}{C \vdash \neg(A \wedge \neg C)}\,(\neg R)}{A \Rightarrow B, B \Rightarrow C \vdash \neg(A \wedge \neg C), \neg(A \wedge \neg C)}\,(\Rightarrow L)}{A \Rightarrow B, B \Rightarrow C \vdash \neg(A \wedge \neg C)}\,CR$$