5 kapitola

Prepisy:

$(A \Rightarrow B) = (\neg A \vee B)$

$= \neg(a \wedge \bar{b})$

$A \wedge B = \neg(A \Rightarrow \neg B)$

$A \vee B = (\neg A \Rightarrow B)$

$T_1(a) = a$

$T_2(a) = \neg a$

$T_{12} = \{\emptyset\}$ máme výsledok prázdna klauzula

$T_1 : \emptyset$

$T_2 : c \vee d$

$T_{12} : \emptyset$ prázdna množina klauzúl

DNF $\emptyset$ prázdna množina klauzúl → nieje tautológia

$\{\emptyset\}$ prázdna klauzula → tautológia

KNF $\{\emptyset\}$ → kontradikcia $|T_1| \cdot |T_2| \neq \emptyset$

$\emptyset$ → splniteľná $|T_1| \cdot |T_2| = \emptyset$

5.2. a) $(a \vee \neg b) \wedge (\neg a \vee c) \wedge (a \vee b \vee c)$

$T_0(a) : \emptyset$

$T_1(a) : a \vee \neg b,\ a \vee b \vee c$

$T_2(a) : \neg a \vee c$

$T_{12}(a) : \neg b \vee c,\ b \vee c$

$\tilde{T}(a) : \neg b \vee c,\ b \vee c$

$T_0(b) : \emptyset$

$T_1(b) : b \vee c$

$T_2(b) : \neg b \vee c$

$T_{12}(b) : c$

$\tilde{T}(b) : c$

$T_0 : \emptyset$

$T_1 : c$

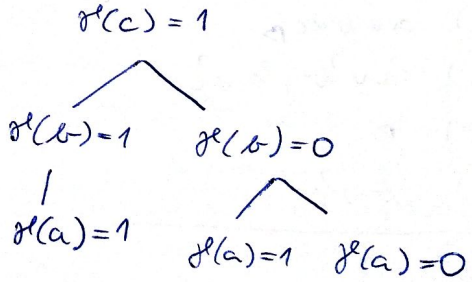
$T_2 : \emptyset$

$T_{12} : \emptyset$

$\tilde{T} : \emptyset$ množina prázdna klauzúl → splniteľná

modely:

0	0	1
1	0	1
1	1	1
#	0	1
1	1	1



b) $(a \vee \neg c) \wedge (\neg b \vee c) \wedge (\neg a \vee \neg b) \wedge b$

$T_0(a)$: $\neg b \vee c$, b

$T_1(a)$: $a \vee \neg c$,

$T_2(a)$: $\neg a \vee \neg b$

$T_{12}(a)$: $\neg b \vee \neg c$

$\tilde{T}(a)$: $\neg b \vee \neg c$, $\neg b \vee c$, b

$T_0(b)$: $\emptyset$

$T_1(b)$: b

$T_2(b)$: $\neg b \vee \neg c$, $\neg b \vee c$

$T_{12}(b)$: $\neg c$, c

$\tilde{T}$: $\neg c$, c

$T_0(c)$: $\emptyset$

$T_1(c)$: c

$T_2(c)$: $\neg c$

T_{12}: $\{\emptyset\}$ prázdna množina klauzúl

formula je kontradikcia

c) $(a \vee b \vee c) \wedge (\neg a \vee \neg b) \wedge (\neg b \vee \neg c) \wedge (\neg a \vee \neg c)$

$T_0(a)$: $\neg b \vee \neg c$

$T_1(a)$: $a \vee b \vee c$

$T_2(a)$: $\neg a \vee \neg b$, $\neg a \vee \neg c$

$T_{12}(a)$: $\emptyset$

$\tilde{T}(a)$: $\neg b \vee \neg c$

$T_0(b)$: $\emptyset$

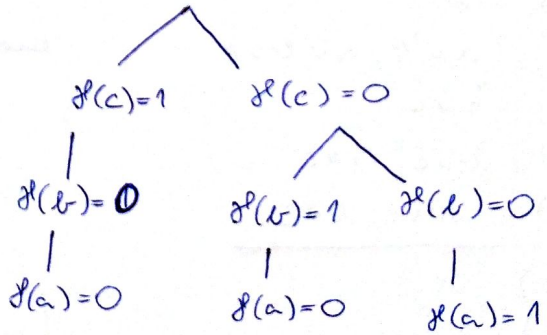
$T_1(b)$: $\emptyset$

$T_2(b)$: $\neg b \vee \neg c$

$T_{12}(b)$: $\emptyset$

$\tilde{T}(b)$: $\emptyset$ množina prázdna klauzúl,

formula je splniteľná



modely: 100
010
001

d) $(a \vee \neg b) \wedge (b \vee \neg c) \wedge (c \vee \neg a) \wedge (a \vee b \vee c) \wedge (\neg a \vee \neg b \vee \neg c)$

$T_0(a)$: $b \vee \neg c$,

$T_1(a)$: $a \vee \neg b$, $a \vee b \vee c$

$T_2(a)$: $c \vee \neg a$, $\neg a \vee \neg b \vee \neg c$

$T_{12}(a)$: $\neg b \vee c$, $\neg b \vee \neg c$, $b \vee c$

$\tilde{T}(a)$: $\neg b \vee c$, $\neg b \vee \neg c$, $b \vee c$

$\mathcal{I}(c) = 1$

$\mathcal{I}(b) = 0$

$\mathcal{I}(a) = 1$ $\mathcal{I}(a) = 0$

#01

$T_0(b)$: $\varnothing$

$T_1(b)$: $b \vee c$

$T_2(b)$: $\neg b \vee c$, $\neg b \vee \neg c$

$T_{12}(b)$: c

$\tilde{T}(b)$: c

$T_0(c)$: $\varnothing$

$T_1(c)$: c

$T_2(c)$: $\varnothing$

$\tilde{T}$: $\varnothing$ množina prázdna klauzule, splniteľná

5.2. e) $(a \vee b) \wedge (c \vee d) \wedge (\neg a \vee \neg c) \wedge (\neg b \vee \neg d) \wedge (\neg a \vee \neg d) \wedge (\neg b \vee \neg c)$

$T_0(a)$: $c \vee d$, $\neg b \vee \neg d$, $\neg b \vee \neg c$

$T_1(a)$: $a \vee b$, KNF

$T_2(a)$: $\neg a \vee \neg c$, $\neg a \vee \neg d$

$T_{12}(a)$: $b \vee \neg c$, $b \vee \neg d$

$\tilde{T}(a)$: $c \vee d$, $\neg b \vee \neg d$, $\neg b \vee \neg c$, $b \vee \neg c$, $b \vee \neg d$

$T_0(b)$: $c \vee d$,

$T_1(b)$: $b \vee \neg c$, $b \vee \neg d$

$T_2(b)$: $\neg b \vee \neg d$, $\neg b \vee \neg c$

$T_{12}(b)$: $\neg c \vee \neg d$, $\neg c$, $\neg d$

$\tilde{T}(b)$: $c \vee d$, $\neg c \vee \neg d$, $\neg c$, $\neg d$

$T_0(c)$: $\neg d$

$T_1(c)$: $c \vee d$

$T_2(c)$: $\neg c \vee \neg d$, $\neg c$

$T_{12}(c)$: $\{d,\ d \vee \neg d\}$

$\tilde{T}(c) = \{d, \neg d\}$

$T_0(d)$: $\varnothing$

$T_1(d)$: d

$T_2(d)$: $\neg d$

$T_{12}(d)$: $\{\square\}$ prázdna klauzula kontradikcia

2.5. b) $T_0(a): c \vee d,\ b \vee \neg d$

$T_1(a): a \vee \neg b \vee c \vee d,\ a \vee \neg c \vee d,\ a \vee b$

$T_2(a): \neg a \vee b \vee c,$

$T_{12}(a): b \vee c$

$\tilde{T}(a): b \vee c,\ c \vee d,\ b \vee \neg d$

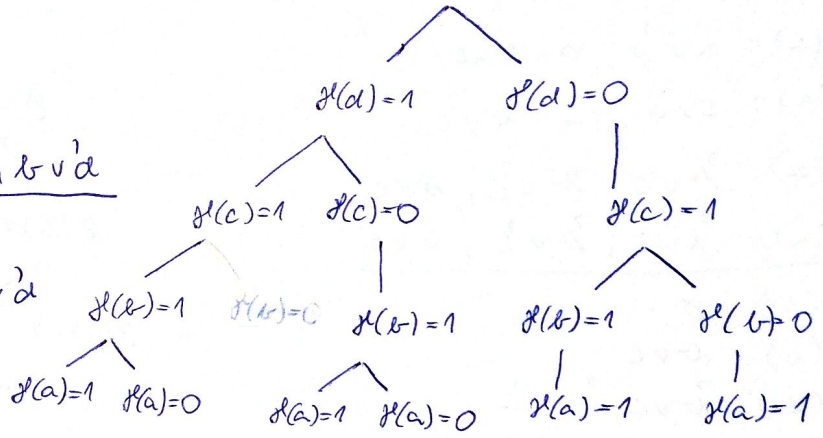
$T_0(b): c \vee d$

$T_1(b): b \vee c,\ b \vee \neg d$

$T_2(b): \emptyset$

$T_{12}: \emptyset$

$\tilde{T}: (c \vee d)$



$T_0(c): \emptyset$

$T_1(c): c \vee d$

$T_2(c): \emptyset$

$T_{12}: \emptyset$

$\tilde{T}(c): \emptyset$ množina prázdna klauzúl, formula je splniteľná

modely: 1010, 1110, 0101, 1101, 0111, 1111

1#10, #101, #111 => 1#10 => #1#1

2.6. a) $T \models A$

$T = \{a \vee b,\ a \vee c,\ b \vee c\}$ (KNF)

$A: \neg(A \vee b \vee c)$

je $T \models A$ kontradikcia?

$(a \vee b) \wedge (a \vee c) \wedge (b \vee c) \wedge (a \vee b \vee c)$

$T_0(a): \{b \vee c\}$

$T_1(a): \{a \vee b,\ a \vee c,\ a \vee b \vee c\}$

$T_2(a): \emptyset$

$T_{12}(a): \emptyset$

$\tilde{T}(a): \{b \vee c\}$

$T_0(b): \emptyset$

$T_1(b): \{b \vee c\}$

$T_2(b): \emptyset$

$T_{12}(b): \emptyset$

$\tilde{T}(b): \emptyset$ množina prázdna klauzúl, skúmaná KNF je splniteľná a teda nieje tautologickým dôsledkom

(splniteľné formuly v negácii by nebolo splnené v pôvodnej KNF)

2.G. b) $T = \{a \vee b,\ b \vee c,\ a \vee c\}$

$A = a \vee b \vee c$

$(a \vee b) \wedge (b \vee c) \wedge (a \vee c) \wedge \neg(a \vee b \vee c)$

$\neg a \wedge \neg b \wedge \neg c$

$T_0(a)$: $b \vee c,\ \neg b,\ \neg c$

$T_1(a)$: $a \vee b,\ a \vee c$

$T_2(a)$: $\neg a$

$T_{12}(a)$: $\{b, c\}$

$\hat{T}(a)$: $\{b \vee c,\ b,\ \neg b,\ c,\ \neg c\}$

$T_0(b)$: $c, \neg c$

$T_1(b)$: $b \vee c,\ b$

$T_2(b)$: $\neg b$

$T_{12}(b)$: $c,\ b$

$\tilde{T}(b)$: $\{c, \neg c\}$

$T_0(c)$: $\emptyset$

$T_1(c)$: c

$T_2(c)$: $\neg c$

$T_{12}(c)$: $\{\emptyset\}$ prázdna množina

$\tilde{T}(c)$: $\{\emptyset\}$ vlastnosť, KNF je kontradikcia

teda A je taut. dôsledkom T

2.6 c) $T = \{ a \vee \neg b,\ c \vee \neg d,\ b \vee d \}$

$A = (d \Rightarrow c)$

$A = (\neg d \vee c)$

$T \models \neg A \qquad \neg A = \neg(\neg d \vee c) = \neg c \wedge d$

$T_0(a) : c \vee \neg d,\ b \vee d,\ \neg c,\ d$

$T_1(a) : a \vee \neg b$

$T_2(a) : \emptyset$

$T_{12}(a) : \emptyset$

$\tilde{T}(a) = T_0(a)$

$T_0(b) : c \vee \neg d,\ \neg c,\ d$

$T_1(b) : b \vee d \qquad T_2(b) : \emptyset \qquad T_{12}(b) : \emptyset$

$\tilde{T}(b) = T_0(b)$

$T_0(c) : d$

$T_1(c) : c \vee \neg d \qquad T_2(c) : \neg c \qquad T_{12}(c) : \neg d$

$\tilde{T}(c) : \{ d, \neg d \}$

$T_0(d) : \emptyset$

$T_1(d) : d \qquad T_2(d) : \neg d \qquad T_{12}(d) : \{ \square \}$

prázdna klauzula

tautologický dôsledok

4. d)

$T = \{ a \Rightarrow b,\ b \Rightarrow \neg a,\ b \Rightarrow a \}$

$A : \neg a \wedge \neg b \longrightarrow \neg A : (a \vee b)$

$(\neg a \vee b) \wedge (\neg b \vee \neg a) \wedge (\neg b \vee a) \wedge (a \vee b)$

$T_0(a) : \varnothing$

$T_1(a) : a \vee \neg b,\ a \vee b$

$T_2(a) : \neg a \vee b,\ \neg a \vee \neg b$

$T_{12}(a) : \neg b,\ b$

$\tilde{T}(a) : \{ b, \neg b \}$

$T_0(b) : \varnothing$

$T_1(b) : b$

$T_2(b) : \neg b$

$T_{12}(b) : \{ \varnothing \}$

$\tilde{T}(b) : \{ \varnothing \}$ prázdna klauzula

$T \vdash A$

5. 4 a)

$(a \wedge b) \vee (a \wedge \neg b) \vee (\neg a)$

$T_0(a) : \varnothing$

$T_1(a) : a \wedge b,\ a \wedge \neg b$

$T_2(a) : \neg a$

$T_{12}(a) : b, \neg b$

$\tilde{T}(a) : \{ b, \neg b \}$

$T_0(b) : \varnothing$ $T_1(b) : b$ $T_2(b) : \neg b$

$T_{12}(b) : \{ \varnothing \}$

$\tilde{T}(b) : \{ \varnothing \}$ prázdna klauzula → taut.

b) $(a \wedge \neg b) \vee (\neg a \wedge b) \vee (a \wedge b \wedge c) \vee (\neg a \wedge \neg b \wedge \neg c) \vee (a \wedge \neg b \wedge c)$

$T_0(a) : \varnothing$

$T_1(a) : a \wedge \neg b,\ a \wedge b \wedge c,\ a \wedge \neg b \wedge c$

$T_2(a) : \neg a \wedge b,\ \neg a \wedge \neg b \wedge \neg c$

$T_{12}(a) : \neg b \wedge \neg c,\ b \wedge c,$

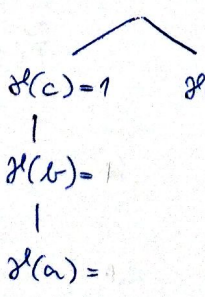
$\tilde{T}(a) : \{ \neg b \wedge \neg c,\ b \wedge c \}$

$T_0(b) : \varnothing$ $T_1(b) : b \wedge c$ $T_2(b) : \neg b \wedge \neg c$

$T_{12}(b) : \{ \varnothing \}$ $\tilde{T}(b) : \{ \varnothing \}$

prázdna na klauzúl => nieje taut.

! $\tilde{T}(b) = 0 \cdot 0 = 0 = \varnothing$!



kedy neplatí?

110
011

✓ 5.4 c)

$(a \wedge c) \vee (\neg a \wedge \neg b) \vee (\neg b \wedge \neg c) \vee (\neg a \wedge \neg b \wedge c) \vee (a \wedge b \wedge \neg c)$

$T_0(a)$: $\neg b \wedge \neg c$

$T_1(a)$: $a \wedge c$, $a \wedge b \wedge \neg c$

$T_2(a)$: $\neg a \wedge \neg b$, $\neg a \wedge \neg b \wedge c$

$T_{12}(a)$: $\neg b \wedge c$, ~~$\neg b \wedge c$~~

$\tilde{T}(a)$: $\neg b \wedge c$

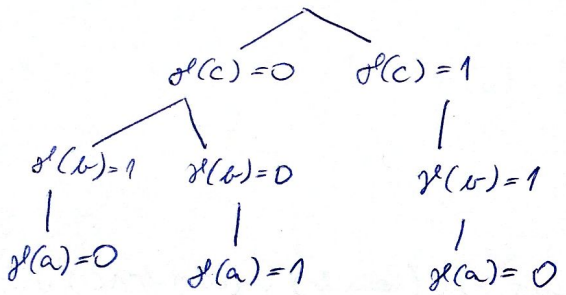
$T_0(b)$: $\emptyset$

$T_1(b)$: $\emptyset$

$T_2(b)$: $\neg b \wedge c$

$T_{12}(b)$: $\emptyset$

$\tilde{T}(b)$: $\emptyset$ prázdna množina klauzúl - nieje tautológia



✓ 5.4. d) $(a \wedge c) \vee (b \wedge c) \vee (\neg b \wedge \neg c) \vee (\neg a \wedge b) \vee (\neg b \wedge c) \vee (\neg a \wedge \neg b \wedge \neg c)$

$T_0(a)$: $b \wedge c$, $\neg b \wedge \neg c$, $\neg b \wedge c$

$T_1(a)$: $(a \wedge c)$,

$T_2(a)$: $\neg a \wedge b$, $\neg a \wedge \neg b \wedge \neg c$

$T_{12}(a)$: $b \wedge c$, ~~$\neg b \wedge c \wedge \neg c$~~

$\tilde{T}$: $b \wedge c$, $\neg b \wedge \neg c$, $\neg b \wedge c$

$T_0(b)$: $\emptyset$

$T_1(b)$: $b \wedge c$

$T_2(b)$: $\neg b \wedge \neg c$, $\neg b \wedge c$

$T_{12}(b)$: c

$\tilde{T}(b)$: c

$T_0(c)$: $\emptyset$

$T_1(c)$: c

$T_2(c)$: $\emptyset$

$T_{12}(c)$: $\emptyset$

$\tilde{T}(c)$: $\emptyset$ prázdna mn. klauzúl - nieje tautológia