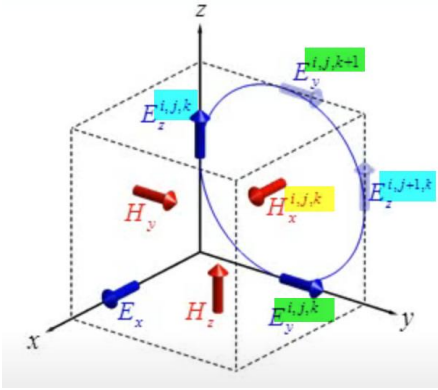
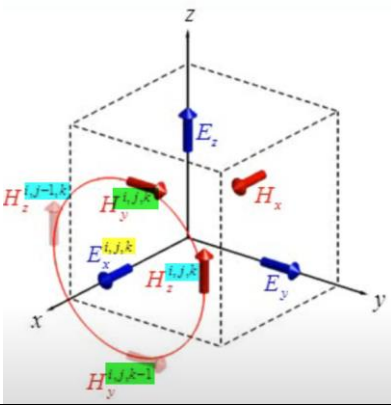
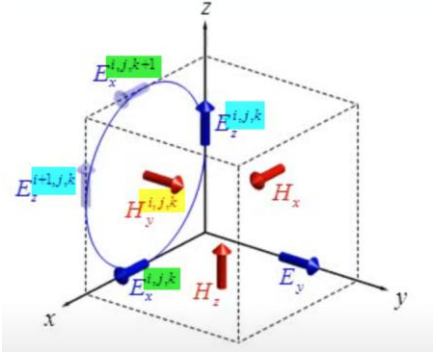
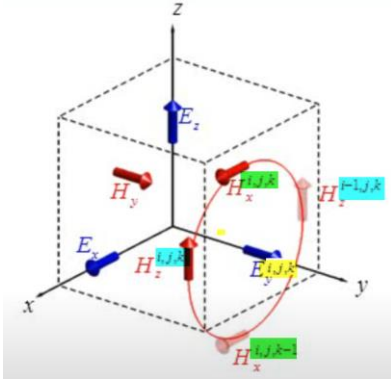
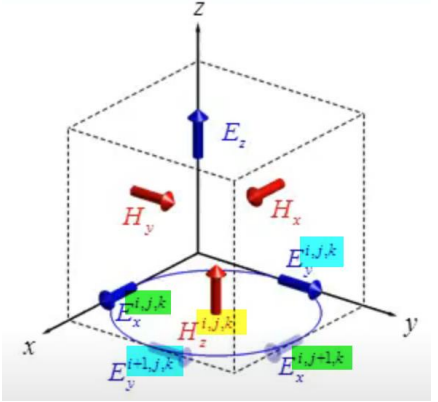
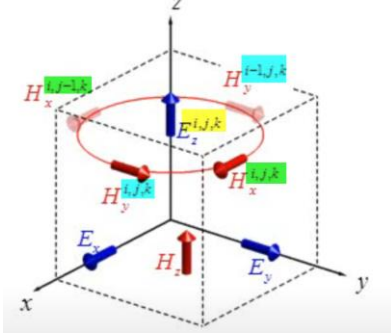


Normalize Maxwell's Equations	$[a] - Tensor$ $\vec{v} - vector$		
$\nabla \times \vec{E} = -[\mu] \frac{\partial \vec{H}}{\partial t}$	$\nabla \times \vec{H} = [\epsilon] \frac{\partial \vec{E}}{\partial t}$	$\nabla \times \vec{E} = -[\mu] \frac{\partial \vec{H}}{\partial t}$	$\nabla \times \vec{H} = [\epsilon] \frac{\partial \vec{E}}{\partial t}$
let: $\vec{\tilde{H}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \vec{H}$	or: $\vec{H} = \sqrt{\frac{\epsilon_0}{\mu_0}} \vec{\tilde{H}}$	let: $\vec{\tilde{E}} = \sqrt{\frac{\epsilon_0}{\mu_0}} \vec{E}$	or: $\vec{E} = \sqrt{\frac{\mu_0}{\epsilon_0}} \vec{\tilde{E}}$
Substitute: $\nabla \times \vec{E} = -[\mu] \frac{\partial \left( \sqrt{\frac{\epsilon_0}{\mu_0}} \vec{\tilde{H}} \right)}{\partial t}$	And: $\nabla \times \left( \sqrt{\frac{\epsilon_0}{\mu_0}} \vec{\tilde{H}} \right) = [\epsilon] \frac{\partial \vec{E}}{\partial t}$	Substitute: $\nabla \times \sqrt{\frac{\mu_0}{\epsilon_0}} \vec{\tilde{E}} = -[\mu] \frac{\partial (\vec{H})}{\partial t}$	And: $\nabla \times \vec{H} = [\epsilon] \frac{\partial \left( \sqrt{\frac{\mu_0}{\epsilon_0}} \vec{\tilde{E}} \right)}{\partial t}$
$\nabla \times \vec{E} = -[\mu_r \mu_0] \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{\partial (\vec{\tilde{H}})}{\partial t}$	$\nabla \times \vec{\tilde{H}} = -\sqrt{\frac{\mu_0}{\epsilon_0}} [\epsilon_r \epsilon_0] \frac{\partial \vec{E}}{\partial t}$	$\nabla \times \vec{\tilde{E}} = -[\mu_r \mu_0] \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{\partial (\vec{H})}{\partial t}$	$\nabla \times \vec{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} [\epsilon_r \epsilon_0] \frac{\partial \vec{\tilde{E}}}{\partial t}$
$\nabla \times \vec{E} = -[\mu_r] \sqrt{\mu_0 \epsilon_0} \frac{\partial (\vec{\tilde{H}})}{\partial t}$	$\nabla \times \vec{\tilde{H}} = \sqrt{\mu_0 \epsilon_0} [\epsilon_r] \frac{\partial \vec{E}}{\partial t}$	$\nabla \times \vec{\tilde{E}} = -[\mu_r] \sqrt{\mu_0 \epsilon_0} \frac{\partial (\vec{H})}{\partial t}$	$\nabla \times \vec{H} = \sqrt{\mu_0 \epsilon_0} [\epsilon_r] \frac{\partial \vec{\tilde{E}}}{\partial t}$
recall: $c_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$		recall: $c_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$	Also let: $\vec{\tilde{E}} = \frac{\vec{\tilde{D}}}{\epsilon_r}$
$\nabla \times \vec{E} = -\frac{[\mu_r]}{\underset{c_0}{c_0}} \frac{\partial (\vec{\tilde{H}})}{\partial t}$	$\nabla \times \vec{\tilde{H}} = \frac{[\epsilon_r]}{\underset{c_0}{c_0}} \frac{\partial \vec{E}}{\partial t}$	$\nabla \times \vec{\tilde{E}} = -\frac{[\mu_r]}{\underset{c_0}{c_0}} \frac{\partial (\vec{H})}{\partial t}$	$\nabla \times \vec{H} = \frac{[\epsilon_r]}{\underset{c_0}{c_0}} \frac{\partial \vec{\tilde{E}}}{\partial t} = \frac{1}{\underset{c_0}{c_0}} \frac{\partial \vec{\tilde{D}}}{\partial t}$

Expansion of spatial component (Analytic)				Expansion of temporal component (Analytic)			
$\nabla \times \vec{\tilde{E}} = \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{bmatrix}$		$\hat{x} \left[ \frac{\partial \tilde{E}_z}{\partial y} - \frac{\partial \tilde{E}_y}{\partial z} \right] - \hat{y} \left[ \frac{\partial \tilde{E}_z}{\partial x} - \frac{\partial \tilde{E}_x}{\partial z} \right] + \hat{z} \left[ \frac{\partial \tilde{E}_y}{\partial x} - \frac{\partial \tilde{E}_x}{\partial y} \right]$ or $\begin{bmatrix} \frac{\partial \tilde{E}_z}{\partial y} - \frac{\partial \tilde{E}_y}{\partial z} \\ \frac{\partial \tilde{E}_z}{\partial x} - \frac{\partial \tilde{E}_x}{\partial z} \\ \frac{\partial \tilde{E}_y}{\partial x} - \frac{\partial \tilde{E}_x}{\partial y} \end{bmatrix}$		$-\frac{[\mu_r]}{c_0} \frac{\partial (\vec{H})}{\partial t} = -\frac{\begin{bmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{bmatrix} \begin{bmatrix} \frac{\partial H_x}{\partial t} \\ \frac{\partial H_y}{\partial t} \\ \frac{\partial H_z}{\partial t} \end{bmatrix}}{c_0} = -\frac{1}{c_0} \begin{bmatrix} \mu_{xx} \frac{\partial H_x}{\partial t} + \mu_{xy} \frac{\partial H_y}{\partial t} + \mu_{xz} \frac{\partial H_z}{\partial t} \\ \mu_{yx} \frac{\partial H_x}{\partial t} + \mu_{yy} \frac{\partial H_y}{\partial t} + \mu_{yz} \frac{\partial H_z}{\partial t} \\ \mu_{zx} \frac{\partial H_x}{\partial t} + \mu_{zy} \frac{\partial H_y}{\partial t} + \mu_{zz} \frac{\partial H_z}{\partial t} \end{bmatrix}$			
$\nabla \times \vec{H} = \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{bmatrix}$		$\hat{x} \left[ \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right] - \hat{y} \left[ \frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} \right] + \hat{z} \left[ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right]$ or $\begin{bmatrix} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \\ \frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \end{bmatrix}$		$\frac{[\epsilon_r]}{c_0} \frac{\partial \vec{\tilde{E}}}{\partial t} = \frac{\begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \begin{bmatrix} \frac{\partial \tilde{E}_x}{\partial t} \\ \frac{\partial \tilde{E}_y}{\partial t} \\ \frac{\partial \tilde{E}_z}{\partial t} \end{bmatrix}}{c_0} = \frac{1}{c_0} \begin{bmatrix} \epsilon_{xx} \frac{\partial \tilde{E}_x}{\partial t} + \epsilon_{xy} \frac{\partial \tilde{E}_y}{\partial t} + \epsilon_{xz} \frac{\partial \tilde{E}_z}{\partial t} \\ \epsilon_{yx} \frac{\partial \tilde{E}_x}{\partial t} + \epsilon_{yy} \frac{\partial \tilde{E}_y}{\partial t} + \epsilon_{yz} \frac{\partial \tilde{E}_z}{\partial t} \\ \epsilon_{zx} \frac{\partial \tilde{E}_x}{\partial t} + \epsilon_{zy} \frac{\partial \tilde{E}_y}{\partial t} + \epsilon_{zz} \frac{\partial \tilde{E}_z}{\partial t} \end{bmatrix}$			
Combine Like terms							
1)	$\frac{\partial \tilde{E}_z}{\partial y} - \frac{\partial \tilde{E}_y}{\partial z} =$	$\frac{\mu_{xx}}{c_0} \frac{\partial H_x}{\partial t} + \frac{\mu_{xy}}{c_0} \frac{\partial H_y}{\partial t} + \frac{\mu_{xz}}{c_0} \frac{\partial H_z}{\partial t}$		$\overbrace{\frac{\partial \tilde{E}_z}{\partial y} - \frac{\partial \tilde{E}_y}{\partial z}}^{Spatial} =$	$\overbrace{\frac{\mu_{xx}}{c_0} \frac{\partial H_x}{\partial t}}^{Temporal}$	Temporal Derivative: <a href="https://youtu.be/TbzFef5gUms?si=RskghzimP8cbLjer&amp;t=2488">https://youtu.be/TbzFef5gUms?si=RskghzimP8cbLjer&amp;t=2488</a> Spatial Derivative: <a href="https://youtu.be/sxv_L4rscL0?si=CZmMD1BDGB6J-39e&amp;t=1150">https://youtu.be/sxv_L4rscL0?si=CZmMD1BDGB6J-39e&amp;t=1150</a>	
2)	$\frac{\partial \tilde{E}_x}{\partial z} - \frac{\partial \tilde{E}_z}{\partial x} =$	$\frac{\mu_{yx}}{c_0} \frac{\partial H_x}{\partial t} + \frac{\mu_{yy}}{c_0} \frac{\partial H_y}{\partial t} + \frac{\mu_{yz}}{c_0} \frac{\partial H_z}{\partial t}$		$\frac{\partial \tilde{E}_x}{\partial z} - \frac{\partial \tilde{E}_z}{\partial x} =$	$\frac{\mu_{yy}}{c_0} \frac{\partial H_y}{\partial t}$		
3)	$\frac{\partial \tilde{E}_y}{\partial x} - \frac{\partial \tilde{E}_x}{\partial y} =$	$\frac{\mu_{zx}}{c_0} \frac{\partial H_x}{\partial t} + \frac{\mu_{zy}}{c_0} \frac{\partial H_y}{\partial t} + \frac{\mu_{zz}}{c_0} \frac{\partial H_z}{\partial t}$	Assume Diagonal $\epsilon$ & $\mu$ Tensor $\rightarrow$	$\frac{\partial \tilde{E}_y}{\partial x} - \frac{\partial \tilde{E}_x}{\partial y} =$	$\frac{\mu_{zz}}{c_0} \frac{\partial H_z}{\partial t}$		
4)	$\frac{\partial \tilde{H}_z}{\partial y} - \frac{\partial \tilde{H}_y}{\partial z} =$	$\frac{\epsilon_{xx}}{c_0} \frac{\partial \tilde{E}_x}{\partial t} + \frac{\epsilon_{xy}}{c_0} \frac{\partial \tilde{E}_y}{\partial t} + \frac{\epsilon_{xz}}{c_0} \frac{\partial \tilde{E}_z}{\partial t}$		$\frac{\partial \tilde{H}_z}{\partial y} - \frac{\partial \tilde{H}_y}{\partial z} =$	$\frac{\epsilon_{xx}}{c_0} \frac{\partial \tilde{E}_x}{\partial t}$		
5)	$\frac{\partial \tilde{H}_x}{\partial z} - \frac{\partial \tilde{H}_z}{\partial x} =$	$\frac{\epsilon_{yx}}{c_0} \frac{\partial \tilde{E}_x}{\partial t} + \frac{\epsilon_{yy}}{c_0} \frac{\partial \tilde{E}_y}{\partial t} + \frac{\epsilon_{yz}}{c_0} \frac{\partial \tilde{E}_z}{\partial t}$		$\frac{\partial \tilde{H}_x}{\partial z} - \frac{\partial \tilde{H}_z}{\partial x} =$	$\frac{\epsilon_{yy}}{c_0} \frac{\partial \tilde{E}_y}{\partial t}$		
6)	$\frac{\partial \tilde{H}_y}{\partial x} - \frac{\partial \tilde{H}_x}{\partial y} =$	$\frac{\epsilon_{zx}}{c_0} \frac{\partial \tilde{E}_x}{\partial t} + \frac{\epsilon_{zy}}{c_0} \frac{\partial \tilde{E}_y}{\partial t} + \frac{\epsilon_{zz}}{c_0} \frac{\partial \tilde{E}_z}{\partial t}$		$\frac{\partial \tilde{H}_y}{\partial x} - \frac{\partial \tilde{H}_x}{\partial y} =$	$\frac{\epsilon_{zz}}{c_0} \frac{\partial \tilde{E}_z}{\partial t}$		

Expansion of Temporal component (numeric)			
$\frac{\partial \tilde{E}_z(t)}{\partial y} - \frac{\partial \tilde{E}_y(t)}{\partial z} = \frac{\overbrace{\mu_{xx} \frac{\partial H_x(t)}{\partial t}}^{\text{Temporal}}}{c_0}$	$\frac{\partial \tilde{E}_z(t)}{\partial y} - \frac{\partial \tilde{E}_y(t)}{\partial z} = \frac{\mu_{xx} \overbrace{H_x(t + \Delta t/2) - H_x(t - \Delta t/2)}^{\text{Future value}}}{c_0 \Delta t}$	Solve for the future value of the field →	$H_x(t + \Delta t/2) = \frac{c_0 \Delta t}{\mu_{xx}} \left[ \frac{\partial \tilde{E}_z(t)}{\partial y} - \frac{\partial \tilde{E}_y(t)}{\partial z} \right] + H_x(t - \Delta t/2)$
$\frac{\partial \tilde{E}_x(t)}{\partial z} - \frac{\partial \tilde{E}_z(t)}{\partial x} = \frac{\mu_{yy} \frac{\partial H_y(t)}{\partial t}}{c_0}$	$\frac{\partial \tilde{E}_x(t)}{\partial z} - \frac{\partial \tilde{E}_z(t)}{\partial x} = \frac{\mu_{yy} H_y(t + \Delta t/2) - H_y(t - \Delta t/2)}{c_0 \Delta t}$		$H_y(t + \Delta t/2) = \frac{c_0 \Delta t}{\mu_{yy}} \left[ \frac{\partial \tilde{E}_x(t)}{\partial z} - \frac{\partial \tilde{E}_z(t)}{\partial x} \right] + H_y(t - \Delta t/2)$
$\frac{\partial \tilde{E}_y(t)}{\partial x} - \frac{\partial \tilde{E}_x(t)}{\partial y} = \frac{\mu_{zz} \frac{\partial H_z(t)}{\partial t}}{c_0}$	$\frac{\partial \tilde{E}_y(t)}{\partial x} - \frac{\partial \tilde{E}_x(t)}{\partial y} = \frac{\mu_{zz} H_z(t + \Delta t/2) - H_z(t - \Delta t/2)}{c_0 \Delta t}$		$H_z(t + \Delta t/2) = \frac{c_0 \Delta t}{\mu_{zz}} \left[ \frac{\partial \tilde{E}_y(t)}{\partial x} - \frac{\partial \tilde{E}_x(t)}{\partial y} \right] + H_z(t - \Delta t/2)$
$\frac{\partial H_z(t + \Delta t/2)}{\partial y} - \frac{\partial H_y(t + \Delta t/2)}{\partial z} = \frac{\epsilon_{xx} \frac{\partial \tilde{E}_x(t + \Delta t/2)}{\partial t}}{c_0}$	$\frac{\partial H_z(t + \Delta t/2)}{\partial y} - \frac{\partial H_y(t + \Delta t/2)}{\partial z} = \frac{\epsilon_{xx} \tilde{E}_x(t + \Delta t) - \tilde{E}_x(t)}{c_0 \Delta t}$		$\tilde{E}_x(t + \Delta t) = \frac{c_0 \Delta t}{\epsilon_{xx}} \left[ \frac{\partial H_z(t + \Delta t/2)}{\partial y} - \frac{\partial H_y(t + \Delta t/2)}{\partial z} \right] + \tilde{E}_x(t)$
$\frac{\partial H_x(t + \Delta t/2)}{\partial z} - \frac{\partial H_z(t + \Delta t/2)}{\partial x} = \frac{\epsilon_{yy} \frac{\partial \tilde{E}_y(t + \Delta t/2)}{\partial t}}{c_0}$	$\frac{\partial H_x(t + \Delta t/2)}{\partial z} - \frac{\partial H_z(t + \Delta t/2)}{\partial x} = \frac{\epsilon_{yy} \tilde{E}_y(t + \Delta t) - \tilde{E}_y(t)}{c_0 \Delta t}$		$\tilde{E}_y(t + \Delta t) = \frac{c_0 \Delta t}{\epsilon_{yy}} \left[ \frac{\partial H_x(t + \Delta t/2)}{\partial z} - \frac{\partial H_z(t + \Delta t/2)}{\partial x} \right] + \tilde{E}_y(t)$
$\frac{\partial H_y(t + \Delta t/2)}{\partial x} - \frac{\partial H_x(t + \Delta t/2)}{\partial y} = \frac{\epsilon_{zz} \frac{\partial \tilde{E}_z(t + \Delta t/2)}{\partial t}}{c_0}$	$\frac{\partial H_y(t + \Delta t/2)}{\partial x} - \frac{\partial H_x(t + \Delta t/2)}{\partial y} = \frac{\epsilon_{zz} \tilde{E}_z(t + \Delta t) - \tilde{E}_z(t)}{c_0 \Delta t}$		$\tilde{E}_z(t + \Delta t) = \frac{c_0 \Delta t}{\epsilon_{zz}} \left[ \frac{\partial H_y(t + \Delta t/2)}{\partial x} - \frac{\partial H_x(t + \Delta t/2)}{\partial y} \right] + \tilde{E}_z(t)$

Expansion of Spatial component (numeric)					
Analytical	Numerical	Yee Grid Cell	Analytical	Numerical	Yee Grid
$\frac{\partial \tilde{E}_z(t)}{\partial y} - \frac{\partial \tilde{E}_y(t)}{\partial z} = \mu_{xx} \frac{\partial H_x^{i,j,k}(t)}{\partial t}$	$\frac{\tilde{E}_z^{i,j,k+1}(t) - \tilde{E}_z^{i,j,k}(t)}{\Delta y} - \frac{\tilde{E}_y^{i,j,k+1}(t) - \tilde{E}_y^{i,j,k}(t)}{\Delta z}$		$\frac{\partial H_z\left(t+\frac{\Delta t}{2}\right)}{\partial y}-\frac{\partial H_y\left(t+\frac{\Delta t}{2}\right)}{\partial z}=\epsilon_{xx} \frac{\partial \tilde{E}_x^{i,j,k}\left(t+\frac{\Delta t}{2}\right)}{\partial t}$	$\frac{H_z^{i,j,k}\left(t+\frac{\Delta t}{2}\right)-H_z^{i,j,k-1}\left(t+\frac{\Delta t}{2}\right)}{\Delta y}-\frac{H_y^{i,j,k}\left(t+\frac{\Delta t}{2}\right)-H_y^{i,j,k-1}\left(t+\frac{\Delta t}{2}\right)}{\Delta z}$	
$\frac{\partial \tilde{E}_x(t)}{\partial z}-\frac{\partial \tilde{E}_z(t)}{\partial x}=\mu_{yy} \frac{\partial H_y^{i,j,k}(t)}{\partial t}$	$\frac{\tilde{E}_x^{i,j,k+1}(t)-\tilde{E}_x^{i,j,k}(t)}{\Delta z}-\frac{\tilde{E}_z^{i+1,j,k}(t)-\tilde{E}_z^{i,j,k}(t)}{\Delta x}$		$\frac{\partial H_x\left(t+\frac{\Delta t}{2}\right)}{\partial z}-\frac{\partial H_z\left(t+\frac{\Delta t}{2}\right)}{\partial x}=\epsilon_{yy} \frac{\partial \tilde{E}_y^{i,j,k}\left(t+\frac{\Delta t}{2}\right)}{\partial t}$	$\frac{H_x^{i,j,k}\left(t+\frac{\Delta t}{2}\right)-H_x^{i,j,k-1}\left(t+\frac{\Delta t}{2}\right)}{\Delta z}-\frac{H_z^{i,j,k}\left(t+\frac{\Delta t}{2}\right)-H_z^{i-1,j,k}\left(t+\frac{\Delta t}{2}\right)}{\Delta x}$	
$\frac{\partial \tilde{E}_y(t)}{\partial x}-\frac{\partial \tilde{E}_x(t)}{\partial y}=\mu_{zz} \frac{\partial H_z^{i,j,k}(t)}{\partial t}$	$\frac{\tilde{E}_y^{i+1,j,k}(t)-\tilde{E}_y^{i,j,k}(t)}{\Delta x}-\frac{\tilde{E}_x^{i,j,k+1}(t)-\tilde{E}_x^{i,j,k}(t)}{\Delta y}$		$\frac{\partial H_y\left(t+\frac{\Delta t}{2}\right)}{\partial x}-\frac{\partial H_x\left(t+\frac{\Delta t}{2}\right)}{\partial y}=\epsilon_{zz} \frac{\partial \tilde{E}_z^{i,j,k}\left(t+\frac{\Delta t}{2}\right)}{\partial t}$	$\frac{H_y^{i,j,k}\left(t+\frac{\Delta t}{2}\right)-H_y^{i-1,j,k}\left(t+\frac{\Delta t}{2}\right)}{\Delta x}-\frac{H_x^{i,j,k}\left(t+\frac{\Delta t}{2}\right)-H_x^{i,j,k-1}\left(t+\frac{\Delta t}{2}\right)}{\Delta y}$	

Create Short-Hand Variable for Curl terms		
Analytical	Numerical	
$C_x^{\tilde{E}}(t) = \frac{\partial \tilde{E}_z(t)}{\partial y} - \frac{\partial \tilde{E}_y(t)}{\partial z}$	$C_x^{\tilde{E}}(t) = \frac{\tilde{E}_z^{i,j+1,k}(t) - \tilde{E}_z^{i,j,k}(t)}{\Delta y} - \frac{\tilde{E}_y^{i,j,k+1}(t) - \tilde{E}_y^{i,j,k}(t)}{\Delta z}$	
$C_y^{\tilde{E}}(t) = \frac{\partial \tilde{E}_x(t)}{\partial z} - \frac{\partial \tilde{E}_z(t)}{\partial x}$	$C_y^{\tilde{E}}(t) = \frac{\tilde{E}_x^{i,j,k+1}(t) - \tilde{E}_x^{i,j,k}(t)}{\Delta z} - \frac{\tilde{E}_z^{i+1,j,k}(t) - \tilde{E}_z^{i,j,k}(t)}{\Delta x}$	
$C_z^{\tilde{E}}(t) = \frac{\partial \tilde{E}_y(t)}{\partial x} - \frac{\partial \tilde{E}_x(t)}{\partial y}$	$C_z^{\tilde{E}}(t) = \frac{\tilde{E}_y^{i+1,j,k}(t) - \tilde{E}_y^{i,j,k}(t)}{\Delta x} - \frac{\tilde{E}_x^{i,j+1,k}(t) - \tilde{E}_x^{i,j,k}(t)}{\Delta y}$	
$C_x^H(t + \Delta t/2) = \frac{\partial H_z(t + \Delta t/2)}{\partial y} - \frac{\partial H_y(t + \Delta t/2)}{\partial z}$	$C_x^H(t + \Delta t/2) = \frac{H_z^{i,j,k}(t + \Delta t/2) - H_z^{i,j-1,k}(t + \Delta t/2)}{\Delta y} - \frac{H_y^{i,j,k}(t + \Delta t/2) - H_y^{i,j,k-1}(t + \Delta t/2)}{\Delta z}$	
$C_y^H(t + \Delta t/2) = \frac{\partial H_x(t + \Delta t/2)}{\partial z} - \frac{\partial H_z(t + \Delta t/2)}{\partial x}$	$C_y^H(t + \Delta t/2) = \frac{H_x^{i,j,k}(t + \Delta t/2) - H_x^{i,j,k-1}(t + \Delta t/2)}{\Delta z} - \frac{H_z^{i,j,k}(t + \Delta t/2) - H_z^{i-1,j,k}(t + \Delta t/2)}{\Delta x}$	
$C_z^H(t + \Delta t/2) = \frac{\partial H_y(t + \Delta t/2)}{\partial x} - \frac{\partial H_x(t + \Delta t/2)}{\partial y}$	$C_z^H(t + \Delta t/2) = \frac{H_y^{i,j,k}(t + \Delta t/2) - H_y^{i-1,j,k}(t + \Delta t/2)}{\Delta x} - \frac{H_x^{i,j,k}(t + \Delta t/2) - H_x^{i,j-1,k}(t + \Delta t/2)}{\Delta y}$	
$\tilde{D}_x^{i,j,k}(t) = \epsilon_{xx}^{i,j,k} \tilde{E}_x^{i,j,k}(t + \Delta t)$	$\tilde{D}$ is going to make our code more modular	
$\tilde{D}_y^{i,j,k}(t) = \epsilon_{yy}^{i,j,k} \tilde{E}_y^{i,j,k}(t + \Delta t)$		
$\tilde{D}_z^{i,j,k}(t) = \epsilon_{zz}^{i,j,k} \tilde{E}_z^{i,j,k}(t + \Delta t)$		

Rewrite Maxwell's Equations with Short-Hand Curl Terms		
Analytical	Numerical	Solve for future value: <b>FDTD UPDATE EQUATIONS</b>
$C_x^{\tilde{E}}(t) = -\frac{\mu_{xx}}{c_0} \frac{\partial H_x(t)}{\partial t}$	$C_x^{\tilde{E}}(t) = -\frac{\mu_{xx}}{c_0} \frac{\overbrace{H_x(t + \Delta t/2)}^{Future\ value} - H_x(t - \Delta t/2)}{\Delta t}$	$H_x(t + \Delta t/2) = \frac{c_0 \Delta t}{\mu_{xx}} C_x^{\tilde{E}}(t) + H_x(t - \Delta t/2)$
$C_y^{\tilde{E}}(t) = -\frac{\mu_{xx}}{c_0} \frac{\partial H_y(t)}{\partial t}$	$C_y^{\tilde{E}}(t) = -\frac{\mu_{yy}}{c_0} \frac{H_y(t + \Delta t/2) - H_y(t - \Delta t/2)}{\Delta t}$	$H_y(t + \Delta t/2) = \frac{c_0 \Delta t}{\mu_{yy}} C_y^{\tilde{E}}(t) + H_y(t - \Delta t/2)$
$C_z^{\tilde{E}}(t) = -\frac{\mu_{xx}}{c_0} \frac{\partial H_z(t)}{\partial t}$	$C_z^{\tilde{E}}(t) = -\frac{\mu_{zz}}{c_0} \frac{H_z(t + \Delta t/2) - H_z(t - \Delta t/2)}{\Delta t}$	$H_z(t + \Delta t/2) = \frac{c_0 \Delta t}{\mu_{zz}} C_z^{\tilde{E}}(t) + H_z(t - \Delta t/2)$
$C_x^H(t + \Delta t/2) = \frac{\epsilon_{xx}}{c_0} \frac{\partial \tilde{E}_x(t + \Delta/2)}{\partial t} = \frac{1}{c_0} \frac{\partial \tilde{D}_x(t + \Delta/2)}{\partial t}$	$C_x^H(t + \Delta t/2) = \frac{1}{c_0} \frac{\tilde{D}_x(t + \Delta t) - \tilde{D}_x(t)}{\Delta t}$	$\tilde{D}_x(t + \Delta t) = \frac{c_0 \Delta t}{1} C_x^H(t + \Delta t/2) + \tilde{D}_x(t)$
$C_y^H(t + \Delta t/2) = \frac{\epsilon_{yy}}{c_0} \frac{\partial \tilde{E}_y(t + \Delta/2)}{\partial t} = \frac{1}{c_0} \frac{\partial \tilde{D}_y(t + \Delta/2)}{\partial t}$	$C_y^H(t + \Delta t/2) = \frac{1}{c_0} \frac{\tilde{D}_y(t + \Delta t) - \tilde{D}_y(t)}{\Delta t}$	$\tilde{D}_y(t + \Delta t) = \frac{c_0 \Delta t}{1} C_y^H(t + \Delta t/2) + \tilde{D}_y(t)$
$C_z^H(t + \Delta t/2) = \frac{\epsilon_{zz}}{c_0} \frac{\partial \tilde{E}_z(t + \Delta/2)}{\partial t} = \frac{1}{c_0} \frac{\partial \tilde{D}_z(t + \Delta/2)}{\partial t}$	$C_z^H(t + \Delta t/2) = \frac{1}{c_0} \frac{\tilde{D}_z(t + \Delta t) - \tilde{D}_z(t)}{\Delta t}$	$\tilde{D}_z(t + \Delta t) = \frac{c_0 \Delta t}{1} C_z^H(t + \Delta t/2) + \tilde{D}_z(t)$
$\tilde{D}_x^{i,j,k}(t) = \epsilon_{xx}^{i,j,k} \tilde{E}_x^{i,j,k}(t + \Delta t)$	$\tilde{D}_x^{i,j,k}(t + \Delta t) = \epsilon_{xx}^{i,j,k} \tilde{E}_x^{i,j,k}(t + \Delta t)$	$\tilde{E}_x^{i,j,k}(t + \Delta t) = \frac{1}{\epsilon_{xx}^{i,j,k}} \tilde{D}_x^{i,j,k}(t + \Delta t)$
$\tilde{D}_y^{i,j,k}(t) = \epsilon_{yy}^{i,j,k} \tilde{E}_y^{i,j,k}(t + \Delta t)$	$\tilde{D}_y^{i,j,k}(t + \Delta t) = \epsilon_{yy}^{i,j,k} \tilde{E}_y^{i,j,k}(t + \Delta t)$	$\tilde{E}_y^{i,j,k}(t + \Delta t) = \frac{1}{\epsilon_{yy}^{i,j,k}} \tilde{D}_y^{i,j,k}(t + \Delta t)$
$\tilde{D}_z^{i,j,k}(t) = \epsilon_{zz}^{i,j,k} \tilde{E}_z^{i,j,k}(t + \Delta t)$	$\tilde{D}_z^{i,j,k}(t + \Delta t) = \epsilon_{zz}^{i,j,k} \tilde{E}_z^{i,j,k}(t + \Delta t)$	$\tilde{E}_z^{i,j,k}(t + \Delta t) = \frac{1}{\epsilon_{zz}^{i,j,k}} \tilde{D}_z^{i,j,k}(t + \Delta t)$

Update Equations: <https://youtu.be/f0orcF2ubls?si=eFNDKVIReuYxd584&t=1326>

Variable List	Matlab variable	Dimension	Matlab Initialization
	t		
	T	1×1	
	Nx	1×1	
	Ny	1×1	
	Nz	1×1	
$C_x^E$	CEx	Nx × Ny × Nz	
$C_y^E$	CEy	Nx × Ny × Nz	
$C_z^E$	CEz	Nx × Ny × Nz	
$C_x^H$	CHx	Nx × Ny × Nz	
$C_y^H$	CHy	Nx × Ny × Nz	
$C_z^H$	CHz	Nx × Ny × Nz	
$H_x$	Hx	Nx × Ny × Nz	
$H_y$	Hy	Nx × Ny × Nz	
$H_z$	Hz	Nx × Ny × Nz	
$\bar{D}_x$	Dx	Nx × Ny × Nz	
$\epsilon_{xx}^{i,j,k}$	exx	Nx × Ny × Nz	
$\epsilon_{yy}^{i,j,k}$	eyy	Nx × Ny × Nz	
$\epsilon_{zz}^{i,j,k}$	ezz	Nx × Ny × Nz	
$\mu_{xx}^{i,j,k}$	uxx	Nx × Ny × Nz	
$\mu_{yy}^{i,j,k}$	uyy	Nx × Ny × Nz	
$\mu_{zz}^{i,j,k}$	uzz	Nx × Ny × Nz	
$c_0$	c0	1×1	
$\epsilon_0$	e0	1×1	
$\mu_0$	u0	1×1	
$\Delta t$	dt	1×1	
$\Delta x$	dx	1×1	
$\Delta y$	dy	1×1	
$\Delta z$	dz	1×1	

Solve for future value: <b>FDTD UPDATE EQUATIONS (...same ones highlighted above.)</b>	
Written	Matlab Code: Update Equations
$C_x^{\tilde{E}}(t) = \frac{\tilde{E}_z^{i,j+1,k}(t) - \tilde{E}_z^{i,j,k}(t)}{\Delta y} - \frac{\tilde{E}_y^{i,j,k+1}(t) - \tilde{E}_y^{i,j,k}(t)}{\Delta z}$	
$C_y^{\tilde{E}}(t) = \frac{\tilde{E}_x^{i,j,k+1}(t) - \tilde{E}_x^{i,j,k}(t)}{\Delta z} - \frac{\tilde{E}_z^{i+1,j,k}(t) - \tilde{E}_z^{i,j,k}(t)}{\Delta x}$	
$C_z^{\tilde{E}}(t) = \frac{\tilde{E}_y^{i+1,j,k}(t) - \tilde{E}_y^{i,j,k}(t)}{\Delta x} - \frac{\tilde{E}_x^{i,j+1,k}(t) - \tilde{E}_x^{i,j,k}(t)}{\Delta y}$	
$C_x^H(t + \Delta t/2) = \frac{H_z^{i,j,k}(t + \Delta t/2) - H_z^{i,j-1,k}(t + \Delta t/2)}{\Delta y} - \frac{H_y^{i,j,k}(t + \Delta t/2) - H_y^{i,j,k-1}(t + \Delta t/2)}{\Delta z}$	
$C_y^H(t + \Delta t/2) = \frac{H_x^{i,j,k}(t + \Delta t/2) - H_x^{i,j,k-1}(t + \Delta t/2)}{\Delta z} - \frac{H_z^{i,j,k}(t + \Delta t/2) - H_z^{i-1,j,k}(t + \Delta t/2)}{\Delta x}$	
$C_z^H(t + \Delta t/2) = \frac{H_y^{i,j,k}(t + \Delta t/2) - H_y^{i-1,j,k}(t + \Delta t/2)}{\Delta x} - \frac{H_x^{i,j,k}(t + \Delta t/2) - H_x^{i,j-1,k}(t + \Delta t/2)}{\Delta y}$	
$H_x(t + \Delta t/2) = \frac{c_0 \Delta t}{\mu_{xx}} C_x^{\tilde{E}}(t) + H_x(t - \Delta t/2)$	
$H_y(t + \Delta t/2) = \frac{c_0 \Delta t}{\mu_{yy}} C_y^{\tilde{E}}(t) + H_y(t - \Delta t/2)$	
$H_z(t + \Delta t/2) = \frac{c_0 \Delta t}{\mu_{zz}} C_z^{\tilde{E}}(t) + H_z(t - \Delta t/2)$	
$\tilde{D}_x(t + \Delta t) = \frac{c_0 \Delta t}{1} C_x^H(t + \Delta t/2) + \tilde{D}_x(t)$	
$\tilde{D}_y(t + \Delta t) = \frac{c_0 \Delta t}{1} C_y^H(t + \Delta t/2) + \tilde{D}_y(t)$	
$\tilde{D}_z(t + \Delta t) = \frac{c_0 \Delta t}{1} C_z^H(t + \Delta t/2) + \tilde{D}_z(t)$	
$\tilde{E}_x^{i,j,k}(t + \Delta t) = \frac{1}{\epsilon_{xx}^{i,j,k}} \tilde{D}_x^{i,j,k}(t + \Delta t)$	
$\tilde{E}_y^{i,j,k}(t + \Delta t) = \frac{1}{\epsilon_{yy}^{i,j,k}} \tilde{D}_y^{i,j,k}(t + \Delta t)$	
$\tilde{E}_z^{i,j,k}(t + \Delta t) = \frac{1}{\epsilon_{zz}^{i,j,k}} \tilde{D}_z^{i,j,k}(t + \Delta t)$	

Boundary Conditions: <https://youtu.be/f0orcF2ubls?si=DO1kOVgE6b5TPSln&t=1553>

Full Finite Difference Equations	
Human Readable Math Form	Matlab Code Form
$H_x^{i,j,k}(t + \Delta t/2) = \frac{\Delta t}{\mu_{xx}} \left[ \frac{\tilde{E}_z^{i,j+1,k}(t) - \tilde{E}_z^{i,j,k}(t)}{\Delta y} - \frac{\tilde{E}_y^{i,j,k+1}(t) - \tilde{E}_y^{i,j,k}(t)}{\Delta z} \right] + H_x(t - \Delta t/2)$	<pre>% Calculate Magnetic field (for next integer half step) Hx(1:(Nz-1)) = Hx(1:(Nz-1)) + squeeze(mH(30,30,1:(Nz-1))) .* (Ey(2:Nz) - Ey(1:(Nz-1)));</pre>
$H_y^{i,j,k}(t + \Delta t/2) = \frac{\Delta t}{\mu_{yy}} \left[ \frac{\tilde{E}_x^{i,j,k+1}(t) - \tilde{E}_x^{i,j,k}(t)}{\Delta z} - \frac{\tilde{E}_z^{i+1,j,k}(t) - \tilde{E}_z^{i,j,k}(t)}{\Delta x} \right] + H_y(t - \Delta t/2)$	
$H_z^{i,j,k}(t + \Delta t/2) = \frac{\Delta t}{\mu_{zz}} \left[ \frac{\tilde{E}_y^{i+1,j,k}(t) - \tilde{E}_y^{i,j,k}(t)}{\Delta x} - \frac{\tilde{E}_x^{i,j+1,k}(t) - \tilde{E}_x^{i,j,k}(t)}{\Delta y} \right] + H_z(t - \Delta t/2)$	
$\tilde{E}_x^{i,j,k}(t + \Delta t) = \frac{\Delta t}{\epsilon_{xx}} \left[ \frac{H_z^{i,j,k} \left( t + \frac{\Delta t}{2} \right) - H_z^{i,j-1,k} \left( t + \frac{\Delta t}{2} \right)}{\Delta y} - \frac{H_y^{i,j,k} \left( t + \frac{\Delta t}{2} \right) - H_y^{i,j,k-1} \left( t + \frac{\Delta t}{2} \right)}{\Delta z} \right] + \tilde{E}_x(t)$	
$\tilde{E}_y^{i,j,k}(t + \Delta t) = \frac{\Delta t}{\epsilon_{yy}} \left[ \frac{H_x^{i,j,k} \left( t + \frac{\Delta t}{2} \right) - H_x^{i,j,k-1} \left( t + \frac{\Delta t}{2} \right)}{\Delta z} - \frac{H_z^{i,j,k} \left( t + \frac{\Delta t}{2} \right) - H_z^{i-1,j,k} \left( t + \frac{\Delta t}{2} \right)}{\Delta x} \right] + \tilde{E}_y(t)$	
$\tilde{E}_z^{i,j,k}(t + \Delta t) = \frac{\Delta t}{\epsilon_{zz}} \left[ \frac{H_y^{i,j,k} \left( t + \frac{\Delta t}{2} \right) - H_y^{i-1,j,k} \left( t + \frac{\Delta t}{2} \right)}{\Delta x} - \frac{H_x^{i,j,k} \left( t + \frac{\Delta t}{2} \right) - H_x^{i,j-1,k} \left( t + \frac{\Delta t}{2} \right)}{\Delta y} \right] + \tilde{E}_z(t)$	

Full Finite Difference Equations	
Human Readable Math Form	Matlab Code Form
$H_x^{i,j,k}(t + \Delta t/2) = \frac{\Delta t}{\mu_{xx}} \left[ \frac{\tilde{E}_z^{i,j+1,k}(t) - \tilde{E}_z^{i,j,k}(t)}{\Delta y} - \frac{\tilde{E}_y^{i,j,k+1}(t) - \tilde{E}_y^{i,j,k}(t)}{\Delta z} \right] + H_x(t - \Delta t/2)$	<pre>% Calculate Magnetic field (for next integer half step) Hx(1:(Nz-1)) = Hx(1:(Nz-1)) + squeeze(mH(30,30,1:(Nz-1))) .* (Ey(2:Nz) - Ey(1:(Nz-1)));</pre>
$H_y^{i,j,k}(t + \Delta t/2) = \frac{\Delta t}{\mu_{yy}} \left[ \frac{\tilde{E}_x^{i,j,k+1}(t) - \tilde{E}_x^{i,j,k}(t)}{\Delta z} - \frac{\tilde{E}_z^{i+1,j,k}(t) - \tilde{E}_z^{i,j,k}(t)}{\Delta x} \right] + H_y(t - \Delta t/2)$	
$H_z^{i,j,k}(t + \Delta t/2) = \frac{\Delta t}{\mu_{zz}} \left[ \frac{\tilde{E}_y^{i+1,j,k}(t) - \tilde{E}_y^{i,j,k}(t)}{\Delta x} - \frac{\tilde{E}_x^{i,j+1,k}(t) - \tilde{E}_x^{i,j,k}(t)}{\Delta y} \right] + H_z(t - \Delta t/2)$	
$\tilde{D}_x^{i,j,k}(t + \Delta t) = \frac{\Delta t}{c_0} \left[ \frac{H_z^{i,j,k} \left( t + \frac{\Delta t}{2} \right) - H_z^{i,j-1,k} \left( t + \frac{\Delta t}{2} \right)}{\Delta y} - \frac{H_y^{i,j,k} \left( t + \frac{\Delta t}{2} \right) - H_y^{i,j,k-1} \left( t + \frac{\Delta t}{2} \right)}{\Delta z} \right] + \tilde{D}_x(t)$	
$\tilde{D}_y^{i,j,k}(t + \Delta t) = \frac{\Delta t}{c_0} \left[ \frac{H_x^{i,j,k} \left( t + \frac{\Delta t}{2} \right) - H_x^{i,j,k-1} \left( t + \frac{\Delta t}{2} \right)}{\Delta z} - \frac{H_z^{i,j,k} \left( t + \frac{\Delta t}{2} \right) - H_z^{i-1,j,k} \left( t + \frac{\Delta t}{2} \right)}{\Delta x} \right] + \tilde{D}_y(t)$	
$\tilde{D}_z^{i,j,k}(t + \Delta t) = \frac{\Delta t}{c_0} \left[ \frac{H_y^{i,j,k} \left( t + \frac{\Delta t}{2} \right) - H_y^{i-1,j,k} \left( t + \frac{\Delta t}{2} \right)}{\Delta x} - \frac{H_x^{i,j,k} \left( t + \frac{\Delta t}{2} \right) - H_x^{i,j-1,k} \left( t + \frac{\Delta t}{2} \right)}{\Delta y} \right] + \tilde{D}_z(t)$	