# C3 Coursework Guide

## Numerical Solution of Equations

**2014-15**

**This document contains explicit instructions on what is required for each mark. We have written it to help you to produce a good piece of work and to help you ensure you have included everything necessary.**

# <u>Expectations</u>

**<u>The deadlines:</u>**

In the summer term you will have worked on the examples from the text book in C3 chapter 6. This was to familiarise you with the three methods that will be used in the coursework in the autumn term of year 13.

**The FINAL deadline for the completed work is by Friday 7th November, the end of the week after half-term.** This is designed to spread the work out and give you plenty of time to get it done well. The final submission MUST be printed out in colour. If you cannot do this at home it can be done in school but you will be charged for it. There are NO extensions. We give you plenty of time to complete this work. THE FINAL DEADLINE IS FOR THE FINISHED THING. It is NOT a draft deadline. You will NOT get your work back with advice on how to improve it. What you hand in at the final deadline is what will be marked. **IT IS YOUR RESPONSIBILITY TO ASK YOUR TEACHER TO CHECK YOUR WORK DURING THE TIME YOU ARE WORKING ON THIS ASSIGNMENT. IF YOU ARE AT ALL UNSURE WHETHER YOU HAVE DONE EVERYTHING, ARE NOT SURE IF YOU HAVE DONE IT CORRECTLY, OR NEED HELP.**

**<u>Getting Help</u>**

We will answer any questions you have while you are doing your coursework so please don't hesitate to ask for help. This document outlines EXACTLY what you need to do to gain every mark. There are also videos on Fronter which take you through what to do.

Do not leave the work till the last minute as it takes quite a long time to produce a good piece of work that will get all the marks. Don't believe the MEI Coursework Guide's estimate of 6-8 hours as most students in the past have found it takes a lot longer to do well. Check your work through before handing it in – there is no such thing as a typo in a number!

**Layout**

**You must produce a word-processed document typed in Times New Roman size 12 and you should submit it in hard copy in colour – your teacher will not print it for you, or mark a computer file.**
Your name and candidate number should appear at the top in the headers and page numbers should appear at the bottom in the footers.

You should also give in a mark sheet with the details filled in and page references showing where you think you have earned each mark. If you do this as you go along it will help you to check you have done everything.

The centre name is Graveney School and the number is 11022.

Part of our expectation for this work is that you get to grips with a standard piece of software, a spreadsheet, in this case Excel, and that you improve your presentation skills. Most businesses have spreadsheets, most do not have Geogebra. Please do, however, use Geogebra to check that your spreadsheet has been set up properly and is producing the correct results. Graphs inserted in your work from Geogebra need to be annotated. You can do this in Word using text boxes.

**Function advice**

**We will provide you with a list of functions to choose from**. We are suggesting only cubics as these provide the least amount of difficulty.

On no account should you ever use a quadratic in this coursework (as all quadratics can be solved using the formula even if they can't be factorised), or any other function that can be solved by factorising. Nor should you use any function for which you know the solution. Don't use a cubic with one integer or rational root, since you could write this as a linear factor times a quadratic, or a quartic with two integer roots etc.

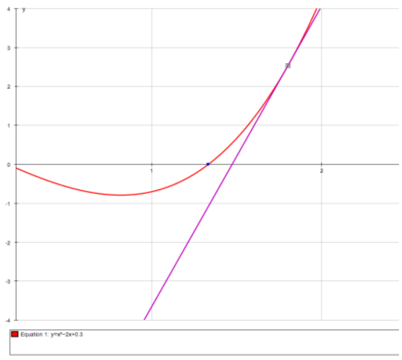
No function appearing in Chapter 6 of the textbook, or in class examples can be used.

Functions should be multiplied out, not appear in wholly or partially factorised form.

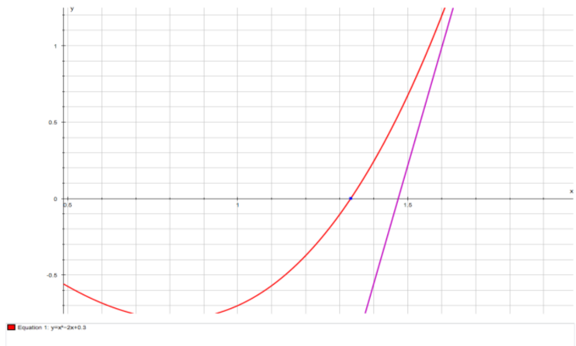
**You need a different equation for each method. Each one should have 2 or 3 roots** (you could use one with more but you'll be making extra work for yourself).

Each function used should be drawn so that both the roots and the turning points are shown and the graph annotated to show the roots you are talking about at the start of each section, then again during the body of the work to illustrate the method. The roots should be shown to be between two integer values on the graph.

Tangents when required must be shown actually touching the curve.




**NOT**




**You must provide a running commentary/explanation of what is going on, written in a textbook style, NOT I did this, next I did that.**

Use equation editor in Word to make the maths look right. If it isn't, you'll be penalised. Do not use computer notation such as * for multiplication, or ^ for powers.

## Use YOUR functions to explain what is going on with each method.

**Accuracy**

If all methods are calculated accurate to **5 significant figures** then there is less scope for confusion and it will mean that any function you use in the comparison will have the same degree of accuracy.

## Change of Sign: Decimal Search

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| Change of sign method (3) | 1<br>1<br>1 | The method is applied successfully to find one root of an equation.<br>Error bounds are stated and the method is illustrated graphically.<br>An example is given of an equation where one of the roots cannot be found by the chosen method. There is an illustrated explanation of why this is the case. |

Use the decimal search method using Excel. Ignore references in the textbook and the coursework guide to linear interpolation and interval bisection.
Draw a graph of your function and label all the roots.
Do a table of integer values for your function. None of the results should be 0. If they are, change the function!
Decide which root to find using this method.
Choose the integers between which the result changes sign.
Do another table using 1dp values. Draw another graph showing the root clearly between two values of your table where the result changes sign. You should be able to see the 1dp values on the x-axis with the line clearly crossing the axis between them.
Repeat until you have the required degree of accuracy (max error ±0.000005). **Include a graph** with the x values from you table clearly shown on the x axis **for each table** and some commentary explaining what is going on and you will have covered the "illustrated graphically" bit of mark 2.

State the error bound either as an error bound (e.g. x = 0.623225±0.000005) or as solution bounds (e.g. 0.62322<x<0.62323). Your table in this case should have shown that when x was 0.62322, f(x) was positive and when x was 0.62323, f(x) was negative (or vice versa). NB be careful what you say - a root between 5.62322 and 5.62323 has a maximum error of ±0.000005 but is only accurate to 4dp/5sf, not 5dp/6sf. Check you have written down the correct number of 0s as this is a frequent cause of lost marks. Your root should be half way between the values which show the sign change, i.e. one more dp than the table shows. Your ±value should have the same number of decimal places as your root, so that when you add and subtract it you get the numbers in your table.

Failure – use a different equation. Still no quadratics or ones where you know the root. Try to find one with 2 roots between a pair of integers.
Do a table of integer values and draw the graph. You should see no change of sign so apparently no roots.
Then do another table of values to 1dp and show that there are in fact 2 changes of sign.

## Newton-Raphson

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| Newton-Raphson method (5) | 1<br>1<br>1<br>1<br>1 | The method is applied successfully to find one root of a second equation.<br>All the roots of the equation are found<br>The method is illustrated graphically for one root.<br>Error bounds are established for one of the roots.<br>An example is given of an equation where this method fails to find a particular root despite taking a starting value close to it.<br>There is an illustrated explanation why this has happened. |

Must be a different function from the decimal search.
Your function MUST have more than one root, or you cannot earn mark 2.
Illustrations of tangents must show the point at which the tangent touches the curve AND the point at which it crosses the x axis, which should be marked $x_1$ if it comes from your start point of $x_0$. Illustrations only need to be done for one root. Annotated graphs needed – use text boxes and arrows!
Make sure you differentiate your function correctly. If you make an error here you will mess the whole thing up!
**You must show the calculations (i.e. the maths) for your first iteration.**
At least 5sf needed. NB 0.1234 is 4sf, not 5 and 0.00012 is 2sf.
Make sure your starting point is close to the root you are looking for, including when you are illustrating failure – it must be no further away than the closest integer on either side.
Error bounds must be “established”, not just stated. I.e. substitute your upper and lower bounds into the function to show one gives a positive result and the other a negative one. i.e. f(lower bound)=negative, f(upper bound)=positive or vice-versa. **Warning! This is a common place for a lost mark!**

Failure comes from either not finding a root at all (as the graph goes into an infinite loop or has a gradient of zero thus never meeting the x-axis again), or finding the wrong one, when you take a starting point close to the desired root. The starting point should not be closer to another root. Your annotated graph should show why this is happening.

## Rearranging Method

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| Rearranging $f(x)=0$ in the form $x=g(x)$ (4) | 1<br>1<br><br>1<br><br>1 | A rearrangement is applied successfully to find a root of a third equation.<br>Convergence of this rearrangement to the root is demonstrated graphically and the magnitude of $g'(x)$ is discussed.<br>A rearrangement of the same equation is applied in a situation where the iteration fails to converge to the required root.<br>This failure is demonstrated graphically and the magnitude of $g'(x)$ is discussed. |

Another new equation needed.
Make sure you rearrange it correctly. Make sure you enter the rearrangement correctly into Geogebra for your illustration. Use of brackets is necessary, be sure you have the right amount. Try to be taking an odd root if you must take a root as this avoids graphing problems. **Warning! This is a common place for a lost mark!**
You don't need to find all the roots.
Your starting value must be close to the root you are looking for.
Discussion of g'(x) (applies to both success and failure): This should not be exclusively algebraic (ie if you can't differentiate your function you don't have to) but should refer to the gradient of the graph at the root. If it is close your graphic illustration could show either y = x + c or y = -x + c or both to show the gradient is inside/outside the required range. It may be beneficial to draw this line in, to make it clear. Students often are quoted as saying "obviously from the graph" when it isn't obvious at all. You need to arrange for your line to be crossing your y = g(x) graph at the root. Making the x and y scales the same will help. If you do decide to do this algebraically, make sure you differentiate your function properly and substitute the root or a value very close to it in.

### Failure: You MUST use the same equation. BUT you could use the same rearrangement to find a different root, or a different rearrangement to find the same root or a different rearrangement to find a different root.

## Comparison of Methods

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| Comparison of methods (3) | 1<br>1<br>1 | One of the equations used above is selected and the other two methods are applied successfully to find the same root.<br>There is a sensible comparison of the relative merits of the three methods in terms of speed of convergence.<br>There is a sensible comparison of the relative merits of the three methods in terms of ease of use with available hardware and software. |

Choose one of the methods you've already done, choose one root you've already found, and use the other two methods to find the same root **to the same degree of accuracy**. It doesn't matter which one you choose but a wise choice will make for less work! You should also use the **same starting point** for N-R and Rearranging for a fair test.

Speed of convergence – how many calculations does it take. Each new x value is one iteration. Be particularly careful with decimal search! Each table is not one iteration but several. All the input x values should be counted from the top of each table down to the one where the sign changes but not below this. You may have calculated them in Excel but you wouldn't have done so if you did it manually would you?! For Rearranging and N-R the last iteration to count is the one where the output matches the input to the degree of accuracy you are looking for.

Hardware & Software – Explain that you have used Excel and Geogebra as tools to help you in the coursework. People don't want to read that you clicked the black cross then dragged down to copy a function in Excel. You are explaining what you would have had to have done without the software and how this has sped up the process. The use of Geogebra software meant you didn't need to do successive iterations by hand too.

When deciding which method you found best, actually refer to your conclusions. If N-R took less iterations, then that's better isn't it? Also mention the difficulty and skills required in each method. e.g. N-R may have taken less iterations but was required to have knowledge of differentiation etc.

## Written and Oral Communication

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| Written Communication (1) | 1 | Correct notation and terminology are used | | |
| Oral Communication (2) | 2 | Presentation | | Please tick at least one box and give a brief report. |
| | | Interview | | |
| | | Discussion | | |

NB
A function f(x) = …. has roots where its graph crosses the x axis.
You solve an equation …. = 0 to find the value of x at these roots, and the solution is the set of all the roots.
You can illustrate a function with a graph of y = f(x) or y = ………
An expression has no = sign.
Your use of these will be assessed in the written communication mark, as will your use of other mathematical notation!
Make sure you use subscripts correctly and give mathematical notation for powers, not computer notation.

Oral communication will be dealt with by discussion with you throughout the coursework lessons and also on the standard of your argument in the work itself. We cannot be seen to give you a favourable mark if your coursework doesn't really make sense.