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# Table of some multiplicative functions and their symbols

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Symbol	Function name	$f(p^e)$ for $e \geq 1$	Function description.
$\frac{\emptyset}{\emptyset}$	Unit for Dirichlet conv.	0	Tells you whether your number is 1
$\frac{\{1\}}{\emptyset}$	Constant function	1	Tells you whether your number is a number
$\frac{\{1\}}{\emptyset}$	Möbius mu function	-1 if $e = 1$ , 0 if $e \geq 2$	The Dirichlet inverse of the constant value function
$\frac{\{p\}}{\emptyset}$	Identity function	$p^e$	Tells you what your number is
$\frac{\{p^2\}}{\emptyset}$	Square function	$p^{2e}$	Tells you the square of your number
$\frac{\{p^3\}}{\emptyset}$	Cube function	$p^{3e}$	Tells you the cube of your number
$\frac{\{p^k\}}{\emptyset}$	$k$ 'th power function	$p^{ke}$	Raises your number to the $k$ -th power
$\frac{\{1,1\}}{\emptyset}$	Number of divisors	$e + 1$	Counts the number of divisors of your number
$\frac{\{1,p\}}{\emptyset}$	Sum of divisors	$\frac{p^{e+1}-1}{p-1}$	Computes the sum of all the divisors of your number
$\frac{\{1,p^k\}}{\emptyset}$	$k$ 'th divisor function	$\frac{p^{k(e+1)}-1}{p^k-1}$	Sums all the divisors of your number, each raised to the power of $k$
$\frac{\{p\}}{\{1\}}$	Euler totient function	$p^{e-1}(p-1)$	Counts the numbers below your number that share no factors with it
$\frac{\{-1\}}{\emptyset}$	Liouville function	$(-1)^e$	Tells you the parity of the number of prime factors of your number
$\frac{\{1\}}{\{2\}}$	The $\gamma$ -function	-1	Tells you the parity of the number of distinct prime factors

Symbol	Function name	$f(p^e)$ for $e \geq 1$	Function description.
$\frac{\{1, -1\}}{\emptyset}$	Characteristic function of squares	1 if $e \equiv 0 \pmod{2}$	Tells you whether the number is a square number
$\frac{\{1, \omega, \omega^2\}}{\emptyset}$	Characteristic function of cubes	1 if $e \equiv 0 \pmod{3}$	Tells you whether the number is a cube number
$\frac{\{1, \dots, \omega^{k-1}\}}{\emptyset}$	Char. function of k-th powers	1 if $e \equiv 0 \pmod{k}$	Tells you if the number is a number to the $k$ 'th power
$\frac{\emptyset}{\{-1\}}$	Char. func. of square-free numbers	1 if $e < 2$	Tells you if the number has any factors repeated twice
$\frac{\{1, 1, \dots, 1\}}{\emptyset}$	No standard name?	$\binom{e+k-1}{e}$	Number of ordered factorisations of $n$ into $k$ factors
$\frac{\{p^k\}}{\{1\}}$	Jordan totient function	$p^{ek} \left(1 - \frac{1}{p^k}\right)$	Number of elements in a reduced $(n, k)$ -residue system
$\frac{\{p, -1\}}{\emptyset}$	No standard name?	–	Number of $x$ with $1 \leq x \leq n$ and $\gcd(x, n)$ a square
$\frac{\{1\}}{\{1-p\}}$	Core function	$p$	The product of all distinct primes dividing your number
$\frac{\{1\}}{\{-1\}}$	No standard name?	2	Number of square free divisors of $n$ .
$\frac{\{p\}}{\{-1\}}$	Dedekind's $\psi$ function	$p^e + p^{e-1}$	Equals $J_2/J_1$ , where $J_k$ is the Jordan totient function
$\frac{\{p^k\}}{\{-1\}}$	$k$ 'th Dedekind's function	$p^{ek} \left(1 + \frac{1}{p^k}\right)$	Equals $J_{2k}/J_k$