4) coin denominations: $c_1 = 24$, $c_2 = 11$, $c_3 = 5$, $c_4 = 1$

a) what combo would algo use to make 33?

$c_1 + c_3 + c_1 + c_1 + c_1 + c_1 = 33$, 6 coins.

$24 + 5 + 1 + 1 + 1 + 1$

b) least coin count?

$c_2 + c_2 + c_2 = 33$, 3 coins

$11 + 11 + 11$

7) Determine whether sets are Finite, countably infinite, or uncountable.

a) $\bigcap_{i=1}^{5} A_i$ where $A_i = \{i, i+1, i+2, \dots i+5\}$

$A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5$

$A_1 = \{1,2,3,4,5,6\}$

$A_2 = \{2,3,4,5,6,7\}$

$A_3 = \{3, \dots, 8\}$

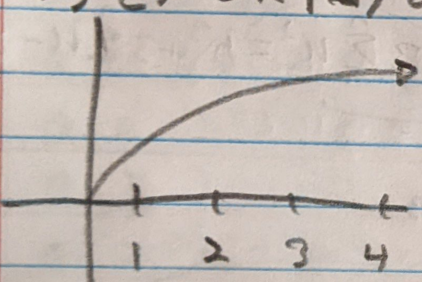
$A_4 = \{4, \dots, 9\}$

$A_5 = \{5, \dots, 10\}$

elements 5 and 6 in common

$\therefore \bigcap_{i=1}^{5} A_i = \{5,6\}$ Finite

b) $\{x \in \mathbb{R} \mid \exists y \in \mathbb{Z}^+ (x = \log_2 y)\}$



There is a bijection between the set of values x and the set of all positive int. Therefore the set is countably infinite

c) $[-2,-1] - (-2,-1) = \{-2,-1\}$

$[-2,-1] = \{-2, (-2,-1), -1\} - (-2,-1)$ Finite

8) Give an example of two uncountable sets A and B such that $A \cap B$ is

a) Finite.

A = the set of all positive real num

B = the set of all negative real num

$A \cap B = \emptyset$

b) countably infinite

$A = [1,2] \cup \mathbb{Z}^+$

$B = [3,4] \cup \mathbb{Z}^+$

$A \cap B = \mathbb{Z}^+$

c) uncountable

$A = [1,4]$

$B = [3,5]$

$A \cap B = [3,4]$

6) Prove $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

$$\sum_{k=1}^{n} (k^3-(k-1)^3) = n^3 - 0^3 = n^3$$

$$\sum_{k=1}^{n} (k^3-(k^2-2k+1)(k-1)) = n^3$$

$$\sum_{k=1}^{n} (k^3-(k^3-2k^2+k-k^2+2k-1)) = n^3$$

$$\sum_{k=1}^{n} k^3-(k^3-3k^2+3k-1) = n^3$$

$$\sum \cancel{k^3} - \cancel{k^3} + 3k^2-3k+1 = n^3$$

$$\sum^{n} 3k^2 + \sum^{n} -3k + \sum^{n} 1 = n^3 \Rightarrow 3\sum^{n} k^2 = n^3 + 3\sum^{n} k - \sum^{n} 1$$

$$\sum_{j=1}^{n} (a_j - a_{j-1}) = a_n - a_0, \text{ let } a_k = k^3$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \quad \sum_{k=1}^{n} 1 = n$$ known identities

$$\Rightarrow \frac{3\sum_{k=1}^{n} k^2}{3} = \frac{n^3+3\left(\frac{n(n+1)}{2}\right)-n}{3}$$

$$\sum_{k=1}^{n} k^2 = \frac{\left(\frac{2}{3}\right)n^3+\frac{3}{2}(n(n+1))-n\left(\frac{2}{2}\right)}{3} \Rightarrow \sum_{k=1}^{n} k^2 = \frac{2n^3+3(n(n+1$$

$$\Rightarrow \sum_{k=1}^{n} k^2 = \frac{2n^3+3(n^2+n)-2n}{6}$$

$$= \frac{2n^3+3n^2+3n-2n}{6} \Rightarrow \frac{2n^3+3n^2+n}{6}$$

$$\Rightarrow \frac{n(2n^2+3n+1)}{6} \Rightarrow \boxed{\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}}$$

4) Find a closed form for the recurrence relations.

a) $a_n = -a_{n-1}$, $a_0 = 3$

$a_1 = -1^1 \cdot 3^1 = -3$

$a_2 = -1^2 \cdot 3^2 = 9$ $\quad a_n = (-1)^n (3)^n$

$a_3 = -1^3 \cdot 3^3 = -27$ $\quad \boxed{a_n = (-3)^n}$

...

b) $a_n = a_{n-1} - n$, $a_0 = 5$

$a_1 = 5 - 1 = 4$ $\quad -1 \quad \boxed{a_n = 6 - (n^2+n)}$

$a_2 = 4 - 2 = 2$ $\quad -2$

$a_3 = 2 - 3 = -1$ $\quad -3$

$a_4 = -1 - 4 = -5$ $\quad -4$

$a_5 = -5 - 5 = -10$ $\quad -5$

c) $a_n = 2a_{n-1} - 3$, $a_0 = 2$

$a_1 = 2(2) - 3 = 4 - 3 = 1$ $\quad -1 \quad \boxed{a_n = 3 - (2)^n}$

$a_2 = 2(1) - 3 = 2 - 3 = -1$ $\quad -2$

$a_3 = 2(-1) - 3 = -2 - 3 = -5$ $\quad -4$

$a_4 = 2(-5) - 3 = -10 - 3 = -13$ $\quad -8$

5) Find the value of the following summations

a) $\sum_{j=0}^{8} (3^j - 2^j)$

$= (3^0 - 2^0) + (3^1 - 2^1) + (3^2 - 2^2) + (3^3 - 2^3) + (3^4 - 2^4) +$
$(3^5 - 2^5) + (3^6 - 2^6) + (3^7 - 2^7) + (3^8 - 2^8)$

$\boxed{= 9330}$

b) $\sum_{i=1}^{2} \sum_{j=1}^{3} (i - j) = \sum_{i=1}^{2} (i-1) + (i-2) + (i-3)$

$= \sum_{i=1}^{2} 3i - 6 = (3(1) - 6) + (3(2) - 6)$

$-3 + 0$

$\boxed{= -3}$

Homework 5

1) Prove that $\lceil x \rceil \leq \lfloor \lfloor x \rfloor + 1 \rfloor$

let $x = n + \varepsilon$, where n is int and $0 \leq \varepsilon < 1$

case 1: $n = 2$, $\varepsilon = 0$: $\lceil 2+0 \rceil \leq \lfloor \lfloor 2+0 \rfloor + 1 \rfloor$

$= 2 \leq 3$ TRUE

case 2: $n = 2$, $\varepsilon = 1/4$ $\lceil 2 + 1/4 \rceil \leq \lfloor \lfloor 2 + 1/4 \rfloor + 1 \rfloor$

$3 \leq \lfloor 2+1 \rfloor \Rightarrow 3 \leq 3$ TRUE

case 3: $n = 2$, $\varepsilon = 3/4$ $\lceil 2 + 3/4 \rceil \leq \lfloor \lfloor 2 + 3/4 \rfloor + 1 \rfloor$

$3 \leq \lfloor 2+1 \rfloor \Rightarrow 3 \leq 3$ TRUE

2)

n	s	
1	1	1
2	2	1,1 2
3	4	1,1,1 2,1 3
4	7	1,1,1,1 2,2 1,1,2 1,3
5	14	1,1,1,1,1 1,1,3 1,1,1,2 2,3

$n > 3 : a_n = 2^{n-1} - (n-3)$

$n \leq 3 : a_n = 2^{n-1}$

3) Find the First Five terms

a) $a_n = -2a_{n-1}$, $a_0 = -1$

$a_1 = 2$ $a_2 = -4$ $a_3 = 8$ $a_4 = -16$ $a_5 = 32$

b) $a_n = 3a_{n-1}^2$ $a_0 = 1$

$a_1 = 3$ $a_2 = 27$ $a_3 = 2,187$ $a_4 = 14,348,907$ $a_5 = 617,673,396,283,947$