2017 IEEE International Conference on Computational Science and Engineering (CSE) and IEEE International Conference on Embedded and Ubiquitous Computing (EUC)

A Survey of Multobjective Evolutionary Algorithms

Jiawei Zhang
School of Management Science and Engineering
National University of Defense Technology
Changsha, China

Lining Xing
School of Management Science and Engineering
National University of Defense Technology
Changsha, China

***Abstract*—Multiobjective optimization aims to simultaneously optimize two or more objectives for a problem, with multiobjective evolutionary algorithms (MOEAs) having become a popular research topic in evolutionary multiobjective optimization. We first define the multiobjective optimization problem and briefly summarize multiobjective optimization methods based on the evolutionary algorithm. Representative MOEAs from three categories are then introduced in detail, and we discuss some of the problems and challenges in improving MOEAs. Finally, future research directions for MOEAs are proposed.**

***Keywords*-multobjective optimization, multobjective evolutionary algorithm, evolutionary multobjective optimization.**

I. Introduction

A multiobjective optimization problem (MOP) can be described as follows:

$$\begin{aligned} minF(x) = (f_1(x), f_2(x), \cdots, f_n(x))^T \\ subject\ to\ x \in \Omega, \end{aligned} \tag{1}$$

where Ω is the search space, x is the decision variable vector called an individual, $F : \Omega \rightarrow R^n$ is the function to be optimized, n is the number of objective functions, and R^n is the objective space.

In most practical problems, the objective functions conflict with each other. Thus, there rarely exists a unique global solution that simultaneously optimizes all the functions. Instead, by considering trade-offs among the conflicting objective functions, the concept of Pareto dominance seeks a set of good compromise solutions from multiobjective evolutionary algorithms (MOEAs). We require some basic definitions.

Definition 1. *An individual $x_1 \in \Omega$ dominates another individual $x_2 \in \Omega$, denoted by $x_1 \prec x_2$, if $f_i(x_1) \leq f_i(x_2)$ for all $i \in \{1, 2, \cdots, n\}$ and there exists some $j \in \{1, 2, \cdots, n\}$ such that $f_j(x_1) < f_j(x_2)$.*

Definition 2. *A solution x is said to be Pareto optimal if and only if there is no $x^{'} \in \Omega$ such that $x^{'} \prec x$.*

Definition 3. *The set of Pareto-optimal solutions is called the P-O set, and the corresponding image of the P-O set in the search space is called the Pareto front (PF).*

The evolutionary algorithm (EA) is a stochastic optimization method that simulates the natural evolution process. Compared with traditional optimization techniques, the two prominent characteristics are the population search strategy and the information exchange between the individuals. The EA uses natural evolution mechanisms to reproduce complex phenomena, making full use of the fitness functions without the help of prior knowledge, and solves traditional problems quickly [1].

In the early 1960s, Rosenberg first proposed EAs for solving multiobjective optimization problems. The study of EAs was largely neglected until the middle of the 1980s. Since then, evolutionary computational methods for multiobjective optimization have been given increasing attention. Evolutionary algorithms, which are heuristic search algorithms, have been successfully applied to multiobjective optimization problems, and evolutionary multiobjective optimization (EMO) and multiobjective evolutionary algorithms (MOEAs) have become promising areas of research in evolutionary computation [2].

Traditional multiobjective optimization methods based on EAs can be grouped into three stages. In early EAs (1993$\sim$1998), the individual selection method was based on non-dominated sorting and the population diversity strategy was based on a fitness sharing mechanism. Examples are the MOGA proposed by Fonseca and Fleming [3], the NPGA proposed by Horn et al. [4], and the NSGA proposed by Srinivas and Deb [5]. As EAs improved (1995$\sim$2002), individual selection methods based on external archive strategies, with fast non-dominated sorting and elitism, were developed. For example, NSGA-II proposed by Deb et al. [6], the SPEA [7] and SPEA-II [8] proposed by Zitzler and Thiele, and the PAES proposed by Knowles and Corne [9].

Since 2003, many new as well as improved MOEAs have been proposed to solve MOPs more efficiently and accurately.

Zhang and Li proposed an MOEA based on decomposition (MOEA/D) [10] by combining traditional mathematical programming methods with EAs. A dynamic weight design method based on the projections of the current non-dominated solutions and equidistant interpolation was presented in [11], and was demonstrated to obtain evenly distributed Pareto-optimal solutions. Zitzler et al. provided a rigorous analysis of the limitations underlying different types of quality assessment in [12]. A novel approach to non-dominated sorting, called efficient non-dominated sort, was proposed in [13]. A new opposition-based self-adaptive hybridized differential evolution algorithm was proposed in [14] to deal with continuous MOPs. A selection mechanism based on the -constraint method was introduced in [15] to solve constrained MOPs by extending MOEA/D. Under the premise of strictly controlling the computational budget, a machine learning mechanism for

978-1-5386-3221-5/17 $31.00 © 2017 IEEE
DOI 10.1109/CSE-EUC.2017.27

Authorized licensed use limited to: Southern University of Science and Technology. Downloaded on March 21,2024 at 10:22:32 UTC from IEEE Xplore. Restrictions apply.

MOEA/D was proposed in [16] to solve expensive MOPs. In [17], a reference vector guided evolutionary algorithm for many-objective optimization was presented that balances convergence and diversity of solutions in a high-dimensional objective space. Deb and Jain [18] suggested a many-objective optimization method (NSGA-III), which they extended to solve generic constrained many-objective optimization problems [19]. In [20], experiments were performed to examine the performance of different MOEAs in noisy environments.

Studies of MOEAs have made great progress over the past two decades. This paper presents a comprehensive description of the most important and widely used MOEAs. Section II summarizes the most important MOEAs in three categories. A detailed discussion of the algorithms structure and characteristics is given, including implementation and theoretical aspects. Section III outlines the main conclusions of this work, and Section IV presents directions for future research.

II. Main MOEAs

A. MOEA based on Dominance

Two key problems in the application of dominance-based MOEAs are: 1) how to make the population search approach the PF as soon as possible, which relates to the convergence of the population; and 2) how to obtain uniformly distributed non-dominated solutions, which relates to the diversity of the population. To find a diverse set of solutions that converges to the true PF, concepts such as non-dominated sorting, fitness sharing, the elitist strategy, and crowding distance have been proposed, leading to many different dominance-based MOEAs.

1) NSGA-II: Non-dominated Sorting Genetic Algorithm II

The NSGA proposed by Srinivas and Deb in 1995 is an early dominance-based EA. The basic ideas of NSGA are to identify excellent solutions by the naive and slow procedure of sorting a population into different non-domination levels, and to maintain the diversity of population by a sharing function method. However, because non-dominated sorting has a high computational complexity of $\mathcal{O}(MN^3)$, where M is the number of objectives and N is the population size, NSGA is computationally expensive for large population sizes. Further, without an elite selection strategy NSGA fails to achieve the convergence rates of other MOEAs. In addition, NSGA needs a sharing parameter to calculate the sharing fitness, which ensures the diversity of the population.

Deb et al. [6] proposed NSGA-II to overcome the drawbacks of NSGA. Specifically, NSGA-II presents a fast non-dominated sorting approach with worst-case computational complexity $\mathcal{O}(M(2N)^2)$. This is an iterative process that searches for non-dominated solutions in different levels. First, for each solution x in the population, we calculate two attribute values: 1) n_x, the number of solutions dominating x, and 2) S_x, a set of solutions dominated by x. The solutions for which $n_x = 0$ belong to the first level. Second, we consider each member y in the set S_x for each solution in the first level, and reduce the value of n_y by one. If any n_y is reduced to zero during this stage, the corresponding member y is put in

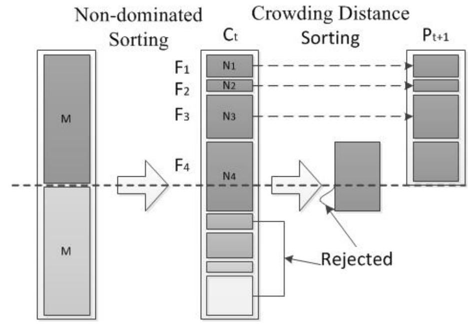

Fig. 1. NSGA-II procedure.

the second level. The above process is continued with each member in the second level to identify the third level and so on. This process continues until all solutions are sorted.

Furthermore, NSGA-II uses the concept of crowding distance with worst-case computational complexity of $\mathcal{O}(M(2N)log(2N))$. The introduction of crowding distance replaces the fitness sharing strategy that requires a sharing parameter to be set by the user. The new approach does not require a user-defined parameter to maintain diversity among solutions. The crowding distance selects solutions located in less-crowded regions after fast non-dominated sorting, which can extend the solution set to an entire PF and ensures the diversity of the population.

Finally, NSGA-II introduces an elitist strategy with worst-case computational complexity of $\mathcal{O}(2Nlog(2N))$. In [21] and [22], the elitist strategy is used to enhance the convergence of an MOEA and avoid the loss of optimal solutions after they are found. As shown in Figure 1, the selection, recombination, and mutation operators are first applied to create an offspring population O_t of size M. A combined population C_t of size $2M$ is then formed, consisting of the current population P_t and the offspring population O_t. By using fast non-dominated sorting, C_t is divided into levels $F_1, F_2, \cdots, F_n$, with the number of solutions in each level F_i denoted by N_i. Next, we choose members for the new population P_{t+1} from the level F_1 to level F_{t-1}, noting that $N_1 + N_2 + \cdots N_t > M$ and $N_1 + N_2 + \cdots N_{t-1} \leq M$. Thereafter, to obtain exactly M population members in P_{t+1}, we sort the solutions in level F_t using the crowding distance and select the best solutions to fill any empty slots in the new population P_{t+1}. This process is continued until the termination condition is satisfied.

2) SPEA-II: Strength-Pareto Evolutionary Algorithm II

The SPEA proposed by Zitzler and Thiele [7] is an early technique for approximating Pareto-optimal solutions of MOPs. The SPEA uses an archive (external population) to preserve the current non-dominated solutions and introduces the concept of a strength value to assign fitness values to both archive and population members. The size of the archive is controlled by a clustering technique to a predefined limit $N^{'}$.

Given an archive $P^{'}$ and a current population P, the fitness value f_m of an individual m in the archive $P^{'}$ is defined as

Authorized licensed use limited to: Southern University of Science and Technology. Downloaded on March 21,2024 at 10:22:32 UTC from IEEE Xplore. Restrictions apply.

$$f_m = s_m,\ m \in P^{'}, \tag{2}$$

the fitness value f_n of an individual n in the current population P is calculated as

$$f_n = 1 + \sum\nolimits_{m \prec n} s_m,\ m \in P^{'},\ n \in P, \tag{3}$$

and the strength value s_m is defined as

$$s_m = \frac{i}{N+1},\ s_m \in [0,1), \tag{4}$$

where i denotes the number of population members dominated by or equal to m, and N is the size of population P.

However, when searching in larger spaces, there appear to be some defects in that the fitness assignment may not be accurate and the convergence rate is not fast. In addition, the clustering technique based on density estimation is only applied to the archive but not to the population, so outer solutions may be lost and a bad spread of non-dominated solutions may be obtained, which may result in poor diversity of solutions.

An improved version, SPEA-II, proposed by Zitzler et al. [8] adjusted the fitness assignment strategy, the density estimation technique, and the archive truncation method. The fitness assignment strategy was improved by redefining the concept of the strength value and introducing a nearest-neighbor density estimation technique. The new strength value s_m is defined as

$$s_m = |\{n | n \in (P + P^{'}) \wedge m \succ n\}|, \tag{5}$$

where n is an individual in the union of P and $P^{'}$. This formulation takes each individual into account for both dominating and dominated solutions. The raw fitness r_m of the individual m is defined as

$$r_m = \sum\nolimits_{n \in (P+P^{'}), n \succ m} s_n. \tag{6}$$

However, when most individuals do not dominate each other, the raw fitness value may be invalid because many individuals may have identical raw fitness values. Therefore, the density estimation technique is used to compute the density d_m of individual m:

$$d_m = \frac{1}{\delta_m^k + 2}, \tag{7}$$

where δ_m^k represents the distance from the individual m to its kth nearest neighbor. In general, we set k to be the square root of the sum of the population size N and the archive size $N^{'}$:

$$k = \sqrt{N + N'}. \tag{8}$$

The fitness f_m is calculated as

$$f_m = r_m + d_m. \tag{9}$$

A new archive truncation method is used to preserve boundary solutions. When the size of the current non-dominated set exceeds the archive size $N^{'}$, the archive truncation process is performed iteratively to remove individuals from the current archive until the size of non-dominated set is reduced to $N^{'}$. An individual that has the minimum distance to another individual is selected for removal. If there are several individuals with the same minimum distance, then the second smallest distances are considered, and so on until two different distances are found. This ensures that an individual with a relatively small distance to others in the archive is chosen for removal.

B. MOEA based on Performance Indicator

In recent years, indicator-based MOEAs, which allow the implicit incorporation of user preferences in a search, have become widely used for solving MOPs. Several performance indicators are used to measure the quality of a Pareto-optimal set. The hypervolume, also known as the hyperarea [23], the S-metric [24] and the Lebesgue measure [25], [26], is of particular importance. This is a comprehensive evaluation indicator that synthetically rewards convergence and the diversity of the PF by computing the volume of the objective space dominated by the approximation set. When the population approximates the real PF and is distributed uniformly, the indicator value is large. Fleischer [26] proved that the hypervolume is maximized if and only if the set of solutions contains only Pareto optima.

Because of the superiority of the hypervolume indicator in quality evaluation of populations, some researchers have embedded it into the environmental selection operator in MOEAs to guide the search process. Some representative indicator-based MOEAs are the IBEA [27], the SIBEA [28], and the SMS-EMOA [24]. For the convenience of discussion, we require several further definitions.

Definition 4. *Let $S \in Z$ be a Pareto set approximation and let $q \in Z$ be an individual. The objective space dominated by q is called the independent-dominating space.*

Definition 5. *Let $A \in Z$ be a Pareto set approximation and let $R \in Z$ be a reference set of mutually non-dominating objective vectors. Then the hypervolume indicator $H(A,R)$ is*

$$Hv(A,R) = Leb(H(A,R)), \tag{10}$$

where

$$H(A,R) := \{z \in Z; \exists a \in A, \exists r \in R : f(a) \leq z \leq r\} \tag{11}$$

and Leb is the Lebesgue measure. The set $H(A,R)$ denotes the objective vectors that are enclosed by the front $f(A)$ given by A and the reference set R.

Definition 6. *According to Definitions 4 and 5 , the space $H(A,R)$ can be partitioned into sets $H(S,A,R)$ each associated with a specific subset $S \in A$:*

Authorized licensed use limited to: Southern University of Science and Technology. Downloaded on March 21,2024 at 10:22:32 UTC from IEEE Xplore. Restrictions apply.

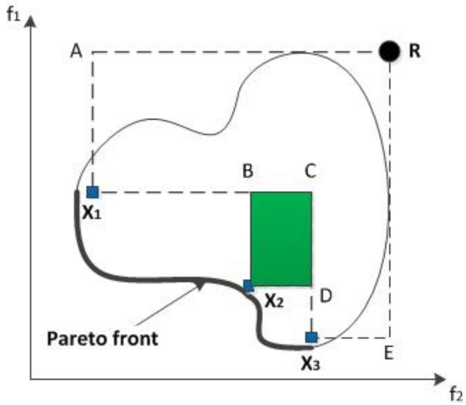

Fig. 2. The hypervolume dominated by $\{X_1, X_2, X_3\}$ is the area enclosed by the set $\{A, X_1, B, X_2, D, X_3, E, R\}$ where R is the reference point, and the independent-dominating space of X_2 is the rectangle area enclosed by the set $\{B, C, D, X_2\}$.

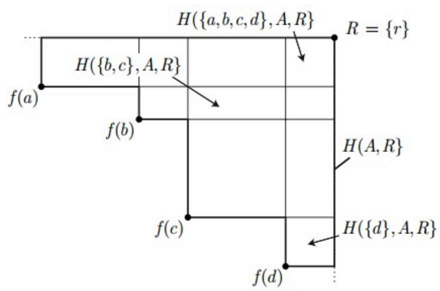

Fig. 3. The relationship between $H(A, R)$ and $H(S, A, R)$.

$$H(S, A, R) := [\cap_{s \in S} H(\{s\}, R)] \backslash [\cup_{a \in A \backslash S} H(\{a\}, R)]. \quad (12)$$

The set $H(S, A, R)$ represents the region of the objective space that is jointly dominated by the solutions in S but is not weakly dominated by any other solution in A. Note that:

$$\cup_{S \subseteq A} H(S, A, R) = H(A, R). \quad (13)$$

1) SMS-EMOA: S Metric Selection Evolutionary Multiobjective Algorithm

The SMS-EMOA was designed to find a maximal hypervolume of the objective space covered by a finite number of points. It combines fast non-dominated sorting with the selection operator by referring to the advantages of other MOEAs such as NSGA-II and archiving strategies. The SMS-EMOA is a steady-state algorithm with two important features: 1) the non-dominated sorting approach is applied as a ranking criterion, and 2) the hypervolume is used as the selection criterion to remove individuals that contribute the least hypervolume to the worst-ranked front.

In the SMS-EMOA algorithm, a random population is first initialized. Subsequently new individuals are generated by a randomized variation operation and is combined with the current population to form a temporary population. The fast non-dominated sorting approach is then used to partition the temporary population into different non-domination levels indexed by hierarchical order. Each non-domination level is called a front, within which the solutions are mutually non-dominating. An individual contributing the least hypervolume to the worst-ranked front is discarded. The algorithm repeats this procedure until the termination condition is satisfied. The algorithm structure of SMS-EMOA is as follows.

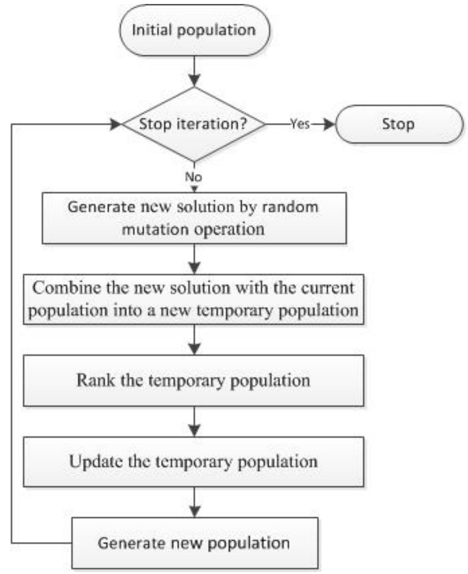

Fig. 4. The SMS−EMOA procedure.

Algorithm 1 SMS-EMOA

$S_0 \leftarrow init()$
$t \leftarrow 0$
repeat
  $q_{t+1} \leftarrow generate(S_t)$
  $\{P_0, \cdots, P_n\} \leftarrow fast\ nondominated\ sort(S_t + \{q_{t+1}\})$
  $r \leftarrow argmin_{x \in P_n}[\triangle Hv(x, P_n)]$
  $S_{t+1} \leftarrow (S_t + \{q_{t+1}\} - \{r\})$
  $t \leftarrow t + 1$
until $termination\ condition\ fulfilled$

The expression $\triangle Hv()$ is the contributing hypervolume, which can be interpreted as the exclusive contribution of x to the hypervolume of the front, and is defined by

$$\triangle Hv(x, P_n) = Hv(P_n, R) - Hv((P_n - \{x\}), R). \quad (14)$$

From the definition of $\triangle Hv(x, P_n)$, an individual that dominates another individual is always retained, and it is impossible for a dominated individual to take the place of a non-dominated individual. Those individuals that maximize the populations hypervolume are always retained, which implies that the covered hypervolume of a population will not

Authorized licensed use limited to: Southern University of Science and Technology. Downloaded on March 21,2024 at 10:22:32 UTC from IEEE Xplore. Restrictions apply.

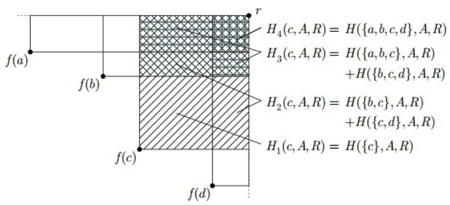

Fig. 5. The relationship between $H_i(a, A, R)$ and $H(S, A, R)$.

decrease. Thus $Hv(P_n) \leq Hv(P_{n+1})$, so the population gradually evolves in the direction of optimization.

2) HypE: Hypervolume Estimation Algorithm

In hypervolume-based MOEAs, the high computational complexity of hypervolume calculation seriously hinders the effective performance of this indicator. Thus this search algorithm is only applied to problems with few objectives [29], [30]. To overcome this drawback, researchers have proposed a number of improvements ([31], [32]), in particular approximating the hypervolume by Monte Carlo simulation ([33], [34]). We present a hypervolume estimation algorithm (HypE) based on this idea.

Using the partitioning of the objective space in ***Definition 6***, it is infeasible to determine all the sets $H(S, A, R)$ because of combinatorial explosion. Instead, we consider a more concise splitting of the objective space with regard to single solutions. That is,

$$H_i(a, A, R) := \cup_{\substack{S \subseteq A \\ a \in S \\ |S|=i}} H(S, A, R), \tag{15}$$

where $|S|$ is the number of elements in the set S.

From the above formula, $H_i(a, A, R)$ represents the region of the objective space that is jointly dominated by a and any $i-1$ other solutions in A (see Figure 5). This partition method decreases the number of subspaces to be considered from $2^{|A|}$ to $|A|^2$.

For a population P, each solution $a \in P$ corresponds to a vector $(Leb(H_1(a, P, R)), \cdots, Leb(H_{|P|}(a, P, R)))$ of hypervolume contributions, which is used to assign fitness values to solutions. Note that

$$Hv(a, P, R) := \sum\nolimits_{i=1}^{|A|} \frac{1}{i} Leb(H_i(a, P, R)). \tag{16}$$

Because $H_i(a, P, R)$ is integrable, its Lebesgue measure can be approximated by Monte Carlo simulation, thus also approximating the fitness values for each solution a in the population P. Thus, a sample space $S \subseteq Z$ needs to be defined that satisfies the following three conditions: 1) the hypervolume of S can be computed easily, 2) samples from S can be chosen quickly, and 3) S is a superset of $H_i(a, P, R)$.

Through comprehensive consideration of these factors, we suggest defining the sample space S by using an axis-aligned bounding box [35] that contains the $H_i(a, P, R)$ subspaces:

$$S := \{(z_1, z_2, \cdots, z_n) \in Z | \forall 1 \leq i \leq n : l_i \leq z_i \leq u_i\}, \tag{17}$$

where

$$l_i := min_{a \in P} f_i(a), \tag{18}$$

$$u_i := max_{(r_1, r_2, \cdots, r_n) \in R} r_i, \tag{19}$$

for $1 \leq i \leq n$. Hence, the hypervolume V of the sample space S is

$$V = \prod_{i=1}^{n} max(0, u_i - l_i). \tag{20}$$

The objective vectors $s_1, s_2, \cdots, s_M$ are obtained as points sampled uniformly at random from S. Next, it is necessary to determine whether each sampled point s_j lies in a partition $H_i(a, P, R)$, which can be checked in two ways: 1) whether there exists an $r_i \in R$ dominated by s_j, and 2) whether the set A of population members that dominate s_j is not empty. Thus, we introduce the variable $X_j^{(i,a)}$ that equals 1 if s_j lies in all partitions $H_i(a, P, R)$ where $a \in A, i = |A|$ and equals 0 therwise. Finally, an estimate of the Lebesgue measure for each $H_i(a, P, R)$ is given by multiplying the proportion of the sampled points that are in $H_i(a, P, R)$ by the hypervolume of the sample space S:

$$\hat{Leb}(H_i(a, P, R)) = \frac{\sum_{j=1}^{M} x_j^{(i,a)}}{M} V. \tag{21}$$

Applying the above idea to fitness assignment and the selection strategy in an MOEA can significantly reduce the computational time and cost and can improve the performance of the algorithms. Experiments show that HypE can solve MOPs with an arbitrary number of objective functions and that it is highly effective compared with existing indicator-based MOEAs.

C. MOEA based on Decomposition

Decomposition is widely used to solve traditional MOPs. Since the proposal of MOEA/D by Zhang and Li [10], the approach has attracted significant attention from many researchers. Although domination-based MOEAs and indicator-based MOEAs remain very popular, they both have limitations. First, MOEAs based on Pareto domination are not applicable to many-objective optimization problems because almost all solutions in the population become non-dominated relative to each other, thereby reducing the selection pressure and hindering the process of evolution [36]. Second, MOEAs based on performance indicators always use the hypervolume indicator, which is computationally expensive because of the exponential with the number of objectives. Some studies have suggested alternative indicators with lower computational costs and good theoretical properties to take the place of the hypervolume indicator, such as R2 [37]. Nevertheless, more attention has been given to MOEA/D in the past few years. Important research directions include improving weight vector generation methods, applying new multiobjective decomposition methods, efficient allocation of computational resources, modifying the reproduction operators, and enhancing mating selection and the replacement procedure.

Authorized licensed use limited to: Southern University of Science and Technology. Downloaded on March 21,2024 at 10:22:32 UTC from IEEE Xplore. Restrictions apply.

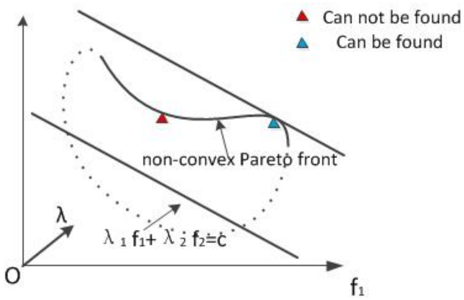

Fig. 6. Illustration of the WS approach.

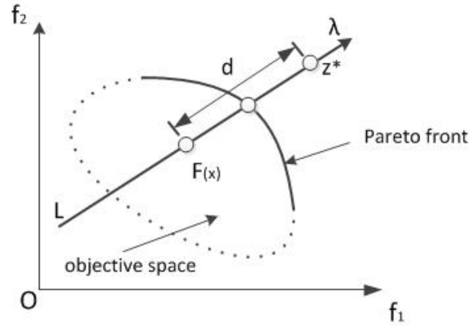

Fig. 7. Illustration of BI approach.

In MOEA/D, an MOP is decomposed into several subproblems by using a decomposition method such as a weighted sum (WS) approach, and a suitable EA is used to solve the subproblems simultaneously by evolving a population of solutions. In fact, the optimal solutions are obtained by defining a neighborhood relationship based on the distance between weight vectors for different subproblems. Therefore, the optimal solutions for two adjacent subproblems are very similar, and each subproblem is optimized with the help of the optimal solution from the adjoint subproblem.

In this section, five methods of decomposing an MOP into N subproblems are introduced. Other aspects of research in MOEA/D are not discussed. We define a set of uniformly spread weight vectors $\lambda_1, \lambda_2, \cdots, \lambda_N$ for an optimization problem with m objectives, where $\lambda_i = (\lambda_i^1, \lambda_i^2, \cdots, \lambda_i^m)$ and $\sum_{j=1}^{m} \lambda_i^j = 1 (i = 1, 2, \cdots, N)$.

1) WS: Weighted Sum Approach

This approach employs a convex combination of different objective functions, where the ith subproblem is formulated as follows:

$$minimize\ g^{ws}(x|\lambda_i) = \sum\nolimits_{j=1}^{m} \lambda_i^j f_j(x), x \in \Omega. \tag{22}$$

The WS decomposition approach obtains a set of Pareto-optimal solutions by generating a set of weight vectors for the optimization problem, and is suitable for convex problems (for minimization problems). Thus, as shown in Figure 6, it cannot deal with non-convex PFs as it is quite difficult to find Pareto solutions with a uniform distribution.

2) TCH: Tchbycheff Approach

TCH decomposition approach adds a set of ideal reference points $z_1^*, \cdots, z_m^*$ on the basis of WS, where the ith subproblem can is formulated as follows:

$$minimize\ g^{te}(x|\lambda_i, z^*) = max_{1 \le j \le m}\{\lambda_i^j |f_j(x) - z_j^*|\}. \tag{23}$$

Experiments show that TCH and NSGA-II perform similarly.

3) BI: Boundary Intersection Approach

From the geometric point of view, the Pareto-optimal solution of the MOP can be approximated as the intersection of a set of uniformly distributed straight lines and the PF. Therefore, BI decomposition approach based on this concept can handle the problem with the non-convex PFs. In this approach, we usually select a set of rays L from the reference point z (the direction vector of L is λ), then the ith subproblem is formulated as follows:

$$minimize\ g^{bi}(x|\lambda_i, z^*) = d_i, x \in \Omega. \tag{24}$$

As shown in Figure 7, the constraint $z^* - F(x) = d\lambda$ ensures that $F(x)$ is located on the line L. The purpose of BI approach is to make $F(x)$ as close as possible to the PF, so that the optimal solution of the feasible objective space can be effectively searched.

4) PBI: Penalty-based Boundary Intersection Approach

The drawback of the boundary intersection approach is that it must deal with equality constraints. In practice, the penalty-based boundary intersection decomposition approach is often used to solve multiple constrained MOPs. The ith subproblem is formulated as follows:

$$minimize\ g^{pbi}(x|\lambda_i, z^*) = d_1 + \theta d_2, x \in \Omega, \theta > 0, \tag{25}$$

$$d_1 = \frac{||(F(x) - z^*)^T \lambda_i||}{||\lambda_i||}, \tag{26}$$

$$d_2 = ||F(x) - (z^* - d_1 \frac{\lambda_i}{||\lambda_i||})||, \tag{27}$$

where θ is a default penalty parameter, and z is the reference point as defined in the TCH approach.

As shown in Figure 8, if we let y be the projection of $F(x)$ onto the line L, then d_1 represents the distance between z^* and y, and d_2 represents the distance between $F(x)$ and L. We can infer that the optimal solutions will gradually approach the PF by considering the distances between the individuals and the weight vectors along with the distances between the weight vectors and the Pareto-optimal boundary.

5) WL_p: Weighted L_p Approach

In the weighted L_p approach, the ith subproblem is formulated as follows:

Authorized licensed use limited to: Southern University of Science and Technology. Downloaded on March 21,2024 at 10:22:32 UTC from IEEE Xplore. Restrictions apply.

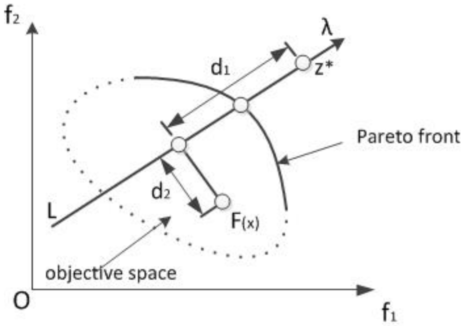

Fig. 8. Illustration of PBI approach.

$$minimize\ g^{L_p}(x|\lambda_i, z^*) = (\sum_{j=1}^{m} \lambda_i^j |f_j(x) - z_j^*|^p)^{\frac{1}{p}}\ , x \in \Omega, \quad (28)$$

where z^* is the ideal point, and p is the key parameter. The WS approach and the TCH approach can be derived by assigning $p = 1$ and $p = \infty$, respectively.

By analyzing the properties of the L_p approach for different p values, it has been found that there is a trade-off between the search ability of the L_p approach and its robustness on PF geometries [38]. As the p value increases, the search ability of the L_p approach weakens but it becomes more robust on PF geometries. Therefore, it is very important to select an appropriate p value for each weight vector and for different PF geometries. To avoid estimating the geometric shape of the PF, Wang [38] proposed MOEA/D-PaS which uses a simple and effective method called Pareto adaptive scalarizing (PaS) to obtain a suitable p value. The algorithm MOEA/D-PaS has been demonstrated to outperform MOEA/D with adaptive use of WS and TCH [39], and MOEA/D with simultaneous use of WS and TCH [40].

III. Conclusion

This paper presents a comprehensive survey of research on MOEAs, which shows that the study of MOEAs has progressed significantly in the past two decades and continues to remain popular as a research topic. There is no clear indication of how long this research interest will be sustained. Most studies use simulation methods to verify the performance of the algorithms through different types of test functions, but there are still few strict theoretical proofs. Moreover, compared with the large number of algorithms proposed each year, relatively few commercial EMO software tools have been developed.

At present, MOEAs based on Pareto domination are the most advanced and perform best at solving general MOPs. However, there are obvious limitations on their application to high-dimensional problems, so it is necessary to consider new domination mechanisms and to explore efficient heuristic algorithms.

The prospects for further developing MOEAs based on performance indicators are relatively narrow because of limitations on the types of performance indicators and the high computational complexity of the algorithms. Thus, the effectiveness of different indicators needs further study, and new performance indicators need to be proposed.

Decomposition-based MOEAs now have good performance in solving high-dimension problems. However, because the approaches to generating the new population, fitness distribution, solution evaluation and so on are still in the early stages of development, more research is needed to explore these ideas and to improve the performance of the algorithms.

IV. Future Directions

Our survey indicates that there are several future research directions to be pursued regarding MOEAs. These are summarized below.

1) The ultimate goal of MOEAs is to solve practical problems. By investigating the common properties of real-world MOPs, it may be possible to combine these properties and design a universal MOEA.

2) Researchers should consider applying well-developed single-objective optimization methods to MOPs.

3) The evaluation of solutions needs further improvement, including the assessment of both convergence and diversity.

4) In general, the algorithms have been developed for certain problems. Thus, it is crucial to propose a wide variety of test problems with more features by examining real-world problems.

5) The robustness of MOEAs for various test problems needs further analysis.

6) In most MOEAs, it is essential to improve the adaptability of the search mechanism to each subproblem.

7) Only a few theoretical studies have focused on analyzing MOEAs. Thus, the theoretical aspects of MOEAs need to be studied further.

8) Currently, many-objective optimization problems , which refers to a series of problems with four or more objectives, present several challenges for MOEA research. Thus, MOEAs need to be extended to solve many-objective optimization problems.

The MOEAs are designed to solve complex real-world optimization problems, so the interaction between MOEAs and practical engineering problems is of paramount importance. However, there are many difficulties with this, such as how to reasonably distinguish between Multiobjective and single-objective engineering problems and how to exploit both realistic and available constraints. Attempts to establish an effective correspondence between algorithms and engineering practices are certainly worth considering.

Acknowledgment

This work is supported by the National Natural Science Foundation of China (No.71331008, 61473301, 71501180, 71501179 and 61603400).

Authorized licensed use limited to: Southern University of Science and Technology. Downloaded on March 21,2024 at 10:22:32 UTC from IEEE Xplore. Restrictions apply.

References

[1] V. L. Vachhani, V. K. Dabhi, and H. B. Prajapati, "Survey of multi objective evolutionary algorithms," in *2015 International Conference on Circuits, Power and Computing Technologies [ICCPCT-2015]*, March 2015, pp. 1–9.

[2] M. G. Gong, L. C. Jiao, D. D. Yang, and W. P. Ma, "Research on evolutionary multi-objective optimization algorithms," *Journal of Software*, vol. 20, no. 20, pp. 271–289, 2009.

[3] C. M. Fonseca and P. J. Fleming, "Multiobjective genetic algorithms," in *IEE Colloquium on Genetic Algorithms for Control Systems Engineering*, May 1993, pp. 6/1–6/5.

[4] J. Horn, N. Nafpliotis, and D. E. Goldberg, "A niched pareto genetic algorithm for multiobjective optimization," in *Proceedings of the First IEEE Conference on Evolutionary Computation. IEEE World Congress on Computational Intelligence*, Jun 1994, pp. 82–87 vol.1.

[5] N. Srinivas and K. Deb, "Muiltiobjective optimization using nondominated sorting in genetic algorithms," *IEEE Transactions on Evolutionary Computation*, vol. 2, no. 3, pp. 221–248, Sept 1994.

[6] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: Nsga-ii," *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, pp. 182–197, Apr 2002.

[7] E. Zitzler and L. Thiele, "Multiobjective evolutionary algorithms: a comparative case study and the strength pareto approach," *IEEE Transactions on Evolutionary Computation*, vol. 3, no. 4, pp. 257–271, Nov 1999.

[8] M. L. E. Zitzler and L. Thiele, "Spea2: Improving the strength pareto evolutionary algorithm," 2001.

[9] J. Knowles and D. Corne, "The pareto archived evolution strategy: a new baseline algorithm for pareto multiobjective optimisation," in *Proceedings of the 1999 Congress on Evolutionary Computation-CEC99 (Cat. No. 99TH8406)*, vol. 1, 1999, p. 105 Vol. 1.

[10] Q. Zhang and H. Li, "Moea/d: A multiobjective evolutionary algorithm based on decomposition," *IEEE Transactions on Evolutionary Computation*, vol. 11, no. 6, pp. 712–731, Dec 2007.

[11] H. L. F. Gu, Fangqing and K. C. Tan, "A multiobjective evolutionary algorithm using dynamic weight design method," *International Journal of Innovative Computing Information & Control Ijicic*, vol. 8, no. 5, pp. 3677–3688, 2012.

[12] E. Zitzler, L. Thiele, M. Laumanns, C. M. Fonseca, and V. G. da Fonseca, "Performance assessment of multiobjective optimizers: an analysis and review," *IEEE Transactions on Evolutionary Computation*, vol. 7, no. 2, pp. 117–132, April 2003.

[13] X. Zhang, Y. Tian, R. Cheng, and Y. Jin, "An efficient approach to nondominated sorting for evolutionary multiobjective optimization," *IEEE Transactions on Evolutionary Computation*, vol. 19, no. 2, pp. 201–213, April 2015.

[14] J. K. Chong and X. Qiu, "An opposition-based self-adaptive differential evolution with decomposition for solving the multiobjective multiple salesman problem," in *2016 IEEE Congress on Evolutionary Computation (CEC)*, July 2016, pp. 4096–4103.

[15] S. Z. Martinez and C. A. C. Coello, "A multi-objective evolutionary algorithm based on decomposition for constrained multi-objective optimization," in *2014 IEEE Congress on Evolutionary Computation (CEC)*, July 2014, pp. 429–436.

[16] Y. S. Liau, K. C. Tan, J. Hu, X. Qiu, and S. B. Gee, *Machine Learning Enhanced Multi-Objective Evolutionary Algorithm Based on Decomposition*. Springer Berlin Heidelberg, 2013.

[17] R. Cheng, Y. Jin, M. Olhofer, and B. Sendhoff, "A reference vector guided evolutionary algorithm for many-objective optimization," *IEEE Transactions on Evolutionary Computation*, vol. 20, no. 5, pp. 773–791, Oct 2016.

[18] K. Deb and H. Jain, "An evolutionary many-objective optimization algorithm using reference-point-based nondominated sorting approach, part i: Solving problems with box constraints," *IEEE Transactions on Evolutionary Computation*, vol. 18, no. 4, pp. 577–601, Aug 2014.

[19] H. Jain and K. Deb, "An evolutionary many-objective optimization algorithm using reference-point based nondominated sorting approach, part ii: Handling constraints and extending to an adaptive approach," *IEEE Transactions on Evolutionary Computation*, vol. 18, no. 4, pp. 602–622, Aug 2014.

[20] C. K. Goh, S. C. Chiam, and K. C. Tan, "An investigation on noisy environments in evolutionary multi-objective optimization," in *2006 IEEE Conference on Cybernetics and Intelligent Systems*, June 2006, pp. 1–7.

[21] E. Zitzler, K. Deb, and L. Thiele, "Comparison of multiobjective evolutionary algorithms: Empirical results," *Evolutionary Computation*, vol. 8, no. 2, pp. 173–195, June 2000.

[22] ——, "Comparison of multiobjective evolutionary algorithms: Empirical results," *Evolutionary Computation*, vol. 8, no. 2, pp. 173–195, June 2000.

[23] S. S. G. A. C. C. A. Lara, S. Alvarado and O. Schutze, "The gradient free directed search method as local search within multi-objective evolutionary algorithms," in *Evolve*, 2013.

[24] B. N. N. Beume and M. Emmerich, "Sms-emoa: Multiobjective selection based on dominated hypervolume," *European Journal of Operational Research*, vol. 181, no. 3, pp. 1653–1669, 2007.

[25] M. Laumanns, E. Zitzler, and L. Thiele, "A unified model for multi-objective evolutionary algorithms with elitism," in *Proceedings of the 2000 Congress on Evolutionary Computation. CEC00 (Cat. No.00TH8512)*, vol. 1, 2000, pp. 46–53 vol.1.

[26] F. L. G. R. Zavala, A. J. Nebro and C. A. C. Coello, "A survey of multi-objective metaheuristics applied to structural optimization," *Structural & Multidisciplinary Optimization*, vol. 49, no. 4, pp. 537–558, 2014.

[27] E. Zitzler and S. Knzli, *Indicator-Based Selection in Multiobjective Search*, 2004, vol. 3242.

[28] D. Brockhoff and E. Zitzler, "Improving hypervolume-based multiobjective evolutionary algorithms by using objective reduction methods," in *2007 IEEE Congress on Evolutionary Computation*, Sept 2007, pp. 2086–2093.

[29] L. While, "A new analysis of the lebmeasure algorithm for calculating hypervolume," in *International Conference on Evolutionary Multi-Criterion Optimization*, 2005, pp. 326–340.

[30] J. Bader and E. Zitzler, *A Hypervolume-Based Optimizer for High-Dimensional Objective Spaces*. Springer Berlin Heidelberg, 2010.

[31] C. Igel, N. Hansen, and S. Roth, "Covariance matrix adaptation for multi-objective optimization," *Evolutionary Computation*, vol. 15, no. 1, pp. 1–28, March 2007.

[32] T. C. Wang, R. T. Liaw, and C. K. Ting, "Moea/d using covariance matrix adaptation evolution strategy for complex multi-objective optimization problems," in *2016 IEEE Congress on Evolutionary Computation (CEC)*, July 2016, pp. 983–990.

[33] J. Bader and E. Zitzler, "Hype: An algorithm for fast hypervolume-based many-objective optimization," *Evolutionary Computation*, vol. 19, no. 1, pp. 45–76, March 2011.

[34] J. Bader, K. Deb, and E. Zitzler, "Faster hypervolume-based search using monte carlo sampling," *Lecture Notes in Economics & Mathematical Systems*, vol. 634, no. 32, pp. 313–326, 2010.

[35] H. A. Sulaiman, M. A. Othman, M. M. Ismail, M. A. M. Said, A. Ramlee, M. H. Misran, A. Bade, and M. H. Abdullah, "Distance computation using axis aligned bounding box (aabb) parallel distribution of dynamic origin point," in *2013 Annual International Conference on Emerging Research Areas and 2013 International Conference on Microelectronics, Communications and Renewable Energy*, June 2013, pp. 1–6.

[36] N. T. H. Ishibuchi and Y. Nojima, "Evolutionary many-objective optimization: A short review," in *2008 IEEE Congress on Evolutionary Computation (IEEE World Congress on Computational Intelligence)*, June 2008, pp. 2419–2426.

[37] D. Brockhoff, T. Wagner, and H. Trautmann, "2 indicator-based multiobjective search," *Evolutionary Computation*, vol. 23, no. 3, pp. 369–395, Sept 2015.

[38] R. Wang, Q. Zhang, and T. Zhang, "Decomposition-based algorithms using pareto adaptive scalarizing methods," *IEEE Transactions on Evolutionary Computation*, vol. 20, no. 6, pp. 821–837, Dec 2016.

[39] N. T. H. Ishibuchi, Y. Sakane and Y. Nojima, "Adaptation of scalarizing functions in moea/d: An adaptive scalarizing function-based multiobjective evolutionary algorithm," in *Evolutionary Multi-Criterion Optimization, International Conference, Emo 2009, Nantes, France, April 7-10, 2009. Proceedings*, 2009, pp. 438–452.

[40] K. D. H. Ishibuchi and Y. Nojima, *Use of Piecewise Linear and Nonlinear Scalarizing Functions in MOEA/D*. Springer International Publishing, 2016.

Authorized licensed use limited to: Southern University of Science and Technology. Downloaded on March 21,2024 at 10:22:32 UTC from IEEE Xplore. Restrictions apply.