Oefeningen Wiskunde II 2013, 1e Bach Informtica

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Hoofdstuk 1

1.1

a)

$$\begin{pmatrix} 1 & 1 & -2 & 10 \\ 4 & 2 & 2 & 1 \\ 6 & 2 & 3 & 4 \end{pmatrix} \xrightarrow[R3 = R3 - 6R1]{R2 = R2 - 4R1} \begin{pmatrix} 1 & 1 & -2 & 10 \\ 0 & -2 & 10 & -39 \\ 0 & -4 & 15 & -56 \end{pmatrix} \xrightarrow[R3 = -\frac{3}{4}R3]{R2 = -\frac{1}{2}R2} \begin{pmatrix} 1 & 1 & -2 & 10 \\ 0 & 1 & -5 & \frac{39}{2} \\ 0 & 1 & -\frac{15}{4} & 14 \end{pmatrix}$$

$$\xrightarrow[R3 = R3 - R2]{R1 = R1 - R2} \begin{pmatrix} 1 & 0 & 3 & -\frac{19}{2} \\ 0 & 1 & 3 & \frac{39}{2} \\ 0 & 0 & \frac{5}{4} & -\frac{11}{2} \end{pmatrix} \xrightarrow{R3 = \frac{4}{5}R3} \begin{pmatrix} 1 & 0 & 3 & -\frac{19}{2} \\ 0 & 1 & 3 & \frac{39}{2} \\ 0 & 0 & 1 & -\frac{22}{5} \end{pmatrix} \xrightarrow[R2 = R2 - 3R3]{R1 = R1 - 3R3} \begin{pmatrix} 1 & 0 & 0 & \frac{37}{10} \\ 0 & 1 & 0 & -\frac{5}{2} \\ 0 & 0 & 1 & -\frac{22}{5} \end{pmatrix}$$

Antwoord:
$x = \frac{37}{10}, y = -\frac{5}{2}, z = -\frac{22}{5}$

Correcter is:
$V = \left\{\left(\frac{37}{10}, -\frac{5}{2}, -\frac{22}{5}\right)\right\}$

b)

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 3 & -1 & 2 & 7 \\ 5 & 3 & -4 & 2 \end{array}\right) \xrightarrow[R3 = R3 - 5R1]{R2 = R2 - 3R1} \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & -7 & 11 & 10 \\ 0 & -7 & 11 & 7 \end{array}\right)$$

Antwoord:
Het stelsel is strijdig.

Correcter:

$V = \emptyset$

1.4

$(Ax+B)(x^2-1)+C(x^2+4)(x+1)+D(x^2+4)(x-1)$
$= Ax^3 - Ax + Bx^2 - B + cx^3 + cx^2 + 4Cx + 4C + Dx^3 - Dx^2 + 4Dx - 4D$

$$\begin{cases} A+C+D=0 \\ B+C-D=0 \\ -A+4C+4D=0 \\ -B+4C-4D=1 \end{cases} \longrightarrow \left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ -1 & 0 & 4 & 4 & 0 \\ 0 & -1 & 4 & -4 & 1 \end{array}\right) \overset{GRM}{\rightarrow} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{5} \\ 0 & 0 & 1 & 0 & \frac{1}{10} \\ 0 & 0 & 0 & 1 & -\frac{1}{10} \end{array}\right)$$

Antwoord:

$$\begin{cases} A=0 \\ B=-\frac{1}{5} \\ C=\frac{1}{10} \\ D=-\frac{1}{10} \end{cases}$$

1.6

a)

$$\begin{pmatrix} 1 & 2 & -3 & 6 \\ 2 & -1 & 4 & 2 \\ 4 & 3 & -2 & a \end{pmatrix} \overset{\substack{R2=R2-2R1 \\ R3=R3-4R1}}{\rightarrow} \begin{pmatrix} 1 & 2 & -3 & 6 \\ 0 & -5 & 10 & -10 \\ 0 & -5 & 10 & (a-24) \end{pmatrix} \overset{\substack{R2=-\frac{1}{5}R2 \\ R3=-\frac{1}{5}R3}}{\rightarrow} \begin{pmatrix} 1 & 2 & -3 & 6 \\ 0 & 1 & -2 & 2 \\ 0 & 1 & -2 & -\frac{(a-24)}{5} \end{pmatrix}$$

Antwoord:
Voor $a \neq 14$ zal het stelsel geen oplossingen hebben. Voor $a = 14$ zal het stelsel oneindig veel oplossingen hebben.

(voor $a = 14$)

$$\begin{pmatrix} 1 & 2 & -3 & 6 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Wat we nog kunnen rijreduceren tot:

$$\begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

We constateren dat z hier een vrije variabele is en wijzen haar de waarde λ toe. Dan is onze oplossingsverzameling de volgende:

$V = \{(2-\lambda, 2+2\lambda, \lambda)\,|\lambda \in \mathbb{R}\}$

b)

$$\left(\begin{array}{cc|c} 1 & a & (a+1) \\ a & 1 & 2 \end{array}\right) \overset{R2 = R2 - aR1}{\rightarrow} \left(\begin{array}{cc|c} 1 & a & (a+1) \\ 0 & (1-a^2) & (a^2-a+2) \end{array}\right)$$

Antwoord:
Voor $a = 1$: oneindig veel oplossingen, $\begin{cases} x = 2\lambda \\ y = \lambda \end{cases}$
Voor $a = -1$: geen oplossingen.
Voor $a \neq 1 \wedge a \neq -1$: precies 1 oplossing, $\begin{cases} x = \frac{1}{a+1} \\ y = \frac{a+2}{a+1} \end{cases}$

c)

$$\left(\begin{array}{ccc|c} a & (a+1) & 1 & 0 \\ a & 1 & (a+1) & 0 \\ 2a & 1 & 1 & (a+1) \end{array}\right) \overset{\substack{R2 = R2 - R1 \\ R3 = R3 - 2R1}}{\rightarrow} \left(\begin{array}{ccc|c} a & (a+1) & 1 & 0 \\ 0 & -a & a & 0 \\ 0 & -(2a+1) & -1 & (a+1) \end{array}\right)$$

$$\overset{R2 = -\frac{1}{a}R2}{\rightarrow} \left(\begin{array}{ccc|c} a & (a+1) & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -(2a+1) & -1 & (a+1) \end{array}\right) \overset{\substack{R1 = R1 - (a+1)R2 \\ R3 = R3 + (2a+1)R2}}{\rightarrow} \left(\begin{array}{ccc|c} a & 0 & (a+2) & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -2(a+1) & (a+1) \end{array}\right)$$

$$\overset{R3 = -\frac{2}{(a+1)}R3}{\rightarrow} \left(\begin{array}{ccc|c} a & 0 & (a+2) & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} \end{array}\right) \overset{\substack{R1 = R1 - (a+2)R3 \\ R2 = R2 + R3}}{\rightarrow} \left(\begin{array}{ccc|c} a & 0 & 0 & \frac{a+2}{2} \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} \end{array}\right)$$

$$\overset{R1 = \frac{1}{a}R1}{\rightarrow} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{a+2}{2a} \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} \end{array}\right)$$

Antwoord:
Voor $a = 0$: geen oplossingen.
Voor $a = -1$: oneindig veel oplossingen: $\begin{cases} x = \lambda \\ y = \lambda \\ z = \lambda \end{cases}$
Voor $a \neq -1 \wedge a \neq 0$: precies 1 oplossing: $\begin{cases} x = \frac{a+2}{2a} \\ y = -\frac{1}{2} \\ z = -\frac{1}{2} \end{cases}$

1.8

$$\begin{pmatrix} k & 5 & 3 \\ 5 & 1 & -1 \\ k & 2 & 1 \end{pmatrix} \xrightarrow[\;]{\substack{R1 = R1 - \frac{k}{5}R2 \\ R3 = R3 - \frac{k}{5}R2}} \begin{pmatrix} 0 & 5 - \frac{k}{5} & 3 + \frac{k}{5} \\ 5 & 1 & -1 \\ 0 & 2 - \frac{k}{5} & 1 + \frac{k}{5} \end{pmatrix}$$

Antwoord:
Enkel als $k = 1$ zullen R1 en R3 lineair afhankelijk zijn en zullen er niet-triviale oplossingen zijn, anders is er enkel de triviale nuloplossing.

1.10

a)

$$\left(\begin{array}{ccc|c} 3 & 1 & 1 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 3 & 0 \end{array}\right) \xrightarrow{\substack{R1 = R1 - 3R2 \\ R3 = R3 - R2}} \left(\begin{array}{ccc|c} 0 & -8 & -2 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & -2 & 2 & 0 \end{array}\right) \xrightarrow{\substack{R1 = R2 \\ R2 = -\frac{1}{2}R3 \\ R3 = -\frac{1}{2}R1}} \left(\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 4 & 1 & 0 \end{array}\right)$$

$$\xrightarrow{\substack{R1 = R1 - 3R2 \\ R3 = R3 - 4R2}} \left(\begin{array}{ccc|c} 1 & 0 & 4 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 5 & 0 \end{array}\right) \xrightarrow{R3 = \frac{1}{5}R3} \left(\begin{array}{ccc|c} 1 & 0 & 4 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \xrightarrow{\substack{R1 = R1 - 4R3 \\ R2 = R2 + R3}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

Antwoord:
Enkel de tiriviale oplossing $\vec{x} = \vec{0}$, want $Rang(A) = n$

c)

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \\ 2 & -1 & 7 \\ 5 & 5 & 4 \\ 9 & 7 & 6 \end{pmatrix} \xrightarrow{\substack{R2 = R2 - 2R1 \\ R3 = R3 - 2R1 \\ R4 = R4 - 5R1 \\ R5 = R5 - 9R1}} \begin{pmatrix} 1 & 2 & -1 \\ 0 & -3 & 3 \\ 0 & -5 & 5 \\ 0 & -5 & 9 \\ 0 & -11 & 15 \end{pmatrix} \xrightarrow{\substack{R2 = -\frac{1}{3}R2 \\ R2\ en\ R3\ zijn\ lineair \\ afhankelijk,\ schrap\ R3}} \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 0 & -5 & 9 \\ 0 & -11 & 15 \end{pmatrix}$$

$$\xrightarrow{\substack{R1 = R1 - 2R2 \\ R3 = R3 + 5R2 \\ R4 = R4 + 11R2}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 4 \\ 0 & 0 & 4 \end{pmatrix} \xrightarrow{\substack{R3 = \frac{1}{4}R3 \\ R3\ en\ R4\ zijn\ lineair \\ afhankelijk,\ schrap\ R4}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{\substack{R1 = R1 - R3 \\ R2 = R2 + R3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Antwoord:

$$\begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases}$$

1.12

a)

$$\begin{pmatrix} 1 & 5 & 2 & 3 \\ 4 & 2 & -1 & -6 \\ -5 & 1 & 3 & 11 \end{pmatrix} \xrightarrow[\rightarrow]{\substack{R2 = R2 - 4R1 \\ R3 = R3 + 5R1}} \begin{pmatrix} 1 & 5 & 2 & 3 \\ 0 & -18 & -9 & -18 \\ 0 & 26 & 13 & 26 \end{pmatrix} \xrightarrow{\substack{R2 = -\frac{1}{9} \\ R2\ en\ R3\ zijn\ lineair \\ afhankelijk,\ schrap\ R3}} \begin{pmatrix} 1 & 5 & 2 & 3 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Antwoord:
Lineair afhankelijk. Een zo groot mogelijke lineair onafhankelijke deelverzamelijk wordt bv. gegeven door de eerste 2 vectoren.
$\lambda_3 = pmetp$ willekeurig en $\lambda_4 = t$ met t willekeurig.
$2\lambda_2 = -p - 2t \Rightarrow \lambda_2 = \frac{1}{2}p - t$
$\lambda_1 = -5(-\frac{1}{2}p - t) - 2p - 3t = \frac{5}{2}p + 5t - 2p = \frac{1}{2}p - 2t$

$$(\frac{1}{2} - 2t)\begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix} + (\frac{1}{2}p - t)\begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} + p\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 3 \\ -6 \\ 11 \end{pmatrix} = \vec{0}$$

1.13

a)

$$\begin{pmatrix} 1 & -4 & 2 \\ -4 & 1 & -2 \\ 2 & -2 & -2 \end{pmatrix} \xrightarrow{\substack{R2 = R2 + 4R1 \\ R3 = R3 - 2R1}} \begin{pmatrix} 1 & -4 & 2 \\ 0 & -15 & 6 \\ 0 & 6 & -6 \end{pmatrix} \xrightarrow{R3 = \frac{6}{15}R2} \begin{pmatrix} 1 & -4 & 2 \\ 0 & -15 & 6 \\ 0 & 0 & -\frac{36}{10} \end{pmatrix}$$

Antwoord:
Deze drie vectoren zijn lineair onafhankelijk.

b)

$$\begin{pmatrix} 1 & 1 & -1 \\ 2 & -1 & -5 \\ -5 & 4 & 14 \\ 3 & -1 & -17 \end{pmatrix} \xrightarrow{\substack{R2 = R2 - 2R1 \\ R3 = R3 + 5R1 \\ R4 = R4 - 3R1}} \begin{pmatrix} 1 & 1 & -1 \\ 0 & -3 & -3 \\ 0 & 9 & 9 \\ 0 & -4 & 14 \end{pmatrix}$$

Antwoord:
Lineair afhankelijk, een zo groot mogelijk lineair onafhakelijke deelverzameling wordt bv. gegeven door de eerste 3 vectoren.

c)

$$\begin{pmatrix} 1 & -1 & 1 & 2 \\ -2 & 5 & 0 & -15 \\ 3 & 3 & 2 & 1 \end{pmatrix} \xrightarrow{\substack{R2 = R2 + 2R1 \\ R3 = R3 - 3R1}} \begin{pmatrix} 1 & -1 & 1 & 2 \\ 0 & 3 & 2 & -11 \\ 0 & 6 & -1 & -5 \end{pmatrix} \xrightarrow{R3 = R3 - 2R2} \begin{pmatrix} 1 & -1 & 1 & 2 \\ 0 & 3 & 2 & -11 \\ 0 & 0 & -5 & 17 \end{pmatrix}$$

Antwoord:
Lineair onafhankelijk?

1.14

$$\begin{pmatrix} cos(\theta) & 1 \\ 1 & 2cos(\theta) \end{pmatrix} \xrightarrow{R1 = R1 - cos(\theta)R2} \begin{pmatrix} 0 & 1 - 2cos^2(\theta) \\ 1 & 2cos(\theta) \end{pmatrix}$$

Lineair afhankelijk asa

$$\begin{aligned} 1 - 2cos^2(\theta) &= 0 \\ cos^2(\theta) &= \frac{1}{2} \\ cos(\theta) &= \pm\frac{\sqrt{2}}{2} \\ \theta &= \pm\frac{\pi}{4} \vee \theta = \pm\frac{3\pi}{4} \end{aligned}$$

Hoofdstuk 2

2.1

a)

$$A + B = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 3 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 1 & -4 \\ 2 & -3 & 0 \end{pmatrix} = \begin{pmatrix} 1 & -1 & -1 \\ 2 & 0 & 4 \end{pmatrix}$$

f)

$$2P - Q = 2\begin{pmatrix} 1 & -2 \\ 0 & 4 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & -4 \\ 0 & 7 \end{pmatrix}$$

k)

$$P.C = \begin{pmatrix} 1 & -2 \\ 0 & 4 \end{pmatrix} . \begin{pmatrix} -5 & 3 \\ 4 & -1 \\ 2 & -1 \end{pmatrix} Onmogelijk$$

p)

$$\vec{b}.C = \begin{pmatrix} 2 & 5 & -2 \end{pmatrix} . \begin{pmatrix} -5 & 3 \\ 4 & -1 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} (-10+20-4) & (6-5+2) \end{pmatrix} = \begin{pmatrix} 6 & 3 \end{pmatrix}$$

2.3

a)

$$A.B = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} . \begin{pmatrix} -1 & 3 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} -5 & 4 \\ 4 & -1 \end{pmatrix} = A.B \ (toevallig)$$

b)

$$A.B = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} . \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$$

$$A.B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} . \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix}$$

2.4

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} . \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \rightarrow \left(\begin{array}{cccc|c} 2 & 1 & 0 & 0 & 3 \\ 3 & 2 & 0 & 0 & 2 \\ 0 & 0 & 2 & 3 & 1 \\ 0 & 0 & 1 & 2 & 4 \end{array}\right) \overset{GRM}{\rightarrow} \begin{cases} a = 0 \\ b = 1 \\ c = -10 \\ d = 7 \end{cases}$$

2.7

b)

$$\begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} \Leftrightarrow \begin{cases} 2c = 2b \\ b + 2d = 2a \\ 2a = c + 2d \\ 2b = 2c \end{cases} \leftrightarrow \begin{cases} a = d \\ b = 2a - 2d \\ c = 2d - 2a \\ d = a \end{cases}$$

2.8

a)

Neen, want $2AB$ komt van $AB + BA$ en $AB \neq BA$.

b)

Ja.

2.9

a)

$$\left(\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 4 & 1 & 0 & 1 \end{array}\right) \overset{R2 = R2 - 2R1}{\rightarrow} \left(\begin{array}{cccc|c} 2 & 1 & 0 & 0 & 3 \\ 3 & 2 & 0 & 0 & 2 \\ 0 & 0 & 2 & 3 & 1 \\ 0 & 0 & 1 & 2 & 4 \end{array}\right) \left(\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 0 & 7 & -2 & 1 \end{array}\right) \overset{R1 = R1 + \frac{3}{7}R2}{\rightarrow} \left(\begin{array}{cc|cc} 2 & 0 & \frac{1}{7} & \frac{3}{7} \\ 0 & 7 & -2 & 1 \end{array}\right)$$

$$\overset{\substack{R1 = \frac{1}{2}R1 \\ R2 = \frac{1}{7}R2}}{\rightarrow} \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{14} & \frac{3}{14} \\ 0 & 1 & -\frac{2}{7} & \frac{1}{7} \end{array}\right) \quad Antwoord: \begin{pmatrix} \frac{1}{14} & \frac{3}{14} \\ -\frac{2}{7} & \frac{1}{7} \end{pmatrix}$$

2.10

$$\left(\begin{array}{ccc|ccc} 2 & 5 & 1 & 1 & 0 & 0 \\ 3 & 1 & 2 & 0 & 1 & 0 \\ -2 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \overset{GRM}{\rightarrow} \left(\begin{array}{ccc|ccc} 19 & 0 & 0 & 2 & -1 & -9 \\ 0 & 19 & 0 & 5 & -2 & 1 \\ 0 & 0 & 19 & -5 & 12 & 13 \end{array}\right)$$

Antwoord:

$$\frac{1}{19} \begin{pmatrix} 2 & -1 & -9 \\ 4 & -2 & 1 \\ -5 & 12 & 13 \end{pmatrix}$$

2.14

a)

$$\begin{pmatrix} cos(\theta_1) & -sin(\theta_1) \\ sin(\theta_1) & cos(\theta_1) \end{pmatrix} \cdot \begin{pmatrix} cos(\theta_2) & -sin(\theta_2) \\ sin(\theta_2) & cos(\theta_2) \end{pmatrix}$$
$$= \begin{pmatrix} cos(\theta_1)cos(\theta_2) - sin(\theta_1)sin(\theta_2) & -(cos(\theta_1)sin(\theta_2) + cos(\theta_2)sin(\theta_1)) \\ sin(\theta_1)cos(\theta_2) + cos(\theta_1)sin(\theta_2) & cos(\theta_1)cos(\theta_2) - -sin(\theta_1)sin(\theta_2) \end{pmatrix}$$
$$= \begin{pmatrix} cos(\theta_1 + \theta_2) & -sin(\theta_1 + \theta_2) \\ sin(\theta_1 + \theta_2) & cos(\theta_1 + \theta_2) \end{pmatrix}$$

2.17

$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \quad want \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} (1 * 1 - 1 * 0) & (1 * 1 - 1 * 0) \\ (0 * 1 - 0 * 0) & (0 * 1 - 0 * 0) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

Hoofdstuk 3

3.2

a)

$$\begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = 6$$

b)

$$\begin{vmatrix} 0 & 1 \\ -2 & 3 \end{vmatrix} = 2$$

c)

$$\begin{vmatrix} cos(n\theta) & -sin(n\theta) \\ sin(n\theta) & cos(n\theta) \end{vmatrix} = cos^2(n\theta) + sin^2(n\theta) = 1$$

3.3

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{vmatrix} = 2$$

$$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 14 & 2 & 3 \\ 36 & 4 & 9 \end{vmatrix} = 2,\ D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 14 & 3 \\ 1 & 36 & 9 \end{vmatrix} = 4,\ D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 14 \\ 1 & 4 & 36 \end{vmatrix} = 6$$

$$x_1 = \frac{D_1}{D},\ x_2 = \frac{D_2}{D},\ x_3 = \frac{D_3}{D}$$

Antwoord:

$$\begin{cases} x_1 = 1 \\ x_2 = 2 \\ x_3 = 3 \end{cases}$$

3.4

a)

$$\begin{vmatrix} 2 & -1 & 3 \\ 4 & 1 & -2 \\ -3 & 2 & 1 \end{vmatrix} = 2.\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} - (-1).\begin{vmatrix} 4 & -2 \\ -3 & 1 \end{vmatrix} + 3.\begin{vmatrix} 4 & 1 \\ -3 & 2 \end{vmatrix} = 10-2+33 = 41$$

$$\begin{vmatrix} 2 & -1 & 3 \\ 4 & 1 & -2 \\ -3 & 2 & 1 \end{vmatrix} = -(-1).\begin{vmatrix} 4 & -2 \\ 3 & 1 \end{vmatrix} + 1.\begin{vmatrix} 2 & 3 \\ -3 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 3 \\ 4 & -2 \end{vmatrix} = -2+11+32 = 41$$

e)

$$\begin{vmatrix} 0 & 3 & 2 \\ 2 & 0 & 1 \\ 2 & 6 & 0 \end{vmatrix} = 0.\begin{vmatrix} 0 & 1 \\ 6 & 0 \end{vmatrix} - 3.\begin{vmatrix} 2 & 1 \\ 2 & 0 \end{vmatrix} + 2.\begin{vmatrix} 2 & 0 \\ 2 & 6 \end{vmatrix} = 18$$

$$\begin{vmatrix} 0 & 3 & 2 \\ 2 & 0 & 1 \\ 2 & 6 & 0 \end{vmatrix} = -3.\begin{vmatrix} 2 & 1 \\ 2 & 0 \end{vmatrix} + 0.\begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} - 6.\begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix} = 18$$

3.6

c)

$$\begin{vmatrix} -2 & 6 & 17 & -5 \\ 0 & 3 & 22 & -12 \\ 0 & 0 & 4 & -12 \\ 0 & 0 & 0 & -6 \end{vmatrix} = (-2).3.4.(-6) = 144$$

d)

$$\begin{vmatrix}0&2&0&0\\2&0&2&0\\0&2&0&2\\0&0&2&0\end{vmatrix} = -2\begin{vmatrix}2&2&0\\0&0&2\\0&2&0\end{vmatrix} = -2.2\begin{vmatrix}0&2\\2&0\end{vmatrix} = 16$$

3.8

b)

$$\begin{vmatrix}1&2x&3x^2\\2x^3&3x^4&4x^5\\3x^6&4x^7&6x^8\end{vmatrix} \overset{\substack{R2=R2-2x^3R1\\R3=R3-3x^6R1}}{=} \begin{vmatrix}1&2x&3x^2\\0&-x^4&-x^5\\0&-2x^7&-3x^8\end{vmatrix} = 0 \;\Leftrightarrow\; \begin{vmatrix}-x^4&-x^5\\-2x^7&-3x^8\end{vmatrix} = 0$$

$$\Leftrightarrow 3x^12-2x^12 = x^12 = 0 \;\Leftrightarrow\; x=0$$

3.9

a)

$$\begin{vmatrix}3&2&-2\\6&1&5\\-9&3&4\end{vmatrix} \overset{\substack{R2=R2-2R\\R3=R3+3R1}}{=} \begin{vmatrix}3&2&-2\\0&-3&9\\0&9&-2\end{vmatrix} \overset{R3=-\frac{1}{3}R3}{=} -3.\begin{vmatrix}3&2&-2\\0&1&-3\\0&9&-2\end{vmatrix} \overset{R3=R3-9R2}{=} -3.\begin{vmatrix}3&2&-2\\0&1&-3\\0&0&25\end{vmatrix}$$

$$=(-3)*3*25=-225$$

b)

$$\begin{vmatrix}1&0&1&1\\0&1&1&0\\1&0&0&1\\1&1&1&0\end{vmatrix} \overset{\substack{R3=R3-R1\\R4=R4-R1}}{=} \begin{vmatrix}1&0&1&1\\0&1&1&0\\0&0&-1&0\\0&1&0&-1\end{vmatrix} \overset{R4=R4-R2}{=}$$

$$\begin{vmatrix}1&0&1&1\\0&1&1&0\\0&0&-1&0\\0&0&-1&-1\end{vmatrix} \overset{R4=R4-R3}{=} \begin{vmatrix}1&0&1&1\\0&1&1&0\\0&0&-1&0\\0&0&0&-1\end{vmatrix} = 1$$

3.11

$$
\begin{aligned}
A^{\tau} &= A^{-1} && \text{(1. Definitie orthogonale matrix)}\\
A.A^{\tau} &= I = A^{\tau}.A && \text{(2. Direct gevolg van definitie)}\\
det(A^{\tau}) &= det(A^{-1}) && \text{(3. (uit 1.))}\\
\frac{det(A^{-1})}{det(A)} &= 1 && \text{(4. (uit 3.))}\\
det(A.A^{-1}) &= det(I) = 1 && \text{(5. (uit 2.))}\\
det(A.A^{-1}) &= det(A).det(A^{-1}) && \text{(6. eigenschap determinant)}\\
\frac{det(A^{-1})}{det(A)} &= det(A).det(A^{-1}) && \text{(7. (uit 4. en 5.))}\\
\frac{1}{det(A)} &= det(A) && \text{(8.)}\\
det(A) &= 1 \vee det(A) = -1 && \text{(9.)}
\end{aligned}
$$

Hoofdstuk 4

4.1

$$\vec{x} = \vec{p} + t(\vec{q} - \vec{p})$$

b)

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

d)

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 0-1 \\ 1-2 \\ 2-3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$$

4.2

a)

$$V : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + t1 \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + t2 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

4.4

a)

We berekenen een normaalvector:

$$\begin{pmatrix}1\\2\\2\end{pmatrix} \times \begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}-4\\5\\-3\end{pmatrix}$$

Er bestaat een a zodat dit een eenheidsnormaal is:

$$a.\begin{pmatrix}-4\\5\\-3\end{pmatrix}$$

$$\sqrt{(-4a)^2+(5a)^2+(-3a)^2} = \sqrt{50a^2} = 1$$

Antwoord:

$$\begin{pmatrix}\frac{-4}{\sqrt{50}}\\\frac{5}{\sqrt{50}}\\\frac{-3}{\sqrt{50}}\end{pmatrix}$$

4.5

Stel:

$$Normaal:\ \vec{n} = \begin{pmatrix}a\\b\\c\end{pmatrix},\ Positievector:\ \vec{p} = \begin{pmatrix}x_0\\y_0\\z_0\end{pmatrix},\ en\ \vec{x} = \begin{pmatrix}x\\y\\z\end{pmatrix}$$

Carthesische vergelijking van een vlak:

$$V:\ a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

Parametervergelijking van een vlak:

$$V:\ \vec{x} = \vec{p} + t_1\vec{r_1} + t_2\vec{r_2}$$

waarbij:

$$\vec{r_1} = \begin{pmatrix}-b\\a\\0\end{pmatrix},\ en\ \vec{r_2} = \begin{pmatrix}-c\\0\\a\end{pmatrix}$$

a)

$$V:\ 2(x-1)+1(y-1)+3(z-2)=0$$
$$\Leftrightarrow 2x+y+3z=9$$
$$V:\ \begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\1\\2\end{pmatrix}+t1\begin{pmatrix}-1\\2\\0\end{pmatrix}+t2\begin{pmatrix}-3\\0\\2\end{pmatrix}$$

4.5

b)

$$V:\ -1(x-2)+4(y+1)+5(z-3)=0$$
$$\Leftrightarrow -x+4y+5z=9$$
$$V:\ \begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2\\-1\\3\end{pmatrix}+t1\begin{pmatrix}-4\\-1\\0\end{pmatrix}+t2\begin{pmatrix}-5\\0\\-1\end{pmatrix}$$

4.7

$$r_1=\begin{pmatrix}2\\-1\\3\end{pmatrix},\ richtvector:\ r_2=\begin{pmatrix}2-1\\4-2\\2-3\end{pmatrix}=\begin{pmatrix}1\\2\\-1\end{pmatrix}$$
$$normaal:\ \begin{pmatrix}0\\1\\0\end{pmatrix}\times\begin{pmatrix}1\\2\\-1\end{pmatrix}=\begin{pmatrix}-1\\0\\-1\end{pmatrix}$$
$$V:\ -(x-1)-(z-3)=0$$
$$\Leftrightarrow x+z=4$$

4.8

Stel:

$$Positievectoren:\ \vec{p},\ en\ \vec{q}\ ,\ \ Richtvector:(\vec{q}-\vec{p}),\ en\ Normalen:\ n_1=\begin{pmatrix}a_1\\b_1\\c_1\end{pmatrix},\ en\ n_2=\begin{pmatrix}a_2\\b_2\\c_2\end{pmatrix}$$

Carthesische vergelijking van een rechte:

$$L:\ \begin{cases}a_1x+b_1y+c_1z=d_1\\a_2x+b_2y+c_2z=d_2\end{cases}$$

Parametervergelijking van een rechte:

$$L: \ \vec{x} = \vec{p} + t(\vec{q} - \vec{p})$$

a)

$$L: \ \vec{x} = \begin{pmatrix} 1 \\ 2 \\ 8 \end{pmatrix} + t \begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix}$$

$$\begin{pmatrix} a \\ 3 \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 8 \end{pmatrix} + t \begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix} \Leftrightarrow \begin{cases} a = 1 + 3t \\ 3 = 2 + (-1)t \\ b = 8 - 4t \end{cases} \Leftrightarrow \begin{cases} a = 1 + 3(-\frac{1}{3}) \\ t = -\frac{1}{3} \\ b = 8 - 4(-\frac{1}{3}) \end{cases} \Leftrightarrow \begin{cases} a = \frac{1}{3} \\ t = -\frac{1}{3} \\ b = 12 \end{cases}$$

b)

$$V: \ 3(x+4) - 2(y+0) + 6(z-3) = 0$$
$$\Leftrightarrow 3x - 2y + 6z = 6$$
$$\Leftrightarrow 3(1+3t) - 2(2-t) + 6(8-4t) = 6$$
$$\Leftrightarrow 9t + 2t - 24t + 3 - 4 + 48 = 6 \ \Leftrightarrow \ 9t + 2t - 24t + 3 - 4 + 48 = 6$$
$$\Leftrightarrow t = \frac{41}{13}$$

Antwoord:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 8 \end{pmatrix} + \frac{41}{13} \begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix} = \begin{pmatrix} \frac{136}{13} \\ -\frac{15}{13} \\ -\frac{60}{13} \end{pmatrix}$$

4.18

a)

$$\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \times \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ -1 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -9 \\ 1 \\ -3 \end{pmatrix}$$

d)

$$\left(\begin{pmatrix}1\\3\\-2\end{pmatrix}\times\begin{pmatrix}0\\-1\\2\end{pmatrix}\right).\begin{pmatrix}0\\3\\1\end{pmatrix}=\begin{pmatrix}8\\-2\\1\end{pmatrix}.\begin{pmatrix}0\\3\\1\end{pmatrix}=-7$$

e)

$$\begin{pmatrix}1\\3\\-2\end{pmatrix}\times\left(\begin{pmatrix}0\\3\\1\end{pmatrix}\times\begin{pmatrix}0\\-1\\2\end{pmatrix}\right)=\begin{pmatrix}1\\3\\-2\end{pmatrix}\times\begin{pmatrix}7\\0\\0\end{pmatrix}=\begin{pmatrix}0\\14\\-21\end{pmatrix}$$

4.22

a)

$$d(\vec{p},\vec{v}))=\frac{|10-2.1-3.1-(-1).2|}{\sqrt{2^2+3^3+1^2}}=\frac{7}{\sqrt{14}}$$

b)

$$\vec{r_1}=\begin{pmatrix}-1\\0\\1\end{pmatrix},\ \vec{r_2}=\begin{pmatrix}-1\\1\\0\end{pmatrix},\ \vec{p}=\begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\vec{r_1}\times\vec{r_2}=\begin{pmatrix}-1\\0\\1\end{pmatrix}\times\begin{pmatrix}-1\\1\\0\end{pmatrix}=\begin{pmatrix}-1\\-1\\-1\end{pmatrix}$$

$$-(x-1)-y-z=0$$

$$x+y+z=1$$

afstand tot $(2,1,1)^\tau$:

$$d(\vec{p},v)=\frac{|1-2-1-1|}{\sqrt{3}}=\sqrt{3}$$

4.30

a)

$\left|det\begin{pmatrix}1&-1&2\\1&4&2\\0&0&2\end{pmatrix}\right|$