(1)

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THUIS

Oefeningen hoofdstuk 1

e ?
$41 - 38i$

1.1

a) $z = 15 + 9i - 5i - 3i^2$

$z = 18 + 4i$ ✓

b) $z = 1 - 6i + 9i^2 - 1 - 6i - 9i^2$

$z = -12i$ ✓

c) $z = \frac{1-i}{1+i} \cdot \frac{1-i}{1+i}$

$= \frac{1-2i+i^2}{1-i^2} = \frac{-2i}{2} = -i$ ✓

d) $z = 1 + 9 = 10$ ✓

f) $z = \frac{1}{5} - \frac{3-4i}{3+4i} \cdot \frac{3-4i}{3-4i}$

$= \frac{1}{5} - \frac{9 - 24i + 16i^2}{9 + 16}$

$= \frac{1}{5} - \frac{-7-24i}{25}$

$= \frac{12}{25} + \frac{24i}{25}$ ✓

$e = 41 - 38i$

e) $(1+2i).(1+2i).$
$(1+2i).(1+2i).(1+2i)$

$= [1 + 2i + 2i + 4i^2]^2 . (1+2i)$

$= [1 + 4i - 4]^2 . (1+2i)$

$= (-3 + 4i)^2 . (1+2i)$

$= (9 - 24i + 16i^2) . (1+2i)$

$= (9 - 24i - 16) . (1+2i)$

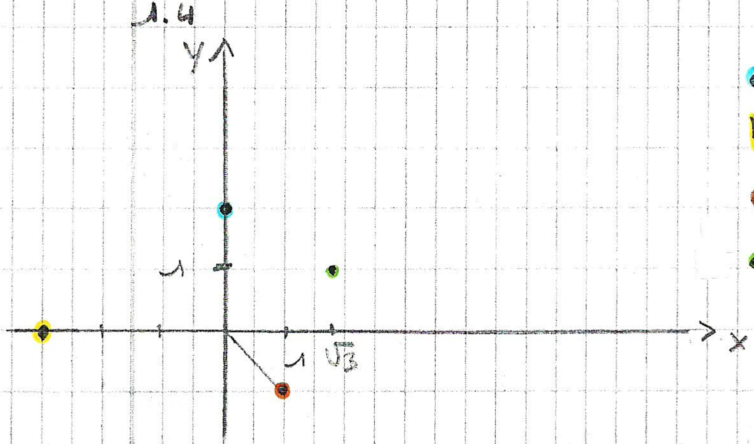
$= (7 - 24i)(1-2i)$

$= 7 - 24i - 14i + 48i^2$

$= 7 - 38i - 48$

$= -41 - 38i$ ✓

1.4



a) $|z| = 2$ ✓ $\arg z = \frac{\pi}{2} + 2k\pi$ $k \in \mathbb{Z}$ $\frac{\pi}{2}$ (mod 2π)

b) $|z| = 3$ ✓ $\arg z = \pi + 2k\pi$ $k \in \mathbb{Z}$

c) $|z| = \sqrt{2}$ ✓ $\arg z = -\frac{\pi}{4} + 2k\pi$ $k \in \mathbb{Z}$

d) $|z| = 2$ ✓ $\arg z = 0{,}5235 + 2k\pi$ $k \in \mathbb{Z}$ $= \pi/6$ ✓

1.6

$\frac{1}{2}(x+iy+x-iy) = \frac{1}{2}(2x) = x = \mathrm{Re}\, z$ ✓
$\frac{1}{2i}(x+iy-x+iy) = \frac{1}{2i}(2iy) = y = \mathrm{Im}\, z$ ✓
} als $z = x + iy$

1.7

a) $e^{i\pi/4} = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} \stackrel{?}{=} \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$ ✓

b) $6e^{i\frac{2\pi}{3}} = \overbrace{6\cos\frac{2\pi}{3}}^{-3} + \overbrace{6i\sin\frac{2\pi}{3}}^{3\sqrt{3}i}$ ✓

c) $e^{-\frac{\pi}{4}i + \ln 2} = e^{\ln 2}\left(\cos -\frac{\pi}{4} + i\sin -\frac{\pi}{4}\right)$

$= \underbrace{2\cos\frac{\pi}{4}}_{\sqrt{2}} - \underbrace{2i\sin\frac{\pi}{4}}_{= \sqrt{2}i}$ ✓

d) $e^{-2\pi i} + e^{4\pi i} = \underbrace{\cos -2\pi}_{1} + i\underbrace{\sin -2\pi}_{0} + \underbrace{\cos 4\pi}_{1} + i\underbrace{\sin 4\pi}_{0}$

$= 2$ ✓

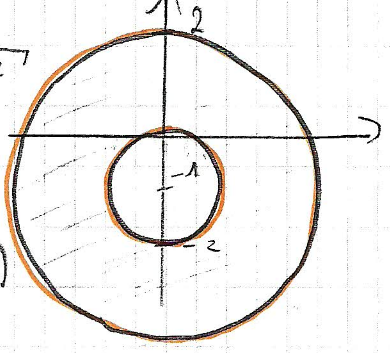
geen antwoorden van 1.8

a) $|z_1 - z_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

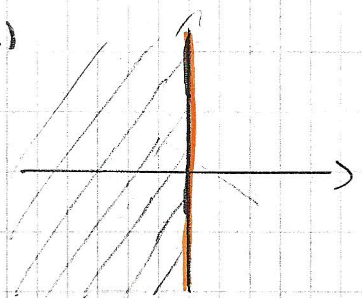
$z_1 = x + iy$ $\quad z_2 = -i$

$\sqrt{x^2 + (y+1)^2}$

⇒ straal van cirkel met $M(0,-1)$



c)

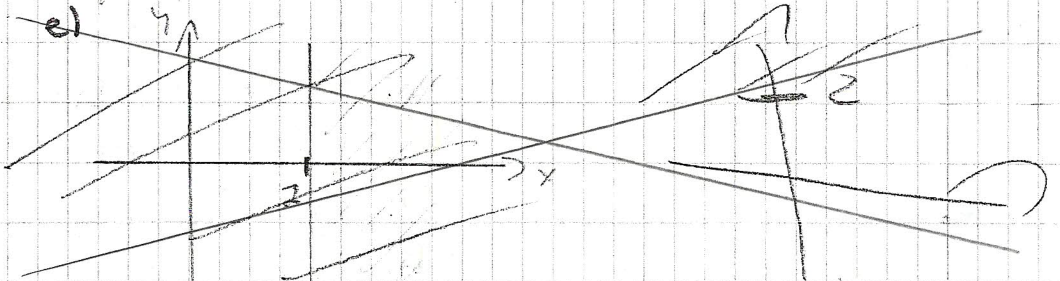


$|e^z| = e^x$

$e^x < 1$

$x < \ln 1$

? e)



1.9

a) $z_1 = 2e^{i\frac{\pi}{3}} \quad z_2 = 3e^{i\frac{\pi}{2}}$

$\sqrt{3} + 3i$ ✓ $= z_1 z_2 = 6\left(\cos\frac{5}{6}\pi + i\sin\frac{5}{6}\pi\right)$

$\frac{\sqrt{3}}{3} + \frac{1}{3}i$ ✓ $= \frac{z_1}{z_2} = \frac{2}{3}\left(\cos -\frac{\pi}{6} + i\sin -\frac{\pi}{6}\right)$

$\frac{3\sqrt{3}}{4} + \frac{3}{4}i$ ✓ $= \frac{z_2}{z_1} = \frac{3}{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

b) $z_1 = 2e^{i\frac{3\pi}{4}} \quad z_2 = 2e^{-i\pi/4}$

$z_1 z_2 = 4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = +4$ $= 4i$

$z_1/z_2 = \cos\pi + i\sin\pi = -1$ ✓

$z_2/z_1 = \cos -\pi + i\sin -\pi = -1$ ✓

1.11

a) $e^z = 0$ -> kan niet $V = \emptyset$ geen oplossingen

b) $e^{iz} = 1$

$z = 2\pi k$ ✓

1.13

a) $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

$(\cos\theta + i\sin\theta)^4 = \cos 4\theta + i\sin 4\theta$

$(\cos\theta + i\sin\theta)^4 = \cos^4\theta + 4\cos^3\theta\, i\sin\theta + 6\cos^2\theta\, i^2\sin^2\theta$
$+ 4\cos\theta\, i^3\sin^3\theta + \sin^4\theta\, i^4$
$= \cos^4\theta + \sin^4\theta - 6\cos^2\theta\sin^2\theta + i(4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta)$

$\cos^2\theta - \sin^2\theta = \cos 2\theta$
$\cos\theta\sin\theta = \sin 2\theta$

$\Rightarrow \underline{\cos 4\theta + i\sin 4\theta} = \underline{\cos^4\theta + \sin^4\theta - 6\cos^2\theta\sin^2\theta} + i4\cos\theta\sin\theta(\cos^2\theta - \sin^2\theta)$

$\rightarrow \sin 4\theta \overset{?}{=} 4\cos\theta\sin\theta(\cos^2\theta - \sin^2\theta)$

$= 2\sin 2\theta \cdot \cos 2\theta = \sin 4\theta$ => klopt ✓

1.18

$\omega^2 = -i$

$|-i| = 1$ en $\arg(-i) = -\frac{\pi}{2}$

$-i = e^{-\pi/2\, i}$

stel $\omega = re^{i\theta}$

$\Rightarrow r^2 = 1$ en $2\theta = -\frac{\pi}{2} + 2k\pi$

$\Rightarrow r = 1$ en $\theta = -\frac{\pi}{4} + k\pi$

$\omega_1 = e^{-\pi/4\, i}$ en $\omega_2 = -e^{-\pi/4\, i}$

$= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ ✓ $\cancel{= -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i}$

$= -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

OEFENZITTING

$z = x + iy$

$z = r e^{i\theta}$

$e^{ix} = \cos x + i \sin x$

$e^{z} = e^{x+iy} = e^{x} (\cos y + i \sin y)$

1.3

b) $z^3 = 8$

stel $z = r e^{i\theta}$

dan is $z^3 = r^3 e^{i3\theta} = 8 = \underbrace{2^3}_{reëel}$

Re: $r^3 = 2^3 \Rightarrow r = 2$

Im: $e^{i3\theta} = 1 \Rightarrow 3\theta = 2\pi k$ $\qquad e^{i2\pi} = 1$ en ook $e^{i2\pi k} = 1 \quad k \in \mathbb{Z}$

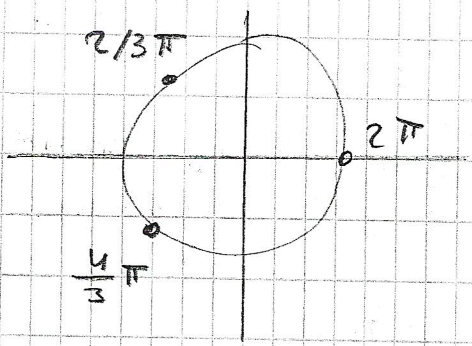
$\theta = \frac{2}{3}\pi k$

$k=1: \theta = \frac{2}{3}\pi$

$k=2: \theta = \frac{4}{3}\pi$

$k=3: \theta = 2\pi$

$k=4: \theta = \frac{8}{3}\pi$ $=$



oplossing: $2 e^{i2/3\pi}$ ✓

$2 e^{i4/3\pi}$ ✓

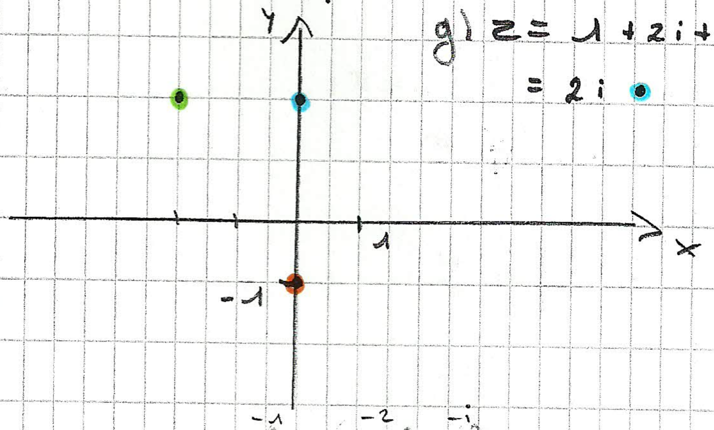
$2 e^{i2\pi} = 2$ ✓

1.4

f) $\frac{1}{i} \cdot \frac{-i}{-i} = \frac{-i}{-i^2} = -i$ $\qquad |z| = 1$ ✓ $\qquad \arg z = -\frac{\pi}{2} + 2k\pi \quad k \in \mathbb{Z}$

g) $z = 1 + 2i + i^2$ $\qquad |z| = 2$ ✓

$= 2i$ $\qquad \arg z = \frac{\pi}{2} + 2k\pi \quad k \in \mathbb{Z}$



h) $z = 1 + 2i + \overset{-1}{i^2} + i + \overset{-2}{2i^2} + \overset{-i}{i^3}$ $\qquad |z| = \sqrt{8} = 2\sqrt{2}$

$= -2 + 2i$ $\qquad \arg z = \frac{3}{4}\pi + 2k\pi \quad k \in \mathbb{Z}$

1.5

d) $e^{z} = e^{x+iy} = e^{x}(\cos y + i \sin y)$

$\overline{e^{z}} = e^{x}(\cos y - i \sin y) = LL$

$e^{\bar z} = e^{x-iy}$

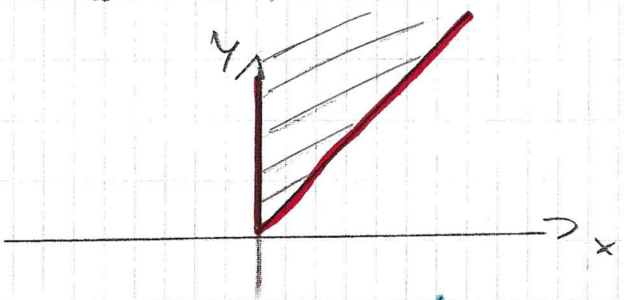
$= e^{x}(\cos(-y) + i \sin(-y))$

$= e^{x}(\cos y - i \sin y) = RL$

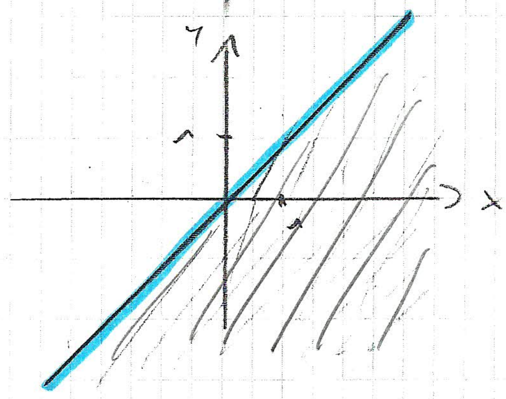
$\Rightarrow \overline{e^{z}} = e^{\bar z}$ ✓

1.8

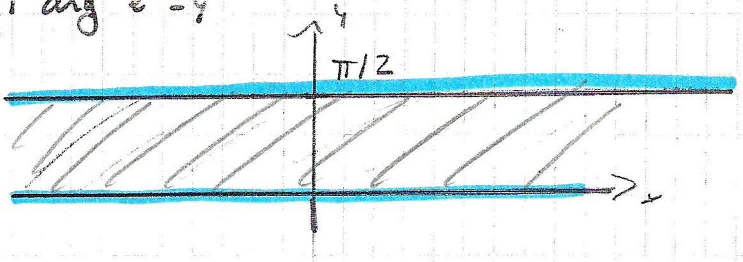
b)



d)



f) $\arg e^{z} = y$



1.10

a) serie: $z_1 + z_2 = 3 - 2i$ ✓

parallel: $\left(\frac{1}{2+3i} + \frac{1}{1-5i}\right)^{-1} = \left(\frac{1-5i+2+3i}{2-10i+3i-15i^2}\right)^{-1}$

$= \left(\frac{3-2i}{17-7i}\right)^{-1} = \frac{17-7i}{3-2i} \cdot \frac{3+2i}{3+2i}$

$= \frac{51-21i-14i^2+34i}{9+4} = \frac{13i+65}{13} = 5+i$ ✓

1.19

a) $z^2 + 2iz - 1 = 0$

$D = b^2 - 4ac$

$= 4i^2 + 4 = 0$

$x = \frac{-b}{2a} = \frac{-2i}{2} = -i$ ✓

c) $(z^2+1)^2 = 0$

$z^2 = -1$

$z = \pm\sqrt{-1} = \pm\sqrt{i^2}$

$= \pm i$ ✓

1.20

$\frac{1}{a} + \frac{1}{b} = \frac{b+a}{ab} = \frac{1}{a+b}$

$ab = b^2 + 2ab + a^2$

$0 = b^2 + ab + a^2$

$0 = \frac{a^2}{b^2} + \frac{ab}{b^2} + \frac{b^2}{b^2}$

$0 = \frac{a^2}{b^2} + \frac{a}{b} + 1$ Stel $\frac{a}{b} = z$

$0 = z^2 + z + 1$

$D = b^2 - 4ac$

$= 1 - 4 = -3$ $\sqrt{-3} = \sqrt{i^2 3} = i\sqrt{3}$

$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm i\sqrt{3}}{2}$ ✓

1.21

$e^{ix} = \cos x + i \sin x$

$e^{inx} = \cos nx + i \sin nx$

$\overline{e^{imx}} = \overline{\cos mx - i \sin mx} = e^{-imx}$

$e^{inx}\, \overline{e^{imx}} = e^{inx - imx} = e^{ix(n-m)}$

$\int e^{ix(n-m)}\, dx = \frac{e^{ix(n-m)}}{i(n-m)} + c$

als $m \neq n$ $\int_0^{2\pi} e^{ix(n-m)}\, dx = \frac{e^{i2\pi(n-m)}}{i(n-m)} - \frac{e^{i0(n-m)}}{i(n-m)} = \frac{1}{i(n-m)} - \frac{1}{i(n-m)} = 0$ ✓

! als $m = n$ $\int_0^{2\pi} e^{ix(0)}\, dx = \int_0^{2\pi} 1\, dx = [x]_0^{2\pi} = 2\pi - 0 = 2\pi$ ✓

1.22

a) $\int e^{-t+2it} dt = \frac{e^{-t+2it}}{-1+2i}$

eig mit $\lim_{a \to \infty}$

$\int_0^\infty e^{-t+2it} dt = \left[\frac{e^{-t+2it}}{-1+2i} \right]_0^\infty$

$= \frac{e^{-\infty+2i\infty}}{-1+2i} - \frac{e^{0+2i0}}{-1+2i}$

$Re \rightarrow 0 - \frac{1}{-1+2i} \cdot \frac{-1-2i}{-1-2i}$

$= \frac{1+2i}{1+4} = \frac{1}{5} \cancel{+ \frac{2i}{5}}$

①

THUIS

Gefeningen hoofdstuk 2

2.1

a) $\left(\begin{array}{ccc|c} 1 & 1 & -2 & 10 \\ 4 & 2 & 2 & 1 \\ 6 & 2 & 3 & 4 \end{array}\right) \xrightarrow[R_3 \mapsto R_3 - 6R_1]{R_2 \mapsto R_2 - 4R_1} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 10 \\ 0 & -2 & 10 & -39 \\ 0 & -4 & 15 & -56 \end{array}\right)$

$\xrightarrow{R_3 \mapsto R_3 - 2R_2} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 10 \\ 0 & -2 & 10 & -39 \\ 0 & 0 & -5 & 22 \end{array}\right)$

$-5z = 22 \quad \Rightarrow z = -\frac{22}{5}$ ✓

$-2y + 10\left(-\frac{22}{5}\right) = -39 \quad \Rightarrow y = -\frac{5}{2}$ ✓

$x + \frac{-5}{2} - 2\,\frac{-22}{5} = 10 \quad \Rightarrow x = \frac{37}{10}$ ✓

2.2

geen opl. ? a) $\left(\begin{array}{ccc|c} 2 & -1 & 1 & 2 \\ 3 & 1 & -2 & 1 \\ 1 & -3 & 4 & k \end{array}\right) \xrightarrow[R_3 \mapsto 2R_3 - R_1]{R_2 \mapsto 2R_2 - 3R_1} \left(\begin{array}{ccc|c} 2 & -1 & 1 & 2 \\ 0 & 5 & -7 & -4 \\ 0 & -5 & 7 & 2k-2 \end{array}\right)$

$\xrightarrow{R_3 \mapsto R_3 + R_2} \left(\begin{array}{ccc|c} 2 & -1 & 1 & 2 \\ 0 & 5 & -7 & -4 \\ 0 & 0 & 0 & 2k-6 \end{array}\right)$ $\Rightarrow$ stelsel heeft enkel oplossingen als $2k-6 = 0 \Rightarrow k = 3$

b) als $k = 3$ $\left(\begin{array}{ccc|c} 2 & -1 & 1 & 2 \\ 0 & 5 & -7 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right)$

stel $z = \lambda \Rightarrow 5y - 7\lambda = -4 \quad \Rightarrow y = \frac{7}{5}\lambda - \frac{4}{5}$

$2x - y + \lambda = 2 \quad \Rightarrow x = \frac{7}{10}\lambda - \frac{4}{10} - \frac{\lambda}{2} + 1$

$x = \frac{1}{5}\lambda + \frac{3}{5}$

$\Rightarrow x = \frac{1}{5}\lambda + \frac{3}{5}, \quad y = \frac{7}{5}\lambda - \frac{4}{5}, \quad z = \lambda$ ✓

met $\lambda \in \mathbb{R}$

2.3

stel $x = \#$ koeien en $y = \#$ kippen

$$\begin{cases} x + y = 60 \\ 4x + 2y = 200 \end{cases}$$

$$\left(\begin{array}{cc|c} 1 & 1 & 60 \\ 4 & 2 & 200 \end{array}\right) \xrightarrow{R_2 \mapsto R_2 - 4R_1} \left(\begin{array}{cc|c} 1 & 1 & 60 \\ 0 & -2 & -40 \end{array}\right)$$

$y = 20$ ✓ $\Rightarrow$ 20 kippen en 40 koeien ✓

$x = 40$ ✓

2.6

a) $$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 2 & -1 & 4 & 2 \\ 4 & 3 & -2 & a \end{array}\right) \xrightarrow[R_3 \mapsto R_3 - 4R_1]{R_2 \mapsto R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 0 & -5 & 10 & -10 \\ 0 & -5 & 10 & a-24 \end{array}\right)$$

$$\xrightarrow{R_2 \mapsto \frac{1}{5}R_2} \left(\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 0 & -1 & 2 & -2 \\ 0 & -5 & 10 & a-24 \end{array}\right) \xrightarrow{R_3 \mapsto R_3 - 5R_2} \left(\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 0 & -1 & 2 & -2 \\ 0 & 0 & 0 & a-14 \end{array}\right)$$

$a \neq 14$: strijdig stelsel : geen oplossingen ✓

$a = 14$: $$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 0 & -1 & 2 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

stel $z = \lambda$

$\rightarrow -y = -2\lambda - 2 \Rightarrow y = 2\lambda + 2$

$\rightarrow x + 2y - 3\lambda = 6 \Rightarrow -x = 4\lambda + 4 - 3\lambda - 6$

$x = -\lambda + 2$

$\Rightarrow x = -\lambda + 2,\ y = 2\lambda + 2,\ z = \lambda$ ✓

$x = \frac{1}{a+1}$

$y = \frac{a+2}{a+1}$

b) $$\left(\begin{array}{cc|c} 1 & a & a+1 \\ a & 1 & 2 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & a & a+1 \\ 0 & 1-a^2 & 2-a^2-a \end{array}\right)$$

$*\ 1 - a^2 = 0 \rightarrow a = 1$ of $a = -1$

$*\ a = 1$: $\left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right)$ stel $y = \lambda \rightarrow x = 2 - \lambda$

$\Rightarrow x = 2 - \lambda,\ y = \lambda$ ✓

$*\ a = -1$: $\left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 2 \end{array}\right) \rightarrow$ strijdig : geen oplossingen ✓

$*\ a \neq 1$ en $a \neq -1$ en $a \neq -2$

$\left(\begin{array}{cc|c} 1 & a & a+1 \\ 0 & 1-a^2 & 2-a-a^2 \end{array}\right)$

$y = \frac{-a^2 - a + 2}{1 - a^2}$

$x = a + 1 + \frac{a^3 + a^2 - 2a}{1 - a^2}$

$*\ a = -2$: $\left(\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & -3 & 0 \end{array}\right) \rightarrow y = 0$ en $x = -1$

2.10

a) $\begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 1 \\ 1 & 1 & 3 \\ 3 & 1 & 1 \end{pmatrix} \xrightarrow[R_3 \mapsto R_3 - 3R_1]{R_2 \mapsto R_2 - R_1} \begin{pmatrix} 1 & 3 & 1 \\ 0 & -2 & 2 \\ 0 & -8 & -2 \end{pmatrix}$

$\to \begin{pmatrix} 1 & 3 & 1 \\ 0 & -1 & 1 \\ 0 & -4 & -1 \end{pmatrix} \xrightarrow{R_3 \mapsto R_3 - 4R_2} \begin{pmatrix} 1 & 3 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & -5 \end{pmatrix}$

$\Rightarrow$ enkel triviale oplossing : $x=0,\ y=0,\ z=0$ ✓

b) $\begin{pmatrix} 1 & 3 & -2 \\ 1 & -8 & 8 \\ 3 & -2 & 4 \end{pmatrix} \xrightarrow[R_3 \mapsto R_3 - 3R_1]{R_2 \mapsto R_2 - R_1} \begin{pmatrix} 1 & 3 & -2 \\ 0 & -11 & 10 \\ 0 & -11 & 10 \end{pmatrix}$

$\xrightarrow{R_3 \mapsto R_3 - R_2} \begin{pmatrix} 1 & 3 & -2 \\ 0 & -11 & 10 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow$ oneindig veel oplossingen

stel $z = \lambda \to -11y + 10\lambda = 0 \Rightarrow y = \frac{10}{11}\lambda$

$\to x + 3y - 2\lambda = 0 \Rightarrow x = -\frac{8}{11}\lambda$

$x = \frac{-8}{11}\lambda,\ y = \frac{10}{11}\lambda,\ z = \lambda$ ✓

$\vec{x} = \lambda \begin{pmatrix} -8/11 \\ 10/11 \\ 1 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \\ 2 & -1 & 7 \\ 5 & 5 & 4 \\ 9 & 7 & 6 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & -1 \\ 0 & -3 & 3 \\ 0 & -5 & 9 \\ 0 & -5 & 9 \\ 0 & -11 & 15 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 0 & -5 & 9 \\ 0 & -11 & 15 \\ 0 & 0 & 0 \end{pmatrix}$

$\to \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 4 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\Rightarrow$ enkel triviale oplossing : $x=0,\ y=0,\ z=0$ ✓

2.13

a) $\begin{vmatrix} 1 & -4 & 2 \\ -4 & 1 & -2 \\ \cancel{2}_{1} & \cancel{-2}_{-1} & \cancel{-2}_{-1} \end{vmatrix} \Rightarrow \begin{pmatrix} 1 & -4 & 2 \\ 0 & -15 & 6 \\ 0 & 3_{1} & -3_{-1} \end{pmatrix} \begin{pmatrix} 1 & -4 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & -9 \end{pmatrix}$

$\Rightarrow$ enkel triviale oplossing $\Rightarrow$ lineair onafh. ✓

b) lineair onafh.

OEFENZITTING

2.1

b) $\left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 3 & -1 & 2 & 7 \\ 5 & 3 & -4 & 2 \end{array}\right) \xrightarrow[R_2 \mapsto R_2 - 3R_1]{R_3 \mapsto R_3 - 5R_1} \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & -7 & 11 & 10 \\ 0 & -7 & 11 & 7 \end{array}\right)$

$\xrightarrow{R_3 \mapsto R_3 - R_2} \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & -7 & 11 & 10 \\ 0 & 0 & 0 & 3 \end{array}\right) \Rightarrow$ strijdig: geen opl. ✓

2.4

$(Ax+B)(x^2-1) + C(x^2+4)(x+1) + D(x^2+4)(x-1)$

$\underline{Ax^3} - Ax + \underline{Bx^2} - B + \underline{Cx^3} + \underline{Cx^2} + 4Cx + 4C + \underline{Dx^3} - \underline{Dx^2} + 4Dx - 4D$

$(A+C+D)x^3 + (B+C-D)x^2 + (-A+4C+4D)x + (-B+4C-4D)$

$A+C+D=0$
$B+C-D=0$
$-A+4C+4D=0$
$-B+4C-4D=1$

$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ -1 & 0 & 4 & 4 & 0 \\ 0 & -1 & 4 & -4 & 1 \end{array}\right)$

$A = 0$ ✓
$B = -1/5$ ✓
$C = 1/10$ ✓
$D = -1/10$ ✓

2.6

c) $\left(\begin{array}{ccc|c} a & a+1 & 1 & 0 \\ a & 1 & a+1 & 0 \\ 2a & 1 & 1 & a+1 \end{array}\right) \xrightarrow[R_2 \mapsto R_2 - R_1]{R_3 \mapsto R_3 - 2R_1}$

$\left(\begin{array}{ccc|c} a & a+1 & 1 & 0 \\ 0 & -a & a & 0 \\ 0 & -2a-1 & -1 & a+1 \end{array}\right)$

$a=0$: $\left(\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 1 \end{array}\right) \rightarrow$ strijdig: geen opl. ✓

$a=-1$ $\left(\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

stel $z = \lambda \rightarrow x = \lambda$ en $y = \lambda$

$\Rightarrow (\lambda, \lambda, \lambda)$ ✓

voor $a \neq -1$ en $a \neq 0$

$R_3 \mapsto R_3(-a) + R_2(-2a-1)$ ->
$$\left(\begin{array}{ccc|c} a & a+1 & 1 & 0 \\ 0 & -a & a & 0 \\ 0 & 0 & 2a^2+2a & -a^2-a \end{array}\right)$$

$$z = -\frac{a^2+a}{2(a^2+a)} = \frac{-1}{2} \checkmark$$

$$y(-a) - \frac{a}{2} = 0 \rightarrow y = -\frac{1}{2} \checkmark$$

$$ax + (a+1)y + z = 0$$
$$x = \frac{a+1}{2a} + \frac{1}{2a} = \frac{a+2}{2a} \checkmark$$

2.8
$$\begin{pmatrix} k & 5 & 3 \\ 5 & 1 & -1 \\ k & 2 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 5 & 1 & -1 \\ k & 5 & 3 \\ k & 2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 1 & -1 \\ 0 & 25-k & 15+k \\ 0 & 10-k & 5+k \end{pmatrix}$$

$\ast$ $25-k \neq 0 \rightarrow k \neq 25$

$$\begin{pmatrix} 5 & 1 & -1 \\ 0 & 25-k & 15+k \\ 0 & 0 & 25k-25 \end{pmatrix} \rightarrow \begin{pmatrix} 5 & 1 & -1 \\ 0 & 25-k & 15+k \\ 0 & 0 & k-1 \end{pmatrix}$$

$\ast$ $k-1 \neq 0 \rightarrow k \neq 1$

$\ast$ $k=1$ $\begin{pmatrix} 5 & 1 & -1 \\ 0 & 24 & 16 \\ 0 & 0 & 0 \end{pmatrix}$ -> niet triviale opl. ✓

$\ast$ $k=25$ $\begin{pmatrix} 5 & 1 & -1 \\ 0 & 0 & 40 \\ 0 & 0 & 24 \end{pmatrix}$ -> enkel triviale opl.

$\ast$ $k \neq 1$ en $k \neq 25$ -> enkel triviale opl.

EX

$\boxed{2.12}$

a) $\begin{pmatrix} 1 & 5 & 2 & 3 \\ 4 & 2 & -1 & -6 \\ -5 & 1 & 3 & 11 \end{pmatrix} \xrightarrow[R_3 \leftrightarrow R_3 + 5R_1]{R_2 \leftrightarrow R_2 - 4R_1} \begin{pmatrix} 1 & 5 & 2 & 3 \\ 0 & -18 & -9 & -18 \\ 0 & 26 & 13 & 26 \end{pmatrix}$

$$\rightarrow \begin{array}{cccc} \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\ \end{array}$$
$$\begin{pmatrix} 1 & 0 & -1/2 & -2 \\ 0 & 1 & 1/2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \text{lineair afhankelijk}$$

$\lambda_1 a + \lambda_2 b + \lambda_3 c + \lambda_4 d = 0$

Stel $u = \lambda_3$ en $\lambda_4 = v$

$\lambda_2 = -\frac{1}{2}u - v$

$\lambda_1 = \frac{1}{2}u + 2v$

b) $\vec{a}$ en $\vec{b}$ zijn lineair onafhankelijk

$\hookrightarrow$ rang 2 : max 2 lin. onafh.

$\vec{c}$ in functie van $\vec{a}$ en $\vec{b}$ schrijven

$v=0$ en $u=1$

$\lambda_1 \vec{a} + \lambda_2 \vec{b} = -\vec{c}$

$\frac{1}{2}\vec{a} - \frac{1}{2}\vec{b} = -c \Rightarrow c = -\frac{1}{2}\vec{a} + \frac{1}{2}\vec{b}$

$v=1$ en $u=0$

$\lambda_1 \vec{a} + \lambda_2 \vec{b} = -\vec{d}$

$2\vec{a} - \vec{b} = -\vec{d}$

$\Rightarrow \vec{d} = -2\vec{a} + \vec{b}$

2.13

c) $\begin{pmatrix} 1 & -1 & 1 & 2 \\ -2 & 5 & 0 & -15 \\ 3 & 3 & 2 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -7/5 \\ 0 & 0 & 1 & -17/5 \end{pmatrix}$

$\Rightarrow$ lineair afhankelijk ✓

$\rightarrow$ lineair onafh. : $\vec{a}, \vec{b}, \vec{c}$

geen antw 2.15

$$\begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & b^2-a^2 & c^2-a^2 \end{pmatrix} \cdot (b+a)$$

$$\to \begin{pmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & 0 & \underbrace{bc(c-b)+ab(b-a)+ac(a-c)}_{\neq 0} \end{pmatrix}$$

$(c-a)(c-b)$

$\Rightarrow$ vectoren zijn lineair onafh.

1

Oefeningen hoofdstuk 3

THUIS

3.1

a) $A + B = \begin{pmatrix} 1 & -1 & -1 \\ 2 & 0 & 4 \end{pmatrix}$ ✓

b) $A - B = \begin{pmatrix} 1 & -3 & 7 \\ -2 & 6 & 4 \end{pmatrix}$ ✓

c) $C^T = \begin{pmatrix} -5 & 4 & 2 \\ 3 & -1 & -1 \end{pmatrix}$

$B - C^T = \begin{pmatrix} 5 & -3 & -6 \\ -1 & -2 & 1 \end{pmatrix}$ ✓

d) $C + D$: gaat niet ✓ onmogelijk

e) $3\vec{b}^T = \begin{pmatrix} 6 \\ 15 \\ -6 \end{pmatrix}$ $\quad \vec{a} + 3\vec{b}^T = \begin{pmatrix} 6 \\ 12 \\ -5 \end{pmatrix}$ ✓

f) $2P = \begin{pmatrix} 2 & -4 \\ 0 & 8 \end{pmatrix}$ $\quad 2P - Q = \begin{pmatrix} -1 & -4 \\ 0 & 7 \end{pmatrix}$ ✓

g) AB : gaat niet ✓ onmogelijk

h) $A^T = \begin{pmatrix} 1 & 0 \\ -2 & 3 \\ 3 & 4 \end{pmatrix}$ $\quad A^T \cdot B = \begin{pmatrix} 0 & 7 & -4 \\ 6 & -11 & 8 \\ 8 & -9 & -12 \end{pmatrix}$ ✓

j) $C \cdot P = \begin{pmatrix} -5 & 22 \\ 4 & -12 \\ 2 & -8 \end{pmatrix}$ ✓

k) $P \cdot C$: gaat niet ✓ onmogelijk

m) $\vec{a} \cdot \vec{b} = \begin{pmatrix} 0 & 0 & 0 \\ -6 & -15 & 6 \\ 2 & 5 & -2 \end{pmatrix}$ ✓

o) $\vec{b} \cdot \vec{a} = -17$ ✓

p) $\vec{b} \cdot C = \begin{pmatrix} 6 & 3 \end{pmatrix}$ ✓

t) $C \cdot \vec{a}$: gaat niet ✓ onmogelijk

3.3

a) $AB = \begin{pmatrix} -5 & 4 \\ 4 & -1 \end{pmatrix}$ ✓ $BA = \begin{pmatrix} -5 & 4 \\ 4 & -1 \end{pmatrix}$ ✓

b) $AB = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ ✓ $BA = \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix}$ ✓

3.5

a) $AB = \begin{pmatrix} -5 & 4 \\ 4 & -1 \end{pmatrix}$ $BA = \begin{pmatrix} -5 & 4 \\ 4 & -1 \end{pmatrix}$ $AB - BA = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ ✓

b) $AB = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ $BA = \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix}$ $AB - BA = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$ ✓

3.8

c) ja (p 43) ✓

b) neen, vb $A = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$(AB)^2 = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ en $A^2 B^2 = \begin{pmatrix} -1 & 0 \\ -1 & 0 \end{pmatrix}$ ✓

3.9

a) $\left(\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 4 & 1 & 0 & 1 \end{array}\right) \xrightarrow{R_2 \mapsto R_2 - 2R_1} \left(\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 0 & 7 & -2 & 1 \end{array}\right)$

$\rightarrow \left(\begin{array}{cc|cc} 1 & -3/2 & 1/2 & 0 \\ 0 & 1 & -2/7 & 1/7 \end{array}\right) \xrightarrow{R_1 \mapsto R_1 + 3/2 R_2} \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{14} & \frac{3}{14} \\ 0 & 1 & \frac{-2}{7} & \frac{1}{7} \end{array}\right)$

$\Rightarrow$ inverse : $\begin{pmatrix} \frac{1}{14} & \frac{3}{14} \\ \frac{-2}{7} & \frac{1}{7} \end{pmatrix}$ ✓

b) $\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ -2 & 1 & 2 & 0 & 1 & 0 \\ 3 & -1 & -1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_3 \mapsto R_3 - 3R_1]{R_2 \mapsto R_2 + 2R_1} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 5 & 8 & 2 & 1 & 0 \\ 0 & -7 & -10 & -3 & 0 & 1 \end{array}\right)$

$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 8/5 & 2/5 & 1/5 & 0 \\ 0 & 1 & \frac{10}{7} & 3/7 & 0 & -1/7 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 8/5 & 2/5 & 1/5 & 0 \\ 0 & 0 & -6/35 & 1/35 & -1/5 & \frac{1}{7} \end{array}\right)$

$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 8/5 & 2/5 & 1/5 & 0 \\ 0 & 0 & 1 & -1/6 & 7/6 & 5/6 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 3/2 & -7/2 & -5/2 \\ 0 & 1 & 0 & 2/3 & -5/3 & -4/3 \\ 0 & 0 & 1 & -1/6 & 7/6 & 5/6 \end{array}\right)$

$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/6 & -1/6 & 1/6 \\ 0 & 1 & 0 & 2/3 & -5/3 & -4/3 \\ 0 & 0 & 1 & -1/6 & 7/6 & 5/6 \end{array}\right)$

geen opl. 3.14

a) $R_{\theta_1} R_{\theta_2} = \begin{pmatrix} \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 & -\cos\theta_1 \sin\theta_2 - \sin\theta_1 \cos\theta_2 \\ \sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2 & -\sin\theta_1 \sin\theta_2 + \cos\theta_1 \cos\theta_2 \end{pmatrix}$

$= \begin{pmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{pmatrix}$

$R_{(\theta_1+\theta_2)} = \begin{pmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{pmatrix} = R_{\theta_1} R_{\theta_2}$

b) $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow A^{-1} = \dfrac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \Rightarrow R_\theta^{-1} = \dfrac{1}{\cos^2\theta + \sin^2\theta} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

$R_\theta^{-1} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ $\left(\begin{array}{l} \cos\theta = \cos(-\theta) \\ \text{en } \sin(-\theta) = -\sin\theta \end{array} \right)$

$R_\theta^{-1} = \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix} = R_{(-\theta)}$

OEFENZITTING

3.1

i) $BC = \begin{pmatrix} -4 & 3 \\ -22 & 9 \end{pmatrix}$ ✓

l) $A\vec{x} = \begin{pmatrix} 9 \\ -5 \end{pmatrix}$

$Q(A\vec{x}) = \begin{pmatrix} 27 \\ -5 \end{pmatrix}$ ✓

r) $PQ = \begin{pmatrix} 3 & -2 \\ 0 & 4 \end{pmatrix}$ ✓

s) $QP = \begin{pmatrix} 3 & -6 \\ 0 & 4 \end{pmatrix}$ ✓

3.4

$BA = \begin{pmatrix} 2a+3b & a+2b \\ 2c+3d & c+2d \end{pmatrix}$

$2a+3b = 3 \quad \rightarrow -4b+4+3b = 3 \quad \Rightarrow b = 1$

$a+2b = 2 \quad \rightarrow a = 0$

$2c+3d = 1 \quad \rightarrow -4d+8+3d = 1 \quad \Rightarrow d = 7$

$c+2d = 4 \quad \rightarrow c = -10$

$\begin{pmatrix} 0 & 1 \\ -10 & 7 \end{pmatrix}$ ✓

3.7

b) $\begin{pmatrix} a+2c & b+2d \\ 2a & 2b \end{pmatrix} = \begin{pmatrix} a+2b & 2a \\ c+2d & 2c \end{pmatrix}$

$a+2c = a+2b \quad \rightarrow c = b$ Stel a, d

$2a = c+2d \quad \rightarrow 2a = b+2d$ $\quad \rightarrow b = 2a - 2d = c$

$b+2d = 2a$

$2b = 2c \quad \rightarrow c = b$

$\Rightarrow \begin{pmatrix} a & 2a-2d \\ 2a-2d & d \end{pmatrix}$ met a en d willekeurig ✓

3.8

a) $(A+B)(A+B) = A\cdot A + A\cdot B + B\cdot A + B\cdot B$

$= A^2 + A\cdot B + B\cdot A + B^2$ matrixproduct

$\neq A^2 + 2A\cdot B + B^2$ is niet commutatief

=> geldt niet altijd ✓

d) $(A\,B)^T \overset{?}{=} A^T B^T$

stel $A = C^T$ en $B = D^T$

$(A\,B)^T = B^T A^T = D\,C$

$A^T B^T = C\,D$ ≠ niet commutatief

neen ✓

3.9

d) $\left(\begin{array}{cc|cc} 0 & -1 & 1 & 0 \\ 1 & x & 0 & 1 \end{array}\right)$

$A^{-1} = \frac{1}{1}\begin{pmatrix} x & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} x & 1 \\ -1 & 0 \end{pmatrix}$ ✓

f) $\left(\begin{array}{cccc|cccc} 3 & 4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 2 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_4 \mapsto 3R_4 - 4R_3]{R_2 \mapsto R_2 - \frac{1}{3}R_1}$

$\left(\begin{array}{cccc|cccc} 3 & 4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2/3 & 0 & 0 & -1/3 & 1 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & -4 & 3 \end{array}\right)$

$\left(\begin{array}{cccc|cccc} 1 & 4/3 & 0 & 0 & 1/3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 3/2 & 0 & 0 \\ 0 & 0 & 1 & 1/3 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -2 & 3/2 \end{array}\right) \xrightarrow[R_3 \mapsto R_3 - \frac{1}{3}R_4]{R_1 \mapsto R_1 - \frac{4}{3}R_2}$

$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 3/2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -1/2 \\ 0 & 0 & 0 & 1 & 0 & 0 & -2 & 3/2 \end{array}\right)$ ✓

3.10

$$\left(\begin{array}{ccc|ccc} 2 & 5 & 1 & 1 & 0 & 0 \\ 3 & 1 & 2 & 0 & 1 & 0 \\ -2 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_3 \mapsto R_3 + R_1]{R_2 \mapsto 2R_2 - 3R_1}$$

$$\left(\begin{array}{ccc|ccc} 2 & 5 & 1 & 1 & 0 & 0 \\ 0 & -13 & 1 & -3 & 2 & 0 \\ 0 & 6 & 1 & 1 & 0 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 5/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & -1/13 & 3/13 & -2/13 & 0 \\ 0 & 1 & 1/6 & 1/6 & 0 & 1/6 \end{array}\right)$$

$R_3 \mapsto R_3 - R_2$

$$\left(\begin{array}{cccccc} 1 & 5/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & -1/13 & 3/13 & -2/13 & 0 \\ 0 & 0 & 19/78 & & & \end{array}\right)$$

ZRM: $\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2/19 & -1/19 & -9/19 \\ 0 & 1 & 0 & 4/19 & -2/19 & 1/19 \\ 0 & 0 & 1 & -5/19 & 12/19 & 13/19 \end{array}\right)$ ✓

geen opl 3.12

2×2 symm. $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \Rightarrow b = c$

van de vorm $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$

vb $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$

geen opl 3.15

b) $S_\theta^T = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$

$$S_\theta^{-1} = \frac{1}{-\cos^2\theta - \sin^2\theta}\begin{pmatrix} -\cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$= -1 \begin{pmatrix} -\cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} = S_\theta^T$$

↗ h. 4. 2

geen opl 3. 16

a) $A^T = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$

$A \cdot A^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow$ orthogonaal

b) $A^{-1} A \vec{x} = A^{-1} \vec{b}$

$\vec{x} = A^{-1} \vec{b}$

en $A^{-1} = A^T \Rightarrow A^T$ gemakkelijk te berekenen

$\vec{x} = A^T \vec{b}$

3. 17

? a) $\begin{pmatrix} a & c \\ b & d \end{pmatrix} \cdot \begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} a^2+cb & ac+cd \\ ab+bd & bc+d^2 \end{pmatrix}$

$a = a^2 + cb$ $\quad a - a^2 = d - d^2$

$b = ab + bd$ $\quad a + d = 1 \quad 1 - d - 1 + 2d - d^2 = d - d^2$

$c = ac + cd$ $\quad a + d = 1$

$d = bc + d^2$

$\boxed{\text{Nw: } a + d = 1 \quad a - a^2 = cb}$

$\begin{pmatrix} 1/3 & 2/3 \\ 1/3 & 2/3 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

1

Gefeningen hoofdstuk 4

4.1

$D = 5 \qquad x_1 = 2$ ✓

$D_1 = 10 \qquad x_2 = 3 \quad = 4$ ✓

$D_2 = 15$

4.2

a) $\begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = 6$ ✓

c) $\begin{vmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{vmatrix} = \cos^2 n\theta + \sin^2 n\theta = 1$ ✓

4.4

a) $\begin{vmatrix} 2 & -1 & 3 \\ 4 & 1 & -2 \\ -3 & 2 & 1 \end{vmatrix} = 2(-1+4) + (4-6) + 3(8+3)$

$= 41$ ✓

d) $\begin{vmatrix} 1 & 3 & -2 \\ 0 & -1 & 2 \\ 0 & 0 & 4 \end{vmatrix} = -4 - 3\cdot 0 - 2\cdot 0 \quad = -4$ ✓

4.6

b) $\begin{vmatrix} 3 & 4 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 4 & 2 \end{vmatrix} = 3 \begin{vmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 4 & 2 \end{vmatrix} - \begin{vmatrix} 4 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 4 & 2 \end{vmatrix}$

$= 3\cdot 2(6-4) - 4(6-4) = 4$ ✓

c) $\begin{vmatrix} -2 & 6 & 17 & -5 \\ 0 & 3 & 33 & -12 \\ 0 & 0 & 4 & -12 \\ 0 & 0 & 0 & -6 \end{vmatrix} = 144$ ✓

4.8

a) $\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 0 & 0 \\ 1 & 0 & x & 0 \\ 1 & 0 & 0 & x \end{vmatrix} = x \begin{vmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix} - 1 \begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & x \end{vmatrix} + \begin{vmatrix} 1 & x & 0 \\ 1 & 0 & 0 \\ 1 & 0 & x \end{vmatrix} - \begin{vmatrix} 1 & x & 0 \\ 1 & 0 & x \\ 1 & 0 & 0 \end{vmatrix}$

$= x \cdot x \cdot x \cdot x - x \cdot x + x(-x) - x \cdot x$

$= x^4 - 3x^2 = 0$

$x^2(x^2 - 3) = 0$

$\Rightarrow x = 0$ of $x = \sqrt{3}$ of $x = -\sqrt{3}$ ✓

4.9

a) $\begin{vmatrix} 3 & 2 & -2 \\ 6 & 1 & 5 \\ -9 & 3 & 4 \end{vmatrix} \xrightarrow{\substack{R_2 - 2R_1 \\ R_3 + 3R_1}} \begin{vmatrix} 3 & 2 & -2 \\ 0 & -3 & 9 \\ 0 & 9 & -2 \end{vmatrix} = 3(6 - 81) = -225$ ✓

b) $\begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{vmatrix} \xrightarrow{\substack{R_3 - R_1 \\ R_4 - R_2 - R_1}} \begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & -1 \end{vmatrix} \xrightarrow{R_4 - R_3} \begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix}$

$= 1$ ✓

4.10

$D = e^t \sin 2t \, (-2e^t \sin 2t) - e^t \cos 2t \cdot 2e^t \cos 2t$

$= -2e^{2t} \sin^2 2t - 2e^{2t} \cos^2 2t = -2e^{2t}$

$D_1 = t \, (-2e^t \sin 2t) - t^2 e^t \cos 2t$

$= te^t(-2 \sin 2t - t \cos 2t)$

$D_2 = e^t \sin 2t \cdot t^2 - t \, 2e^t \cos 2t$

$= te^t (t \sin 2t - 2 \cos 2t)$

$x = \frac{D_1}{D} = \frac{-t}{2e^t} (-2 \sin 2t - t \cos 2t)$ ✓

$y = \frac{D_2}{D} = -\frac{t}{2e^t} (t \sin 2t - 2 \cos 2t)$ ✓

OEFENZITTING

Regel van Cramer voor oplossen van stelsels

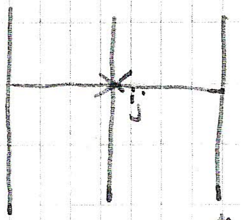
$$A\begin{pmatrix}x_1\\ \vdots\\ x_n\end{pmatrix}=\vec{b}$$

D = determinant van A

D_i := determinant van matrix A met i-de kolom vervangen door $\vec{b}$

$$\Rightarrow x_i=\frac{D_i}{D}$$

Determinant van $n\times n$ door ontwikkelen naar rijen of kolommen



= Minor M_{ij} -> cofactor $C_{ij}=\pm M_{ij}$

$$\det A=\sum_{j=1}^{n} a_{ij}C_{ij} \quad \text{of} \quad \det A=\sum_{i=1}^{n} a_{ij}C_{ij}$$

4.2

b) $\begin{vmatrix}0 & 1\\ -2 & 3\end{vmatrix}=2$ ✓

4.3

$$\begin{vmatrix}1&1&1\\1&2&3\\1&4&9\end{vmatrix}=(2\cdot 9-3\cdot 4)-(9-4)+(3-2)$$
$$=2=D$$

$$\begin{vmatrix}6&1&1\\14&2&3\\36&4&9\end{vmatrix}=6(2\cdot 9-4\cdot 3)-1(14\cdot 9-3\cdot 36)+1(14\cdot 4-2\cdot 36)$$
$$=2=D_1$$

$$\begin{vmatrix}1&6&1\\1&14&3\\1&36&9\end{vmatrix}=(14\cdot 9-3\cdot 36)-(6\cdot 9-36)+(6\cdot 3-14)$$
$$=4=D_2$$

$$\begin{vmatrix}1&1&6\\1&2&14\\1&4&36\end{vmatrix}=(2\cdot 36-4\cdot 14)-(36-4\cdot 6)+(14-2\cdot 6)$$
$$=6=D_3$$

$x=1$ ✓ $y=2$ ✓ $z=3$ ✓

4.4

c) $\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = a(a^2-bc) - b(ca-b^2) + c(c^2-ab)$
$= a^3 - abc - bca + b^3 + c^3 - cab$
$= a^3 + b^3 + c^3 - 3abc$ ✓

e) $\begin{vmatrix} 0 & 3 & 2 \\ 2 & 0 & 1 \\ 2 & 6 & 0 \end{vmatrix} = -3(-2) + 2(-12) = 30$ ✓

geen opl. 4.5

$C_{11} = -1$ 1° rij: $\det A = 1(-1) - 3 \cdot 2 - 2 \cdot (-3)$
$C_{12} = -3$ $= -1$
$C_{13} = -2$ 1° kolom: $\det A = 1(-1) + 1 \cdot 2 - 2 \cdot 1$
$C_{21} = 1$ $= -1$
$C_{22} = +4$ 2° rij: $\det A = 2 \cdot 1 - 1 \cdot 3 = -1$
$C_{23} = 3$ 2° kolom: $\det A = 2(-3) - 1(-5) = -1$
$C_{31} = -2$ 3° rij: $\det A = -2 + 5 - 4 = -1$
$C_{32} = -5$ 3° kolom: $\det A = 6 - 3 - 4 = -1$
$C_{33} = -4$

4.6

d) $\begin{vmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{vmatrix} = -2 \begin{vmatrix} 2 & 0 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & 0 \end{vmatrix} = 2 \cdot 2 \begin{vmatrix} 2 & 0 \\ 2 & 2 \end{vmatrix} = 16$ ✓

4.8

geen opl. b) $\begin{vmatrix} 1 & 2x & 3x^2 \\ 2x^3 & 3x^4 & 4x^5 \\ 3x^6 & 4x^7 & 6x^8 \end{vmatrix} = 3x^4 \cdot 6x^8 - 4x^7 \cdot 4x^5 - 2x(2x^3 \cdot 6x^8 - 4x^5 \cdot 3x^6)$
$+ 3x^2(2x^3 4x^7 - 3x^4 3x^6)$
$= 18x^{12} - 16x^{12} - 24x^{12} + 24x^{12}$
$+ 24x^{12} - 27x^{12}$
$= -x^{12} = 0$
$\Rightarrow x = 0$

geen opl 4.8

d) $\begin{vmatrix} x & a & b & c \\ a & x & b & c \\ a & b & x & c \\ a & b & c & x \end{vmatrix} \xrightarrow{\substack{R_3 \to R_3 - R_2 \\ R_4 \to R_4 - R_2}} \begin{vmatrix} x & a & b & c \\ a & x & b & c \\ 0 & b-x & x-b & 0 \\ 0 & b-x & c-b & x-c \end{vmatrix} \xrightarrow{\substack{R_2 \to R_2 - R_1 \\ R_4 \to R_4 - R_3}}$

$\begin{vmatrix} x & a & b & c \\ a-x & x-a & 0 & 0 \\ 0 & b-x & x-b & 0 \\ 0 & 0 & c-x & x-c \end{vmatrix} = -c \begin{vmatrix} a-x & x-a & 0 \\ 0 & b-x & x-b \\ 0 & 0 & c-x \end{vmatrix} + (x-c) \begin{vmatrix} x & a & b \\ a-x & x-a & 0 \\ 0 & b-x & x-b \end{vmatrix}$

$= -c(a-x)(b-x)(c-x) + (x-c)(x-a)\big(\underbrace{(x-b)(x-a) - b(x-b)}_{=(x-b)(x-a-b)}\big)$

$= (a-x)(b-x)(c-x)(x-a-b+c) = 0$

$\Rightarrow x = a$ of $x = b$ of $x = c$ of $x = a+b-c$

4.14

a) $\operatorname{adj}(A) = \begin{pmatrix} -1 & 1 & -2 \\ -3 & 4 & -5 \\ -2 & 3 & -4 \end{pmatrix}$

b) $A^{-1} = \dfrac{1}{\det A} \operatorname{adj}(A)$ $(\det A = -1)$

$= \begin{pmatrix} 1 & -1 & 2 \\ 3 & -4 & 5 \\ 2 & -3 & 4 \end{pmatrix}$

geen opl. 4.16

$A^T = -A$

Stel n oneven $\Rightarrow \det(A) = 0$ $\quad n = 2k+1$

$\det(\lambda A) = \lambda^n \det A$

$\det(A) = \det(A^T) = \det(-A) = (-1)^{2k+1} \det A$

$= -\det(A)$

$= 0$

THUIS

Oefeningen hoofdstuk 5

5.1

a) $\cancel{p = \begin{pmatrix}1\\2\end{pmatrix} \quad q = \begin{pmatrix}-7\\4\end{pmatrix}}$ $\quad -3 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_2 - 2}{x_2 - 1} \Rightarrow \begin{matrix} x_2 = -7 \\ y_2 = 4 \end{matrix}$

$\cancel{\vec{x} = \begin{pmatrix}1\\2\end{pmatrix} + t\begin{pmatrix}-8\\2\end{pmatrix}} \quad \rightarrow \vec{x} = \begin{pmatrix}1\\2\end{pmatrix} + t\begin{pmatrix}1\\-3\end{pmatrix}$ ✓

b) $\vec{x} = \begin{pmatrix}1\\2\\3\end{pmatrix} + t\begin{pmatrix}1\\0\\0\end{pmatrix}$ ✓

c) $\vec{x} = \begin{pmatrix}1\\2\end{pmatrix} + t\begin{pmatrix}1\\1\end{pmatrix}$ ✓

d) $\vec{x} = \begin{pmatrix}0\\1\\2\end{pmatrix} + t\begin{pmatrix}1\\1\\1\end{pmatrix}$ × $\quad \vec{x} = \begin{pmatrix}1\\2\\3\end{pmatrix} + t\begin{pmatrix}1\\1\\1\end{pmatrix}$

e) $\vec{x} = \begin{pmatrix}0\\-3\\-1\end{pmatrix} + t\begin{pmatrix}3\\2\\7\end{pmatrix}$ × $\quad \vec{x} = \begin{pmatrix}3\\1\\6\end{pmatrix} + t\begin{pmatrix}3\\2\\7\end{pmatrix}$

5.2

b) $\vec{x} = \begin{pmatrix}1\\0\\0\end{pmatrix} + t_1\begin{pmatrix}-1\\1\\0\end{pmatrix} + t_2\begin{pmatrix}-1\\0\\1\end{pmatrix}$ ✓

5.5

a) $2(x-1) + (y-1) + 3(z-2) = 0$

$2x - 2 + y - 1 + 3z - 6 = 0$

$2x + y + 3z = 9$

$\vec{r}_1$ en $\vec{r}_2 \perp \vec{n}$

$\vec{r}_1 = (-1 \quad 2 \quad 0)^T \qquad \vec{r}_2 = (-3 \quad 0 \quad 2)^T$

$\vec{p} = (9/2 \quad 0 \quad 0)^T$

$\vec{x} = \begin{pmatrix}9/2\\0\\0\end{pmatrix} + t_1\begin{pmatrix}-1\\2\\0\end{pmatrix} + t_2\begin{pmatrix}-3\\0\\2\end{pmatrix}$ × $\quad \vec{x} = \begin{pmatrix}1\\1\\2\end{pmatrix} + t_1\begin{pmatrix}-1\\2\\0\end{pmatrix} + t_2\begin{pmatrix}-3\\0\\2\end{pmatrix}$

5.18

a) $\vec{a} \times \vec{b} = \begin{pmatrix}1\\3\\-2\end{pmatrix} \times \begin{pmatrix}0\\3\\1\end{pmatrix} = \begin{pmatrix}9\\-1\\3\end{pmatrix}$ ✓

$\vec{b} \times \vec{a} = \begin{pmatrix}0\\3\\1\end{pmatrix} \times \begin{pmatrix}1\\3\\-2\end{pmatrix} = \begin{pmatrix}-9\\1\\-3\end{pmatrix}$ ✓

b) $\vec{b} \times \vec{b} = \begin{pmatrix}0\\3\\1\end{pmatrix} \times \begin{pmatrix}0\\3\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$ ✓

geen opl

5.21

$$\begin{vmatrix} a_1 & b_1 & a_2b_3 - a_3b_2 \\ a_2 & b_2 & a_3b_1 - a_1b_3 \\ a_3 & b_3 & a_1b_2 - a_2b_1 \end{vmatrix}$$

ontwikkelen naar 3e kolom

$$= (a_2b_3 - a_3b_2)(a_2b_3 - b_2a_3) + (a_1b_3 - a_3b_1)(a_1b_3 - b_1a_3) + (a_1b_2 - a_2b_1)(a_1b_2 - b_1a_2)$$

$$= (a_2b_3 - a_3b_2)^2 + (a_1b_3 - b_1a_3)^2 + (a_1b_2 - a_2b_1)^2 = |\vec{a} \times \vec{b}|^2$$

lengte $|\vec{a} \times \vec{b}| = \sqrt{(a_2b_3 - a_3b_2)^2 + (a_1b_3 - b_1a_3)^2 + (a_1b_2 - a_2b_1)^2}$

5.22

a) $d(P, V) = \dfrac{|10 - 2 - 3 + 2|}{\sqrt{4 + 9 + 1}} = \dfrac{7}{\sqrt{14}}$ ✓

5.31

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 2 & 1 & -1 \end{vmatrix} = -1 + (-2-3) + 2(\cdot 2) = -2$$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ 2 & 0 & 1 \\ 3 & 1 & -1 \end{vmatrix} = \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = -1 - 5 + 4 = -2$$ ✓

OEFENZITTING

I Parametervgl

1 rechte

* richtvector $\vec{r}$ en steunvector $\vec{p}$

$$\vec{x} = \vec{p} + t\,\vec{r}$$

* door 2 punten $\vec{p}$ en $\vec{q}$

$$\vec{x} = \vec{p} + t\,(\vec{q} - \vec{p})$$

2 vlak

→ lineair onafh.

* LO richtvectoren $\vec{r}_1$, $\vec{r}_2$ steunvector $\vec{p}$

$$\vec{x} = \vec{p} + t_1\,\vec{r}_1 + t_2\,\vec{r}_2$$

* door 3 punten $\vec{p}_0$, $\vec{p}_1$, $\vec{p}_2$

$$\vec{x} = \vec{p}_0 + t_1\,(\vec{p}_1 - \vec{p}_0) + t_2\,(\vec{p}_2 - \vec{p}_0)$$

II Cartesische vgl

1 vlak

* door (x_0, y_0, z_0) en normaal $\vec{r}\,(a, b, c)$

met $d = ax_0 + by_0 + cz_0$

$$V \Leftrightarrow ax + by + cz = d$$

* door 3 punten $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \begin{pmatrix} d \\ e \\ f \end{pmatrix} \begin{pmatrix} g \\ h \\ i \end{pmatrix}$

$$V \Leftrightarrow \begin{vmatrix} x & a & d & g \\ y & b & e & h \\ z & c & f & i \\ 1 & 1 & 1 & 1 \end{vmatrix} = 0$$

2 rechte

Als doorsnede van 2 vlakken $\vec{n}_1$, $\vec{n}_2$ en $\vec{p}$: $\vec{n}_1 \cdot \vec{p} = d_1$

$\vec{n}_2 \cdot \vec{p} = d_2$

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \end{cases}$$

III omzetten

1. Parameter → cart

a vlak

$\vec{x} = \vec{p} + t_1 \vec{r}_1 + t_2 \vec{r}_2$

neem $\vec{n} = \vec{r}_1 \times \vec{r}_2$

$V \Leftrightarrow \vec{n} \cdot (\vec{x} - \vec{p}) = 0$

b rechte

→ t wegwerken (p 82)

2 Cart → parameter

a vlak

$ax + by + cy = d$

2 L.O richtvectoren ⊥ $\vec{n}$

$\vec{r}_1 = \begin{pmatrix} -b \\ a \\ 0 \end{pmatrix}$ $\vec{r}_2 = \begin{pmatrix} -c \\ 0 \\ a \end{pmatrix}$

steunvector $\vec{p} = \begin{pmatrix} d/a \\ 0 \\ 0 \end{pmatrix}$

$\vec{x} = \vec{p} + t_1 \vec{r}_1 + t_2 \vec{r}_2$

b rechte

$\begin{cases} a_1 x + b_1 y + c_1 y = d_1 \\ a_2 x + b_2 y + c_2 y = d_2 \end{cases}$

neem $\vec{r} = \vec{n}_1 \times \vec{n}_2$

en $\vec{p}$ particuliere oplossing

IV vectorproduct

$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$\vec{e}_1 = (1, 0, 0)$

$\vec{e}_2 = (0, 1, 0)$

$\vec{e}_3 = (0, 0, 1)$

5.2

a) $\vec{x} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + t_1 \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + t_2 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ ✓

5.4

a) $\Leftrightarrow \begin{vmatrix} x & 1 & 2 & 3 \\ y & -1 & 1 & 0 \\ z & 2 & 4 & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 0 \Leftrightarrow \begin{vmatrix} x & 1 & 1 & 2 \\ y & -1 & 2 & 1 \\ z & 2 & 2 & -1 \\ 1 & 1 & 0 & 0 \end{vmatrix} = 0$

$\begin{matrix} 1 & 0 & 0 \\ -1 & 3 & 3 \\ 2 & 0 & -5 \end{matrix}$

$x(-2-2) - y(-1-4) + z(1-4) - \begin{vmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 2 & 2 & -1 \end{vmatrix} = 0$

$-4x + 5y - 3z + 15 = 0$

$-4x + 5y - 3z = -15$

$(-4 \quad 5 \quad -3) = \vec{n}$

$\left(\frac{-4}{\sqrt{50}} \quad \frac{5}{\sqrt{50}} \quad \frac{-3}{\sqrt{50}}\right) = \vec{e}_1$ ✓

5.5

b) $-1 \cdot 2 + 4 \cdot (-1) + 5 \cdot 3 = 9$

$-x + 4y + 5z = 9$

$\cancel{\vec{r}_1 = \begin{pmatrix} -4 \\ -1 \\ 0 \end{pmatrix} \quad \vec{r}_2 = \begin{pmatrix} -5 \\ 0 \\ -1 \end{pmatrix}}$

$\cancel{\vec{p} = \begin{pmatrix} -9 \\ 0 \\ 0 \end{pmatrix}}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + t_1 \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} + t_2 \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix}$

$\cancel{\vec{x} = \begin{pmatrix} -9 \\ 0 \\ 0 \end{pmatrix} + t_1 \begin{pmatrix} -4 \\ -1 \\ 0 \end{pmatrix} + t_2 \begin{pmatrix} -5 \\ 0 \\ -1 \end{pmatrix}}$

5.7

$(1, 2, 3) \quad (2, 4, 2) \quad (1, 3, 3)$

$\begin{vmatrix} x & 1 & 2 & 1 \\ y & 2 & 4 & 3 \\ z & 3 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 0 \Leftrightarrow \begin{vmatrix} x & 1 & 1 & 0 \\ y & 2 & 2 & 1 \\ z & 3 & -1 & 0 \\ 1 & 1 & 0 & 0 \end{vmatrix}$

$= x(-1) - y(0) + z(1) + 4 = 0$

$\cancel{-x + z = -4}$

$-x + z = 2$

5.8

a) $1 \quad 2 \quad 8$ $\quad \perp + 1$ (= richtvector)

$a \quad 3 \quad b$

$a = -2$, $b = 12$ ✓

b) $-4 \cdot 3 + 3 \cdot 6 = 6$

$3x - 2y + 6z = 6$

$\vec{x} = \begin{pmatrix} 1 \\ 2 \\ 8 \end{pmatrix} + t \begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix}$

$x = 1 + t \cdot 3 \qquad y = 2 + t(-1) \qquad z = 8 + t \cdot 4$

$t = \frac{x-1}{3} \qquad t = -y + 2 \qquad t = \frac{z-8}{-4}$

$\begin{cases} x - 1 = -3y + 6 \\ z - 8 = 4y - 8 \end{cases} \Rightarrow \begin{cases} x + 3y = 7 \\ -4y + z = \cancel{16}\ 0 \end{cases}$

$\begin{cases} x + 3y = 7 \\ -4y + z = 0 \\ 3x - 2y + 6z = 6 \end{cases} \Leftrightarrow \begin{cases} x = \quad 136/13 \\ y = \quad -15/13 \\ z = \quad -60/13 \end{cases}$ ✓ ✓ ✓

5.16

$\begin{vmatrix} x & 1 & 4 & 1 \\ y & 1 & 0 & -5 \\ z & -2 & -3 & 10 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 0 \Leftrightarrow \begin{vmatrix} x & 1 & 3 & 0 \\ y & 1 & -1 & -6 \\ z & -2 & -1 & 12 \\ 1 & 1 & 0 & 0 \end{vmatrix}$

$= x(-12-6) - y(3 \cdot 12) + z(-6 \cdot 3) - [(-12-6) - 3(12-12)] = 0$

$\Rightarrow -18x - 36y - 18z = -18$

$-18 \cdot (-7) - 36 \cdot 2 - 18 \cdot 4 \stackrel{?}{=} -18$

$= -18 \Rightarrow$ liggen in 1 vlak ✓

S. 18

c) $\begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 3 & -2 \\ 0 & -1 & 2 \end{vmatrix} + \begin{vmatrix} e_1 & e_2 & e_3 \\ 0 & -1 & 2 \\ 1 & 3 & -2 \end{vmatrix}$

$= \begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix} + \begin{pmatrix} -4 \\ 2 \\ 1 \end{pmatrix} = \vec{0}$ ✓

d) $\begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 3 & -2 \\ 0 & -1 & 2 \end{vmatrix} = \begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix}$ $\quad \begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} = 0 - 6 - 1 = -7$ ✓

e) $\begin{vmatrix} e_1 & e_2 & e_3 \\ 0 & 3 & 1 \\ 0 & -1 & 2 \end{vmatrix} = \begin{pmatrix} 7 \\ 0 \\ 0 \end{pmatrix}$

$\begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 3 & -2 \\ 7 & 0 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ -14 \\ -21 \end{pmatrix}$ ✓

f) $\vec{a} + \vec{b} = \begin{pmatrix} 9 \\ -1 \\ 3 \end{pmatrix}$

$\begin{vmatrix} e_1 & e_2 & e_3 \\ 9 & -1 & 3 \\ 0 & -1 & 2 \end{vmatrix} = \begin{pmatrix} 1 \\ -18 \\ -9 \end{pmatrix}$ ✓

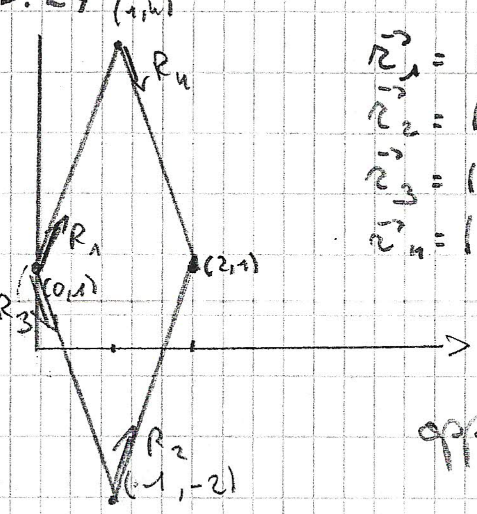
S. 22

b) $\begin{vmatrix} x & 1 & 0 & 0 \\ y & 0 & 1 & 0 \\ z & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 0 \Leftrightarrow x(1) - y(-1) + z(1) - 1 = 0$

$x + y + z = 1$

$d(P,\varepsilon) = \frac{|1 - 2 - 1 - 1|}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$ ✓

5.27



$\vec{r}_1 = (1\ ,3)$ $\Big\{$ evenwijdig
$\vec{r}_2 = (1\ ,3)$
$\vec{r}_3 = (1\ ,-3)$ $\Big\{$ evenwijdig
$\vec{r}_4 = (1\ ,-3)$

$\Rightarrow$ parallellogram

$$\text{opp} = \left| \det \begin{pmatrix} 1 & 1 \\ 3 & -3 \end{pmatrix} \right|$$

$$= |-3-3| = 6 \checkmark$$

5.30

a) $\text{vol} = \begin{vmatrix} 1 & -1 & 2 \\ 1 & 4 & 2 \\ 0 & 0 & 2 \end{vmatrix} = 2(4+1) = 10 \checkmark$

1

Oefeningen hoofdstuk 6

THUIS

6.1

a) $\begin{vmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} = (2-\lambda)(3-\lambda)-2$

$= 6 - 5\lambda + \lambda^2 - 2$

$= \lambda^2 - 5\lambda + 4$

$D = b^2 - 4ac = 25 - 16 = 9$

$\lambda = \frac{5 \pm 3}{2} \qquad \lambda_1 = 4 \qquad \lambda_2 = 1$ ✓

$\underline{\lambda = 4}$

$\left(\begin{array}{cc|c} -2 & 2 & 0 \\ 1 & -1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right) \qquad -x+y=0$

$y = c \Rightarrow x = c$

eigenvectoren : $c\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ✓

$\underline{\lambda = 1}$

$\left(\begin{array}{cc|c} 1 & 2 & 0 \\ 1 & 2 & 0 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right) \qquad x+2y=0$

$y = c \Rightarrow x = -2c$

eigenvectoren: $c\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ ✓

b) $\lambda_1 = 2$ en $\lambda_2 = -3$ ✓

$c\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ✓ $\quad c\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ ✓

? c) $\cancel{\begin{vmatrix} -\lambda & 2 & 2 \\ 2 & -\lambda & 2 \\ 2 & 2 & -\lambda \end{vmatrix} = \begin{vmatrix} -\lambda & 2 & 2 \\ 2+\lambda & -\lambda-2 & 0 \\ 2+\lambda & 0 & -\lambda-2 \end{vmatrix} = \lambda(-\lambda-2)^2 - 2(2+\lambda)(-\lambda-2) - 2(2+\lambda)(-\lambda-2)}$

$\cancel{\lambda^2+4\lambda+4 \quad -2\lambda-4 = \lambda^2 - 2\lambda}$

$\cancel{= \lambda^3 + 4\lambda^2 + 4\lambda + 8\lambda + 16 + 4\lambda^2 + 8\lambda}$

$\cancel{= \lambda^3 + 8\lambda^2 + 20\lambda + 16}$

oplossing: $\lambda = 4 \rightarrow c\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ ✓

$\lambda = -2 \rightarrow c_1\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + c_2\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ ✓

6.2

b) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ en $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$1 \cdot 0 + 0 \cdot 1 = 0 \quad \Rightarrow \perp$ op elkaar

g) $\lambda = 4 \quad \sqrt{1+1+1} = \sqrt{3}$

$\Rightarrow$ genorm. eigenvector $\begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix}$ ✓

$\lambda = -2$: genorm eigenvectoren $\begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}$ en $\begin{pmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$ ✓

6.3

geen opl

a) $\det(A) = \det(A^T)$ (4.4.1 p 66)

$\det(A - \lambda I) = \det(A^T - \lambda I)$

6.7

a) $\begin{vmatrix} 1-\lambda & 0 \\ 6 & -1-\lambda \end{vmatrix} = (1-\lambda)(-1-\lambda) = -1 + \lambda^2$

$\Rightarrow \lambda_1 = 1$ en $\lambda_2 = -1$

$D = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Rightarrow$ diagonaliseerbaar

c) gaat niet, geen $n \times n$ matrix

6.8

a) $\begin{vmatrix} 1+i-\lambda & 2-i \\ 3+i & -i-\lambda \end{vmatrix} = (1+i-\lambda)(-i-\lambda) - (2-i)(3+i)$

$= 1 - \lambda \cancel{-i^2} \cancel{-\lambda i} + \cancel{\lambda i} + \lambda^2 - 6 \cancel{-2i} + \cancel{3i} \cancel{+i^2}$

$= \lambda^2 - \lambda - 6$

$D = b^2 - 4ac = 1 + 24 = 25$

$\lambda = \frac{1 \pm 5}{2} \quad \lambda_1 = 3$ ✓ en $\lambda_2 = -2$ ✓

$\underline{\lambda = 3}$

$\left(\begin{array}{cc|c} i-2 & 2-i & 0 \\ 3+i & -i-3 & 0 \end{array}\right)$

$\rightarrow \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right) \quad x - y = 0 \quad y = c = x \Rightarrow c\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\underline{\lambda = -2}$

$\left(\begin{array}{cc|c} 3+i & 2-i & 0 \\ 3+i & 2-i & 0 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 3+i & 2-i & 0 \\ 0 & 0 & 0 \end{array}\right) \quad x + \frac{2-i}{3+i} y = 0$ stel $y = c \Rightarrow x = \frac{2-i}{3+i} c$

$\Rightarrow c \begin{pmatrix} (2-i)/(3+i) \\ 1 \end{pmatrix}$

6.10

a) $\begin{vmatrix} \alpha-\lambda & \beta & 0 & 0 \\ \beta & \alpha-\lambda & \beta & 0 \\ 0 & \beta & \alpha-\lambda & \beta \\ 0 & 0 & \beta & \alpha-\lambda \end{vmatrix} = (\alpha-\lambda)\left((\alpha-\lambda)((\alpha-\lambda)^2-\beta^2)-\beta(\beta(\alpha-\lambda))\right) - \beta\,\beta\,((\alpha-\lambda)^2-\beta^2)$

$= (\alpha-\lambda)^4 - (\alpha-\lambda)^2\beta^2 - \beta^2(\alpha-\lambda)^2 - \beta^2(\alpha-\lambda)^2 + \beta^4$

$= (\alpha-\lambda)^4 - 3(\alpha-\lambda)^2\beta^2 + \beta^4$

$\Rightarrow$ oplossing: $\lambda = \alpha \pm \left(\frac{1\pm\sqrt{5}}{2}\right)\beta$ ✓

$= \alpha \pm \sqrt{\frac{3\pm\sqrt{5}}{2}}\,\beta$ ✓

geen opl 6.13

$A = xDx^{-1}$

$AxDx^{-1} = xDx^{-1}xDx^{-1}$

$A\,A = xDIDx^{-1}$

$A^2 = xDDx^{-1}$

$A^2 = xD^2x^{-1}$

$A^2xDx^{-1} = xD^2x^{-1}xDx^{-1}$

$A^3 = xD^2Dx^{-1}$

$A^3 = xD^3x^{-1}$

geen gpl

6.15 ab

a) gpl : (0, 0, 0) lokaal min

b) gpl : (0, 0) zadelpunt

geen gpl

6.16

a) $A = \begin{pmatrix} 0 & R \\ 1/4 & 1/2 \end{pmatrix}$ $\det(A - \lambda I) = \begin{vmatrix} -\lambda & R \\ 1/4 & 1/2-\lambda \end{vmatrix}$

$= -\lambda(1/2 - \lambda) - R \cdot \frac{1}{4}$

$= \lambda^2 - \frac{1}{2}\lambda - R\frac{1}{4}$

$D = \frac{1}{4} + R = \frac{1}{4} + \frac{4R}{4}$

$\lambda = \frac{\frac{1}{2} \pm \frac{\sqrt{1+4R}}{2}}{2} \Rightarrow \lambda_1 = \frac{1+\sqrt{1+4R}}{4}$

$\lambda_2 = \frac{1-\sqrt{1+4R}}{4}$

$\lambda_1 = \frac{1+\sqrt{1+4R}}{4}$

$\begin{pmatrix} -\frac{1-\sqrt{1+4R}}{4} & R \\ 1/4 & 1/2 - \frac{1+\sqrt{1+4R}}{4} \end{pmatrix}$

gpl : $R > 2$

1

OEFENZITTING

6.1 + 6.2

d) eigenwaarden:

$$\begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$\Rightarrow 9 - 6\lambda + \lambda^2 - 1 = 0$

$\lambda^2 - 6\lambda + 8 = 0 \Rightarrow \lambda = 2$ en $\lambda = 4$

$\lambda = 4$ $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \Rightarrow -x + y = 0 \Rightarrow x = y$

stel $y = c \Rightarrow \begin{pmatrix} c \\ c \end{pmatrix} = c \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

genormaliseerd : lengte = 1

lengte $\sqrt{1^2 + 1^2} = \sqrt{2}$

genormaliseerde eigenvector : $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ✓

$\lambda = 2$

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow x = -y$

stel $y = c \Rightarrow \begin{pmatrix} -c \\ c \end{pmatrix} = c \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

lengte $\sqrt{1^2 + 1^2} = \sqrt{2}$

genormaliseerde eigenvector : $\frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ ✓

orthogonaal : $\frac{1}{\sqrt{2}} \cdot \left(-\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} = 0 \Rightarrow \perp$

e) $\begin{vmatrix} 1-\lambda & 2 & 0 \\ 2 & 1-\lambda & 0 \\ 0 & 2 & -1-\lambda \end{vmatrix} = 0 \Rightarrow (-1-\lambda)\left((1-\lambda)^2 - 4\right) = 0$

$(-1-\lambda)(1 - 2\lambda + \lambda^2 - 4) = 0$

-15

$\cancel{-1 + 2\lambda - \lambda^2 + 15 - \lambda + 2\lambda^2 - \lambda^3 + 15\lambda = 0}$

$\cancel{-\lambda^3 + \lambda^2 + 16\lambda + 14 = 0}$

$\lambda = -1$

of $-2\lambda + \lambda^2 - 3 = 0$

$\Rightarrow \lambda = -1$ en $\lambda = 3$

$D = 4 + 12$
$= 16$
$= \frac{2 \pm 4}{2} = 3$ en -1

$\lambda = -1$ $\begin{pmatrix} 2 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 2 & 0 \end{pmatrix}$ $y = 0$ en $x + y = 0 \Rightarrow x = 0$

$z = c \Rightarrow c \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ ✓

genormaliseerd $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\lambda = 3$ $\begin{pmatrix} -2 & 2 & 0 \\ 2 & -2 & 0 \\ 0 & 2 & -4 \end{pmatrix}$ $x - y = 0$ $y - 2z = 0$

stel $y = c \Rightarrow x = c$ en $z = \frac{1}{2}c$

$c \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ $\sqrt{2^2 + 2^2 + 1^2} = \sqrt{9} = 3$

genormaliseerd : $\frac{1}{3} \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ ✓

g) $\begin{vmatrix} -\lambda & 3 & 0 \\ 3 & -\lambda & 3 \\ 0 & 3 & -\lambda \end{vmatrix} = 0 \Rightarrow -\lambda(\lambda^2 - 9) - 3(-3\lambda) = 0$

$-\lambda^3 + 9\lambda + 9\lambda = 0$

$-\lambda^3 + 18\lambda = 0 \Rightarrow (\lambda)(-\lambda^2 + 18) = 0$

$\Rightarrow \lambda = 0$ en $\lambda = \sqrt{18} = 3\sqrt{2}$ $\quad \lambda = -\sqrt{18} = -3\sqrt{2}$

$\lambda = 0$

$\begin{vmatrix} 0 & 3 & 0 \\ 3 & 0 & 3 \\ 0 & 3 & 0 \end{vmatrix}$ $\quad y = 0 \quad x + z = 0 \quad c\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$c = x \quad z = -c \quad \sqrt{2} =$ lengte

genormaliseerd : $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ × $\frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

$\lambda = 3\sqrt{2}$

$\begin{pmatrix} -3\sqrt{2} & 3 & 0 \\ 3 & -3\sqrt{2} & 3 \\ 0 & 3 & -3\sqrt{2} \end{pmatrix} \rightarrow \begin{pmatrix} -\sqrt{2} & 1 & 0 \\ 1 & -\sqrt{2} & 1 \\ 0 & 1 & -\sqrt{2} \end{pmatrix} \rightarrow \begin{matrix} x - z = 0 \\ y - \sqrt{2}z = 0 \end{matrix}$

stel $z = c \Rightarrow x = c$ en $y = \sqrt{2}c$

$c\begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}$ $\quad \sqrt{1^2 + 2 + 1^2} = \sqrt{4} = 2$

genormaliseerd : $\frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}$ × $\begin{pmatrix} 1/2 \\ 1/\sqrt{2} \\ 1/2 \end{pmatrix}$

$\lambda = -3\sqrt{2}$

$\begin{pmatrix} 3\sqrt{2} & 3 & 0 \\ 3 & 3\sqrt{2} & 3 \\ 0 & 3 & 3\sqrt{2} \end{pmatrix} \rightarrow \begin{pmatrix} \sqrt{2} & 1 & 0 \\ 1 & \sqrt{2} & 1 \\ 0 & 1 & \sqrt{2} \end{pmatrix} \rightarrow \begin{matrix} x - z = 0 \\ y + \sqrt{2}z = 0 \end{matrix}$

stel $z = c \Rightarrow x = c$ en $y = -\sqrt{2}c$

$c\begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$ $\quad \sqrt{1^2 + 2 + 1^2} = 2$

genormaliseerd : $\frac{1}{2}\begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$ × $\begin{pmatrix} 1/2 \\ -1/\sqrt{2} \\ 1/2 \end{pmatrix}$

orthogonaal?

$\frac{1}{\sqrt{2}}\frac{1}{2} + 0 - \frac{1}{\sqrt{2}}\frac{1}{2} = 0$

$\frac{1}{\sqrt{2}}\frac{1}{2} + 0 - \frac{1}{\sqrt{2}}\frac{1}{2} = 0$

$\frac{1}{2}\frac{1}{2} - \frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2} + \frac{1}{2}\frac{1}{2}$

$= \frac{1}{4} - \frac{2}{4} + \frac{1}{4} = 0$

ja

6.7

$n \times n$ matrix diagonaliseerbaar:
als er n lineaire onafh eigenvectoren zijn

b) $\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 1 \\ 0 & 1 & 1-\lambda \end{vmatrix} = (1-\lambda)\left((1-\lambda)^2 - 1\right)$
$= (1-\lambda)(1 - 2\lambda + \lambda^2 - 1)$

$\Rightarrow \lambda = 1$ of $(-2\lambda + \lambda^2) = 0 = \lambda(-2+\lambda)$

$\lambda = 2$ en $\lambda = 0$

$\Rightarrow$ diagonaliseerbaar ✓ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

d) $\begin{vmatrix} 2-\lambda & 0 \\ 1 & 2-\lambda \end{vmatrix} = (2-\lambda)^2 \quad \rightarrow \lambda = 2$

$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \Rightarrow x = 0 \quad y = c \quad c\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\Rightarrow$ niet diagonaliseerbaar ✓

f) $\begin{vmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 3 & 0 & 1-\lambda \end{vmatrix} = -\lambda(-\lambda(1-\lambda)) = \lambda^2 - \lambda^3$

$\Rightarrow \lambda = 0$ en $\lambda = 1$

$\lambda = 0$

$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{pmatrix} \quad \begin{matrix} 3x + z = 0 \\ y = c_2 \end{matrix} \quad c_1\begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}$ en $c_2\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\lambda = 1$

$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 3 & 0 & 0 \end{pmatrix} \rightarrow x = 0 \quad y = 0 \quad c\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\Rightarrow$ diagonaliseerbaar ✓ $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

6.3

gegeven $\exists \vec{x} : A\vec{x} = \lambda \vec{x}$

TB $A^k \vec{x} = \lambda^k \vec{x}$

$A^k \vec{x} = A^{k-1} A \vec{x} = A^{k-1} \lambda \vec{x} = \lambda A^{k-2} A \vec{x} = \lambda^2 A^{k-2} \vec{x} = \dots = \lambda^k \vec{x}$

mooi bewijs mbv inductie

basisstap $k=1$, $A\vec{x} = \lambda \vec{x}$

inductiestap voor $k-1$: $A^k \vec{x} = A^{k-1} A \vec{x} = \lambda A^{k-1} \vec{x} = \lambda^k \vec{x}$

6.8

(c)
$$\begin{vmatrix} 2-\lambda & i \\ i & 1-\lambda \end{vmatrix} = (2-\lambda)(1-\lambda) - i^2 = 2 - 2\lambda - \lambda + \lambda^2 + 1 = \lambda^2 - 3\lambda + 3$$

$D = 9 - 12 = -3 = 3i^2$

$\sqrt{D} = \sqrt{3i^2} = \sqrt{3}\, i$

$\lambda = \dfrac{3 \pm \sqrt{3} i}{2} = \dfrac{3}{2} + \dfrac{\sqrt{3} i}{2}$ of $\dfrac{3}{2} - \dfrac{\sqrt{3} i}{2}$ ✓

$\lambda = \dfrac{3}{2} + \dfrac{\sqrt{3} i}{2}$

$$\begin{vmatrix} 2 - \frac{3}{2} - \frac{\sqrt{3}i}{2} & i \\ i & 1 - \frac{3}{2} - \frac{\sqrt{3}i}{2} \end{vmatrix} \to \begin{pmatrix} \frac{1}{2} - \frac{\sqrt{3}i}{2} & i \\ i & -\frac{1}{2} - \frac{\sqrt{3}i}{2} \end{pmatrix}$$

$$\begin{pmatrix} 1 - \sqrt{3}i & 2i \\ 2i & -1-\sqrt{3} \end{pmatrix} \xrightarrow{R_2 \to (2i)R_1 - (1-\sqrt{3}i)R_2} \begin{pmatrix} 1-\sqrt{3}i & 2i \\ 0 & -4 + \underbrace{(1-\sqrt{3}i)(1+\sqrt{3}i)}_{4} \end{pmatrix}$$

$(1-\sqrt{3}i)x + 2iy = 0$

$y = \left(\frac{\sqrt{3}}{2} + \frac{1}{2} i\right) x$ stel $x = c$

$$c \begin{pmatrix} 1 \\ \frac{\sqrt{3}}{2} + \frac{1}{2} i \end{pmatrix}$$

$\lambda = \frac{3}{2} - \frac{\sqrt{3}}{2} i$ ... hetzelfde

6.11

geen opl

a) $\begin{vmatrix} \alpha-\lambda & \beta & \beta \\ \beta & \alpha-\lambda & \beta \\ \beta & \beta & \alpha-\lambda \end{vmatrix} = \cancel{(\alpha-\lambda)((\alpha-\lambda)^2-\beta^2)-\beta(\beta(\alpha-\lambda)-\beta^2)}$

$\cancel{+\beta(\beta^2-\beta(\alpha-\lambda))}$

$\cancel{=(\alpha-\lambda)(\alpha^2-2\alpha\lambda-\lambda^2-\beta^2)}$

$\cancel{-2\beta(\beta^2-\beta\alpha-\beta\lambda)}$

$\begin{vmatrix} \alpha-\lambda & \beta & \beta \\ \beta-\alpha+\lambda & \alpha-\lambda-\beta & 0 \\ \beta-\alpha-\lambda & 0 & \alpha-\lambda-\beta \end{vmatrix} = (\alpha-\lambda)(\alpha-\lambda-\beta)^2-\beta(\beta-\alpha+\lambda)(\alpha-\lambda-\beta)$

$-\beta(\alpha-\lambda-\beta)(\beta-\alpha-\lambda)$

$=(\alpha-\lambda)(\alpha-\lambda-\beta)^2+2\beta(\alpha-\lambda-\beta)^2$

$=(\alpha-\lambda-\beta)^2 \quad (\alpha-\lambda+2\beta)$

$\Rightarrow \lambda=\alpha-\beta$ en $\lambda=\alpha+2\beta$

b) $\begin{pmatrix} -2\beta & \beta & \beta \\ \beta & -2\beta & \beta \\ \beta & \beta & -2\beta \end{pmatrix} \rightarrow \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$ $\quad x-z=0$, $y-z=0$

$c=z \rightarrow x=y=c$ $\quad c\begin{pmatrix}1\\1\\1\end{pmatrix}$ $\quad \sqrt{1+1+1}=\sqrt{3}$

$\sqrt{3}\begin{pmatrix}1\\1\\1\end{pmatrix}$ x $\begin{pmatrix}1/\sqrt{3}\\1/\sqrt{3}\\1/\sqrt{3}\end{pmatrix}$

geen opl

c) $\begin{pmatrix} \beta & \beta & \beta \\ \beta & \beta & \beta \\ \beta & \beta & \beta \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\quad x+y+z=0$

d) $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix}$ en $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\-1\end{pmatrix}$ x $\begin{pmatrix}\frac{-1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\\ 0\end{pmatrix}$ en $\begin{pmatrix}-\frac{1}{\sqrt{6}}\\ -\frac{1}{\sqrt{6}}\\ \sqrt{\frac{2}{3}}\end{pmatrix}$

Col 6.15

6.17

a) $\begin{pmatrix} x_{k+1} \\ y_{k+1} \end{pmatrix} = \begin{pmatrix} 1/4 & 1/8 \\ p & 1/2 \end{pmatrix} \begin{pmatrix} x_k \\ y_k \end{pmatrix}$

$\begin{vmatrix} 1/4 & 1/8 \\ p & 1/2 \end{vmatrix} = \frac{1}{8} - \frac{1}{8}p$

$\begin{pmatrix} 1/4-\lambda & 1/8 \\ p & 1/2-\lambda \end{pmatrix} = (\frac{1}{4}-\lambda)(\frac{1}{2}-\lambda) - \frac{1}{8}p$

$= \frac{1}{8} - \frac{1}{4}\lambda - \frac{1}{2}\lambda + \lambda^2 - \frac{1}{8}p$

$= \lambda^2 - \frac{3}{4}\lambda + \frac{1}{8}(1-p)$

$D = \frac{9}{16} - \frac{1}{2}(1-p)$

$\lambda_{1,2} = \frac{\frac{3}{4} \pm \sqrt{\frac{9}{16}-\frac{1}{2}(1-p)}}{2} = \frac{3}{8} \pm \frac{\sqrt{1/16 + 1/2\, p}}{2}$ ✓

b)

$\lambda_1 = 1$

$$\frac{3}{8} + \frac{1}{2}\sqrt{\frac{1}{16} + \frac{1}{2}p} = 1$$

$$\frac{1}{2}\sqrt{\frac{1}{16} + \frac{1}{2}p} = \frac{5}{8}$$

$$\sqrt{\frac{1}{16} + \frac{1}{2}p} = \frac{5}{4}$$

$$\frac{1}{16} + \frac{1}{2}p = \frac{25}{16}$$

$$\frac{1}{2}p = \frac{24}{16}$$

$$p = \frac{24}{8} = 3$$

$\lambda_2 = -1$

$$\frac{3}{8} - \frac{1}{2}\sqrt{\frac{1}{16} + \frac{1}{2}p} = -1$$

$$\frac{1}{2}\sqrt{1/16 + \frac{1}{2}p} = \frac{11}{8}$$

$$1/16 + \frac{1}{2}p = \frac{11^2}{4^2}$$

$$\frac{1}{2}p = \frac{120}{16}$$

$$p = \frac{120}{8} = 16$$

$\lambda_1 = -1$

$$\frac{1}{2}\sqrt{\frac{1}{16} + \frac{1}{2}p} = \frac{-11}{8}$$

$$\sqrt{\frac{1}{16} + \frac{1}{2}p} = -\frac{11}{4}$$

=> kan niet

$\lambda_2 = 1$

$$\frac{3}{8} - \frac{1}{2}\sqrt{1/16 + 1/2\, p} = 1$$

$$-\frac{1}{2}\sqrt{1/16 + 1/2\, p} = \frac{5}{8}$$

$$\sqrt{1/16 + 1/2\, p} = -\frac{5}{4}$$

=> kan niet

uitsterven : $|\lambda_1| < 1$ en $|\lambda_2| < 1$: $\underbrace{1 <}_{\text{gegeven}} p < 3$ ✓

exp groei : $|\lambda_1| > 1$ of $|\lambda_2| > 1$: $p > 3$ ✓

c) voor $p = 3$

$\lambda_1 = 1$ en $\lambda_2 = -1/4$

$$\begin{pmatrix} 1/4 - 1 & 1/8 \\ 3 & 1/2 - 1 \end{pmatrix} = \begin{pmatrix} -3/4 & 1/8 \\ 3 & -1/2 \end{pmatrix} = \begin{pmatrix} -3 & 1/2 \\ 3 & -1/2 \end{pmatrix}$$

$-3x + \frac{1}{2}y = 0$ stel $x = c$

$-x + \frac{1}{6}y = 0$ $y = 6c$

$$c\begin{pmatrix} 1 \\ 6 \end{pmatrix}$$

$\lambda_2 = 1/4$

$$\begin{pmatrix} 1/2 & 1/8 \\ 3 & 3/4 \end{pmatrix} = \begin{pmatrix} 1/2 & 1/8 \\ 1 & 1/4 \end{pmatrix} \qquad x + \frac{1}{4}y = 0$$

$x = c \qquad y = -4c$

$$c\begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 500 \\ 500 \end{pmatrix} = c_2 \begin{pmatrix} 1 \\ 6 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

$$c_2 = 250 = c_1$$

$$\lim_{k \to \infty} \vec{p}_k = c_1 \vec{x}_1 = \begin{pmatrix} 250 \\ 1500 \end{pmatrix} \rightarrow \text{evenwicht voor } p = 3$$

6.18

a) $x_k = z_{k-1} \cdot 6$

$y_k = x_{k-1} \frac{1}{4}$

$z_k = y_{k-1} \frac{1}{2}$

$$x \begin{pmatrix} x_{k+1} \\ y_{k+1} \\ z_{k+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 6 \\ \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} x_{k-1} \\ y_{k-1} \\ z_{k-1} \end{pmatrix}$$

b)
$$\begin{vmatrix} -\lambda & 0 & 6 \\ \frac{1}{4} & -\lambda & 0 \\ 0 & \frac{1}{2} & -\lambda \end{vmatrix} = -\lambda \cdot \lambda^2 + 6\left(\frac{1}{8}\right) = 0$$

$$-\lambda^3 = -\frac{6}{8}$$

$$\lambda^3 = \frac{6}{8} = \frac{3}{4}$$

$e^{k2\pi i} = 1$

$\lambda^3 = \frac{3}{4} e^{k2\pi i}$

$k = 1 \qquad \lambda_1 = \left(\frac{3}{4}\right)^{1/3} \checkmark \; \overbrace{e^{\frac{2}{3}\pi i}}^{= \text{modulus} = 1}$

$k = 2 \qquad \lambda_2 = \left(\frac{3}{4}\right)^{1/3} \checkmark \; \underbrace{e^{4/3 \pi i}}_{= \text{modulus} = 1}$

$k = 3 \qquad \lambda_3 = \left(\frac{3}{4}\right)^{1/3} \checkmark$

c) $|\lambda_i| < 1$: sterft uit ✓

zie toon z'n oefeningen

6.19

a) $\begin{pmatrix} 0 & 0 & 6 \\ \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & p \end{pmatrix} \Rightarrow \begin{vmatrix} -\lambda & 0 & 6 \\ \frac{1}{4} & -\lambda & 0 \\ 0 & \frac{1}{2} & p-\lambda \end{vmatrix} = \lambda^2(p-\lambda) + 6\left(\frac{1}{8}\right)$

$\lambda^2 p - \lambda^3 + \frac{6}{8} = 0$

stel $\lambda = 1$: $1p - 1 + \frac{6}{8} = 0$

$p = \frac{1}{4}$

1

THUIS
geen opl.

Oefeningen hoofdstuk 7

7.1

a) $\frac{dx}{dt} = 2e^{2t}$ $\Rightarrow 2e^{2t} = 2e^{2t} - 4 + 4$

$= 2e^{2t}$ => klopt

$x(0) = e^0 - 2 = 1 - 2 = -1$ => klopt niet

7.3

b) $\frac{dx}{dt} = x(\cos t - t)$

$\frac{dx}{x} = (\cos t - t)\,dt$

$\ln x = \sin t - \frac{t^2}{2} + c$

$x = e^{\sin t - \frac{t^2}{2} + c}$

$x(0) = 1$

$= e^{\sin 0 - 0 + c}$

$= e^c = 1 \Rightarrow c = 0$

$x(t) = e^{\sin t - \frac{t^2}{2}}$ ✓

c) $\frac{dx}{dt} = (t-1)x$

$\frac{dx}{x} = (t-1)\,dt$

$\ln x = \frac{t^2}{2} - t + c$

$x = e^{\frac{t^2}{2} - t + c}$ of $x = c\,e^{\frac{t^2}{2} - t}$ ✓

7.4

a) $\frac{dx}{dt} = \frac{t}{x}$ $\Rightarrow 2x\,dx = 2t\,dt$

$x^2 = t^2 + c$

$x = \pm\sqrt{t^2 + c}$

$x(1) = 3 = +\sqrt{1+c}$ $\Rightarrow c = 8$

$x(t) = \sqrt{t^2 + 8}$ ✓

7.4

b) $\frac{dx}{dy} = -2xy$

$\frac{dx}{x} = -2y\,dy$

$\ln x = -y^2 + c$

$x = e^{-y^2+c}$

$x(0) = 1 = e^c \Rightarrow c = 0$

$x(t) = e^{-y^2}$ ✓

geen opl. 7.5

a) $(x + (t+1))^2 = x^2 + 2x(t+1) + (t+1)^2$

$= x^2 + 2xt + 1 + t^2 + 2t + 1 = \frac{dx}{dt}$

$(2 + t^2 + 2t)\,dt = \frac{1}{x^2 + 2xt}\,dx$ → kan niet gescheiden worden

? b)

geen opl.

7.10

constante oplossingen : $x(t) = 0$ en $x(t) = M$

andere oplossingen :

$$\frac{dx}{dt} = k\,x\,(M-x)$$

$$\frac{1}{x(M-x)}\,dx = k\,dt \quad \rightarrow \text{ zie p 133}$$

$$x(t) = \frac{M}{1 + c_2 e^{-Mkt}}$$

tek. zie p 133

7.11

a) $M = 2t - 2$ $\qquad \dfrac{\partial M}{\partial x} = 0 \quad \Rightarrow \quad \dfrac{\partial M}{\partial x} = \dfrac{\partial N}{\partial t} \Rightarrow$ exact

$N = 8x - 16$ $\qquad \dfrac{\partial N}{\partial t} = 0$

$$\frac{\partial F}{\partial t} = 2t - 2 \qquad \frac{\partial F}{\partial x} = 8x - 16$$

$$F = t^2 - 2t + c \qquad F = 4x^2 - 16x + c$$

$$\Rightarrow F = t^2 - 2t + 4x^2 - 16x = C \quad \checkmark$$

7.12

a) $x' - \frac{1}{t}x = -2$

$$U = e^{\int -\frac{1}{t}dt} = e^{-\ln t} = \frac{1}{t}$$

$$\frac{1}{t}x' - \frac{1}{t^2}x = -2\frac{1}{t}$$

$$\frac{d}{dt}\left(\frac{1}{t}x\right) = -2\frac{1}{t}$$

$$\frac{1}{t}x = -2\ln t + c$$

$$x = -2t\ln|t| + tc \quad \checkmark$$

7.12

d) $x' + \frac{1}{t+1} x = 2$

$$u = e^{\int \frac{1}{t+1} dt} = e^{\ln|t+1|} = (t+1)$$

$$(t+1)x' + x = 2(t+1)$$

$$\frac{d}{dt}((t+1)x) = 2(t+1)$$

$$(t+1)x = t^2 + 2t + c$$

$$x = \frac{t^2}{t+1} + \frac{2t}{t+1} + \frac{c}{t+1}$$

$$x(0) = 2 = 0 + 0 + \frac{c}{1} \Rightarrow c = 2$$

$$x(t) = \frac{t^2 + 2t + 2}{t+1}$$ ✓

1

geen opl.

OEFENZITTING

7.1

b) $x'(t) = 2t$

$2t \neq t\, t^2 - 2t$

c) $x'(t) = e^t$

$x''(t) = e^t$

$e^t - 2e^t + e^t = 0$ klopt $\quad x(0) = e^0 = 1 \neq -1$ klopt niet

~~$e^0 - 2e^0 + e^0 = 1 - 2 + 1 = 0 \neq -1$ klopt niet~~

d) $x'(t) = -3 \sin 2t \cdot 2 = -6 \sin 2t$

$x''(t) = -12 \cos 2t$

$-12 \cos 2t + 12 \cos 2t = 0$ klopt

~~$-12 \cos 0 + 12 \cos 0 = 0$ klopt niet~~ $\quad x(0) = 3 \cos 0 = 3$ klopt

geen opl.

7.2

$x'(t) = 2(t + c\sqrt{t})\left(1 + \frac{c}{2\sqrt{t}}\right)$

$= 2t + \frac{2ct}{2\sqrt{t}} + 2c\sqrt{t} + \frac{c\sqrt{t}\, c \cdot 2}{2\sqrt{t}}$

$= 2t + \frac{ct}{\sqrt{t}} + 2c\sqrt{t} + c^2$

$= 2t + \frac{ct\sqrt{t}}{t} + 2c\sqrt{t} + c^2$

$= 2t + c\sqrt{t} + 2c\sqrt{t} + c^2 = 2t + 3c\sqrt{t} + c^2$

$\frac{(t + c\sqrt{t})^2}{t} + \sqrt{(t + c\sqrt{t})^2} = \frac{t^2 + 2tc\sqrt{t} + c^2 t}{t} + t + c\sqrt{t}$

$= t + 2c\sqrt{t} + c^2 + t + c\sqrt{t}$

$= 2t + 3c\sqrt{t} + c^2$ gelijk aan elkaar

$x(0) = (0 + c\sqrt{0})^2 = 0 \rightarrow$ klopt.

(1) Scheiding van veranderlijken

$$\frac{dx}{dt} = f(x)\, g(t)$$

$$\Rightarrow \frac{1}{f(x)} dx = g(t)\, dt$$

$$\Rightarrow \int \frac{1}{f(x)} dx = \int g(t)\, dt$$

(2) Exacte diff. vgl'en

$M(t,x)\, dt + N(t,x)\, dx = 0$

exact als $\exists F : \frac{\partial F}{\partial t} = M$ en $\frac{\partial F}{\partial x} = N$

oplossing : $F = c$

(3) Lineaire diff vgl van de vorm: (p 138-139)

$x'(t) + p(t)\, x(t) = q(t)$

integrerende factor $u(t) = \exp(\int p(t)\, dt)$

$\Rightarrow ux = \int uq\, dt$

7.3

f) $\frac{dx}{dt} = -kx^{n} \Rightarrow x^{-n} dx = -k\, dt$

$$\frac{x^{-n+1}}{-n+1} = -kt + c$$

$$x^{-n+1} = -(n+1)(kt + c)$$

$$x = \sqrt[-n+1]{-(-n+1)(kt+c)}$$

X

g) $\frac{dx}{dt} = -2xt \Rightarrow \frac{dx}{x} = -2t\, dt$

$$\ln x = -\frac{2t^2 + c}{2} = -t^2 + c$$

$$x = e^{-t^2 + c}$$

$$x(0) = e^{0+c} = 1 \Rightarrow c = 0$$

$$x(t) = e^{-t^2}$$ ✓

h) $\frac{(r^2+1)}{r}\,dr = \frac{\sin\theta}{\cos\theta}\,d\theta$

$(r + \frac{1}{r})\,dr = \tan\theta\,d\theta$

$\frac{r^2}{2} + \ln|r| = -\ln|\cos\theta| + C$

$e^{\frac{1}{2}r^2} \cdot r = \frac{c}{\cos\theta}$ ✓

7.4

d) $\frac{dy}{dx} = e^x \cdot e^y$

$e^{-y}\,dy = e^x\,dx$

$-e^{-y} = e^x + c$

$\ln(e^{-y}) = \ln(-e^x - c)$

$-y = \ln(-e^x - c)$

$0 = \ln(-e^0 - c)$

$0 = \ln(-1 - c)$

$\Rightarrow C = -2$

$y = -\ln(-e^x + 2) = \ln\left(\frac{1}{-e^x + 2}\right)$ ✓

7.6

a) $\frac{da}{dt} = -2\,r\,(a(t))^2$

$\frac{da}{a(t)^2} = -2r\,dt$

$\frac{a(t)^{-1}}{-1} = -2rt + c$

$a(t) = \frac{1}{2rt - c}$ ✓

b) $\lim_{t \to +\infty} a(t) = \frac{1}{\infty - c} = 0$ ✓

geen opl. c) $a(0) = \frac{-1}{c}$ $\rightarrow$ $a(0) < 0$ als $c > 0$

7.8

a) $\frac{dy}{dx} = x^2 - x$

$dy = (x^2 - x)\,dx$

$y = \frac{x^3}{3} - \frac{x^2}{2} + c$ ✓

b) $\frac{dy}{dx} = y^2 - y$

$\frac{dy}{y^2 - y} = dx$ $\qquad \frac{1}{y(y-1)} = \frac{A}{y} + \frac{B}{y-1}$

$\left(-\frac{1}{y} + \frac{1}{y-1}\right) dy = dx$ $\qquad = \frac{Ay - A + By}{y(y-1)}$

$-\ln|y| + \ln|y-1| = x + c$ $\qquad \Rightarrow A + B = 0$ en $-A = 1$

$\ln\left(\frac{|y-1|}{|y|}\right) = x + c$ $\qquad B = 1$ $\qquad \Rightarrow A = -1$

$\frac{y-1}{y} = e^{x+c}$

$y - 1 = y e^{x+c}$

$y(1 - e^{x+c}) = 1$

$y = \frac{1}{1 - e^{x+c}}$ ✗ $\qquad y = \frac{1}{1 + e^{x+c}} = \frac{1}{1 + Ce^{x}}$

7.11

b) $M = 4t e^{x/2} + 4$ $\qquad F = 2t^2 e^{x/2} + 4t + c$

$N = t^2 e^{x/2}$ $\qquad F = 2t^2 e^{x/2} + c$

$F = 2t^2 e^{x/2} + 4t = c$ ✓

c) $M = t$ $\qquad F = \frac{t^2}{2} + c$

$N = x$ $\qquad F = \frac{x^2}{2} + c$

$F = \frac{t^2}{2} + \frac{x^2}{2} = c$

$= t^2 + x^2 = c$

$x(t) = \sqrt{c - t^2}$

$x(3) = \sqrt{c - 9} = 4$ $\qquad \Rightarrow c$ moet 25 zijn

$x(t) = \sqrt{25 - t^2}$ ✓

7.12

b) $x' - 4x = 2t$

$u(t) = e^{\int -4\,dt} = e^{-4t+c} = e^{-4t}$

$e^{-4t} \cdot x' - 4e^{-4t}x = 2e^{-4t}t$

$\frac{d}{dt}\left(e^{-4t}x\right) = 2e^{-4t}t$

$e^{-4t}x = \int 2e^{-4t}t\,dt$

$u = t \qquad du = dt$

$dv = e^{-4t}dt \qquad v = -\frac{1}{4}e^{-4t}$

$= 2\left(-\frac{1}{4}te^{-4t} + \int \frac{1}{4}e^{-4t}dt\right)$

$= 2\left(-\frac{1}{4}te^{-4t} + \frac{1}{4} \cdot -\frac{1}{4}e^{-4t} + c\right)$

$= 2\left(-\frac{1}{4}te^{-4t} - \frac{1}{16}e^{-4t} + c\right)$

$= 2\left(-\frac{1}{4}e^{-4t}\left(t + \frac{1}{4}\right) + c\right)$

C - is toch een constante.

$x = -\frac{1}{2}\left(t + \frac{1}{4}\right) + 2c\,e^{4t}$

f) $\frac{dr}{d\theta} - \tan\theta\, r = \sin\theta$

$u = \exp(\int -\tan\theta\, d\theta)$

$= \exp(\ln|\cos\theta| \cancel{+c})$

$= \cos\theta \cancel{+c}$

$u = \sin\theta \qquad du = \cos\theta\, d\theta$

$(\cos\theta)r = \int(\cos\theta)\sin\theta\, d\theta \qquad dv = \cos\theta \quad v = \sin\theta$

$= \sin^2\theta - \int \sin\theta\cos\theta\, d\theta$

$= \frac{\sin^2\theta}{2}$

$r = \frac{\sin^2\theta}{2\cos\theta} + \frac{C}{\cos\theta}$ ✓

7.14

a) $\frac{dy}{dt} = 2x(t)\frac{dx}{dt}$

$t\frac{dy}{dt} = 4t^2 + 3y(t)$ ✓

b) $\frac{dy}{dt} = 4t + \frac{3}{t}y(t)$

$\frac{dy}{dt} - \frac{3}{t}y(t) = 4t$

$u = \exp(\int -\frac{3}{t}dt)$

$= \exp(-3\ln|t|)$

$= t^{-3}$

$t^{-3}x = \int t^{-3}4t\,dt$

$= \int t^{-2}4\,dt$

$= 4\frac{t^{-1}}{-1} + c$

$x = -4\frac{t^3}{t} + ct^3$

$\overset{y(t)}{\overset{\shortparallel}{x}} = -4t^2 + ct^3$ ✓

7.15

a) $T'(t) = k(200 - T(t))$

$T'(t) + kT(t) = 200k$

$u = \exp(\int k\,dt)$

$= e^{kt}$

$e^{kt}T = \int e^{kt}200k\,dt$

$= \frac{200k}{k}e^{kt} + c$

$= 200e^{kt} + c$

$T = 200 + \frac{c}{e^{kt}}$

$-20 = 200 + \frac{c}{1} \rightarrow c = -220$

$20 = 200 + \frac{-220}{e^{k\cdot 20}}$

$-180 = \frac{-220}{e^{k\cdot 20}}$

$\frac{220}{180} = e^{k\cdot 20}$

$\ln\frac{220}{180} = k\cdot 20$

$k = \frac{\ln\frac{220}{180}}{20}$

$= 0{,}01003$ ✓

b) $60 = 200 + \frac{-220}{e^{t\,0{,}01003}}$

$\frac{-220}{-140} = e^{t\,0{,}01003}$

$\ln \frac{220}{140} = t \cdot 0{,}01003$

$t = 45{,}06 \rightarrow 45$ min. ✓

Oefeningen hoofdstuk 8

THUIS

8.3

a) $\frac{d^2x}{dt^2} - x = 0$

$\lambda^2 - 1 = 0$

$\lambda^2 = 1$

$\lambda = 1$ of -1

$x(t) = c_1 e^{t} + c_2 e^{-t}$ ✓

d) $\lambda^2 + 4\lambda + 4 = 0$

$D = 16 - 16 = 0$

$\lambda = \frac{-4}{2} = -2 \quad \rightarrow x_1(t) = e^{-2t}$

$x_2(t) = t\, e^{-2t}$

$x(t) = c_1 e^{-2t} + c_2 t e^{-2t}$ ✓

8.4

a) $\lambda^2 + \lambda - 2 = 0$

$D = 1 + 8 = 9$

$\lambda_{1,2} = \frac{-1 \pm 3}{2} = -2$ of 1

$x(t) = c_1 e^{-2t} + c_2 e^{t}$

$x'(t) = -2c_1 e^{-2t} + c_2 e^{t}$

$\Rightarrow c_1 e^{0} + c_2 e^{0} = 4 = c_1 + c_2$

$-2c_1 e^{0} + c_2 e^{0} = -5 = -2c_1 + c_2$

$\begin{cases} c_1 = 3 \quad c_2 = 1 \end{cases}$

$\Rightarrow x(t) = 3e^{-2t} + e^{t}$ ✓

8.5

a) $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} - 4x = 0$

$\lambda^2 - 3\lambda - 4 = 0$

$D = 9 + 16 = 25$

$\lambda_{1,2} = \frac{3 \pm 5}{2} = 4$ en -1

$x_H(t) = c_1 e^{4t} + c_2 e^{-t}$

$x(t) = A^{2t} \rightarrow x'(t) = 2Ae^{2t}$ en $x''(t) = 4Ae^{2t}$

$\Rightarrow 4Ae^{2t} - 3 \cdot 2Ae^{2t} - 4 \cdot Ae^{2t} = \underbrace{-6Ae^{2t} = 3e^{2t}}_{A = -\frac{1}{2}}$

$x_p(t) = -\frac{1}{2} e^{2t}$

$x(t) = -\frac{1}{2}e^{2t} + c_1 e^{4t} + c_2 e^{-t}$ ✓

f) $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 2 + 3x$

$\lambda^2 - \lambda = 0$

$D = 1$

$\lambda_{1,2} = \frac{1 \pm 1}{2} = 1$ en 0

$y_h(x) = c_1 e^x + c_2$

~~$y(x) = A_0 + A_1 x \rightarrow y'(x) = A_1$~~ $\rightarrow$ kan niet

$y(x) = A_0 x + A_1 x^2 \rightarrow y'(x) = A_0 + 2A_1 x \quad y''(x) = 2A_1$

$2A_1 - A_0 - 2A_1 x = 2 + 3x$

$\Rightarrow (2A_1 - A_0) = 2 \quad \rightarrow A_0 = -5$

$-2A_1 = 3 \quad \rightarrow A_1 = -\frac{3}{2}$

$y_p(x) = -5x - \frac{3}{2}x^2$

$y(x) = -5x - \frac{3}{2}x^2 + c_1 e^x + c_2$ ✓

8.6

a) $\lambda^2 - \lambda - 2 = 0$

$D = 1 + 8 = 9$

$\lambda_{1,2} = \frac{1 \pm 3}{2} = 2$ en -1

$x_H(t) = c_1 e^{2t} + c_2 e^{-t}$

~~$x(t) = Ae^{2t} \rightarrow x'(t) = 2Ae^{t2} \quad x''(t) = 4Ae^{2t}$~~

~~$4Ae^{2t} - 2Ae^{2t} - 2Ae^{2t} = 0$~~

$x(t) = Ate^{2t} \rightarrow x'(t) = Ae^{2t} + 2Ate^{2t}$ $\quad x''(t) = A(1+2t)2e^{2t} + Ae^{2t} \cdot 2$

$= Ae^{2t}(1+2t)$ $\quad = A \cdot 2e^{2t}(1+2t+1)$

$= 2Ae^{2t}(2+2t)$

$= 4Ae^{2t}(1+t)$

$4Ae^{2t}(1+t) - Ae^{2t}(1+2t) - 2Ate^{2t}$

$= Ae^{2t}(4 + \cancel{4t} - 1 - \cancel{2t} - \cancel{2t})$

$= Ae^{2t} \cdot 3 = 3e^{2t} \Rightarrow A = 1$

$x_p(t) = te^{2t}$

$\boxed{x(t) = te^{2t} - e^{2t} + e^{-t}}$ ✓

$x(t) = te^{2t} + c_1 e^{2t} + c_2 e^{-t} \rightarrow x'(t) = e^{2t} + 2te^{2t} + c_1 \cdot 2e^{2t} - c_2 e^{-t}$

$-2 = 1 + 2c_1 - c_2$

$0 = c_1 + c_2$

$c_1 = -1$ en $c_2 = 1$

8.7

$\lambda^2 - 2\lambda + 2 = 0$

$D = 4 - 8 = -4$

$\sqrt{D} = \pm 2i$

$\mu = 1 \qquad \omega = \frac{1}{2} \cdot 2 = 1$

$y_1(t) = e^{\mu x + i\omega x}$ en $x_2(t) = e^{\mu x - i\omega x}$

$= e^{\mu x}(\cos \omega x + i \sin \omega x) \qquad = e^{\mu x}(\cos \omega x - i \sin \omega x)$

$y(x) = (c_1 \cos x + c_2 \sin x)\, e^x$

$2 = (c_1 \cos 0 + c_2 \sin 0)\, e^0$

$= c_1$

$2 = (c_1 \cos \frac{\pi}{2} + c_2 \sin \frac{\pi}{2})\, e^{\pi/2}$

$= c_2 e^{\pi/2} \quad \Rightarrow \quad c_2 = 2 \cdot e^{-\pi/2}$

$y(x) = (\cancel{2} \cos x + 2e^{-\pi/2} \sin x)\, e^x$ ✓

8.10

$m\lambda^2 + k = 0$

$\lambda^2 = -\frac{k}{m}$

$\Rightarrow \lambda = \pm i\sqrt{k/m}$

$\mu = 0 \qquad \omega = \frac{1}{2}\sqrt{\frac{4k}{m}} = \sqrt{k/m}$

$u_H(t) = (c_1 \cos(\sqrt{\tfrac{k}{m}}\, t) + c_2 \sin(\sqrt{\tfrac{k}{m}}\, t))\, e^0$

$= c_1 \cos(\sqrt{\tfrac{k}{m}}\, t) + c_2 \sin(\sqrt{\tfrac{k}{m}}\, t)$

$u_p(t) = A \cos \omega_0 t + B \sin \omega_0 t \quad \Rightarrow u_p'(t) = -A\omega_0 \sin \omega_0 t + B\omega_0 \cos \omega_0 t$

$u_p''(t) = -A\omega_0^2 \cos \omega_0 t - B\omega_0^2 \sin \omega_0 t$

$m(-A\omega_0^2 \cos \omega_0 t - B\omega_0^2 \sin \omega_0 t) + k(A \cos \omega_0 t + B \sin \omega_0 t) = \cos(\omega_0 t)$

$A \cos \omega_0 t\,(k - m\omega_0^2) + B \sin \omega_0 t\,(k - m\omega_0^2) = \cos \omega_0 t$

$A = \frac{1}{k - m\omega_0^2}$ en $B = 0 \quad \Rightarrow u_p(t) = \frac{1}{k - m\omega_0^2} \cos \omega_0 t$

$$u(t) = \frac{1}{k-\omega_0^2 m} \cos \omega_0 t + c_1 \cos\left(\sqrt{\tfrac{k}{m}}\, t\right) + c_2 \sin\left(\sqrt{\tfrac{k}{m}}\, t\right)$$

$$u'(t) = \frac{-\omega_0}{k-\omega_0^2 m} \sin \omega_0 t - c_1 \sqrt{\tfrac{k}{m}} \sin \sqrt{\tfrac{k}{m}}\, t + c_2 \sqrt{\tfrac{k}{m}} \cos \sqrt{\tfrac{k}{m}}\, t$$

$$0 = \frac{1}{k-\omega_0^2 m} + c_1 \quad \Rightarrow c_1 = -\frac{1}{k-\omega_0^2 m}$$

$$0 = c_2 \sqrt{\tfrac{k}{m}} \quad \Rightarrow c_2 = 0$$

$$u(t) = \frac{1}{k-\omega_0^2 m} \cos \omega_0 t + \frac{-1}{k-\omega_0^2 m} \cos \sqrt{\tfrac{k}{m}}\, t$$

$$= \frac{1}{k-\omega_0^2 m} \left(\cos \omega_0 t - \cos \sqrt{\tfrac{k}{m}}\, t \right)$$

snap opl niet.

onbegrensd als?

"Als $\omega_0 \neq \sqrt{\frac{k}{m}}$ dan $u(t) = \frac{1}{m\omega_2^0 - k} \cos\left(\sqrt{\frac{k}{m}}\, t\right) - \frac{1}{m\omega_0^2 - k} \cos(\omega_0 t)$

Als $\omega_0 = \sqrt{\frac{k}{m}}$ dan $u(t) = \frac{1}{2m\omega_0} t \sin(\omega_0 t)$, dit is een onbegrensde opl.

OEFENZITTING

Differentiaalvgl'en v/d 2de orde

Homogene DV met constante coëfficiënten

$$\frac{d^2x}{dt^2} + a\frac{dx}{dt} + bx = 0$$

oplossingen altijd v/d vorm

$x(t) = C_1 x_1(t) + C_2 x_2(t)$

met $x_1(t)$ en $x_2(t)$ LO oplossingen

oplossingen zoeken:

$\lambda^2 + a\lambda + b = 0$

① $D > 0 \Rightarrow \lambda_1 \neq \lambda_2$ en $\in \mathbb{R}$

opl.: $x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$

② $D = 0 \Rightarrow \lambda \in \mathbb{R}$

opl: $x(t) = C_1 e^{\lambda t} + C_2 t e^{\lambda t}$

③ $D < 0 \Rightarrow \lambda_1 = \mu + i\omega$ en $\lambda_2 = \mu - i\omega$

opl: $x(t) = (C_1 \cos(\omega t) + C_2 \sin(\omega t))\, e^{\mu t}$

$\mu = -\frac{a}{2}$ $\quad \omega = \frac{1}{2}\sqrt{4b - a^2}$

niet-homogene DV

$$\frac{d^2x}{dt^2} + a\frac{dx}{dt} + bx = f(t)$$

algemene opl: $x(t) = x_H(t) + x_p(t)$

8.1

$c_1 \frac{1}{t} + c_2 t = 0$ -> kan enkel als c_1 en c_2 nul zijn

=> lin. onafh

$x_1(t) = \frac{1}{t}$ $x_1'(t) = -\frac{1}{t^2}$ $x_2''(t) = +2\frac{1}{t^3}$

$x_2(t) = t$ $x_1'(t) = 1$ $x_2''(t) = 0$

Voor x_1 -> $t^2\left(2\frac{1}{t^3}\right) + t\left(-\frac{1}{t^2}\right) - \frac{1}{t} = +2\frac{1}{t} - \frac{1}{t} - \frac{1}{t} = 0$

↳ klopt

Voor x_2 -> $t - t = 0$

↳ klopt

algemene opl: $x(t) = c_1 \frac{1}{t} + c_2 t$ -> $x'(t) = -c_1 \frac{1}{t^2} + c_2$

$x(1) = 0$ -> $c_1 + c_2 = 0$ $c_1 = \frac{1}{2}$ $c_2 = -\frac{1}{2}$

$x'(1) = -1$ -> $-c_1 + c_2 = -1$

=> $x(t) = \frac{1}{2}\frac{1}{t} - \frac{1}{2}t$

8.3

b) $\lambda^2 - 7\lambda + 6 = 0$

$D = 49 - 24 = 25$

$\lambda = \frac{7 \pm 5}{2} = 6$ en 1

$x(t) = c_1 e^{6t} + c_2 e^{t}$

8.4

b) $\lambda^2 + 2\lambda + 5 = 0$

$D = 4 - 20 = -16$

$\mu = -1$ $\omega = 2$

$x(t) = (c_1 \cos(2t) + c_2 \sin(2t))\, e^{-t}$

$x'(t) = -(c_1 \cos(2t) + c_2 \sin(2t))\, e^{-t} + e^{-t}(-2c_1 \sin(2t) + 2c_2 \cos(2t))$

$x(0) = 1$ -> $(c_1 \cdot 1 + 0)\, 1 = 1$ -> $c_1 = 1$

$x'(0) = 5$ -> $-(c_1 \cdot 1)\, 1 + 1\,(0 + 2c_2) = 5$

=> $-1 + 2c_2 = 5$ => $c_2 = 3$

$x(t) = (\cos(2t) + 3\sin(2t))\, e^{-t}$

8.4

d) $\lambda^2 - 6\lambda + 9 = 0$

$D = 36 - 36 = 0$

$\lambda = \frac{6}{2} = 3$

$y(t) = c_1 e^{3x} + c_2 x e^{3x}$

$y'(t) = 3c_1 e^{3x} + c_2 e^{3x} + 3c_2 x e^{3x}$

$y(0) = 1 \Rightarrow c_1 = 1$

$y'(0) = 0 \Rightarrow 3c_1 + c_2 = 0 \Rightarrow c_2 = -3$

$y(x) = e^{3x} - 3 c_2 x e^{3x}$

8.8

$\lambda^2 + \lambda - 2 = 0$

$D = 1 + 8 = 9$

$\lambda_{1,2} = \frac{-1 \pm 3}{2} = -2$ en 1

$y(x) = c_1 e^{-2x} + c_2 e^{x}$

$y(0) = 2 \rightarrow c_1 + c_2 = 2 \Rightarrow c_1 = 2$

$\underbrace{c_1 e^{-2(+\infty)}}_{=0} + c_2 e^{\infty} = 0$

$\Rightarrow c_2$ moet nul zijn

$y(x) = 2e^{-2x}$

8.11

$u''(x) + \omega^2 u(x) = 0$ $\qquad u(x) = c_2 \sin(n\pi x)$ met $n \in \mathbb{Z}$

$\lambda^2 + \omega^2 = 0$

$\lambda = \pm i\omega$

$\mu = 0$

$u(x) = (c_1 \cos(\omega x) + c_2 \sin(\omega x))$

$u(0) = 0 \rightarrow c_1 = 0$

$u(1) = 0 \rightarrow \underset{=0}{c_1} \cos\omega + c_2 \sin\omega = 0$

$c_2 \sin\omega = 0$

als c_2 nul is $\Rightarrow$ nulfunctie

ook oplossingen als $\omega = n\pi$ met $n \in \mathbb{Z}$

8.5

d) $\lambda^2 - 2\lambda + 1 = 0$

$D = 4 - 4 = 0$

$\lambda = \frac{2}{2} = 1$

$x_H(t) = c_1 e^t + c_2 t e^t$

$x_p(t) = A_0 + A_1 t$

$x'(t) = A_1$

$x''(t) = 0$

$0 - 2A_1 + A_0 + A_1 t = t$

$\Rightarrow A_1 = 1$

$\Rightarrow -2A_1 + A_0 = 0 \quad \Rightarrow A_0 = 2$

$x_p(t) = 2 + t$

$-b \pm \sqrt{D}$
$\uparrow$
pos

$x(t) = 2 + t + c_1 e^t + c_2 t e^t$

8.14

zorg dat $x_p(t) = 2$ en $x_H(t)$ naar nul gaat

$e^{\frac{-c_1 t}{2}}$ en $e^{\frac{-c_2 t}{2}}$

b moet pos zijn

en $\sqrt{D}$ moet kleiner dan b

stel $b = 5$ en $c = 6$

$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = ?$

$\lambda^2 + 5\lambda + 6 = 0$

$D = 25 - 24 = 1$

$\lambda_{1,2} = \frac{-5 \pm 1}{2} = -3$ en -2

$x_H(t) = c_1 e^{-3t} + c_2 e^{-2t}$

$x_p(t) = 2 \quad x'_p(t) = 0 \quad x''_p(t) = 0$

$0 + 0 + 12 = 12$

$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 12$

$x(t) = c_1 e^{-3t} + c_2 e^{-2t} + 2$

8.15

$y(s) = x(e^s)$

$y'(s) = x'(e^s)\, e^s = \frac{dx}{de^s} \cdot e^s$

$y''(s) = x''(e^s)\, e^s e^s + x'(e^s)\, e^s$

$= \frac{d^2x}{de^s} e^{2s} + \frac{dx}{de^s} \cdot e^s$

$$\frac{d^2x}{de^s} e^{2s} + \frac{dx}{de^s} e^s + (A-1)\frac{dx}{de^s} e^s + B\, x(e^s) = 0$$

$$e^{2s} \frac{d^2x}{de^s} + A e^s \frac{dx}{de^s} + B\, x(e^s) = 0$$

stel $e^s = t$

$$t^2 \frac{d^2x}{dt} + At \frac{dx}{dt} + Bx = 0$$

8.16

a) $y''(s) + y'(s) - 2y = 0$

$\lambda^2 + \lambda - 2 = 0$

$D = 1 + 8 = 9$

$\lambda_{1,2} = \frac{-1 \pm 3}{2} = 1$ en -2

$y(s) = c_1 e^s + c_2 e^{-2s} = x(e^s)$

$\Rightarrow x(t) = c_1 t + c_2 t^{-2}$

Oefeningen hoofdstuk 9

THUIS 9.1

$x(t) = \begin{pmatrix} e^{2t} \\ 2e^{2t} \end{pmatrix}$ $\quad x'(t) = \begin{pmatrix} 2e^{2t} \\ 4e^{2t} \end{pmatrix}$

$\begin{pmatrix} 2e^{2t} \\ 4e^{2t} \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} e^{2t} \\ 2e^{2t} \end{pmatrix}$

$= \begin{pmatrix} 4e^{2t} - 2e^{2t} \\ 2e^{2t} + 2e^{2t} \end{pmatrix} = \begin{pmatrix} 2e^{2t} \\ 4e^{2t} \end{pmatrix}$

$x(t) = \begin{pmatrix} e^{3t} \\ e^{3t} \end{pmatrix}$ $\quad x'(t) = \begin{pmatrix} 3e^{3t} \\ 3e^{3t} \end{pmatrix}$

$\begin{pmatrix} 3e^{3t} \\ 3e^{3t} \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} e^{3t} \\ e^{3t} \end{pmatrix}$

$= \begin{pmatrix} 4e^{3t} - e^{3t} \\ 2e^{3t} + e^{3t} \end{pmatrix} = \begin{pmatrix} 3e^{3t} \\ 3e^{3t} \end{pmatrix}$

algemene oplossing:

$x(t) = c_1 \begin{pmatrix} e^{2t} \\ 2e^{2t} \end{pmatrix} + c_2 \begin{pmatrix} e^{3t} \\ e^{3t} \end{pmatrix}$ $\times$ $\quad x(t) = c_1 e^{2t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 e^{3t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

9.2

b) $x' = x + y$ $\quad A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$

$y' = 4x + y$

$\begin{vmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - 4 = (\lambda - 3)(\lambda + 1)$

$\Rightarrow \lambda = 3$ en $\lambda = -1$

$\lambda = 3$

$\begin{pmatrix} -2 & 1 \\ 4 & -2 \end{pmatrix} \xrightarrow{R_2 + 2R_1} \begin{pmatrix} -2 & 1 \\ 0 & 0 \end{pmatrix} \rightarrow -2x + y = 0 \rightarrow \begin{matrix} x = c \\ y = 2c \end{matrix}$

$\rightarrow c \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\lambda = -1$

$\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} \xrightarrow{R_2 - 2R_1} \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix} \rightarrow 2x + y = 0 \quad \begin{matrix} x = c \\ y = -2c \end{matrix}$

$\rightarrow c \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Hoe noemen? $x(t) = e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\quad y(t) = e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

oplossing: $c_1 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ ✓

$\rightarrow$ geen stabiel evenwicht $(3 > 0)$

9.3

a) $\begin{vmatrix} 4-\lambda & 2 \\ 3 & 3-\lambda \end{vmatrix} = (4-\lambda)(3-\lambda) - 6$

$= 12 - 3\lambda - 4\lambda + \lambda^2 - 6$

$= \lambda^2 - 7\lambda + 6$

$D = 49 - 24 = 25$

$\lambda_{1,2} = \frac{7 \pm 5}{2} = 6$ en 1

$\lambda = 6$

$\begin{pmatrix} -2 & 2 \\ 3 & -3 \end{pmatrix} \to \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix}$ $\quad -x + y = 0 \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad x = c$, $y = c$

$\lambda = 1$

$\begin{pmatrix} 3 & 2 \\ 3 & 2 \end{pmatrix} \to \begin{pmatrix} 3 & 2 \\ 0 & 0 \end{pmatrix}$ $\quad 3x + 2y = 0 \quad \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ $\quad y = -3c$, $x = 2c$

$x_1(t) = e^{6t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad x_2(t) = e^{t}\begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$x(t) = c_1 e^{6t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{t}\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ ✓

9.8

a) $\vec{x}'(t) = \begin{pmatrix} -2x + 4y \\ -4y + x \end{pmatrix}$

$\vec{x}(t) =$

(1)

OEFENZITTING

lineair : $\vec{x}' = A(t)\vec{x}$

Algemene oplossing : $c_1\vec{x}_1 + c_2\vec{x}_2 + \dots + c_n\vec{x}_n$

met $\vec{x}_i$ oplossingen

constante coeff. (A(t) -> niet afh van t

$\vec{x}'(t) = A\vec{x}$

oplossingen : $e^{\lambda t}\vec{v}$: met λ eigenwaarde van A met eigenvector $\vec{v}$

9.2

c) $\begin{pmatrix} 4 & -2 \\ 5 & -2 \end{pmatrix}$ $\begin{vmatrix} 4-\lambda & -2 \\ 5 & -2-\lambda \end{vmatrix} = (4-\lambda)(-2-\lambda)+10$

$= -8+2\lambda-4\lambda+\lambda^2+10$

$= \lambda^2-2\lambda+2$

$D = 4-8=-4$ $\sqrt{D} = \pm i\cdot 2$

$\lambda_{1,2} = \frac{2\pm 2i}{2}$ $1+i=\lambda_1$ en $\lambda_2 = 1-i$

$(3-i)(-3+i) = -9+3i+3i+i^2$
$= -9-1 = -10$

$\lambda = 1+i$

$\begin{pmatrix} 3-i & -2 \\ 5 & -3-i \end{pmatrix} \xrightarrow[R_2 \mapsto 3-i(R_2)]{R_1 \mapsto 5R_1} \begin{pmatrix} 15-5i & -10 \\ 15-5i & -10 \end{pmatrix} \to \begin{pmatrix} 3-i & -2 \\ 0 & 0 \end{pmatrix}$

$(3-i)x \quad -2y=0 \quad \to \begin{pmatrix} 2 \\ 3-i \end{pmatrix}$
$6-2i \quad -6+2i$

$\lambda = 1-i$ complex toegevoegde

$\begin{pmatrix} 3+i & -2 \\ 5 & -3+i \end{pmatrix} \to \begin{pmatrix} 2 \\ 3+i \end{pmatrix}$

$\vec{x}_1(t) = e^{(1+i)t}\begin{pmatrix} 2 \\ 3-i \end{pmatrix}$ $\vec{x}_2(t) = e^{(1-i)t}\begin{pmatrix} 2 \\ 3+i \end{pmatrix}$

$= e^t(i\sin t+\cos t)\begin{pmatrix} 2 \\ 3+i \end{pmatrix}$ -> complex toegevoegde van elkaar

$= e^t\begin{pmatrix} 2\cos t + 2i\sin t \\ 3i\sin t + \sin t + 3\cos t - i\cos t \end{pmatrix}$

$= e^t\left(\begin{pmatrix} 2i\sin t \\ 3i\sin t - i\cos t \end{pmatrix} + \begin{pmatrix} 2\cos t \\ \sin t + 3\cos t \end{pmatrix}\right)$

algemene oplossing: $c_1 e^t \begin{pmatrix} 2\cos t \\ 3\cos t + \sin t \end{pmatrix} + c_2 e^t \begin{pmatrix} 2\sin t \\ 3\sin t - \cos t \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

-> van boven: $t=0 \Rightarrow x_1 = 1$

van onder: $t=0 \Rightarrow x_2 = 2$

$\Rightarrow \vec{x}(t) = e^t\begin{pmatrix} \cos t - \sin t \\ 2\cos t - \sin t \end{pmatrix}$ ✓ -> geen stabiel pt

9.3

geen opl

c) $\begin{vmatrix} 6-\lambda & -8 \\ 4 & -6-\lambda \end{vmatrix} = (\lambda^2 - 36) + 32 \Rightarrow \lambda^2 = 4$

$\lambda = 2$ en -2

$\lambda = 2$

$\begin{pmatrix} 4 & -8 \\ 4 & -8 \end{pmatrix} \rightarrow x - 2y = 0 \rightarrow \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\lambda = -2$

$\begin{pmatrix} 8 & -8 \\ 4 & -4 \end{pmatrix} \rightarrow x - y = 0 \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$x(t) = C_1 e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^{-2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

neem $t = 0$ $\quad C_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$C_1 = 1 \quad C_2 = -2$

$x(t) = e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} - 2e^{-2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

9.5

geen opl

a) $-k_1 a + k_2 b + k_1 a - k_2 b - k_3 b + k_3 b = 0$

$\frac{da}{dt}, \frac{db}{dt}, \frac{dc}{dt} \rightarrow$ Verandering van concentraties

$\rightarrow$ 1 stof wordt in de andere omgezet

$\rightarrow$ altijd samen nul.

b) $\begin{pmatrix} -2 & 1 & 0 \\ 2 & -3 & 0 \\ 0 & 2 & 0 \end{pmatrix}$ $\quad \begin{vmatrix} -2-\lambda & 1 & 0 \\ 2 & -3-\lambda & 0 \\ 0 & 2 & -\lambda \end{vmatrix} = -\lambda\left((-2-\lambda)(-3-\lambda) - 2\right)$

$= -\lambda(6 + 2\lambda + 3\lambda + \lambda^2 - 2)$

$= -\lambda(\lambda^2 + 5\lambda + 4)$

$\lambda = 0$ en $D = 25 - 16 = 9$

$\lambda_{1,2} = \frac{-5 \pm 3}{2} \rightarrow -4, -1$

$\lambda = 0$

$\begin{pmatrix} -2 & 1 & 0 \\ 2 & -3 & 0 \\ 0 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} -2 & 1 & 0 \\ 2 & -1 & 0 \\ 0 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} -2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$b = 0$

$a = 0$

$c = c_1$

$\lambda = -4$

$$\begin{pmatrix} 2 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 4 \end{pmatrix} \to \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{matrix} 2a + b = 0 \\ b + 2c = 0 \end{matrix} \to \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix}$$

$\lambda = -1$

$$\begin{pmatrix} -1 & 1 & 0 \\ 2 & -2 & 0 \\ 0 & 2 & 1 \end{pmatrix} \to \begin{pmatrix} -1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{matrix} -a + b = 0 \\ 2b + c = 0 \end{matrix} \qquad \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$$

$$x(t) = c_1 \underbrace{e^0}_{=1} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + c_2 e^{-4t} \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix} + c_3 e^{-t} \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} a(t) \\ b(t) \\ c(t) \end{pmatrix}$$

$c_1 \cdot 0 + c_2 \cdot 2 + c_3 (-1) = A_0 \to 2c_2 + 4c_2 = A_0 \Rightarrow c_2 = \frac{A_0}{6}$

$c_1 \cdot 0 + c_2 (-4) + c_3 (-1) = 0 \to c_3 = -4c_2 \Rightarrow c_3 = -\frac{4}{6} A_0$

$c_1 + c_2\, 2 + c_3\, 2 = 0 \to c_1 + 2c_2 - 8c_2 \to c_1 = 6c_2 \Rightarrow c_1 = A_0$

$$x(t) = A_0 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \frac{A_0}{6} e^{-4t} \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix} - \frac{4}{6} A_0 e^{-t} \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} \times$$

c) $t \to +\infty \Rightarrow e^{-4t} \to 0$ en $e^{-t} \to 0$

$a(t) \to 0$ ✓

$b(t) \to 0$ ✓

$c(t) \to A_0$ ✓

$$\begin{pmatrix} a(t) \\ b(t) \\ c(t) \end{pmatrix} = \frac{2}{3} A_0 \cdot e^{-t} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} + A_0 \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - \frac{A_0}{3} e^{-4t} \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$$

9.6

$$\begin{vmatrix} 2-\lambda & 1 & 0 \\ 3 & 4-\lambda & 0 \\ 5 & -6 & 3-\lambda \end{vmatrix} = (3-\lambda)\left((2-\lambda)(4-\lambda)-3\right)$$
$$= (3-\lambda)(8-4\lambda-2\lambda+\lambda^2-3)$$
$$= (3-\lambda)(\lambda^2-6\lambda+5)$$

$\Rightarrow \lambda = 3$ en $D = 36 - 20 = 16$

$\lambda_{1,2} = \frac{6 \pm 4}{2} \begin{array}{l} 5 = \lambda \\ 1 = \lambda \end{array}$

$\lambda = 3$

$$\begin{pmatrix} -1 & 1 & 0 \\ 3 & 1 & 0 \\ 5 & -6 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$\lambda = 5$

$$\begin{pmatrix} -3 & 1 & 0 \\ 3 & -1 & 0 \\ 5 & -6 & -2 \end{pmatrix} \rightarrow \begin{pmatrix} -3 & 1 & 0 \\ -13 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{array}{l} -3x+y=0 \\ -13x-2z=0 \end{array} \quad \begin{pmatrix} 1 \\ 3 \\ -13/2 \end{pmatrix}$$

$\lambda = 1$

$$\begin{pmatrix} 1 & 1 & 0 \\ 3 & 3 & 0 \\ 5 & -6 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & -11 & 2 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{array}{l} x+y=0 \\ -11y+2z=0 \end{array} \quad \begin{pmatrix} -1 \\ 1 \\ 11/2 \end{pmatrix}$$

$$x(t) = c_1 e^{3t}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + c_2 e^{5t}\begin{pmatrix} 1 \\ 3 \\ -13/2 \end{pmatrix} + c_3 e^{t}\begin{pmatrix} -1 \\ 1 \\ 11/2 \end{pmatrix}$$

$c_1 \cdot 0 + c_2 - c_3 = 5 \quad \rightarrow c_2 = 5 + c_3 = 2$

$c_1 \cdot 0 + 3c_2 + c_3 = 3 \qquad 3c_3 + 15 + c_3 = 3 \rightarrow 4c_3 = -12 \rightarrow c_3 = -3$

$c_1 - \frac{13}{2}c_2 + c_3 \frac{11}{2} = 4$

$c_1 = 4 + 13 + \frac{33}{2} = 33{,}5 = \frac{67}{2}$

$$x(t) = \frac{67}{2} e^{3t}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + 2e^{5t}\begin{pmatrix} 1 \\ 3 \\ -13/2 \end{pmatrix} - 3e^{t}\begin{pmatrix} -1 \\ 1 \\ 11/2 \end{pmatrix} \quad \checkmark$$

9.8 bc + 9.9

9.7

a) stel $y' = x''$ en $y = x'$

$x'' = -3x' + 2x$

$$\begin{cases} x' = y \\ y' = -3y + 2x \end{cases}$$

...

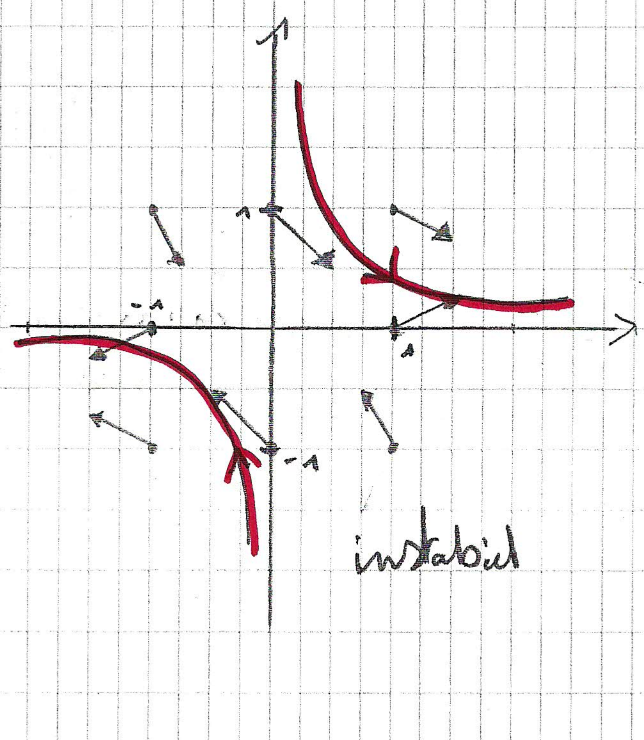
9.8

geen opl

b) $\begin{pmatrix} 2 & 4 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$

neem verschillende punten

- $\begin{pmatrix} 2 & 4 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$
- $\begin{pmatrix} 2 & 4 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$
- $\begin{pmatrix} 2 & 4 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$
- $\begin{pmatrix} 2 & 4 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \end{pmatrix}$
- $\begin{pmatrix} 2 & 4 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$
- $\begin{pmatrix} 2 & 4 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$
- $\begin{pmatrix} 2 & 4 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -6 \end{pmatrix}$
- $\begin{pmatrix} 2 & 4 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} -1 \\ -1 \end{pmatrix} = \begin{pmatrix} -6 \\ 3 \end{pmatrix}$



instabiel

$$\begin{vmatrix} 2-\lambda & 4 \\ 1 & -4-\lambda \end{vmatrix} \Rightarrow \lambda = -1 \pm \sqrt{13}$$

c) $\begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}$

$$\begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$$

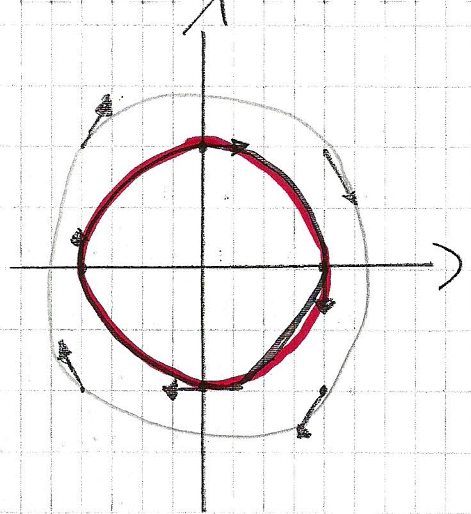
$$\begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} -1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$



$\Rightarrow$ instabiel

mbv discriminant $\Rightarrow \lambda = \pm\sqrt{2}$;

$\Rightarrow$ zegt niets $\Rightarrow$ fase portret

Thuis

9.10

a) $F(x,y) = x + xy$ $\Rightarrow$ evenwicht: $F(x,y)=0$ en $G(x,y)=0$

$G(x,y) = 2y - xy$

$x(1+y) = 0$ $\Rightarrow$ $(0,0)^T$ en $(2,-1)^T$

$y(2-x) = 0$

$\frac{\partial F}{\partial x} = 1+y$ $\quad \frac{\partial F}{\partial y} = x$

$\frac{\partial G}{\partial x} = -y$ $\quad \frac{\partial G}{\partial y} = 2-x$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1+y & x \\ -y & 2-x \end{pmatrix} \begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix}$$

*) $\vec{x}_1 = (0,0)^T$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \qquad \begin{vmatrix} 1-\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda)$$

$= 2 - 2\lambda - \lambda + \lambda^2$

$= \lambda^2 - 3\lambda + 2$

$D = 9 - 8 = 1$

$\lambda = \frac{3 \pm 1}{2} = 2$ en 1

$\lambda_1 = 2$ en $\lambda_2 = 1$

$\Rightarrow$ instabiel ✓

*) $\vec{x}_2 = (2,-1)^T$

$$A = \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} \qquad \begin{vmatrix} -\lambda & 2 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - 2 \quad \Rightarrow \lambda = \pm\sqrt{2}$$

$\lambda_1 = \sqrt{2}$ en $\lambda_2 = -\sqrt{2}$

$\Rightarrow$ instabiel ✓

9.10

b) $F(x,y) = x(1-x-y) = x - x^2 - yx$

$G(x,y) = y(2-y-3x) = 2y - y^2 - 3yx$

$\begin{cases} x(1-x-y)=0 \\ y(2-y-3x)=0 \end{cases}$ $\begin{cases} \vec{x}_0 = (0,0)^T \\ \vec{x}_1 = (0,2)^T \end{cases}$ $\vec{x}_2 = (1,0)^T$ $\vec{x}_3 = (\frac{1}{2}, \frac{1}{2})^T$

$\frac{\partial F}{\partial x} = 1-2x-y \quad \frac{\partial F}{\partial y} = -x$

$\frac{\partial G}{\partial x} = -3y \quad \frac{\partial G}{\partial y} = 2-2y-3x$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1-2x-y & -x \\ -3y & 2-2y-3x \end{pmatrix} \begin{pmatrix} x-x_0 \\ x-x_0 \end{pmatrix}$$

* $\vec{x}_0 = (0,0)^T$

$A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ $\quad \begin{vmatrix} 1-\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} \rightarrow \lambda_1 = 2$ en $\lambda_2 = 1$

$\Rightarrow$ instabiel ✓

* $\vec{x}_1 = (0,2)^T$

$A = \begin{pmatrix} -1 & 0 \\ -6 & -2 \end{pmatrix}$ $\quad \begin{vmatrix} -1-\lambda & 0 \\ -6 & -2-\lambda \end{vmatrix} = (-1-\lambda)(-2-\lambda)$

$= 2 + 2\lambda + \lambda + \lambda^2$

$= \lambda^2 + 3\lambda + 2$

$\lambda_1 = -1$ en $\lambda_2 = -2$

$\rightarrow$ stabiel ✓

$D = 9 - 8 = 1$

$\lambda_{1,2} = \frac{-3 \pm 1}{2} \Leftrightarrow \begin{matrix} -1 \\ -2 \end{matrix}$

* $\vec{x}_2 = (1,0)^T$

$A = \begin{pmatrix} -1 & -1 \\ 0 & -1 \end{pmatrix}$ $\quad \begin{vmatrix} -1-\lambda & -1 \\ 0 & -1-\lambda \end{vmatrix} = (-1-\lambda)^2 = 1 + 2\lambda + \lambda^2$

$\rightarrow$ stabiel ?

$D = 4 - 4$

$\lambda = \frac{-2 \pm 0}{2} = -1$

* $\vec{x}_3 = (\frac{1}{2}, \frac{1}{2})^T$

$A = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \\ -\frac{3}{2} & -\frac{1}{2} \end{pmatrix}$ $\quad \begin{vmatrix} -\frac{1}{2}-\lambda & -\frac{1}{2} \\ -\frac{3}{2} & -\frac{1}{2}-\lambda \end{vmatrix} = (-\frac{1}{2}-\lambda) - \frac{3}{4}$

$= \frac{1}{4} + \lambda + \lambda^2 - \frac{3}{4}$

$= \lambda^2 + \lambda - \frac{1}{2}$

$D = 1 + 2 = 3$

$\lambda_{1,2} = \frac{-1 \pm \sqrt{3}}{2}$

$\lambda_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2} > 0$

$\lambda_1 = -\frac{1}{2} - \frac{\sqrt{3}}{2} < 0$

$\Rightarrow$ instabiel ✓

Thuis

9.11

a) roofdier / prooidier model (p 187) ✓

$\hookrightarrow x \qquad \hookrightarrow y$

$x' = x(-1 + py) \qquad (k = 1 \quad l = p)$

$y' = y\left(1 - \frac{1}{4}y - \frac{1}{4}x\right) \qquad \left(a = 1 \quad b = \frac{1}{4} \quad c = \frac{1}{4}\right)$

b) => 3 evenwichten (p 188)

$\vec{x}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ✓

$\vec{x}_1 = \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ ✓

$\vec{x}_2 = \begin{pmatrix} 1/p \\ \dfrac{p - \frac{1}{4}}{\frac{p}{4}} \end{pmatrix} = \begin{pmatrix} 1/p \\ \dfrac{4p - 1}{p} \end{pmatrix} = \begin{pmatrix} \dfrac{4p-1}{p} \\ 1/p \end{pmatrix}$ ✓

OEFENZITTING

$\vec{x}' = A(t)\,\vec{x}$

stelsels zonder coëfficiënten

$$\begin{cases} x'(t) = F(x,y) \\ x'(t) = G(x,y) \end{cases}$$

evenwichten

$$\begin{cases} F(x,y) = 0 \\ G(x,y) = 0 \end{cases}$$

stabiliteit $\rightarrow$ linearisatie

$$\begin{pmatrix} \frac{\delta F}{\delta x} & \frac{\delta F}{\delta y} \\ \frac{\delta G}{\delta x} & \frac{\delta G}{\delta y} \end{pmatrix} \rightarrow$$ invullen voor evenwicht + eigenwaarden zoeken

$Re\,\lambda_2$ en $Re\,\lambda_1 < 0$ stabiel

$Re\,\lambda_1$ of $Re\,\lambda_2 > 0$ instabiel

$Re\,\lambda_1 = 0$ en $Re\,\lambda_2 < 0$ zegt niets

9.10

geen opl

c) $x' = x(1 - 2y + y^2)$

$y' = y(1 + x)$

$\rightarrow x = 0$ en $y = 0$

$x = -1$ en $y = 1$

$$\begin{pmatrix} \frac{\delta F}{\delta x} & \frac{\delta F}{\delta y} \\ \frac{\delta G}{\delta x} & \frac{\delta G}{\delta y} \end{pmatrix} = \begin{pmatrix} 1 - 2y + y^2 & -2x + 2xy \\ y & 1 + x \end{pmatrix}$$

* $(0,0)$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \rightarrow \begin{vmatrix} 1-\lambda & 0 \\ 0 & 1-\lambda \end{vmatrix} = 1 - 2\lambda + \lambda^2$$

$\Rightarrow D = 4 - 4 = 0$

$\lambda = \frac{2}{2} = 1 \rightarrow$ instabiel

* $(-1,1)$

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \rightarrow \begin{vmatrix} -\lambda & 0 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 = 0 \rightarrow \lambda = 0$$

$\Rightarrow$ we kunnen niets zeggen

9.10

geen opl

d) $\begin{cases} 0 = \sin y \\ 0 = x^2 - y \end{cases}$ $\Rightarrow y = \pi k$

$\Rightarrow x^2 = \pi k \rightarrow x = \pm\sqrt{\pi k}$

$$\begin{pmatrix} 0 & \cos y \\ 2x & -1 \end{pmatrix}$$

① $\propto$ ~~$(\sqrt{\pi k}, \pi k)$~~ $(\sqrt{\pi 2k}, \pi 2k)$

$$\begin{pmatrix} 0 & 1 \\ 2\sqrt{\pi k 2} & -1 \end{pmatrix} \quad \begin{vmatrix} -\lambda & 1 \\ 2\sqrt{\pi k 2} & -1-\lambda \end{vmatrix} = \lambda + \lambda^2 - 2\sqrt{2\pi k}$$

$\rightarrow$ ~~Δ~~ $D = 1 + 8\sqrt{2\pi k}$

~~instabiel~~ $\leftarrow$ $\lambda = \frac{-1 \pm \sqrt{1 + 8\sqrt{2\pi k}}}{2}$

$\Leftrightarrow > 0$ als $\sqrt{D} < 1$ [illegible]

$\begin{cases} 8\sqrt{2k\pi} < 0 \\ \sqrt{2k\pi} < 0 \end{cases}$

$k = 0$: niks zeggen

$k > 0$: instabiel

$k < 0$: stabiel

$K < 0$
$\rightarrow$ onder $\sqrt{\ }$ wordt -
$\Rightarrow$ imaginair
$\Rightarrow$ moeten we enkel kijken naar $-\frac{1}{2} < 0$
$\Rightarrow$ stabiel

② $\propto$ $(-\sqrt{\pi 2k}, \pi 2k)$

$$\begin{pmatrix} 0 & 1 \\ -2\sqrt{\pi k 2} & -1 \end{pmatrix} \quad \begin{vmatrix} -\lambda & 1 \\ -2\sqrt{\pi k 2} & -\lambda - 1 \end{vmatrix} = \lambda + \lambda^2 + 2\sqrt{2\pi k}$$

$\underline{D = 1 - 8\sqrt{2\pi k}}$

imaginair
$\rightarrow$ niet naar kijken

$\lambda = \frac{-1}{2} + i \ldots$

$\Rightarrow$ stabiel

③ $\propto$ $(\sqrt{\pi(2k+1)}, (2k+1)\pi)$

$$\begin{pmatrix} 0 & -1 \\ +2\sqrt{\pi(2k+1)} & -1 \end{pmatrix} \quad \begin{vmatrix} -\lambda & -1 \\ 2\sqrt{\pi(k2+1)} & -1-\lambda \end{vmatrix} = \lambda + \lambda^2 - 2\sqrt{\pi(k2+1)}$$

$D = 1 + 8\sqrt{\pi(2k+1)}$

idem als geval 1 ~~instabiel~~ $\leftarrow$ $\lambda = \frac{-1 \pm \sqrt{1 + 8\sqrt{2\pi k}}}{2}$

④ $\propto$ $(-\sqrt{\pi(2k+1)}, (2k+1)\pi)$

$$\begin{pmatrix} 0 & -1 \\ -2\sqrt{\pi(2k+1)} & -1 \end{pmatrix} \quad \begin{vmatrix} -\lambda & -1 \\ -2\sqrt{\pi(2k+1)} & -1-\lambda \end{vmatrix}$$

$\Rightarrow$ idem als geval 2

9.11

c) $\begin{pmatrix} -1+py & px \\ -\frac{1}{4}y & 1-\frac{1}{2}y-\frac{1}{4}x \end{pmatrix}$

* $\binom{0}{0}$

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ $\begin{vmatrix} -1-\lambda & 0 \\ 0 & 1-\lambda \end{vmatrix} = \lambda^2 - 1 \Rightarrow \lambda^2 = 1$

$\Rightarrow \lambda_{1,2} = \pm 1 \Rightarrow$ instabiel ✓

* $(0, 4)$

$\begin{pmatrix} -1+p4 & 0 \\ -1 & -1 \end{pmatrix}$ $\begin{vmatrix} -1+py-\lambda & 0 \\ -1 & -1-\lambda \end{vmatrix}$

$= 1 - p4 + \lambda + \lambda - p4\lambda + \lambda^2$

$= \lambda^2 + \lambda(2-p4) - p4 + 1$

$D = (2-p4)^2 - 4\cancel{\cdot}(-p4+1)$

$= \cancel{4} - \cancel{16p} + 16p^2 + \cancel{16p} \cancel{-4}$

$\cancel{= 16p(-2+p)} = 16p^2$

$\sqrt{D} = 4p$

$\lambda = \frac{-2+p4 \pm 4p}{2}$ $\lambda_1 = -1$ $\lambda_2 = -1 + 4p$

als $\lambda_2 < 0$ stabiel $\rightarrow$ $p < \frac{1}{4}$ ✓

$\lambda_2 > 0$ onstabiel $\rightarrow$ $p > \frac{1}{4}$

$\lambda_2 = 0$ niets zeggen $\rightarrow$ $\frac{1}{4} = p$

*) $\begin{pmatrix} 4 - \frac{1}{p} \\ 1/p \end{pmatrix}$

$\begin{pmatrix} 0 & 4p-1 \\ -\frac{1}{4p} & \cancel{1} - \frac{1}{2p} - \cancel{1} + \frac{1}{4p} \end{pmatrix}$ $\rightarrow \begin{vmatrix} -\lambda & 4p-1 \\ \frac{-1}{4p} & \frac{-1}{4p} - \lambda \end{vmatrix}$

$\hookrightarrow -\frac{1}{4p}$

$\frac{\lambda}{4p} + \lambda^2 + 1 - \frac{1}{4p} =$

$= \frac{1}{16p^2} - 4 + \frac{1}{p} \Rightarrow \lambda_{1,2} = -\frac{1}{8p} \pm \frac{\sqrt{D}}{2}$ $\Rightarrow$ stabiel als $p > \frac{1}{4}$ ✓

9.13

a) ① $x=0 \Rightarrow y=0$ of $y = p/r$

② $y=0 \Rightarrow x = a/b$

③ $\begin{cases} a - bx - cy = 0 \\ p - qx - ry = 0 \end{cases} \Rightarrow \left(\dfrac{cp - ar}{cq - br}, \dfrac{aq - pb}{cq - br} \right)$

$\Rightarrow$ 4 oplossingen: $(0,0)$, $(0, p/r)$, $(a/b, 0)$ en

b) zie vorige oef. (principe)

c)

9.15

a) neem 1' afgeleide

$x'(t) + y'(t) - \frac{1}{2} \frac{1}{x(t)} x'(t)$

$\cancel{-2xy} + \cancel{2xy} - y - \frac{1}{2} \frac{1}{x} (-2xy) = 0$

$\Rightarrow$ constant

b)