Oefeningen Numerieke Wiskunde: Oefenzitting 2

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1 Probleem 1

De exponent gaat van -126 tot 127. Dit betekent dat hier 8 bits voor nodig zijn. Er blijven dus nog $32 - 8 = 23$ bits over voor de mantisse want er is ook nog een tekenbit. Een andere manier om dit resultaat te bekomen is de formule voor de machine precisie te gebruiken.

$$\epsilon_{mach} = \frac{b^{1-p}}{2}$$

Hierin is b de basis en p het aantal juiste beduidende cijfers. Uit deze formule halen we de p.

$$p1 - \log_b(2\epsilon_{mach})$$

$$p = 1 - \log_2(2 \cdot 2^{-24}) = 23$$

Volgens dezelfde redenering vinden we dat het aantal bits voor het teken, de mantisse en de exponent bij de dubbele-nauwkeurigheidsgetallen respectievelijk 1, 11 en 52 zijn.

2 Probleem 2

- We weten dat 1 en 2 respectievelijk als volgt voorgesteld worden.

  $$1 = .100\ldots00 \cdot 2^1$$

  $$2 = .100\ldots00 \cdot 2^2$$

  Hier zitten dus $2^m - 1 = 2^{52} - 1$ getallen tussen.

- We weten dat 7 en 9 respectievelijk als volgt voorgesteld worden.

  $$7 = .11100\ldots00 \cdot 2^3$$

  $$9 = .10010\ldots00 \cdot 2^4$$

  Hier zitten dus $2^{50} + 2^{49} - 1$ getallen tussen.

3 Probleem 3

Het laatste zinvolle getal in deze rij is 1000. De overgang van 999 naar 1000 resulteert niet in een fout want 1000 wordt als volgt voorgesteld.

$$0.100 \cdot 10^4$$

Als we hier echter nog 1 bij optellen gebeurt er een absolute afrondingsfout van precies -1. Vanaf dan is de beschreven rij dus constant (1000).

4 Probleem 4

- - **bepaal_b** We beginnen met $A = 1$, dan vermenigvuldigen we A met 2 tot er een afrondingsfout gebeurd wanneer je $(A+1)-A = 1$ evalueert. A is nu gelijk aan 2^{10}. Vervolgens initialiseren we i op 1 en hogen i op tot $(A + i) = A$ geen afrondingsfout meer geeft. Dit is wanneer i 6 wordt. b is dus $.103 \cdot 10^4 - .102 \cdot 10^4 = 10$.
  - **bepaal_p** We initialiseren p op 1 en z op 10. Nu hogen we p en de exponent van 10^1 op tot $(z + 1) - z = 1$ een afrondingsfout geeft. Dit is wanneer $p = 3$ geldt.
- - **bepaal_b**
    Te Bewijzen

    Bewijs. TODO □
  - **bepaal_p**
    Te Bewijzen

    Bewijs. TODO □

5 Probleem 5

1.
$$\overline{y} = fl\left(\frac{x}{fl\left(fl\left(\sqrt{fl\,(x+1)}\right)+1\right)}\right)$$
$$= \frac{x(1+\epsilon_1)}{\left(\left(\sqrt{(x+1)\,(1+\epsilon_4)}\right)(1+\epsilon_3)+1\right)(1+\epsilon_2)}$$
$$\overline{y} = y + \sum_{i=1}^{4} \epsilon_i \frac{\delta F}{\delta \epsilon_i}(0,0,0,0)$$

2.
- $$\frac{\delta \overline{y}}{\delta \epsilon_1}(\epsilon_1,0,0,0) = \frac{x}{\sqrt{x+1}+1}$$
- $$\frac{\delta \overline{y}}{\delta \epsilon_2}(0,\epsilon_2,0,0) = \frac{-x}{\left(\sqrt{x+1}+1\right)(1+\epsilon_2)^2}$$
  $$\frac{\delta \overline{y}}{\delta \epsilon_2}(0,0,0,0) = \frac{-x}{\sqrt{x+1}+1}$$
- $$\frac{\delta \overline{y}}{\delta \epsilon_3}(0,0,\epsilon_3,0) = \frac{-x\sqrt{1+x}}{\left((\sqrt{1+x})(1+\epsilon_3)+1\right)^2}$$
  $$\frac{\delta \overline{y}}{\delta \epsilon_3}(0,0,0,0) = \frac{-x\sqrt{1+x}}{\left(\sqrt{1+x}+1\right)^2}$$
- $$\frac{\delta \overline{y}}{\delta \epsilon_4}(0,0,0,\epsilon_4) = \frac{-x(x+1)}{2\sqrt{(x+1)(1+\epsilon_4)}\left(\sqrt{(x+1)(1+\epsilon_4)}+1\right)^2}$$
  $$\frac{\delta \overline{y}}{\delta \epsilon_4}(0,0,0,0) = \frac{-x(x+1)}{2\sqrt{x+1}\left(\sqrt{x+1}+1\right)^2}$$
  $$= \frac{-x(x+1)}{(2\sqrt{(x+1)^3}+4(x+1)+2\sqrt{x+1})}$$

3.

$$\overline{y} \approx y + \frac{x}{\sqrt{x+1}+1}\epsilon_1 + \frac{-x}{\sqrt{x+1}+1}\epsilon_2$$

$$+\frac{-x\sqrt{1+x}}{\left(\sqrt{1+x}+1\right)^2}\epsilon_3 + \frac{-x(x+1)}{(2\sqrt{(x+1)^3}+4(x+1)+2\sqrt{x+1})}\epsilon_4$$

$$\overline{y} \approx y + y\epsilon_1 - y\epsilon_2 + y\frac{\sqrt{1+x}}{\sqrt{1+x}+1}\epsilon_3 - y\frac{1}{2}\frac{\sqrt{1+x}}{\sqrt{1+x}+1}\epsilon_4$$

$$\overline{y} - y = y\epsilon_1 - y\epsilon_2 + y\frac{\sqrt{1+x}}{\sqrt{1+x}+1}\epsilon_3 - y\frac{1}{2}\frac{\sqrt{1+x}}{\sqrt{1+x}+1}\epsilon_4$$

$$\frac{\overline{y} - y}{y} = \epsilon_1 - \epsilon_2 + \frac{\sqrt{1+x}}{\sqrt{1+x}+1}\epsilon_3 - \frac{1}{2}\frac{\sqrt{1+x}}{\sqrt{1+x}+1}\epsilon_4$$

6 Probleem 6

$$y = \frac{1-\cos(x)}{x^2}$$

1.

$$\overline{y} = fl\left(\frac{fl(1 - fl(\cos(x)))}{fl(x^2)}\right)$$

$$\overline{y} = \frac{(1 - (\cos(x))(1+\epsilon_1))(1+\epsilon_2)}{(x^2)(1+\epsilon_3)}(1+\epsilon_4)$$

2.

$$\frac{\delta\overline{y}}{\delta\epsilon_1}(\epsilon_1, 0, 0, 0) = \frac{\delta}{\delta\epsilon_1}\frac{1-(\cos(x))(1+\epsilon_1)}{x^2} = -\frac{\cos(x)}{x^2}$$

$$\frac{\delta\overline{y}}{\delta\epsilon_2}(0, \epsilon_2, 0, 0) = \frac{\delta}{\delta\epsilon_2}\frac{(1-\cos(x))(1+\epsilon_2)}{x^2} = \frac{(1-\cos(x))}{x^2}$$

$$\frac{\delta\overline{y}}{\delta\epsilon_3}(0, 0, \epsilon_3, 0) = \frac{\delta}{\delta\epsilon_3}\frac{1-\cos(x)}{(x^2)(1+\epsilon_3)} = \frac{1-\cos(x)}{(x^2)}\frac{-1}{(1+\epsilon_3)^2}$$

$$\frac{\delta\overline{y}}{\delta\epsilon_3}(0, 0, 0, 0) = -\frac{1-\cos(x)}{x^2}$$

$$\frac{\delta\overline{y}}{\delta\epsilon_4}(0, 0, 0, \epsilon_4) = \frac{\delta}{\delta\epsilon_4}\frac{(1-\cos(x))(1+\epsilon_4)}{x^2} = \frac{1-\cos(x)}{x^2}$$

3.

$$\overline{y} \approx y - \frac{\cos(x)}{x^2}\epsilon_1 + \frac{(1-\cos(x))}{x^2}\epsilon_2 - \frac{1-\cos(x)}{x^2}\epsilon_3 + \frac{1-\cos(x)}{x^2}\epsilon_4$$

$$\overline{y} \approx y - \frac{\cos(x)}{x^2}\epsilon_1 + y\epsilon_2 - y\epsilon_3 + y\epsilon_4$$

4.

$$\overline{y} - y \approx -\frac{\cos(x)}{x^2}\epsilon_1 + y\epsilon_2 - y\epsilon_3 + y\epsilon_4$$

$$\frac{\overline{y} - y}{y} \approx -\frac{\cos(x)}{x(1-\cos(x))}\epsilon_1 + \epsilon_2 - \epsilon_3 + \epsilon_4$$

7 Probleem 7

$$S = \sum_{i=1}^{n} a_i$$

1.

$$\overline{S} = fl(fl(...fl(fl(fl(fl(a_1 + a_2) + a_3) + a_4)...) + a_n)$$
$$= ((...((((a_1 + a_2)(1+\epsilon_2)) + a_3)(1+\epsilon_3))...) + a_n)(1+\epsilon_n)$$

- $$\frac{\delta\overline{y}}{\delta\epsilon_2}(\epsilon_2, 0, ..., 0) = a_1 + a_2$$

- $$\frac{\delta\overline{y}}{\delta\epsilon_3}(0, \epsilon_3, 0, ..., 0) = a_1 + a_2 + a_3$$

- $$\frac{\delta\overline{y}}{\delta\epsilon_i}(0, ..., 0, \epsilon_i, 0, ..., 0) = \sum_{j=1}^{i} a_j$$

2.

$$\overline{S} \approx y + \sum_{k=2}^{n} \epsilon_k \sum_{i=1}^{k} a_i$$

3.

$$\overline{S} - S \approx \sum_{k=2}^{n} \epsilon_k \sum_{i=1}^{k} a_i$$

8 Probleem 8

(a)

$$y = x\sin(x)$$

1.
$$\overline{y} = fl(xfl(\sin(x)))$$
$$\overline{y} = (x\sin(x)(1+\epsilon_1))(1+\epsilon_2)$$

2.
   - $$\frac{\delta\overline{y}}{\delta\epsilon_1}(\epsilon_1, 0) = x\sin(x)$$
   - $$\frac{\delta\overline{y}}{\delta\epsilon_2}(0, \epsilon_2) = x\sin(x)$$

3.
$$\overline{y} \approx y + x\sin(x)\epsilon_1 + x\sin(x)\epsilon_2$$
$$\overline{y} \approx y + y\epsilon_2 + y\epsilon_3$$

4.
$$\overline{y} - y = y\epsilon_1 + y\epsilon_2$$
$$\frac{\overline{y} - y}{y} = \epsilon_1 + \epsilon_2$$

(b)

$$y = \frac{1-\cos(x)}{\sin(x)}$$

1.
$$\overline{y} = fl\left(\frac{fl(1 - fl(\cos(x)))}{fl(\sin(x))}\right)$$
$$\overline{y} = \left(\frac{(1-(\cos(x))(1+\epsilon_1))(1+\epsilon_2)}{(\sin(x))(1+\epsilon_3)}\right)(1+\epsilon_4)$$

2.
   - $$\frac{\delta\overline{y}}{\delta\epsilon_1}(\epsilon_1, 0, 0, 0) = \frac{\delta\overline{y}}{\delta\epsilon_1}\frac{1-\cos(x)(1+\epsilon_1)}{\sin(x)} = \frac{-\cos(x)}{\sin(x)}$$
   - $$\frac{\delta\overline{y}}{\delta\epsilon_2}(0, \epsilon_2, 0, 0) = \frac{\delta\overline{y}}{\delta\epsilon_2}\frac{(1-\cos(x))(1+\epsilon_2)}{\sin(x)} = \frac{1-\cos(x)}{\sin(x)}$$

- $$\frac{\delta \overline{y}}{\delta \epsilon_3}(0,0,\epsilon_3,0) = \frac{\delta \overline{y}}{\delta \epsilon_3} \frac{1-\cos(x)}{(\sin(x))(1+\epsilon_3)} = \frac{\cos(x)-1}{\sin(x)(1+\epsilon_3)^2}$$
$$\frac{\delta \overline{y}}{\delta \epsilon_3}(0,0,0,0) = \frac{\cos(x)-1}{\sin(x)}$$

- $$\frac{\delta \overline{y}}{\delta \epsilon_4}(0,0,0,\epsilon_4) = \frac{\delta \overline{y}}{\delta \epsilon_4} \left(\frac{1-\cos(x)}{\sin(x)}\right)(1+\epsilon_4) = \frac{1-\cos(x)}{\sin(x)}$$

3. $$\overline{y} \approx y + \frac{-\cos(x)}{\sin(x)}\epsilon_1 + \frac{1-\cos(x)}{\sin(x)}\epsilon_2 + \frac{\cos(x)-1}{\sin(x)}\epsilon_3 + \frac{1-\cos(x)}{\sin(x)}\epsilon_4$$
$$\overline{y} \approx y + \frac{-\cos(x)}{\sin(x)}\epsilon_1 + y\epsilon_2 - y\epsilon_3 + y\epsilon_4$$

4. $$\overline{y} - y \approx \frac{-\cos(x)}{\sin(x)}\epsilon_1 + y\epsilon_2 - y\epsilon_3 + y\epsilon_4$$
$$\frac{\overline{y} - y}{y} \approx \frac{-\cos(x)}{1-\cos(x)}\epsilon_1 + \epsilon_2 - \epsilon_3 + \epsilon_4$$

(c)
$$y = \frac{1-e^{-2x}}{x}$$

1. $$\overline{y} = fl\left(\frac{fl\left(1 - fl(e^{fl(-2x)})\right)}{x}\right)$$
$$\overline{y} = \left(\frac{\left(1 - ((e^{(-2x)(1+\epsilon_1)})(1+\epsilon_2))\right)(1+\epsilon_3)}{x}\right)(1+\epsilon_4)$$

2. - $$\frac{\delta \overline{y}}{\delta \epsilon_1}(\epsilon_1,0,0,0) = \frac{\delta \overline{y}}{\delta \epsilon_1} \frac{1-e^{(-2x)(1+\epsilon_1)}}{x} = \frac{2xe^{(-2x)(1+\epsilon_1)}}{x}$$
$$\frac{\delta \overline{y}}{\delta \epsilon_1}(0,0,0,0) = \frac{2xe^{-2x}}{x}$$

   - $$\frac{\delta \overline{y}}{\delta \epsilon_2}(0,\epsilon_2,0,0) = \frac{\delta \overline{y}}{\delta \epsilon_2} \frac{1-(e^{-2x})(1+\epsilon_2)}{x} = \frac{-e^{-2x}}{x}$$

- $$\frac{\delta \overline{y}}{\delta \epsilon_3}(0,0,\epsilon_3,0) = \frac{\delta \overline{y}}{\delta \epsilon_3}\frac{(1-e^{-2x})(1+\epsilon_3)}{x} = \frac{1-e^{-2x}}{x}$$
- $$\frac{\delta \overline{y}}{\delta \epsilon_4}(0,0,0,\epsilon_4) = \frac{\delta \overline{y}}{\delta \epsilon_4}\left(\frac{1-e^{-2x}}{x}\right)(1+\epsilon_4) = \frac{1-e^{-2x}}{x}$$

3. $$\overline{y} \approx y + \frac{2xe^{-2x}}{x} + \frac{-e^{-2x}}{x} + \frac{1-e^{-2x}}{x} + \frac{1-e^{-2x}}{x}$$
$$\overline{y} \approx y + \frac{2xe^{-2x}}{x}\epsilon_1 + \frac{-e^{-2x}}{x}\epsilon_2 + y\epsilon_3 + y\epsilon_4$$

4. $$\overline{y} - y \approx \frac{2xe^{-2x}}{x}\epsilon_1 + \frac{-e^{-2x}}{x}\epsilon_2 + y\epsilon_3 + y\epsilon_4$$
$$\frac{\overline{y} - y}{y} \approx \frac{2xe^{-2x}}{1-e^{-2x}}\epsilon_1 + \frac{-e^{-2x}}{1-e^{-2x}}\epsilon_2 + \epsilon_3 + \epsilon_4$$

(d)
$$y = (1+x)^{\frac{1}{x}}$$

1. $$\overline{y} = fl\left((1+x)^{\frac{1}{x}}\right)$$
$$\overline{y} = \left(((1+x)(1+\epsilon_2))^{\left(\frac{1}{x}\right)(1+\epsilon_1)}\right)(1+\epsilon_3)$$

2. - $$\frac{\delta \overline{y}}{\delta \epsilon_1}(\epsilon_1,0,0) = \frac{\delta \overline{y}}{\delta \epsilon_1}(1+x)^{\left(\frac{1}{x}\right)(1+\epsilon_1)} = \frac{(x+1)^{\frac{\epsilon_1+1}{x}} ln(x+1)}{x}$$
$$\frac{\delta \overline{y}}{\delta \epsilon_1}(0,0,0) = \frac{(x+1)^{\frac{1}{x}} ln(x+1)}{x}$$
   - $$\frac{\delta \overline{y}}{\delta \epsilon_2}(0,\epsilon_2,0) = \frac{\delta \overline{y}}{\delta \epsilon_2}((1+x)(1+\epsilon_2))^{\frac{1}{x}} = \frac{(1+x)}{a}((1+x)(1+\epsilon_2))^{\frac{1}{x}-1}$$
$$\frac{\delta \overline{y}}{\delta \epsilon_2}(0,0,0) = \frac{(1+x)}{a}(1+x)^{\frac{1}{x}-1} = \frac{(1+x)^{\frac{1}{x}}}{a}$$
   - $$\frac{\delta \overline{y}}{\delta \epsilon_3}(0,0,\epsilon_3) = \frac{\delta \overline{y}}{\delta \epsilon_3}\left((1+x)^{\frac{1}{x}}\right)(1+\epsilon_3) = \left((1+x)^{\frac{1}{x}}\right)$$

3.

$$\overline{y} \approx y + \frac{(x+1)^{\frac{1}{x}} ln(x+1)}{x}\epsilon_1 + \frac{(1+x)^{\frac{1}{x}}}{a}\epsilon_2 + \left((1+x)^{\frac{1}{x}}\right)\epsilon_3$$

$$\overline{y} \approx y + \frac{ln(x+1)}{x}y\epsilon_1 + \frac{1}{a}y\epsilon_2 + y\epsilon_3$$

4.

$$\overline{y} - y \approx \frac{ln(x+1)}{x}y\epsilon_1 + \frac{1}{a}y\epsilon_2 + y\epsilon_3$$

$$\frac{\overline{y}-y}{y} \approx \frac{ln(x+1)}{x}\epsilon_1 + \frac{1}{a}\epsilon_2 + \epsilon_3$$

(e)

$$y = \sqrt{e^x - 1}$$

1.

$$\overline{y} = fl\left(\sqrt{fl(fl(e^x) - 1)}\right)$$

$$\overline{y} = \left(\sqrt{((e^x)(1+\epsilon_1) - 1)(1+\epsilon_2)}\right)(1+\epsilon_3)$$

2. •

$$\frac{\delta\overline{y}}{\delta\epsilon_1}(\epsilon_1, 0, 0) = \frac{\delta\overline{y}}{\delta\epsilon_1}\sqrt{(e^x)(1+\epsilon_1) - 1} = \frac{e^x}{2\sqrt{(e^x)(1+\epsilon_1) - 1}}$$

$$\frac{\delta\overline{y}}{\delta\epsilon_1}(0,0,0) = \frac{e^x}{2\sqrt{e^x - 1}}$$

•

$$\frac{\delta\overline{y}}{\delta\epsilon_2}(0, \epsilon_2, 0) = \frac{\delta\overline{y}}{\delta\epsilon_2}\sqrt{(e^x - 1)(1+\epsilon_2)} = \frac{e^x - 1}{2\sqrt{(e^x - 1)(1+\epsilon_2)}}$$

$$\frac{\delta\overline{y}}{\delta\epsilon_2}(0,0,0) = \frac{e^x - 1}{2\sqrt{e^x - 1}} = \frac{1}{2}\sqrt{e^x - 1}$$

•

$$\frac{\delta\overline{y}}{\delta\epsilon_3}(0,0,\epsilon_3) = \frac{\delta\overline{y}}{\delta\epsilon_3}(1+\epsilon_3)\sqrt{e^x - 1} = \sqrt{e^x - 1}$$

3.

$$\overline{y} \approx y + \frac{e^x}{2\sqrt{e^x - 1}}\epsilon_1 + \frac{1}{2}\sqrt{e^x - 1}\epsilon_2 + \sqrt{e^x - 1}\epsilon_3$$

$$\overline{y} \approx y + +y\frac{e^x}{2e^x - 2}\epsilon_1 + y\frac{1}{2}\epsilon_2 + y\epsilon_3$$

4.

$$\overline{y} - y \approx y\frac{e^x}{2e^x - 2}\epsilon_1 + y\frac{1}{2}\epsilon_2 + y\epsilon_3$$
$$\frac{\overline{y} - y}{y} \approx \frac{e^x}{2e^x - 2}\epsilon_1 + \frac{1}{2}\epsilon_2 + \epsilon_3$$

(f)

$$y = \sin\left(\frac{1}{x}\right)$$

1.

$$\overline{y} = \left(\sin\left(\frac{(1+\epsilon_1)}{x}\right)\right)(1+\epsilon_2)$$
$$\overline{y} = ...$$

2. •

$$\frac{\delta\overline{y}}{\delta\epsilon_1}(\epsilon_1, 0) = \frac{\delta\overline{y}}{\delta\epsilon_1}\sin\left(\frac{(1+\epsilon_1)}{x}\right) = \frac{1}{x}\cos\left(\frac{(1+\epsilon_1)}{x}\right)$$
$$\frac{\delta\overline{y}}{\delta\epsilon_1}(0,0) = \frac{1}{x}\cos\left(\frac{1}{x}\right)$$

•

$$\frac{\delta\overline{y}}{\delta\epsilon_2}(0, \epsilon_2) = \frac{\delta\overline{y}}{\delta\epsilon_2}\left(\sin\left(\frac{1}{x}\right)\right)(1+\epsilon_2) = \sin\left(\frac{1}{x}\right)$$

3.

$$\overline{y} \approx y + \frac{1}{x}\cos\left(\frac{1}{x}\right)\epsilon_1 + \sin\left(\frac{1}{x}\right)\epsilon_2$$
$$\overline{y} \approx y + y\frac{1}{x}\cot\left(\frac{1}{x}\right)\epsilon_1 + y\epsilon_2$$

4.

$$\overline{y} - y \approx y\frac{1}{x}\cot\left(\frac{1}{x}\right)\epsilon_1 + y\epsilon_2$$
$$\frac{\overline{y} - y}{y} \approx \frac{1}{x}\cot\left(\frac{1}{x}\right)\epsilon_1 + \epsilon_2$$

(g)

$$y = (1+x^2)^{x^2}$$

1.

$$\overline{y} = fl\left(\left(fl(1 + fl(x^2))\right)^{fl(x^2)}\right)$$
$$\overline{y} = \left(\left((1 + (x^2)(1+\epsilon_2))(1+\epsilon_3)\right)^{(x^2)(1+\epsilon_1)}\right)(1+\epsilon_4)$$

2. •

$$\frac{\delta \overline{y}}{\delta \epsilon_1}(\epsilon_1, 0, 0, 0) = \frac{\delta \overline{y}}{\delta \epsilon_1}(1+x^2)^{(x^2)(1+\epsilon_1)} = x^2(x^2+1)^{x^2(1+\epsilon_1)} ln(x^2+1)$$

$$\frac{\delta \overline{y}}{\delta \epsilon_1}(0, 0, 0, 0) = x^2(x^2+1)^{x^2} ln(x^2+1)$$

•

$$\frac{\delta \overline{y}}{\delta \epsilon_2}(0, \epsilon_2, 0, 0) = \frac{\delta \overline{y}}{\delta \epsilon_2}(1+(x^2)(1+\epsilon_2))^{x^2} = x^4(1+(x^2)(1+\epsilon_2))^{x^2-1}$$

$$\frac{\delta \overline{y}}{\delta \epsilon_2}(0, 0, 0, 0) = x^4(1+x^2)^{x^2-1}$$

•

$$\frac{\delta \overline{y}}{\delta \epsilon_3}(0, 0, \epsilon_3, 0) = \frac{\delta \overline{y}}{\delta \epsilon_3}((1+x^2)(1+\epsilon_3))^{x^2}$$

$$= (1+x^2)x^2((1+x^2)(1+\epsilon_3))^{x^2-1}$$

$$\frac{\delta \overline{y}}{\delta \epsilon_3}(0, 0, 0, 0) = (1+x^2)x^2(1+x^2)^{x^2-1}$$

•

$$\frac{\delta \overline{y}}{\delta \epsilon_4}(0, 0, 0, \epsilon_4) = \frac{\delta \overline{y}}{\delta \epsilon_4}\left((1+x^2)^{x^2}\right)(1+\epsilon_4) = (1+x^2)^{x^2}$$

3.

$$\overline{y} \approx y + x^2(x^2+1)^{x^2} ln(x^2+1)\epsilon_1 + x^4(1+x^2)^{x^2-1}\epsilon_2$$

$$+(1+x^2)x^2(1+x^2)^{x^2-1}\epsilon_3 + (1+x^2)^{x^2}\epsilon_4$$

$$y = (1+x^2)^{x^2}$$

$$\overline{y} \approx y + x^2 y ln(x^2+1)\epsilon_1 + \frac{x^4 y}{1+x^2}\epsilon_2 + yx^2\epsilon_3 + y\epsilon_4$$

4.

$$\overline{y} - y \approx x^2 y ln(x^2+1)\epsilon_1 + \frac{x^4 y}{1+x^2}\epsilon_2 + yx^2\epsilon_3$$

$$\frac{\overline{y} - y}{y} \approx x^2 ln(x^2+1)\epsilon_1 + \frac{x^4}{1+x^2}\epsilon_2 + x^2\epsilon_3$$

(h)

$$y = \frac{e^{x^2} - e^{-x^2}}{2x^2}$$

1.
$$\overline{y} = fl\left(\frac{fl\left(fl\left(e^{fl(x^2)}\right) - fl\left(e^{fl(-fl(x^2))}\right)\right)}{fl(2fl(x^2))}\right)$$
$$\overline{y} = \frac{\left(e^{(x^2)(1+\epsilon_1)} - e^{(-(x^2)(1+\epsilon_1))(1+\epsilon_2)}\right)(1+\epsilon_4)}{(2(x^2)(1+\epsilon_1))(1+\epsilon_3)}(1+\epsilon_5)$$

2. •
$$\frac{\delta\overline{y}}{\delta\epsilon_1}(\epsilon_1, 0, 0, 0, 0) = \frac{\delta\overline{y}}{\delta\epsilon_1}\frac{e^{(x^2)(1+\epsilon_1)} - e^{-(x^2)(1+\epsilon_1)}}{2(x^2)(1+\epsilon_1)}$$
$$= \frac{e^{-x^2(\epsilon_1+1)}\left(x^2(\epsilon_1+1)\left(e^{2x^2(\epsilon_1+1)}+1\right)\right) - e^{2x^2(\epsilon_1+1)} + 1}{2x^2(\epsilon_1+1)^2}$$

AWW HELL NAW

9 Probleem 9

- Product
$$y = \prod_{i=1}^{n}$$

1.
$$\overline{y} = fl(fl(...fl(fl(fl(a_1a_2)a_3)a_4)...a_{n-1})a_n)$$
$$\overline{y} = ((...((a_1a_2)(1+\epsilon_2)a_3)(1+\epsilon_3)...a_{n-1})(1+\epsilon_{n-1})a_n)(1+\epsilon_n)$$
$$= \left(\prod_{i=1}^{n} a_i\right)\left(\prod_{i=2}^{n}(1+\epsilon_i)\right)$$

2. –
$$\frac{\delta\overline{y}}{\delta\epsilon_1}(\epsilon_1, 0, ..., 0) = \frac{\delta\overline{y}}{\delta\epsilon_1} = \left(\prod_{i=1}^{n} a_i\right)(1+\epsilon_1)$$
$$\frac{\delta\overline{y}}{\delta\epsilon_1}(0, ..., 0) = \prod_{i=1}^{n} a_i$$

–
$$\frac{\delta\overline{y}}{\delta\epsilon_2}(0, \epsilon_2, ..., 0) = \frac{\delta\overline{y}}{\delta\epsilon_2} = \left(\prod_{i=1}^{n} a_i\right)(1+\epsilon_2)$$
$$\frac{\delta\overline{y}}{\delta\epsilon_2}(0, ..., 0) = \prod_{i=1}^{n} a_i$$

–

$$\frac{\delta \overline{y}}{\delta \epsilon_j}(0, ..., \epsilon_j, ..., 0) = \frac{\delta \overline{y}}{\delta \epsilon_j} = \left(\prod_{i=1}^{n} a_i \right) (1 + \epsilon_j)$$

$$\frac{\delta \overline{y}}{\delta \epsilon_j}(0, ..., 0) = \prod_{i=1}^{n} a_i$$

3.

$$\overline{y} \approx y + \sum_{j=1}^{n} \epsilon_j \prod_{i=1}^{n} a_i$$

$$\overline{y} \approx y + \sum_{j=1}^{n} \epsilon_j y$$

4.

$$\overline{y} - y \approx \sum_{j=1}^{n} \epsilon_j y$$

$$\frac{\overline{y} - y}{y} \approx \sum_{j=1}^{n} \epsilon_j$$

- Scalair Product
  TODO

$$y = \sum_{i=1}^{n} a_i b_i$$

1.

$$\overline{y} =$$

$$\overline{y} = ...$$

2. –

$$\frac{\delta \overline{y}}{\delta \epsilon_1}(\epsilon_1, 0, ..., 0) = ...$$

$$\frac{\delta \overline{y}}{\delta \epsilon_1}(0, ..., 0) = ..$$

–

$$\frac{\delta \overline{y}}{\delta \epsilon_2}(0, \epsilon_2, ..., 0) = ...$$

$$\frac{\delta \overline{y}}{\delta \epsilon_2}(0, ..., 0) = ...$$

–

$$\frac{\delta\overline{y}}{\delta\epsilon_j}(0,...,\epsilon_j,...,0) = ...$$

$$\frac{\delta\overline{y}}{\delta\epsilon_j}(0,...,0) = ...$$

3.

$$\overline{y} \approx y + ...$$

$$\overline{y} \approx y + ...$$

4.

$$\overline{y} - y \approx ...$$

$$\frac{\overline{y} - y}{y} \approx ...$$

10 Probleem 10

11 Probleem 11