CS7CS4 Machine Learning

The assigned dataset is: # id:16-16-16

Part A

(i) Visualization of Original Data

The provided dataset consists of two numerical features (dubbed X_1 and X_2) and a binary target label (that was -1 or +1). The features were plotted into a scatter plot, with the X-axis representing the X_1 feature and the Y-axis representing the X_2 feature. Data points with label as +1 are represented using cyan circles, while those with label as -1 as orange circles for clarity. Here is the graph produced:

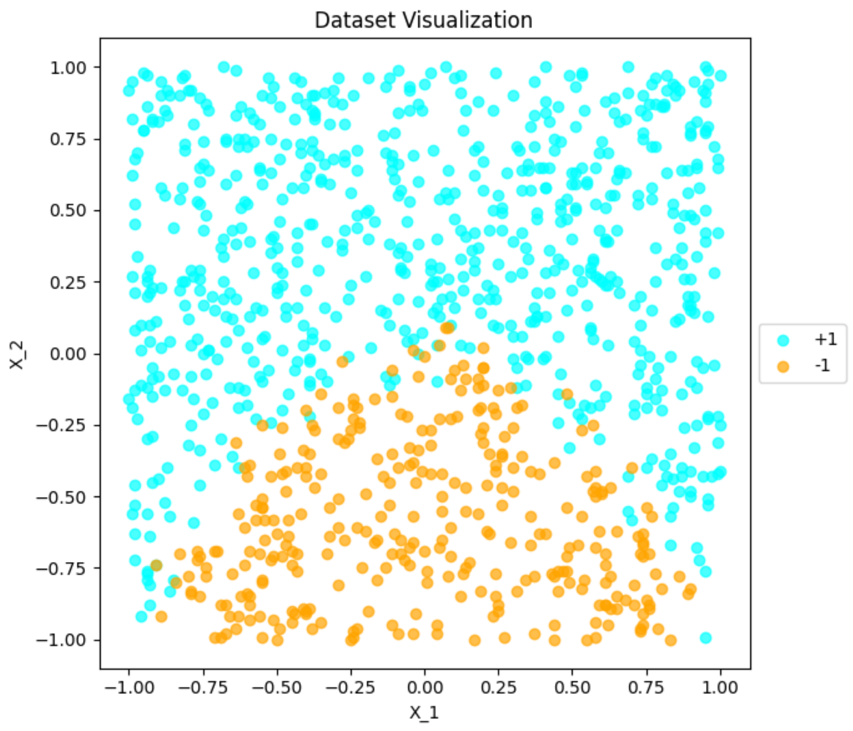


Figure 1. Scatter plot of the given data

The code segment that produced this was using matplotlib's scatter function by providing it the given features, and is present under the plot_given_data() function.

(ii) Training Logistic Regression Model

The Logistic Regression model $\hat{y} = h_\theta(x) = sign(\theta^T x)$ was trained using the given features X_1 and X_2, which computes the probability of a point using the sigmoid function as +1 when $\theta^T x > 0$ and -1 when $\theta^T x < 0$, where $\theta^T x = \theta_0 + \theta_1 x_1 + \theta_2 x_2$ represents the linear combination of the features X_1 and X_2.

After fitting, the feature coefficients that were produced were $\theta_1 = -0.05339$ and $\theta_2 = 5.15001$, while the intercept is $\theta_0 = 1.69049$. The accuracy of this model compared to the actual data produced a score of 0.87888 i.e. approximately 87.9%. The interpretation of the coefficients is as follows:

- θ_1 is negative but small, which implies that increasing feature X_1 would slightly decrease the probability of predicting +1.
- θ_2 is positive and larger, which implies that increasing feature X_2 would increase the probability of predicting +1.
- As such, since $|\theta_1| < |\theta_2|$, feature X_2 would have the most influence on prediction.

This analysis aligns with how the data is structured in Figure 1, where the data points appear to be vertically segregated into two distinct regions.

The code segment that produced this was using sklearn's LogisticRegression() function with default parameters due to less features, and was fitted using the given data as input and target weights, and is present under the train_log_regr() and make_predictions() functions.

(iii) Plotting the Logistic Regression Model's Predictions

The trained model's predictions were plotted on top of the scatter plot produced by the provided data, with the predicted +1 labels appearing as red '+' markers and the predicted -1 labels appearing as green 'x' markers to display a comparison between the two:

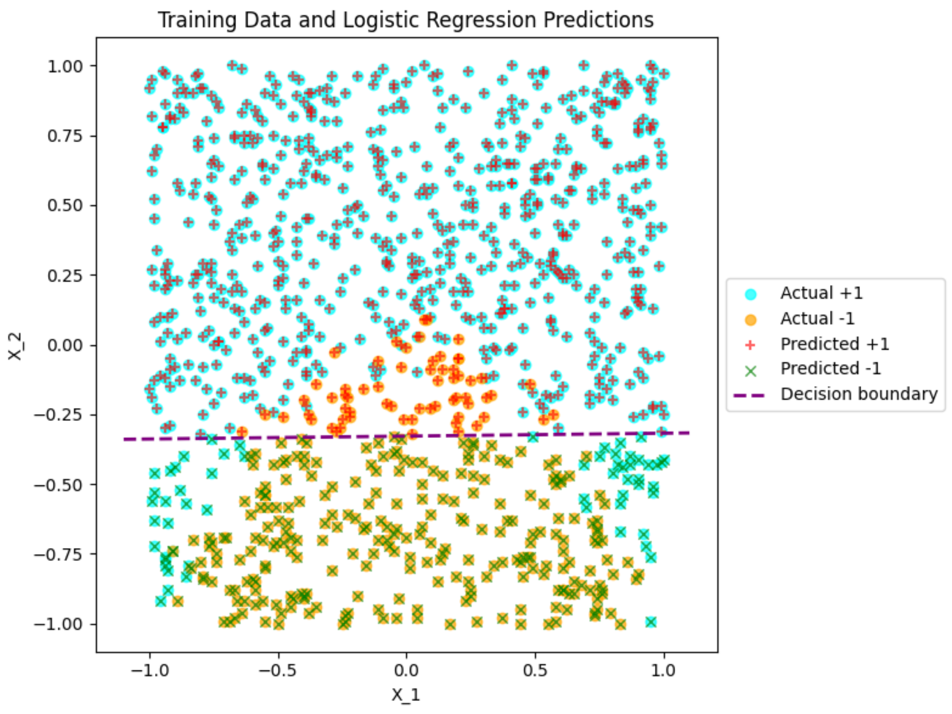


Figure 2. Scatter plot representing given data and predictions of Logistic Regression model

As mentioned in Part A (ii), the tendency of the predicted data showed an 87.9% overlap with the data points from the given dataset, with the predicted data being more distinctively separated into two sections thanks to the decision boundary (represented by the purple line).

The decision boundary was derived from the logistic regression parameters based on the formula mentioned in Part A (ii): $\theta^T x = \theta_0 + \theta_1 x_1 + \theta_2 x_2$ by setting $\theta^T x = 0$ (classifiers are undecided), which then allows us to solve for $x_2 = -\frac{\theta_0 + \theta_1 x_1}{\theta_2}$, which helps us define the decision boundary i.e. line that separates the +1 and -1 classes on the graph shown in Figure 2.

The code segment that produced this was using matplotlib's scatter function with the given features and predicted values, and is present under the plot_log_regr_predictions() function.

(iv) Analysis of Model's Predictions

From Figure 2 and accuracy score of 87.9%, it becomes apparent that the predicted labels align closely with the true labels. The majority of the +1 and -1 data points lie on the correct sides of the decision boundary, with the few miscalculations on either side very close to the boundary line. My observation is that this method relied primarily on the X_2 feature on the vertical axis to separate the two classes, and since only two features were involved in modelling the decision process, the separation line acts as a proper split as compared to the gaussian separation of the given data set. Overall, I would say that the Logistic Regression model provided a good fit for the data with a clear linear decision boundary.

Part B

(i) Training Support Vector Machines on different C Values.

I trained a total of 7 SVM classifiers on the same dataset, with the weighting parameter C having values of 0.001, 0.01, 0.1, 1, 10, 100, 1000, to properly examine the behavior.

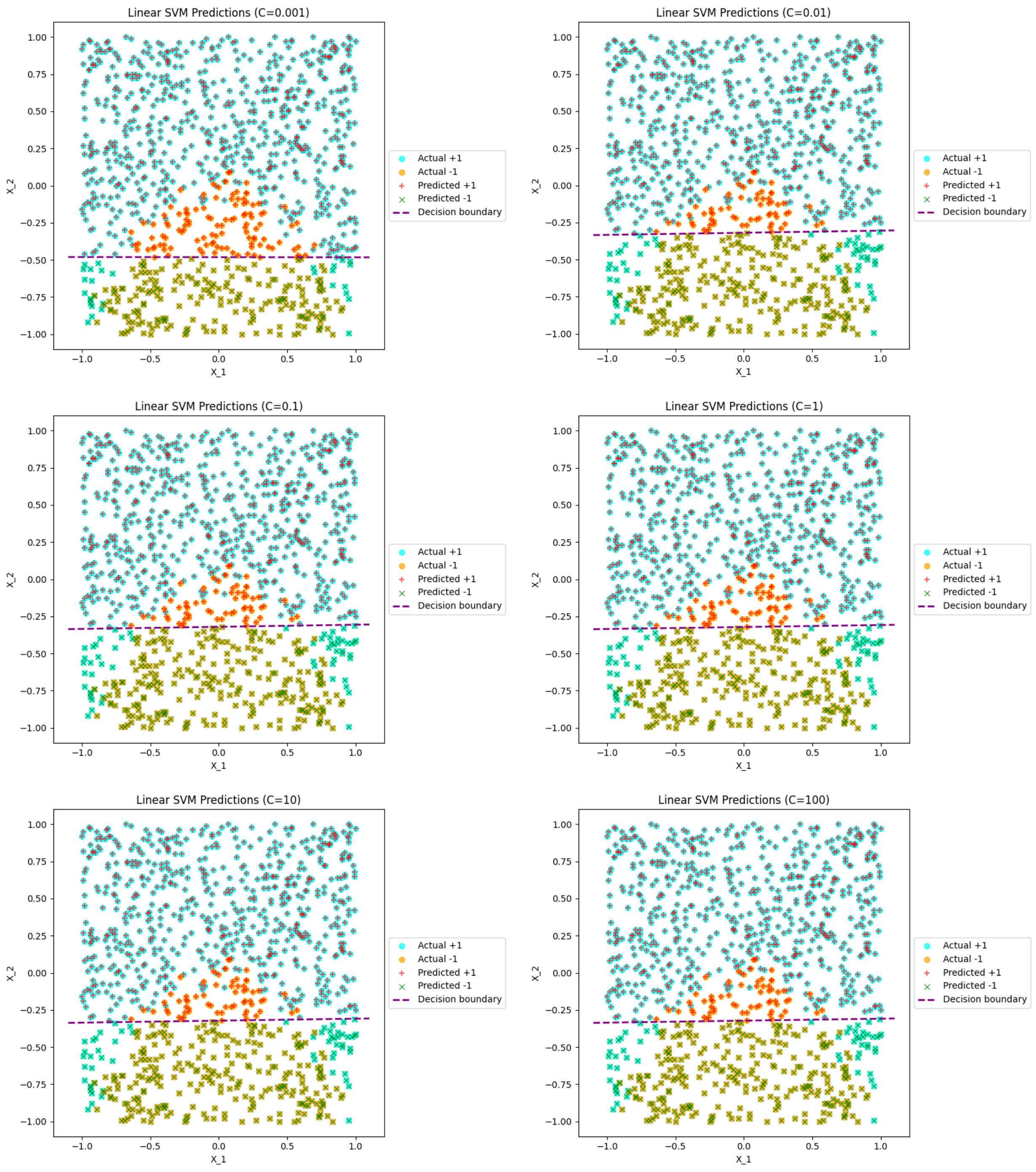
The decision function for just two features (X_1 and X_2) can be computed as $\theta^T x = w_1 x_1 + w_2 x_2 + b$ (conventional notation), where the weights learned by the SVM are w_1 and w_2, while b represents the intercept or bias, and each SVM uses this to predict the class of the data point based on the sign of the calculated $f(x)$. The resulting parameters for all SVMs are as follows:

C	w_1	w_2	bias	accuracy
0.001	0.00040	0.48143	0.23188	0.85686
0.01	-0.01701	1.19233	0.38027	0.87888
0.1	-0.02430	1.72052	0.55092	0.87888
1	-0.02498	1.86583	0.59995	0.87788
10	-0.02524	1.88392	0.60616	0.87788
100	-0.02527	1.88578	0.60679	0.87788
1000	-0.02527	1.88597	0.60685	0.87788

The code segment that produced this was using sklearn's LinearSVC() function with different C values and max iterations increased to 10000 to increase optimization, and was fitted using the given data as input and target weights, and is present under the train_linear_svm() function.

(ii) Plotting the SVM Model's Predictions

The graphs generated using these are as follows:



Figures 3-8. Scatter plot representing given data and predictions of Support Vector Machine for C = 0.001, 0.01, 0.1, 1, 10, 100

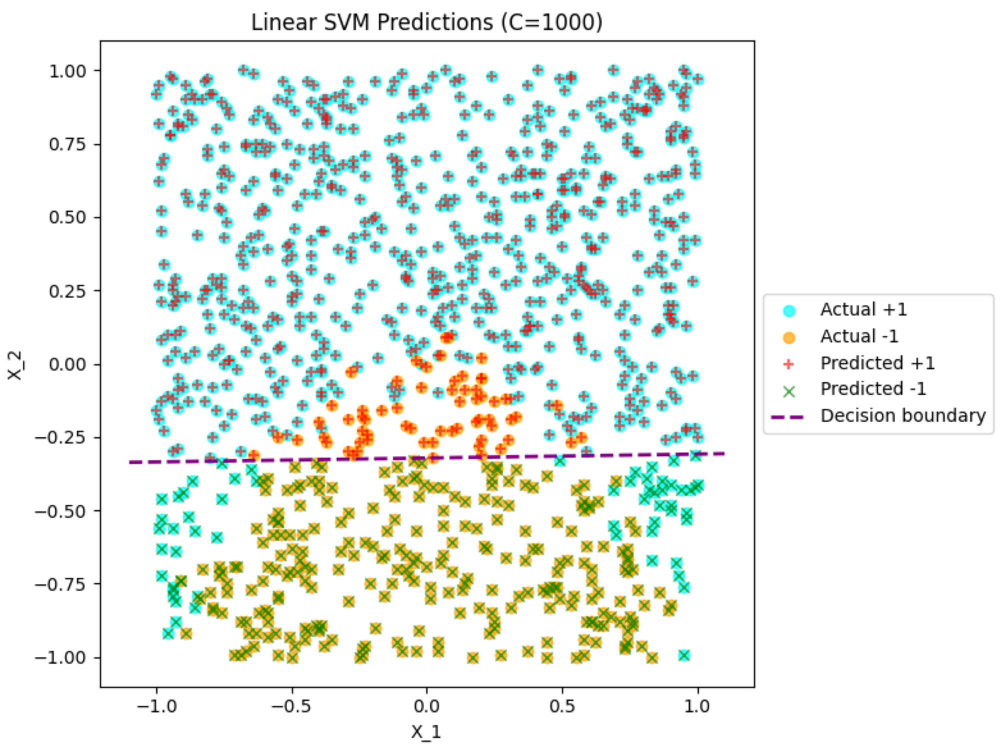


Figure 9. Scatter plot representing given data and predictions of Support Vector Machine for C = 1000

The code segment that produced this was using matplotlib's scatter function with the given features and predicted values, and is present under the plot_svm_predictions() function.

(iii) Analysis of Model's Predictions and Impact of changing C

The weighting parameter C is used in the cost function $J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \max(0, 1 - y^i\theta^T x^i) + \frac{\theta^T\theta}{C}$, where $\frac{\theta^T\theta}{C}$ represents the regularization, that describes an inverse relationship where bigger C makes the penalty less important, and as a result SVM prioritizes correctly classifying points, whereas a smaller C would increase the regularization and result in a wider margin for potential misclassifications.

My interpretation of the table in Part B (i) and Figures 3-9 is as follows:

- Smaller C values such as 0.001 and 0.01 (0.1 as well, to an extent) show smaller weights associated with the features and the biases, signifying the decision boundary to be shallower and some amount of misclassification. This is further represented by the accuracy of these models, especially $C = 0.001$ to be lower than that of the other SVM models with higher C values.
- Higher C values ranging from 1 to 1000 display much less variance in the values of weights and stronger biases than when $C < 1$, and the accuracy scores also appear to converge

into 0.87788 or approximately 87.9%. This shows how the parameters appear to stabilize after a certain point, indicating the data is well separated and further increasing C has very little effect on how the SVM model computes its predictions.

- The decision boundary was computed using $\theta^T x = w_1 x_1 + w_2 x_2 + b$, which can be derived by setting $\theta^T x = 0$ (classifiers are undecided), which then allows us to solve for $x_2 = -\frac{b + w_1 x_1}{w_2}$. The data points that appear close to the decision boundary reflect how the change in variance in the different C values can affect the predicted labels.
- Similar to the Logistic Regression model in Part A (ii), w_1 is mostly negative and small, while w_2 is positive and larger, so increasing X_1 will decrease the probability of predicting +1, while increasing X_2 would increase the probability. Furthermore, $|w_1| < |w_2|$ so once again feature X_2 seems to have a greater influence on prediction.
- To conclude, C offers a unique influence in controlling the model's complexity in an inverse relationship, where the more you increase it, the more the model minimizes misclassification and gives a more complex boundary.

(iv) Comparison of Logistic Regression and SVM Models

Both the Logistic Regression model and the SVM models learn a linear decision boundary, and their orientation is very similar, especially for moderate to large C values, as well as near identical accuracy, with Logistic Regression being incredibly slightly more accurate on my data.

Although the SVM weights are scaled differently due to the hinge loss formulation, with the values themselves being smaller than the Logistic Regression model, their ratio still produces a similar boundary orientation, as well as the trend of the weight associated with X_2 being larger and thus having a greater influence on the prediction remained consistent.

As such, despite the numerical differences, both methods displayed consistency on this dataset for up to 87.9% accuracy, with Logistic Regression providing a probabilistic interpretation and SVM emphasizing margin-based decisions.

Part C

(i) Training Logistic Regression Model with squared features

The Logistic Regression model was trained with two additional features that were the squares of the original input features X_1 and X_2 to allow the model to learn using a quadratic combination of the features as $\theta^T x = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2$.

After fitting, the feature coefficients that were produced were $\theta_1 = -0.1399$, $\theta_2 = 6.82814$, $\theta_3 = 6.28563$ and $\theta_4 = -0.64524$, while the intercept is $\theta_0 = 0.33656$. The accuracy of this model compared to the actual data produced a score of 0.96296 i.e. approximately 96.3%.

The code segment that produced this was using sklearn's LogisticRegression() function, and was fitted using the given data combined with the squared features as input, and the target weights, and is present under the train_log_regr_with_sq() function.

(ii) Plotting the new Logistic Regression Model's Predictions

The trained model's predictions were plotted on top of the scatter plot produced by the provided data, with the predicted +1 labels appearing as red '+' markers and the predicted -1 labels appearing as green 'x' markers to display a comparison between the two:

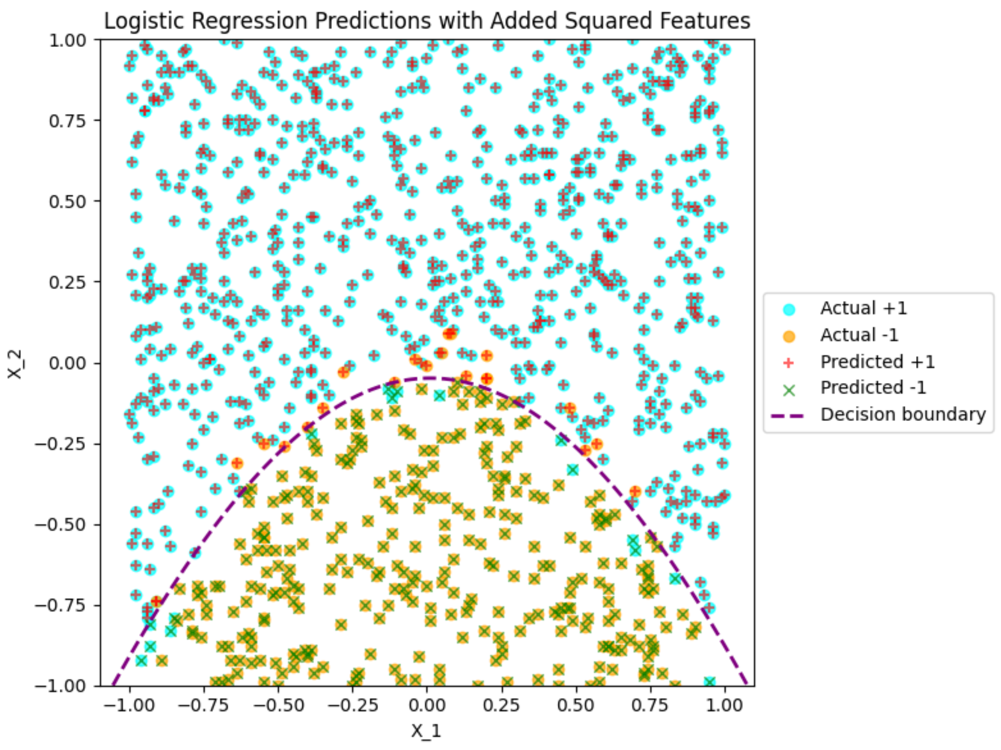


Figure 10. Scatter plot representing given data and predictions of Logistic Regression model involving squared features

The code segment that produced this was using matplotlib's scatter function with the given features and predicted values, and is present under the plot_sq_predictions() function.

The interpretation of the coefficients mentioned in Part C (i) is as follows:

- θ_1 is negative but small, which implies that increasing feature X_1 would slightly decrease the probability of predicting +1.
- θ_2 is positive and larger, which implies that increasing feature X_2 would increase the probability of predicting +1.
- θ_3 is positive and larger, which implies that increasing feature X_1^2 would increase the probability of predicting +1.

- θ_4 is negative but small, which implies that increasing feature X_2^2 would slightly decrease the probability of predicting +1.
- Based on the absolute value of the coefficients, feature X_2 would have the most influence on prediction, followed by X_1^2.

The key difference that the quadratic function introduced for the model to be more refined than the ones in Parts A and B was that the non-linear decision surface allows the classifier to better separate data that wasn't perfectly linearly separable thanks to increased flexibility. This is also reflected in the values of the parameters associated with the weights being much larger, resulting in an improved accuracy score of 96.3% compared to the 87.9% of the previous models. Across all the different models, feature X_2 consistently exhibited the strongest influence on classification, driving the decision boundary primarily along the vertical axis.

One key thing that I observed was how the squared features' coefficients reflected an inverse relationship with their linear counterparts. However, it becomes more intuitive once visualizing them on the graph:

- Data points that are classified as -1 in the given data occur in a small region where $X_2 < 0$ and $-0.85 < X_1 < 0.85$, and other points form the larger region of +1 labels.
- Points that are farther from the central point of X_1 are more likely to be classified as +1, which is reflected in the graph where the given data points at $X_1 < -0.85$ and $X_1 > 0.85$ are indeed labelled as +1.
- Points that are farther from the central point of X_2 are more likely to be classified as -1, which is reflected in the graph where the given data points at $X_2 < 0$ are mostly -1.

This also explains why we yield a curved decision boundary and gives model increased flexibility.

(iii) Comparison of Classifier against a Baseline Predictor

I coded a very trivial baseline predictor that always predicts the most common class in the training data, which was +1, and it produced an accuracy score of 0.68268 i.e. about 68.2%. This showed just how big a jump the current classifier provided in terms of accuracy, as we already experience how the model is able to linearly separate the data into two distinct regions and make predictions, but introducing non-linear features into the Logistic Regression model allows us to demonstrate how powerful it is to distinctively evaluate the decision boundary and make predictions.

This code segment that produced this was by simply calculating the total number of points in the +1 and -1 classes in the given data and then determining which of them is more commonly found and thus computing the accuracy of that set of prediction labels over the entire data set, which gave a simple accuracy evaluation. This is all present in the baseline_accuracy() function.

(iv) Describing the Decision Boundary

Using the formula stated in Part C (i): $\theta^T x = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2$, the decision boundary can be derived by setting $\theta^T x = 0$. We can solve for x_2 using the quadratic formula $ax^2 + bx + c = 0$ which allows us to solve for $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. That would turn our equation into $\theta_4 x_2^2 + \theta_2 x_2 + (\theta_0 + \theta_1 x_1 + \theta_3 x_1^2) = 0$, which would give $x_2 = \frac{-\theta_2 \pm \sqrt{\theta_2^2 - 4\theta_4(\theta_0 + \theta_1 x_1 + \theta_3 x_1^2)}}{2\theta_4}$. In code, this is achieved by developing a 2D mesh grid of the given data points, reshaping the results to match the mesh, using matplotlib's decision_function() to evaluate the model's raw output before applying the sigmoid function, and plotting that as a contour.

This code segment is already present in plot_sq_predictions() function, which was evaluated using the aforementioned methodology, and the decision boundary has been displayed in Figure 10 as a dashed purple curve.

Appendix – Code

```python
'''
Author: Priyansh Nayak
CS7CS4 Machine Learning
Week 2 Assignment
'''

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.linear_model import LogisticRegression
from sklearn.svm import LinearSVC


def plot_given_data(X, Y):
    '''
    Plot the given dataset with +1 and -1 points in different colors.
    '''
    plt.figure(figsize=(7,6))  # larger figure
    # Plot +1 points
    plt.scatter(X[Y == 1, 0], X[Y == 1, 1], marker='o', color='cyan', alpha=0.7, label='+1')

    # Plot -1 points
    plt.scatter(X[Y == -1, 0], X[Y == -1, 1], marker='o', color='orange', alpha=0.7, label='-1')

    # Labels, title, legend
    plt.xlabel("X_1")
    plt.ylabel("X_2")
    plt.title("Dataset Visualization")
    plt.legend(loc='center left', bbox_to_anchor=(1, 0.5))
    plt.tight_layout()
    plt.show()
    #plt.savefig("Week2/given_data.png")


def train_log_regr(X, Y):
    '''
    Train a Logistic Regression model on the given data and print its parameters.
    '''
    # Create and train model
    model = LogisticRegression()
    model.fit(X, Y)

    # relevent parameters
    intercept = model.intercept_[0]
    thetas = model.coef_[0]
    # make a string of all the feature coefficients
    theta_str = ", ".join(f"X{i+1} = {np.round(theta, 5)}" for i, theta in enumerate(thetas))

    accuracy = model.score(X, Y) # no. of correct predictions / total predictions

    # print model parameters
    print("\nLogistic Regression Model Parameters:")
    print(f"Feature coefficients: {theta_str}")  # theta1, theta2
    print(f"Intercept: {intercept:.5f}")        # theta0
    print(f"Accuracy of model on full dataset: {accuracy:.5f}")

    make_predictions(thetas)

    return model


def make_predictions(thetas):
    '''
    Make predictions based on feature coefficients and print interpretations.
    '''
    # interpretation of feature coefficients for predicting +1
    print("\nInterpretation:")
    for i, theta in enumerate(thetas):
        if theta > 0:
            print(f"- Feature X{i+1} increases the probability of predicting +1")
        elif theta < 0:
            print(f"- Feature X{i+1} decreases the probability of predicting +1")
        else:
            print(f"- Feature X{i+1} has no influence")

    # most influential feature
    most_influential = np.argmax(np.abs(thetas))  # index of largest |theta|
    print(f"Feature X{most_influential+1} has the most influence on prediction\n")


def plot_log_regr_predictions(X, Y, model):
    '''
    Plot the logistic regression decision boundary and the data points.
    '''
    # relevent parameters
    intercept = model.intercept_[0]
    thetas = model.coef_[0]
    theta1 = thetas[0]
    theta2 = thetas[1]
    plt.figure(figsize=(8,6))  # larger figure

    # original data points
    plt.scatter(X[Y == 1, 0], X[Y == 1, 1], marker='o', color='cyan', alpha=0.7, label='Actual +1')
    plt.scatter(X[Y == -1, 0], X[Y == -1, 1], marker='o', color='orange', alpha=0.7, label='Actual -1')

    # predicted points
    Y_pred = model.predict(X)
    plt.scatter(X[Y_pred == 1, 0], X[Y_pred == 1, 1], marker='+', color='red', alpha=0.6, linewidths=1.5,
label='Predicted +1')
    plt.scatter(X[Y_pred == -1, 0], X[Y_pred == -1, 1], marker='x', color='green', alpha=0.7, linewidths=1,
label='Predicted -1')

    # decision boundary: X2 = -(theta0 + theta1*X1)/theta2
    x_min, x_max = X[:,0].min(), X[:,0].max()
    margin = 0.05 * (x_max - x_min)  # 5% of data range as margin
    x_values = np.linspace(x_min - margin, x_max + margin, 100)
    y_values = -(intercept + theta1*x_values)/theta2
    plt.plot(x_values, y_values, linestyle='--', color='purple', linewidth=2, label='Decision boundary')

    plt.xlabel("X_1")
    plt.ylabel("X_2")
    plt.title("Training Data and Logistic Regression Predictions")
    plt.legend(loc='center left', bbox_to_anchor=(1, 0.5))
    plt.tight_layout()
    plt.show()
    #plt.savefig("Week2/logistic_regression_predictions.png")


def train_linear_svm(X, Y):
    '''
    Train a Linear SVM model on the given data and print its parameters.
    '''
    # try different C values
    C_values = [0.001, 0.01, 0.1, 1, 10, 100, 1000]
    models = {}

    # print header for the table for visual comparison
    print("Linear SVM Model Parameters for different C values:")
    print(f"{'C':>8} | {'weight_X1':>10} | {'weight_x2':>10} | {'bias':>10} | {'accuracy':>10}")
    print("-"*60)

    for C in C_values:
        model = LinearSVC(C=C, max_iter=10000)
        model.fit(X, Y)

        # extract model parameters
        weights = model.coef_[0]
        weight_X1 = weights[0]  # weight for X1
        weight_X2 = weights[1]  # weight for X2
        bias = model.intercept_[0]     # bias term
        acc = model.score(X, Y)        # accuracy on training data

        # print parameters in tabulated format for visual comparison
        print(f"{C:8.3f} | {weight_X1:10.5f} | {weight_X2:10.5f} | {bias:10.5f} | {acc:10.5f}")

        models[C] = model

    return models


def plot_svm_predictions(X, Y, svm_models):
    '''
    Plot the SVM decision boundaries and the data points.
    '''
    for C, model in svm_models.items():
        # relevent parameters
        weights = model.coef_[0]
        weight_X1 = weights[0]
        weight_x2 = weights[1]
        bias = model.intercept_[0]

        plt.figure(figsize=(8,6)) # larger figure

        # original data points
        plt.scatter(X[Y == 1, 0], X[Y == 1, 1], marker='o', color='cyan', alpha=0.7, label='Actual +1')
        plt.scatter(X[Y == -1, 0], X[Y == -1, 1], marker='o', color='orange', alpha=0.7, label='Actual -1')

        # predicted points
        Y_pred = model.predict(X)
        plt.scatter(X[Y_pred == 1, 0], X[Y_pred == 1, 1], marker='+', color='red', alpha=0.6, linewidths=1.5,
label='Predicted +1')
        plt.scatter(X[Y_pred == -1, 0], X[Y_pred == -1, 1], marker='x', color='green', alpha=0.7, linewidths=1,
label='Predicted -1')

        # decision boundary: X2 = -(bias + weight_X1*X1)/weight_x2
        x_min, x_max = X[:,0].min(), X[:,0].max()
        margin = 0.05 * (x_max - x_min)  # 5% of data range as margin
        x_values = np.linspace(x_min - margin, x_max + margin, 100)
        y_values = -(bias + weight_X1 * x_values) / weight_x2
        plt.plot(x_values, y_values, linestyle='--', color='purple', linewidth=2, label='Decision boundary')

        plt.xlabel("X_1")
        plt.ylabel("X_2")
        plt.title(f"Linear SVM Predictions (C={C})")
        plt.legend(loc='center left', bbox_to_anchor=(1, 0.5))
        plt.tight_layout()
        plt.show()
        #plt.savefig(f"Week2/svm_predictions_C_{C}.png")


def train_log_regr_with_sq(X_sq, Y):
    '''
    Train a Logistic Regression model with squared features on the given data and print its parameters.
    '''
    # create and train model with extended features
    model = LogisticRegression()
    model.fit(X_sq, Y)

    # relevent parameters
    intercept = model.intercept_[0]
    thetas = model.coef_[0]
    # make a string of all the feature coefficients
    theta_str = ", ".join(f"X{i+1} = {np.round(theta, 5)}" for i, theta in enumerate(thetas))

    accuracy = model.score(X_sq, Y) # no. of correct predictions / total predictions

    # print model parameters
    print("\nLogistic Regression with Squared Features Model Parameters:")
    print(f"Feature coefficients: {theta_str}")  # theta1, theta2, theta3, theta4
    print(f"Intercept: {intercept:.5f}")        # theta0
    print(f"Accuracy of model on full dataset: {accuracy:.5f}")

    make_predictions(thetas)

    return model


def plot_sq_predictions(X, Y, X_sq, model):
    '''
    Plot the logistic regression decision boundary and the data points.
    '''
    plt.figure(figsize=(8,6))  # larger figure
    # original data points
    plt.scatter(X[Y == 1, 0], X[Y == 1, 1], marker='o', color='cyan', alpha=0.7, label='Actual +1')
    plt.scatter(X[Y == -1, 0], X[Y == -1, 1], marker='o', color='orange', alpha=0.7, label='Actual -1')

    # predicted points
    Y_pred = model.predict(X_sq)
    plt.scatter(X[Y_pred == 1, 0], X[Y_pred == 1, 1], marker='+', color='red', alpha=0.6, linewidths=1.5,
label='Predicted +1')
    plt.scatter(X[Y_pred == -1, 0], X[Y_pred == -1, 1], marker='x', color='green', alpha=0.7, linewidths=1,
label='Predicted -1')

    # decision boundary: theta0 + theta1*X1 + theta2*X2 + theta3*X1^2 + theta4*X2^2 = 0
    # nonlinear boundary, so we need to plot a contour
    x_min, x_max = X[:, 0].min(), X[:, 0].max()
    y_min, y_max = X[:, 1].min(), X[:, 1].max()

    # 5% of data range as margin
    x_margin = 0.05 * (x_max - x_min)
    x_min, x_max = x_min - x_margin, x_max + x_margin

    # create a grid of points
    xx, yy = np.meshgrid(np.linspace(x_min, x_max, 200), np.linspace(y_min, y_max, 200))

    # compute the decision on a grid
    X_sq_grid = np.column_stack((xx.ravel(), yy.ravel(), xx.ravel()**2, yy.ravel()**2))
    Z = model.decision_function(X_sq_grid)
    Z = Z.reshape(xx.shape)

    # plot decision boundary and margins
    plt.contour(xx, yy, Z, levels=[0], colors='purple', linestyles='--', linewidths=2)

    # adding proxy line because contour doesn't create a legend entry
    plt.plot([], [], color='purple', linestyle='--', linewidth=2, label='Decision boundary')

    plt.xlabel("X_1")
    plt.ylabel("X_2")
    plt.title("Logistic Regression Predictions with Added Squared Features")
    plt.legend(loc='center left', bbox_to_anchor=(1, 0.5))
    plt.tight_layout()
    plt.show()
    #plt.savefig("Week2/logistic_regression_sq_predictions.png")


def baseline_accuracy(Y):
    '''
    Compute and print the baseline accuracy of a model that always predicts the most common class.
    '''
    most_common = np.sign((Y == 1).sum() - (Y == -1).sum()) # +1 if more +1s, -1 if more -1s
    baseline_pred = np.full_like(Y, most_common) # figure out which label is more common
    acc = np.mean(baseline_pred == Y)
    print(f"Baseline (always predicts {most_common}): {acc:.5f}")
    return acc


def main():
    # path for csv file
    csv_path = "Week2/week2.csv"
    print("test")

    # eoad CSV, skip comment line, no header
    df = pd.read_csv(csv_path, comment="#", header=None)
    print(df.head(), end="\n\n")  # preview data

    # extract features and labels
    X1 = df.iloc[:,0] # feature 1
    X2 = df.iloc[:,1] # feature 2
    X = np.column_stack((X1, X2))  # first 2 columns as 2D array
    Y = df.iloc[:, 2] # third column as labels

    plot_given_data(X, Y)

    # train Logistic Regression
    log_model = train_log_regr(X, Y)
    plot_log_regr_predictions(X, Y, log_model)

    # train SVM
    svm_models = train_linear_svm(X, Y)
    plot_svm_predictions(X, Y, svm_models)

    # train Logistic Regression with new features
    # extend features for prediction
    X_sq = np.column_stack((X1, X2, X1**2, X2**2))
    sq_model = train_log_regr_with_sq(X_sq, Y)
    plot_sq_predictions(X, Y, X_sq, sq_model)

    baseline_accuracy(Y) # compare against trivial accuracy


main()
```

Expected Text Output

```
Logistic Regression Model Parameters:
Feature coefficients: X1 = -0.05339, X2 = 5.15001
Intercept: 1.69049
Accuracy of model on full dataset: 0.87888

Interpretation:
- Feature X1 decreases the probability of predicting +1
- Feature X2 increases the probability of predicting +1
Feature X2 has the most influence on prediction

Linear SVM Model Parameters for different C values:
       C |  weight_X1 |  weight_x2 |       bias |   accuracy
------------------------------------------------------------
   0.001 |    0.00040 |    0.48143 |    0.23188 |    0.85686
   0.010 |   -0.01701 |    1.19233 |    0.38027 |    0.87888
   0.100 |   -0.02430 |    1.72052 |    0.55092 |    0.87888
   1.000 |   -0.02498 |    1.86583 |    0.59995 |    0.87788
  10.000 |   -0.02524 |    1.88392 |    0.60616 |    0.87788
 100.000 |   -0.02527 |    1.88578 |    0.60679 |    0.87788
1000.000 |   -0.02527 |    1.88597 |    0.60685 |    0.87788

Logistic Regression with Squared Features Model Parameters:
Feature coefficients: X1 = -0.13999, X2 = 6.82814, X3 = 6.28563, X4 = -0.64524
Intercept: 0.33656
Accuracy of model on full dataset: 0.96296

Interpretation:
- Feature X1 decreases the probability of predicting +1
- Feature X2 increases the probability of predicting +1
- Feature X3 increases the probability of predicting +1
- Feature X4 decreases the probability of predicting +1
Feature X2 has the most influence on prediction

Baseline (always predicts 1): 0.68268
```