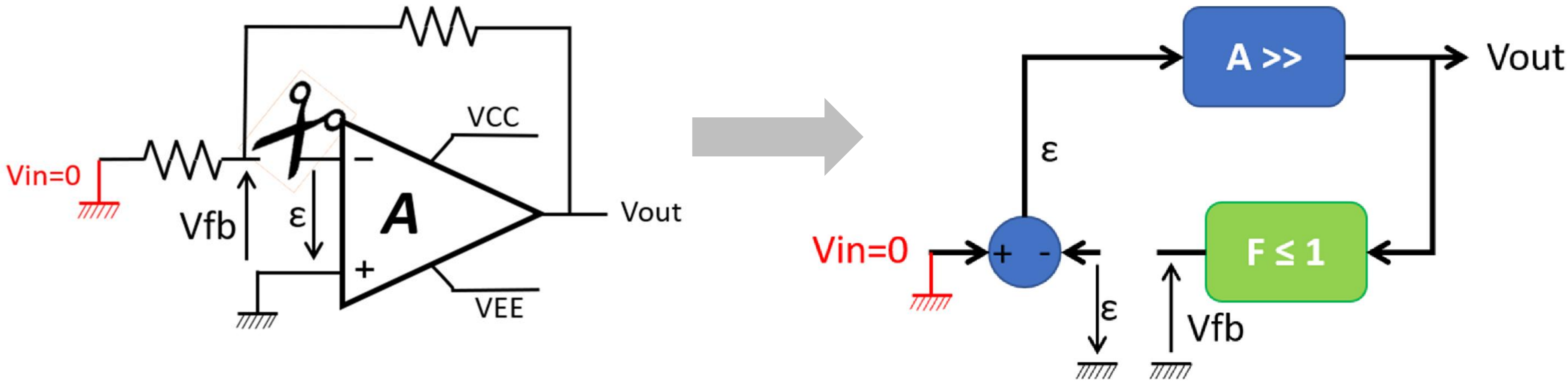
ANALOG CIRCUITS DESIGN

AE4: Operational Amplifiers III, Stability

Course overview

1. Basic concepts:
   - The condition for oscillation
   - The phase margin concept
2. 1^{st} order circuits
3. 2^{nd} order circuits
   - Optimal phase margin
   - Unity-gain stabe OPA
4. Higher order circuits
   - overview
   - Practical examples 1: The inverting/non-inverting amplifier
   - Practical examples 2: The voltage follower
5. Oscillators
   - What for?
   - Relaxation oscillators
   - Harmonic quartz crystal oscillators
6. Appendix A
7. Appendix B
8. Appendix C

- Stability is evaluated in open loop, without signal



- Loop gain: $\frac{Vfb}{\varepsilon} = AF$

$|AF| > 1$ and $\varphi_{AF} \ll 180°$ → stable (negative feedback)

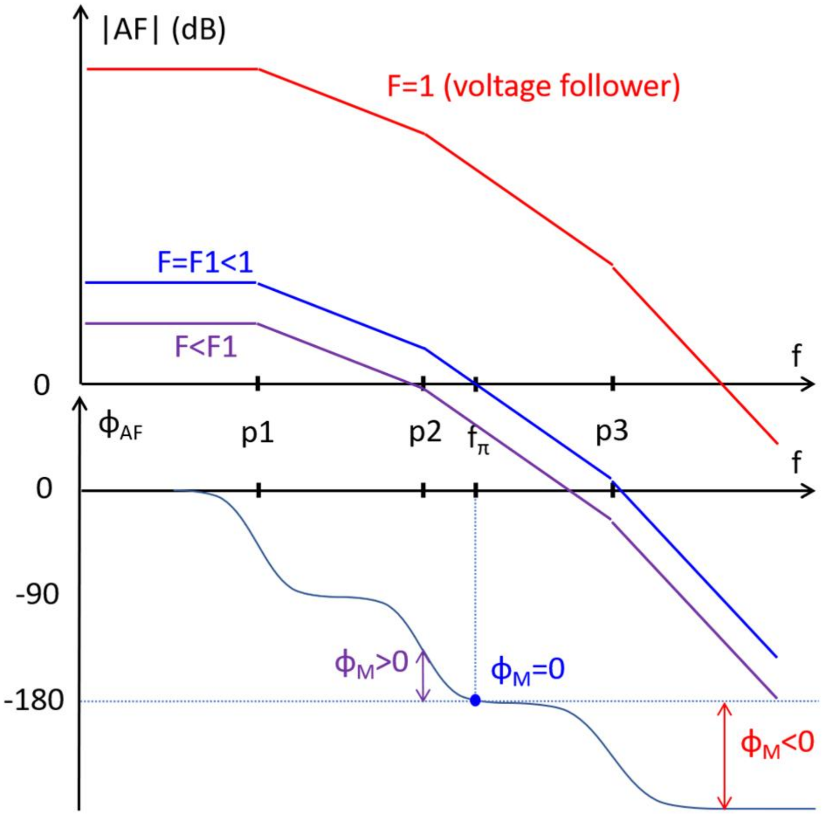
$|AF| = 1$ and $\varphi_{AF} = 180°$ → harmonic oscillator (Barkhausen criterion)

$|AF| \ll 1$ and $\varphi_{AF} > 180°$ → stable

1. Basic concepts: The phase margin oncept

- Stability is ensured if φ_{AF} is « sufficiently far away » from 180° when |AF| = 1

- Phase margin φ_M: difference between circuit phase φ_{AF} and 180°



Example: 3-pole circuit with different values of feedback factor F

- F=1 (worst case): phase margin is negative → UNSTABLE
- F=F1: phase margin is zero, gain is 0dB → OSCILLATOR @ $f=f_\pi$
- F<F1: phase margin is positive, → STABLE

- A maximum value for F (i.e. a minimum closed-loop gain) exists to ensure stability

- How much phase margin is necessary?

- When a pole (p_1) is really dominant (other poles occur when gain is less than one)

- Open-loop gain: $A(s)F \approx \dfrac{A_0\,F}{1+\dfrac{s}{p_1}}$

  (A_0: value of A when f=0)

- One single pole → $\varphi_{AF} \leq 90°$ UNCONDITIONALLY STABLE

- Closed-loop gain: $H(s) \approx \dfrac{A_0}{(1+A_0F)\left(1+\dfrac{s}{(1+A_0F)p_1}\right)} = \dfrac{H_0}{1+\dfrac{s}{p'_1}}$

- Gain-Bandwidth product (GBW) is constant:

$$GBW = H_0\,p'_1 = \frac{A_0}{(1+A_0F)}(1+A_0F)p_1 = A_0\,p_1 = cte$$

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3. 2nd order circuits

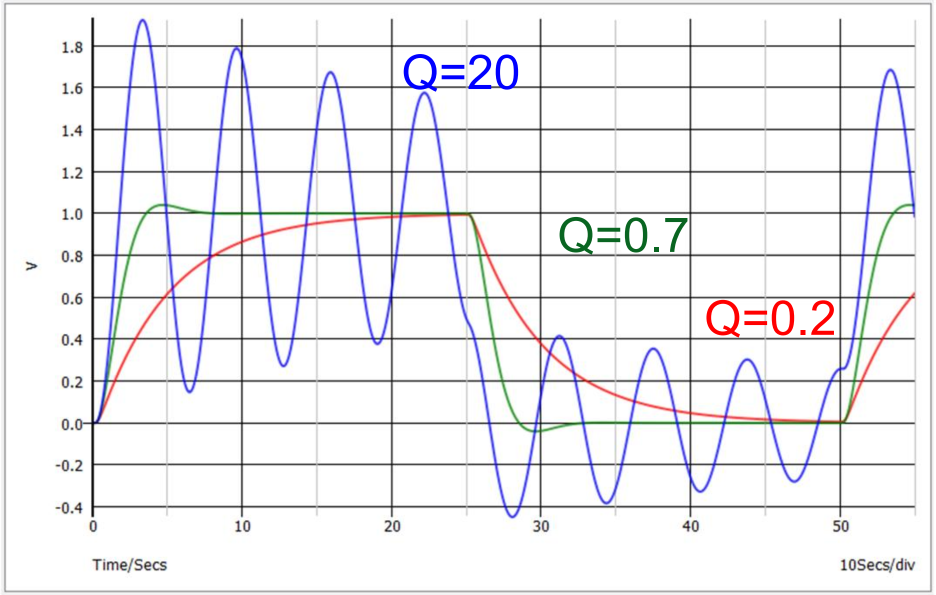
- When two poles (p_1, p_2) exist, p1 is not really dominant

- Open-loop gain: $A(s)F \approx \dfrac{A_0\,F}{\left(1+\dfrac{s}{p_1}\right)\left(1+\dfrac{s}{p_2}\right)}$

- Two poles → φ_{AF} reaches -180° when f = ∞ → MATHEMATICALLY STABLE but….

- Closed-loop gain (ω_p=1rd/s)*:

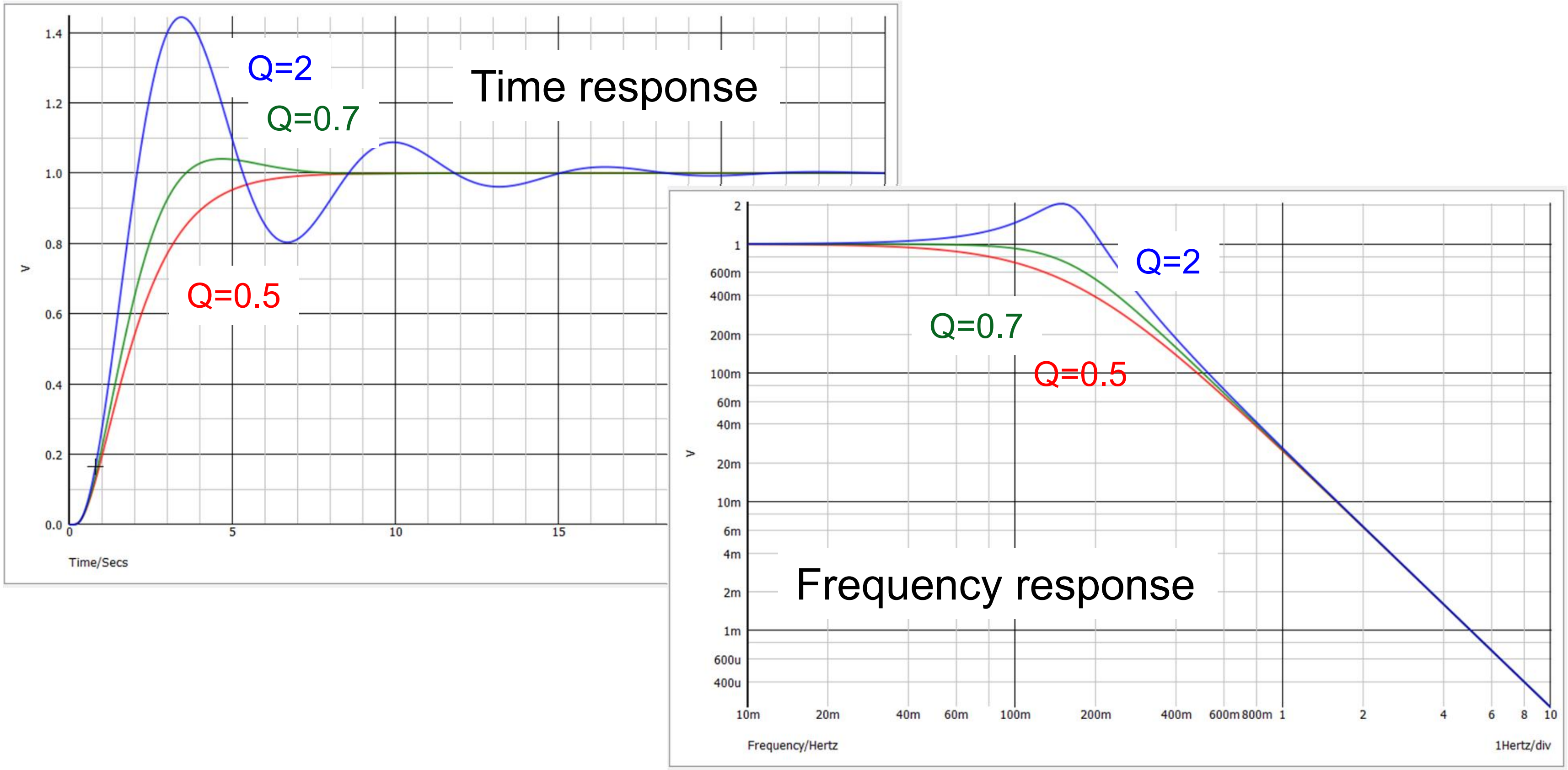
$$H(s) = \frac{H_0}{s^2 + s\dfrac{1}{Q} + 1}$$



Time response may exhibit unacceptable overshoots or oscillations

*: see details in appendix A

3. 2nd order circuits: optimal phase margin

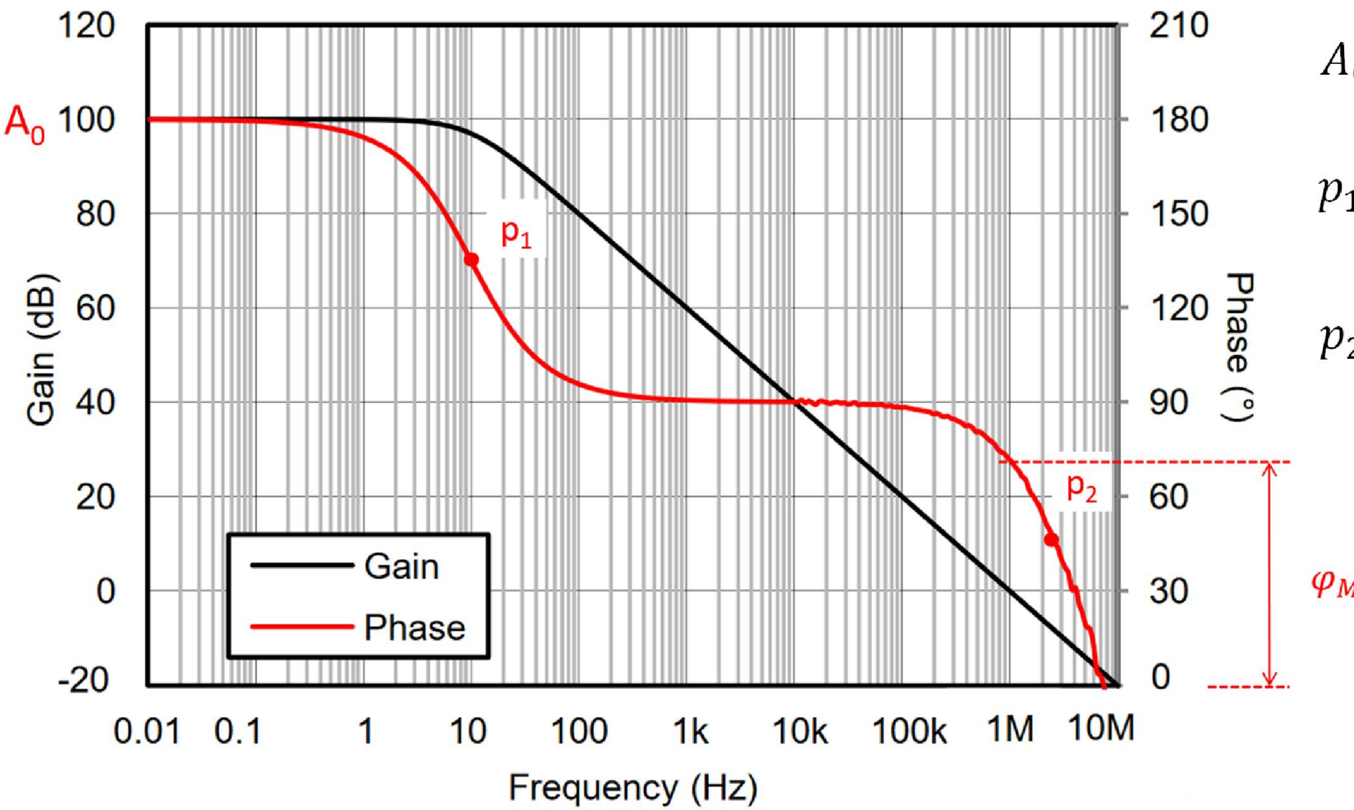


- Optimal response → best trade-off between time and frequency response
- Obtained for (see details in appendix B):

$$\boxed{Q = \frac{\sqrt{2}}{2} \leftrightarrow p_2 = 2A_0Fp_1 \leftrightarrow \varphi_M \approx 60°}$$

3. 2nd order circuits: unity-gain stable OPA

- Worst case for stability: the voltage follower (G =1)
- Corollary: if an OPA is stable for G=1, it is also for any other practical value of G
- WARNING: all OPAs are not unity-gain stable



$A_0 = 100dB \rightarrow 10^5\ {}^{v}/_{v}$

$p_1(@-45°) \approx 10Hz$

$p_2(@-135°) > 2MHz$

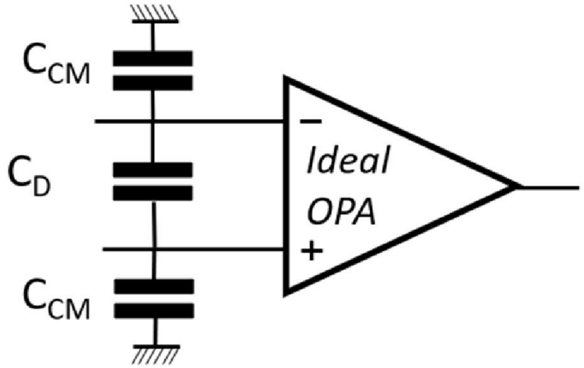
$p_2 > 2A_0p_1$
$(unity-gain\ satble)$

$\varphi_M \approx 60°$

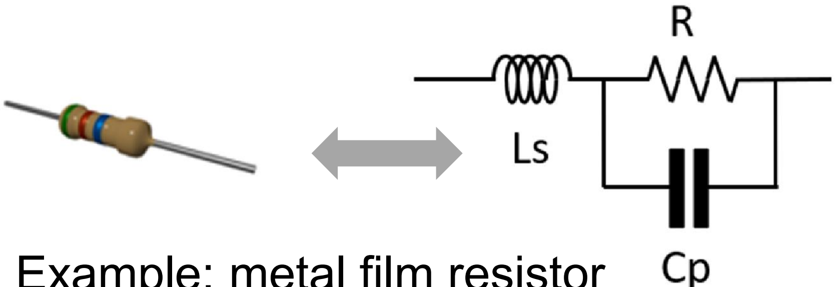
4. Higher order circuits: overview

- More than two poles → phase will cross -180° → POTENTIAL UNSTABILITY

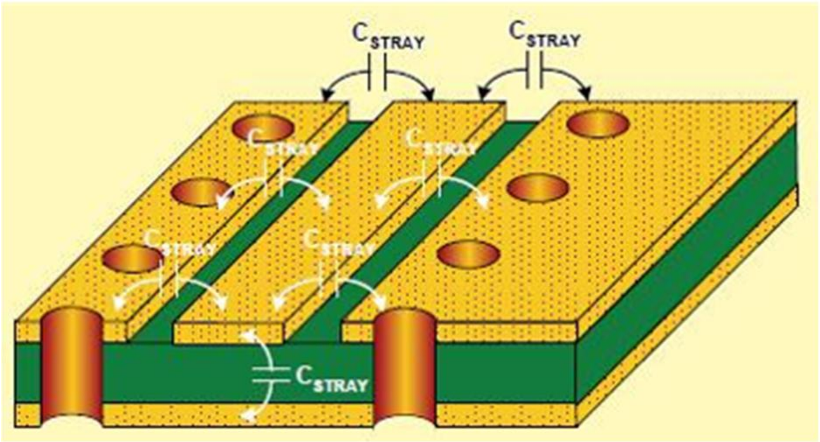
- OPAs are two-pole circuits, so where does the third pole come from?
  - Parasitic OPA capacitors
  - Stray capacitance of external components
  - Parasitic printed-circuit board (PCB) tracks capacitors



INPUT IMPEDANCE	Example: OPA1652	
Differential	100 || 6	MΩ || pF
Common-mode	6000 || 2	GΩ || pF



Example: metal film resistor
R=1MΩ, Ls=5nH, Cp=0.5pF

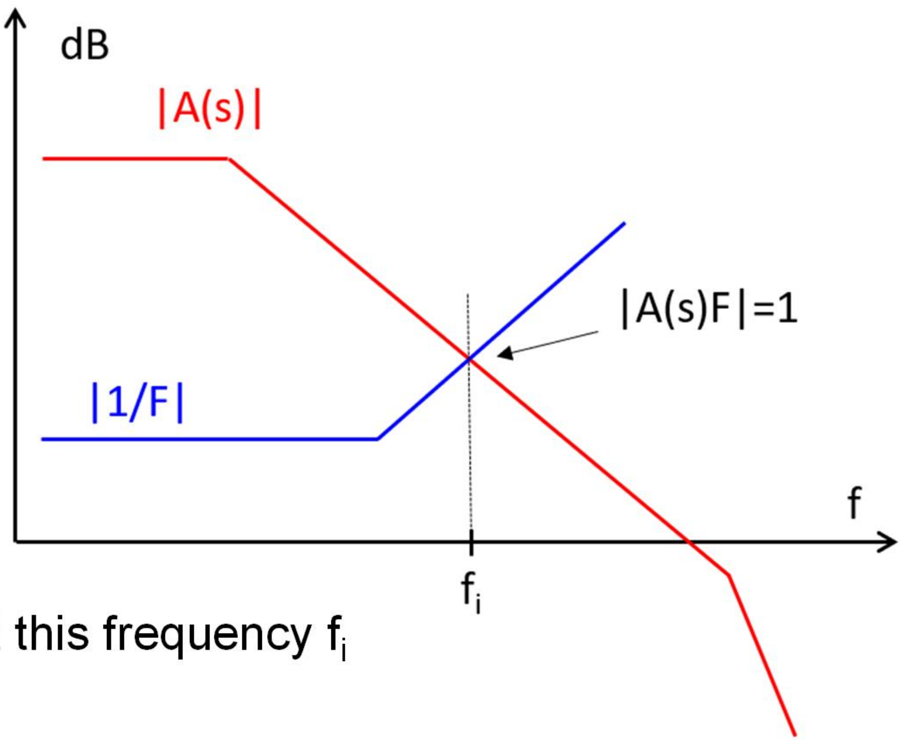


Example: FR4, 1.6mm thickness
$C_{STRAY} \approx 3pF/cm^2$

4. Higher order circuits: overview

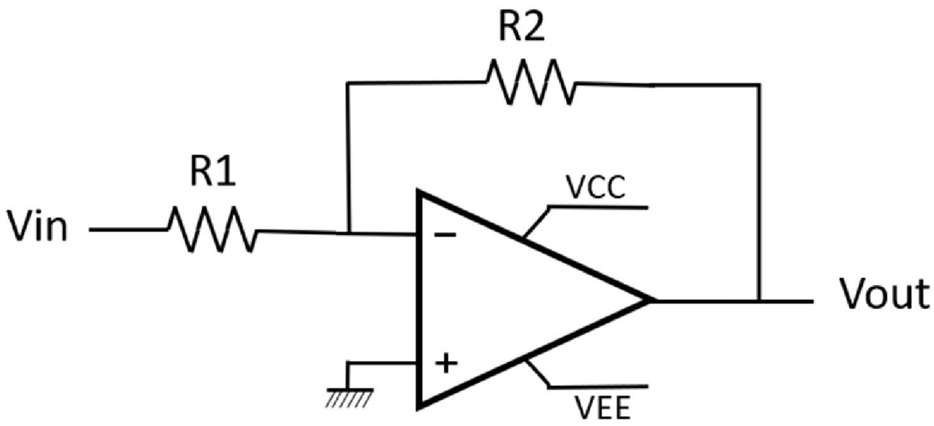
- Oscillation occurs if $|A(s)\ F| = 1$ and $\varphi_{AF}=0$ → what phase margin do we have?

- Stability analysis methodology:
  1. Include all parasitic components in the schematic, open the loop
  2. Isolate the F network, it will contain the parasitic components
  3. Determine the characteristics of the F network
  4. Determine at which frequency f_i $|A(s)\ F| = 1$

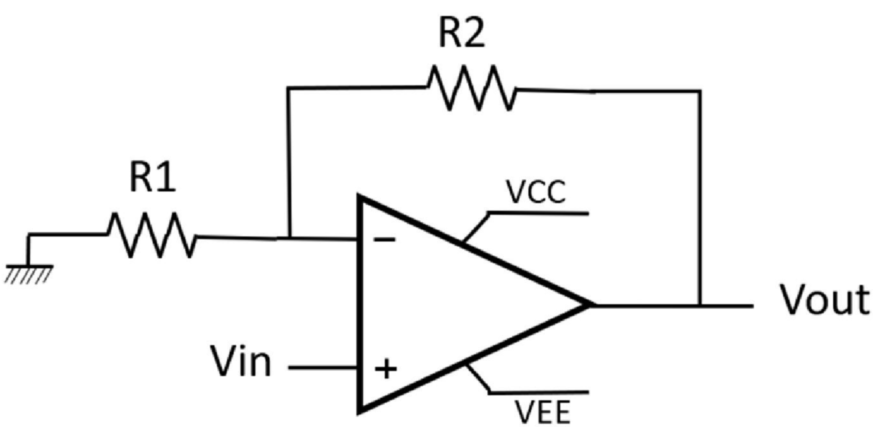


  5. Determine phase margin at this frequency f_i
  6. Conclude on stability

4. Higher order circuits: Practical example 1: the inverting/non-inverting amplifier

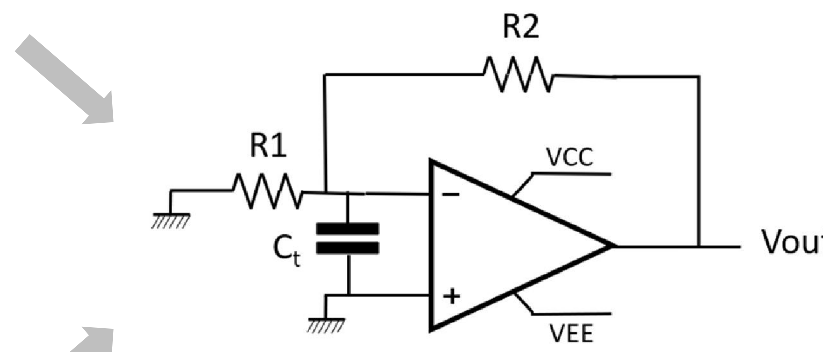


Inverting amplifier



Non-inverting amplifier

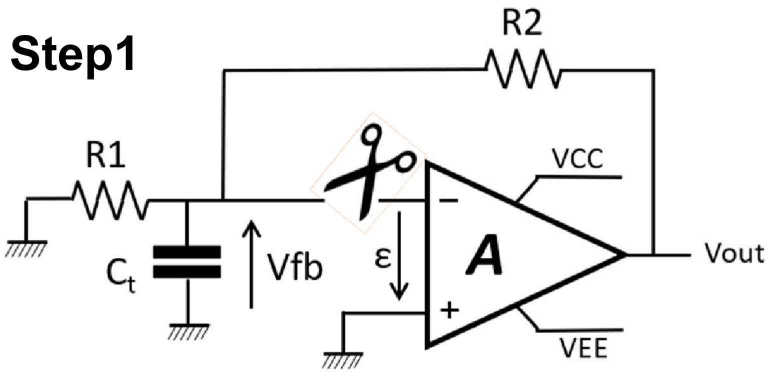
No signal for stability analysis



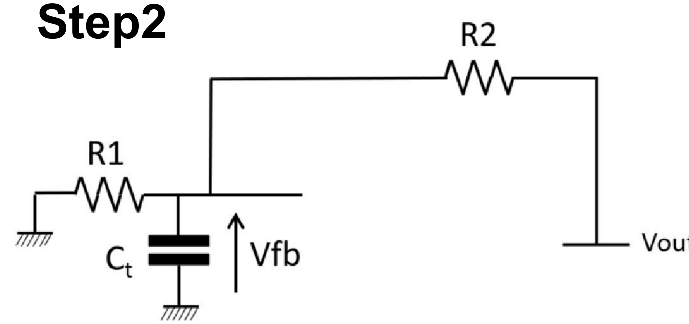
$$C_t = C_D + C_{CM-} + \sum other\ parasitics *$$

*: parasitics due to printed-circuit board (PCB) traces among other things

Step1



Step2



Example:
R1 = 22kΩ
R2 = 220kΩ

C_t = 10pF

OPA:
A_0 = 80dB
GBW = 20MHz
Unity-gain stable

$\left.\right\}$ p_1 = 2kHz
p_2 > 40MHz

Step3

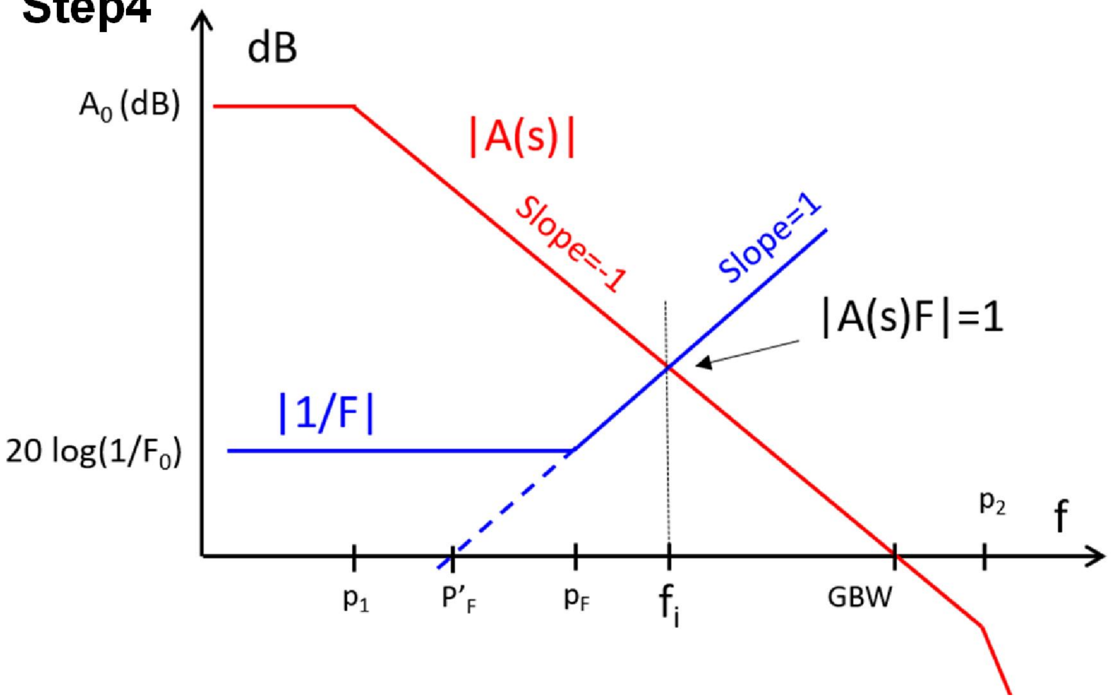
$$F = \frac{Vfb}{Vout} = \underbrace{\frac{R1}{R1 + R2}}_{F_0} \frac{1}{1 + \underbrace{(R1||R2)C_t\, s}_{p_F}}$$

$$F_0 = \frac{1}{11}$$

$$p_F = \frac{1}{2\pi(R1||R2)C_t} \approx 796kHz$$

4. Higher order circuits: Practical example 1: the inverting/non-inverting amplifier

Step4



Because both slopes equal 1:

$$p'_F = p_F\,F_0 = 72.4\text{kHz}$$

$$\frac{f_i}{p'_F} = \frac{GBW}{f_i} \rightarrow f_i = \sqrt{p'_F\,GBW}$$

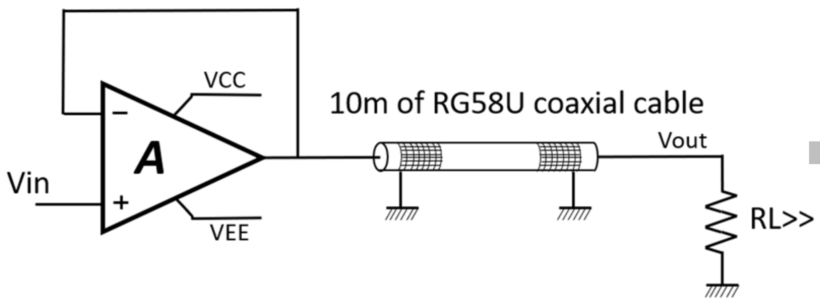
$$f_i = 1.2MHz$$

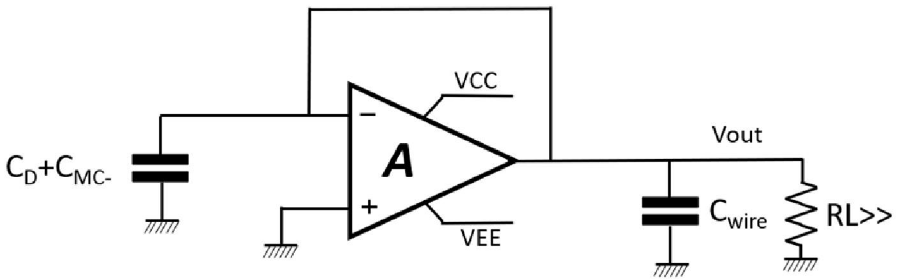
Step5 Amplifier phase shift: $\varphi_A = -90 - \tan^{-1}\frac{f_i}{p_2} = -93.4°$

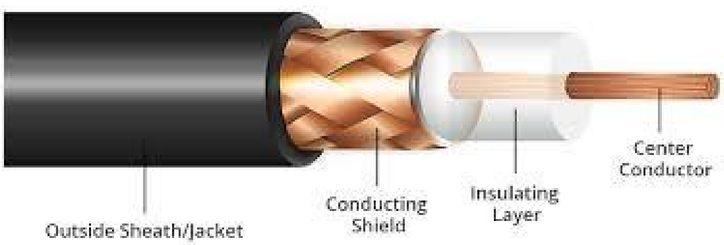
F block phase shift: $\varphi_F = -\tan^{-1}\frac{f_i}{p_F} = -56.3°$

$\varphi_M = (\varphi_A + \varphi_F) - (-180°) = 37°$ → requires compensation → Reduce R1, R2
Reduce C_t

4. Higher order circuits: Practical example 2: the voltage follower





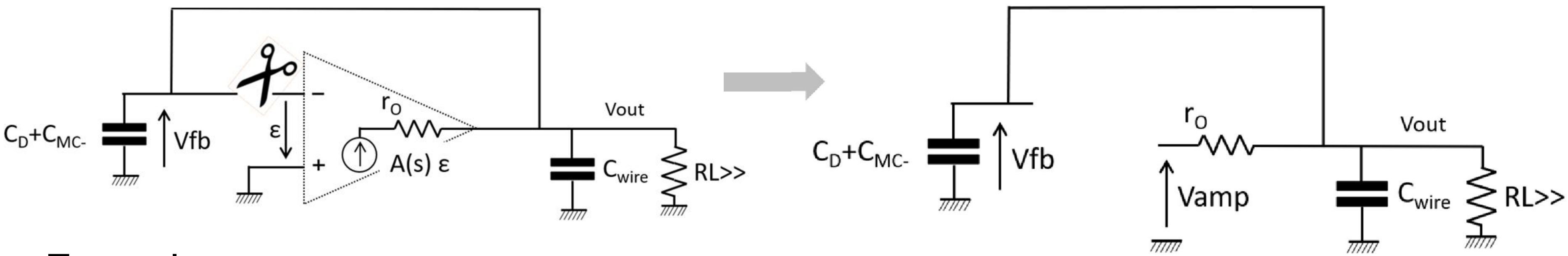


Example:
RG58U → C = 100pF/m

C_{wire} = 10nF

Step1

Step2



Example:
RG58U → C = 100pF/m

C_{wire} = 1nF

OPA:
A_0 = 80dB
GBW = 20MHz
Unity-gain stable
r_0 = 50Ω
$C_D + C_{MC-}$ = 10pF

$\left.\begin{matrix} A_0 = 80\text{dB} \\ \text{GBW} = 20\text{MHz} \\ \text{Unity-gain stable} \end{matrix}\right\}$ p_1 = 2kHz, p_2 > 40MHz

Step3

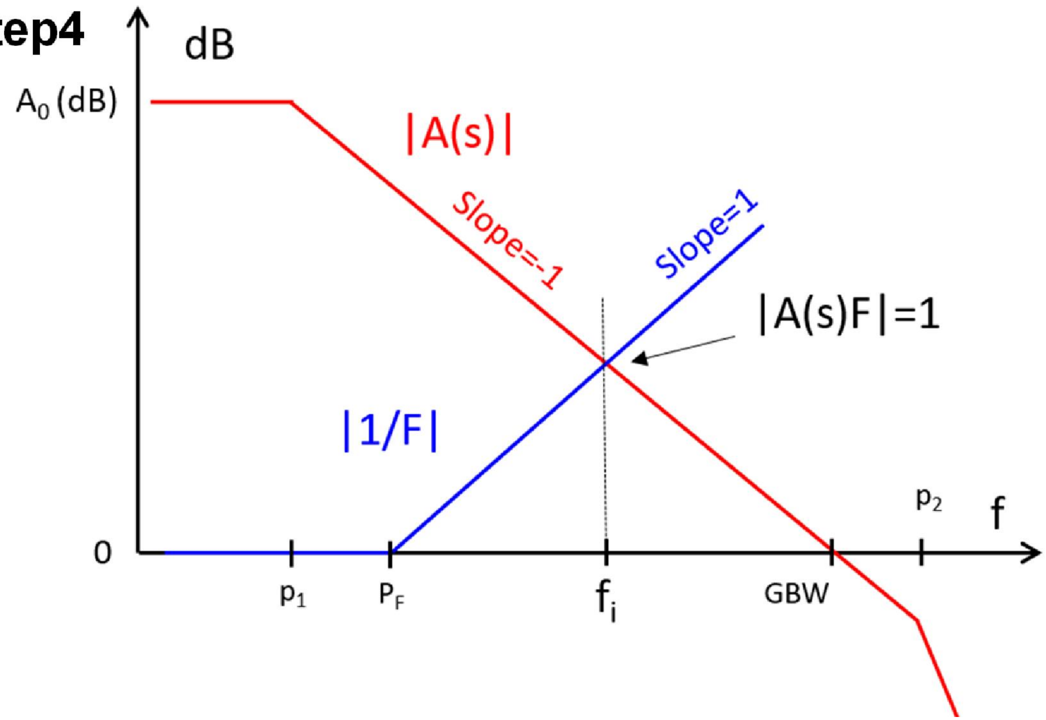
$$F = \frac{Vfb}{Vamp} = \underbrace{\frac{RL}{r_0 + RL}}_{F_0} \frac{1}{\underbrace{1 + (r_0||RL)C_t\, s}_{p_F}}$$

$$C_t = C_D + C_{MC-} + C_{wire} \approx C_{wire}$$

$$F_0 \approx 1$$

$$p_F = \frac{1}{2\pi(r_0||RL)C_t} \approx 3{,}18MHz$$

Step4



$$\frac{f_i}{p_F} = \frac{GBW}{f_i} \rightarrow f_i = \sqrt{p_F\, GBW}$$

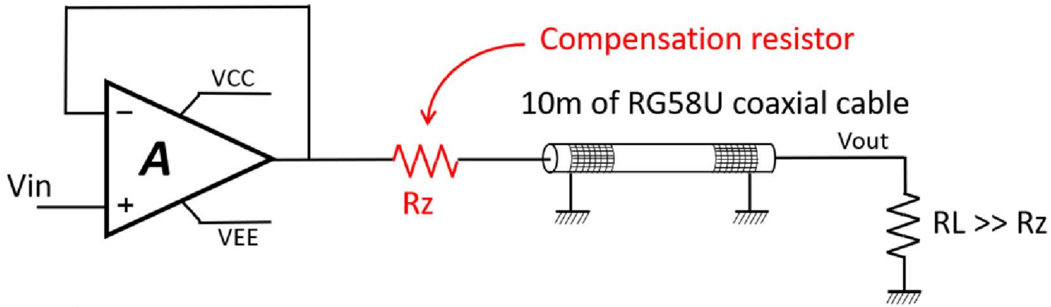
$$f_i \approx 8MHz$$

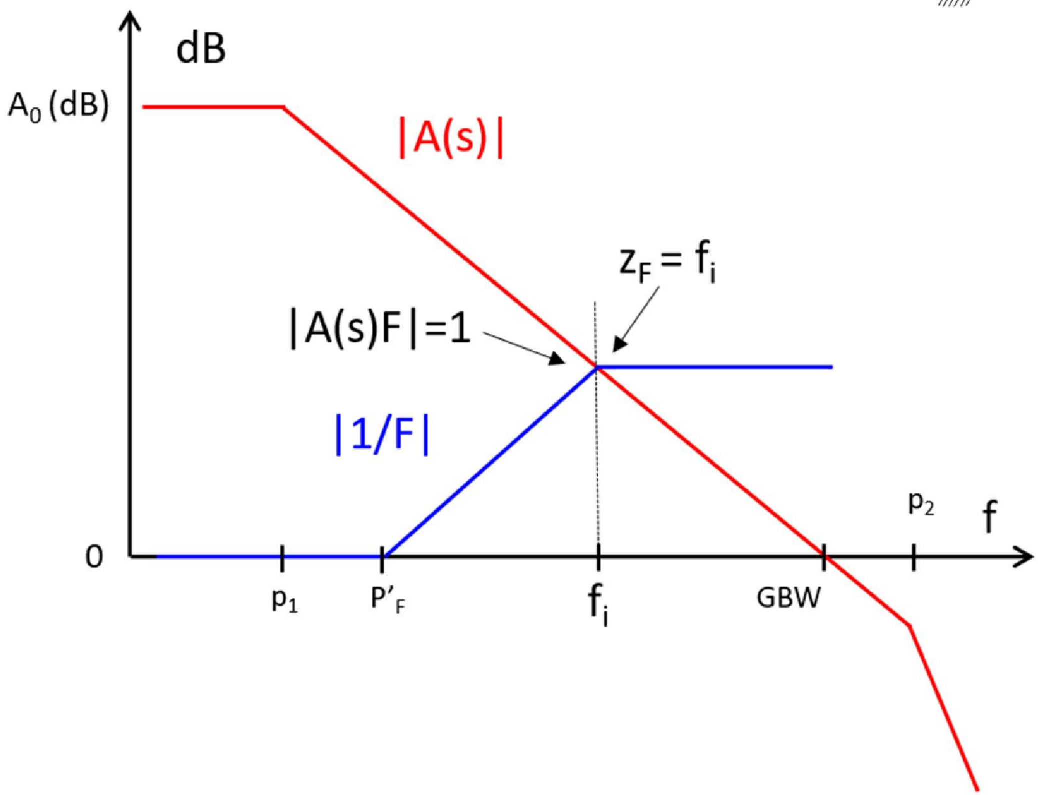
Step5 Amplifier phase shift: $\varphi_A = -90 - \tan^{-1}\frac{f_i}{p_2} = -101.3°$

F block phase shift: $\varphi_F = -\tan^{-1}\frac{f_i}{p_F} = -68.3°$

$\varphi_M = (\varphi_A + \varphi_F) - (-180°) \approx 10° \rightarrow$ requires compensation

4. Higher order circuits: Practical example 2: the voltage follower





Now:

$$F = \frac{Vfb}{Vamp} \approx \frac{\overbrace{Rz\, C_t\ s}^{z_F}}{\underbrace{1 + (r_0 + Rz)C_t\, s}_{p'_F}}$$

$$F_0 \approx 1$$

$$p'_F = \frac{1}{2\pi(r_0 + Rz)C_t}$$

$$z_F = \frac{1}{2\pi\, Rz\, C_t}$$

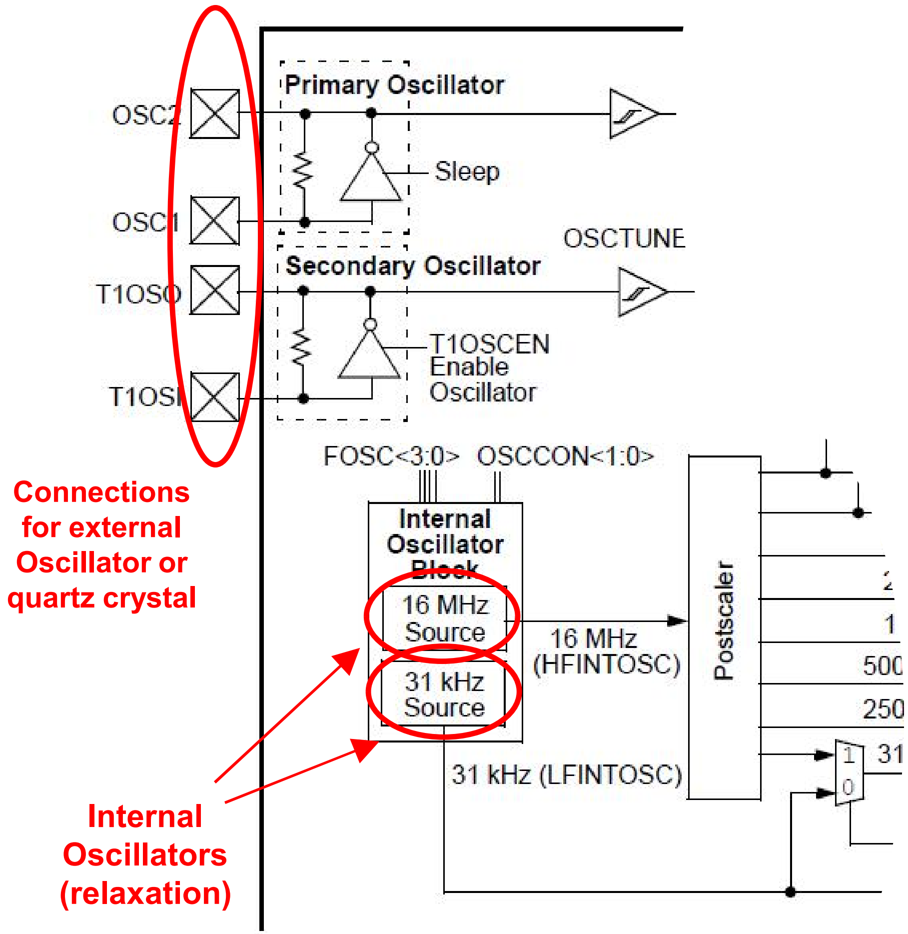
If z_F is set to the previous f_i value:

$$\varphi_M \approx 50°$$

(see details in appendix C)

5. Oscillators: What for?

Example: the PIC18F45K20 clock circuit



- Used whenever a signal with an accurate frequency value is required.

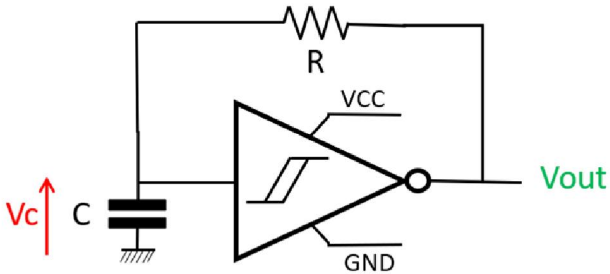
- Main features required:
  - Accurate frequency
  - Stable frequency
  - Spectral purity
  - Stable amplitude

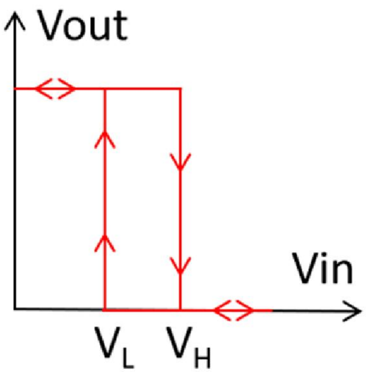
- Two main techniques:
  - Relaxation oscillators
  - Harmonic oscillators

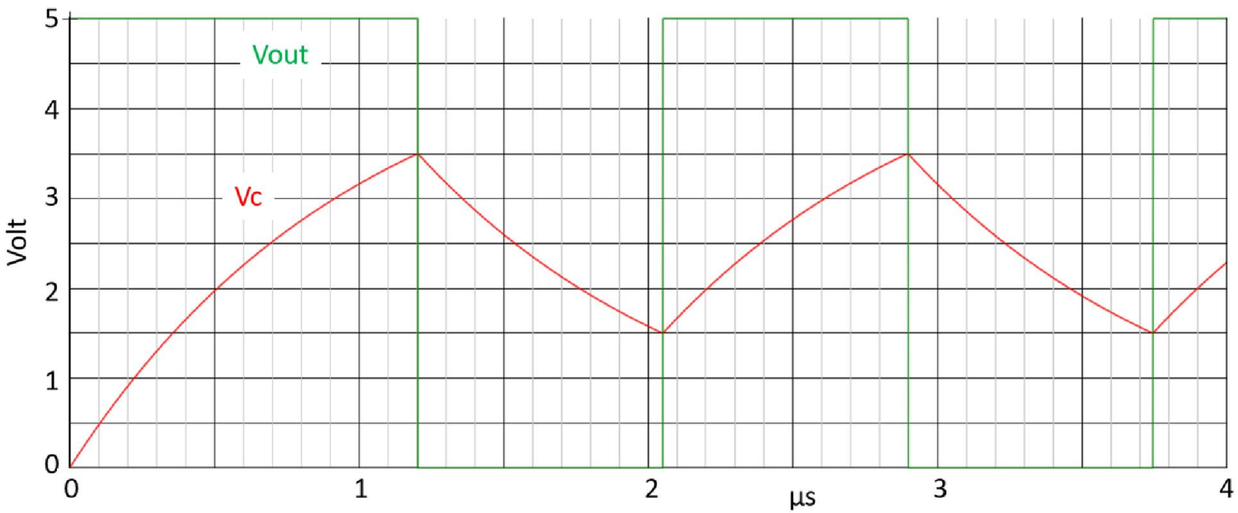
5. Oscillators: Relaxation oscillators

- Based on the charge and discharge of a capacitor between two known thresholds (comparator with hysteresis).

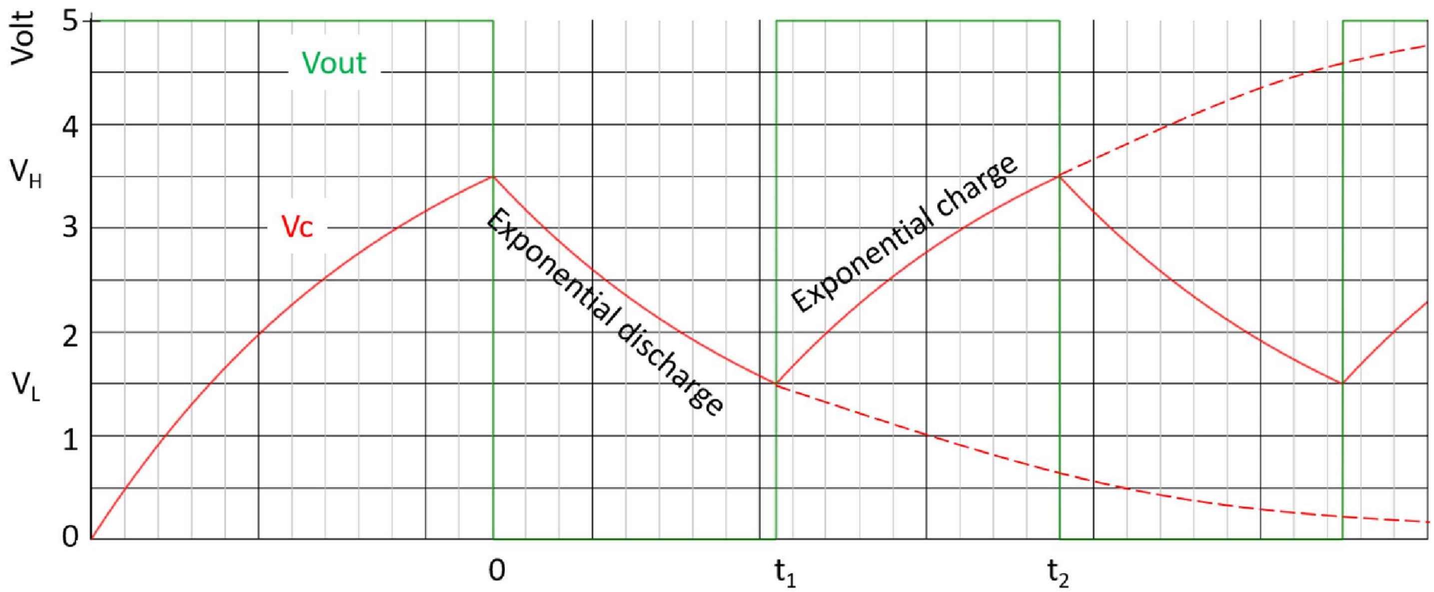
- Charge/discharge can be at constant voltage (exponential variation) or constant current (linear variation)







5. Oscillators: Relaxation oscillators



Discharge: $$V_L = V_H\, e^{\frac{-t}{RC}} \rightarrow t_1 = RC\ \ln\frac{V_H}{V_L}$$

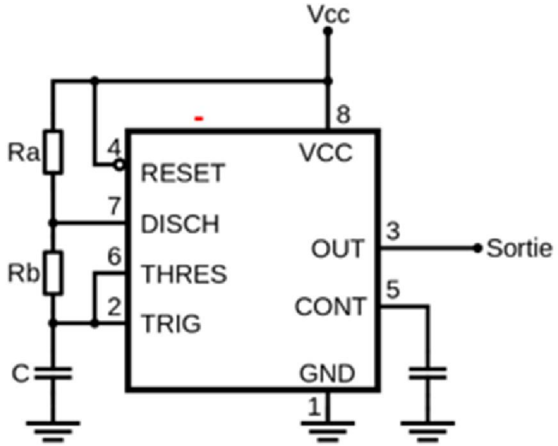
Charge: $$V_H - V_L = (VCC - V_L)(1 - e^{\frac{-t}{RC}}) \rightarrow t_2 - t_1 = RC\ \ln\frac{VCC - V_L}{VCC - V_H}$$

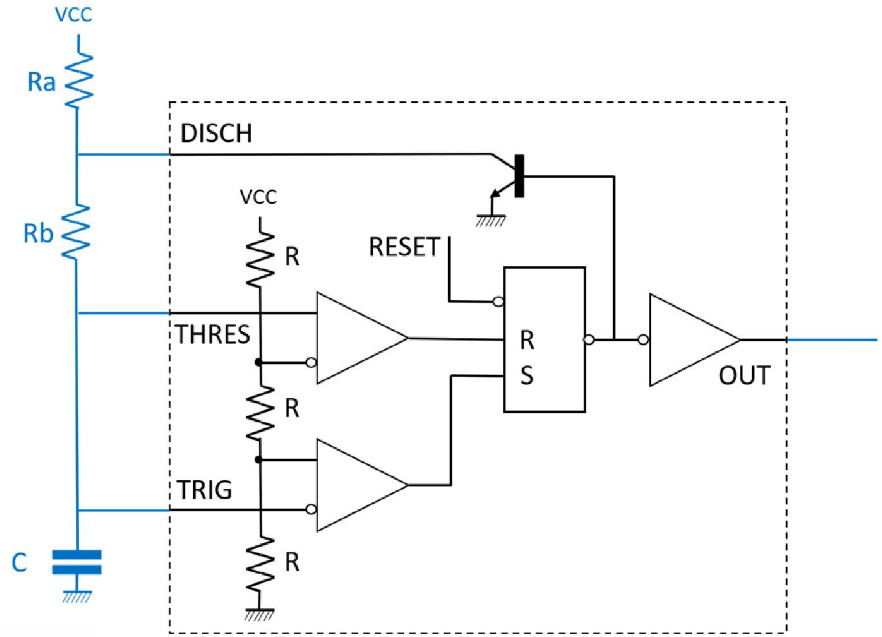
$$T = RC(\ln\frac{V_H}{V_L} + \ln\frac{VCC-V_L}{VCC-V_H})$$

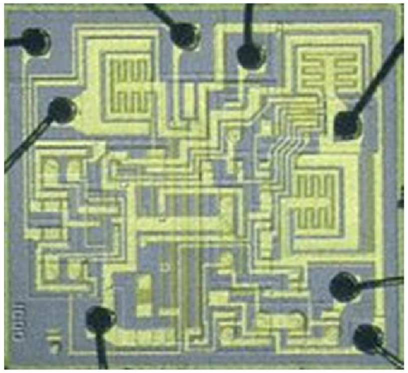
This example: 1.695µs with R=1kΩ, C=1nF

5. Oscillators: Relaxation oscillators

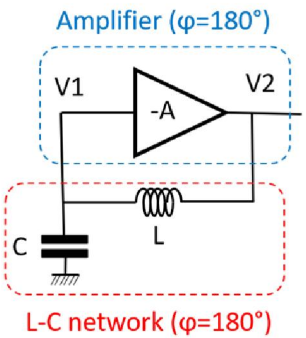
Industrial circuit: the 555 timer







5. Oscillators: Harmonic quartz crystal oscillators



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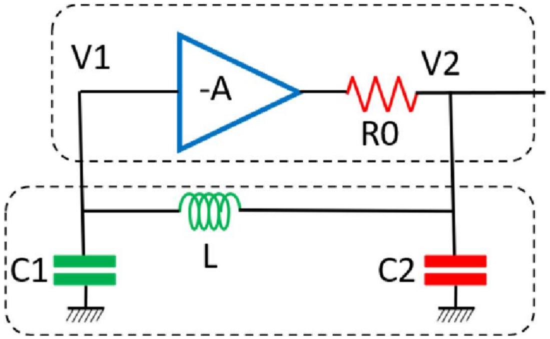
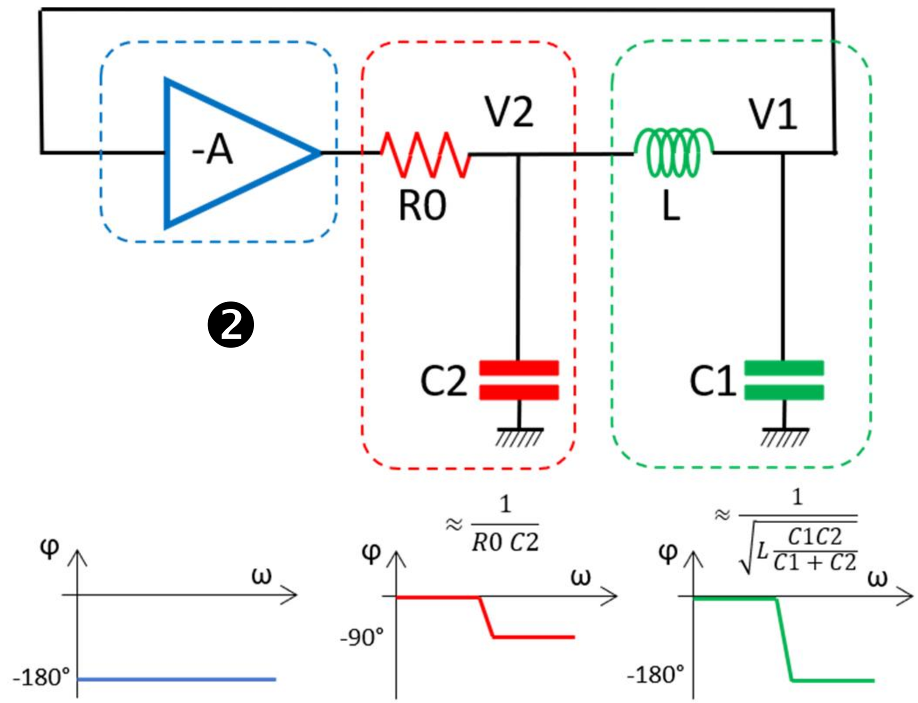


❷

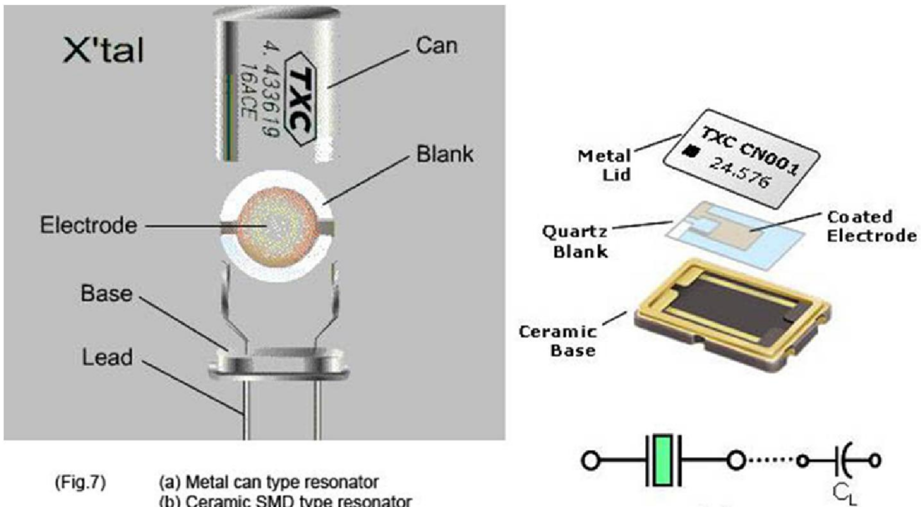
❷ Pierce Oscillator:
an Extra phase shift due to R0 ensure -180° → oscillation occurs at

$$f \approx \frac{1}{2\,\pi\sqrt{L\dfrac{C1C2}{C1+C2}}}$$

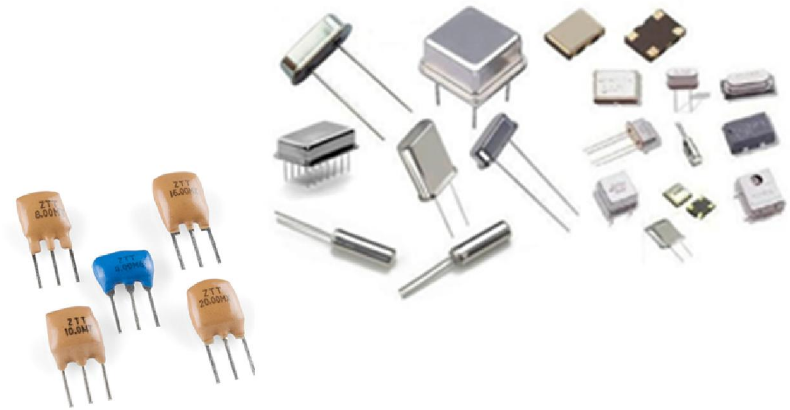
❶ Oscillator principle:
the two phase shifts create positive feedback → oscillation may exist if the gain A is large enough

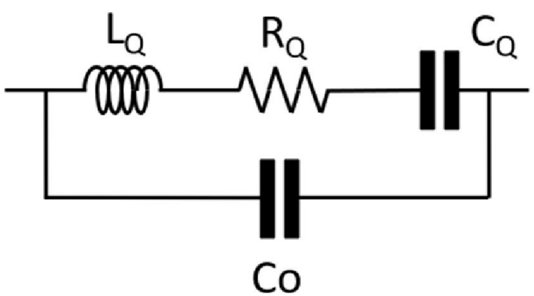


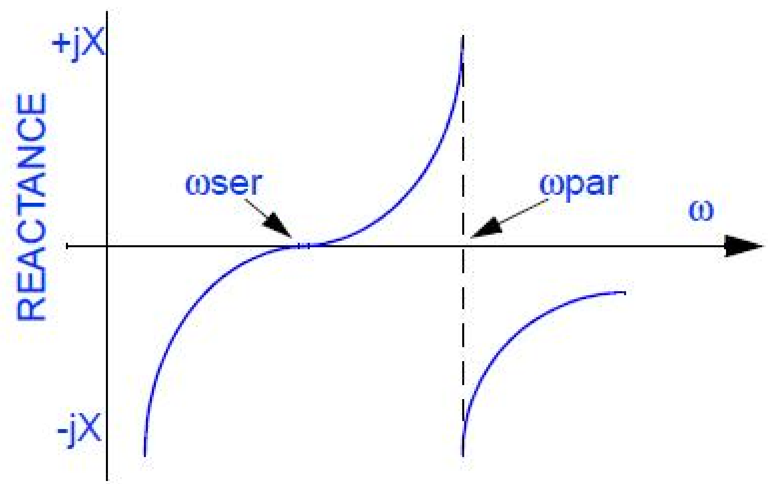
5. Oscillators: Harmonic quartz crystal oscillators



(Fig.7) (a) Metal can type resonator
(b) Ceramic SMD type resonator
(c) Symbol of crystal unit



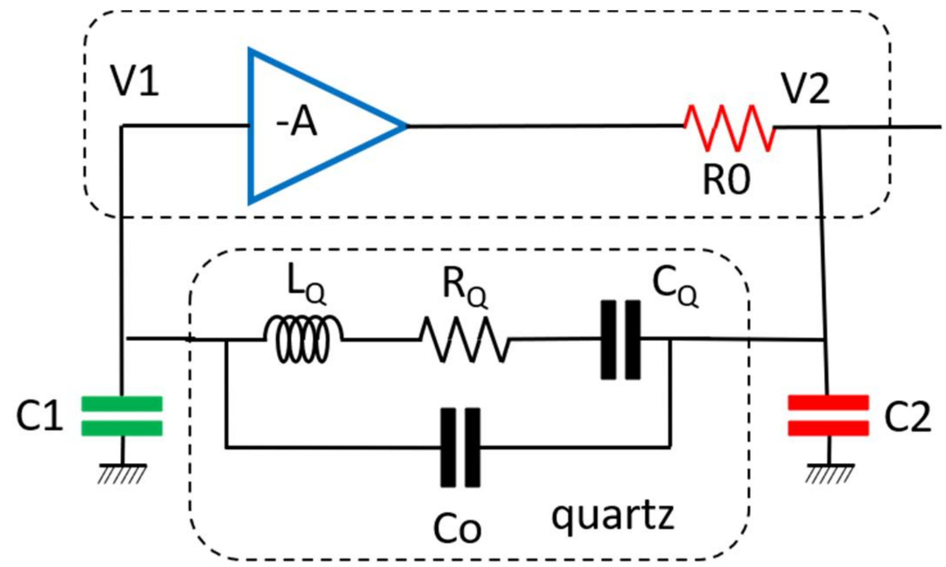


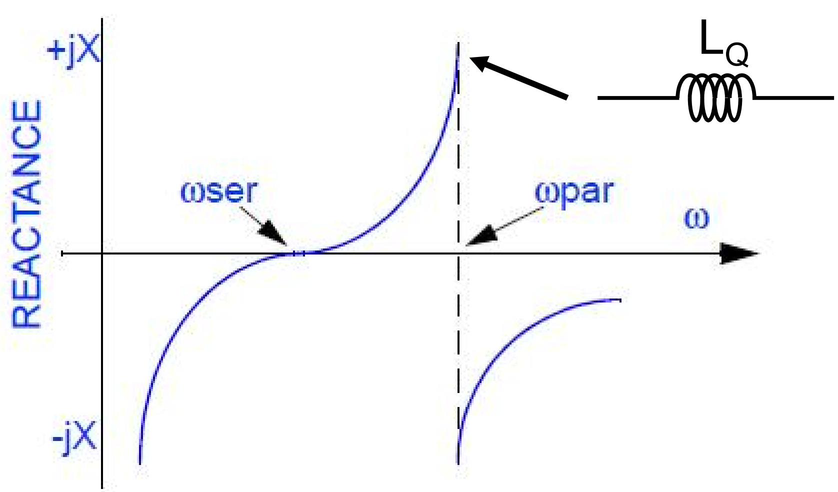


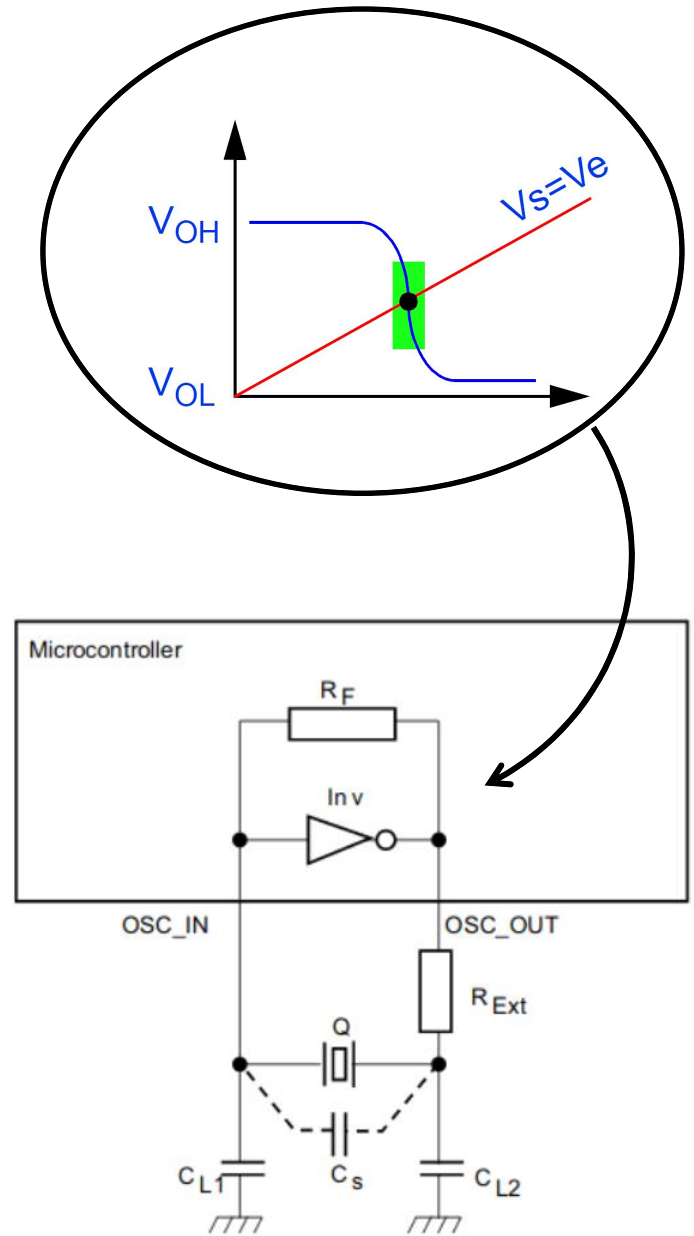
$$\omega_{ser} = \frac{1}{\sqrt{L_Q C_Q}}$$

$$\omega_{par} = \omega_{ser}\sqrt{1 + \frac{C_Q}{Co}}$$

5. Oscillators: Harmonic quartz crystal oscillators







6. Appendix A: closed-loop poles of 2^{nd} order circuits

$$H(s) = \frac{Ho}{s^2 + \frac{\omega_0}{Q} + \omega_0^2}$$

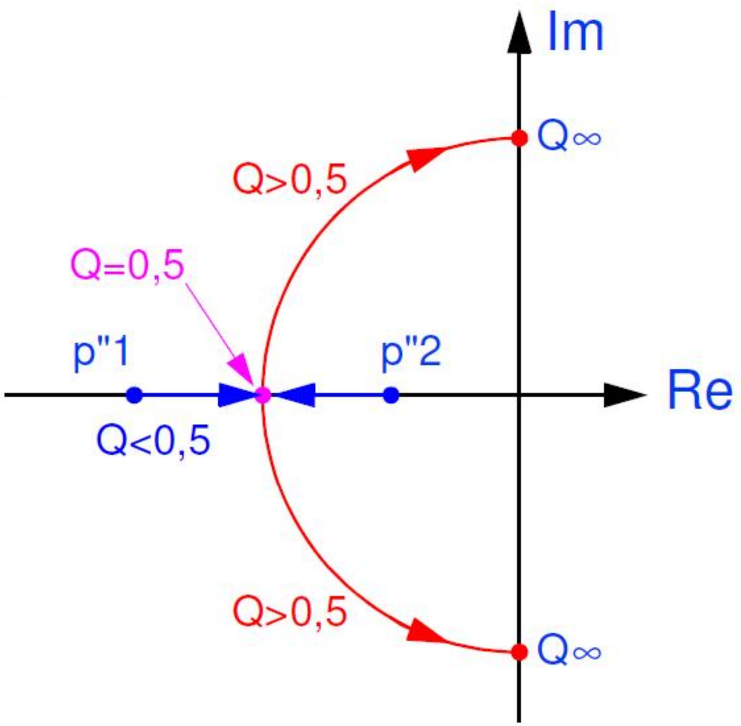
Normalized with $\omega_0 = 1$:

$$H(s) = \frac{Ho}{s^2 + \frac{1}{Q} + 1}$$

Open-loop poles: p1 and p2

Closed-loop poles p'1 and p'2 are given by:

$$s^2 + \frac{1}{Q} + 1 \ = 0 \quad \rightarrow \quad p'_{1,2} = \frac{1}{2Q} \pm \frac{\sqrt{\frac{1}{Q^2} - 4}}{2}$$



7. Appendix B: optimal phase margin in 2nd order circuits

Open-loop poles: p1 and p2

$$H(s) = \frac{Ao}{1 + AoF} = \frac{(1 + AoF)\, p1\, p2}{\underbrace{(1 + AoF)\, p1\, p2}_{\omega_0^2} + s\underbrace{(p1 + p2)}_{\frac{\omega_0}{Q}} + s^2}$$

The optimal response is obtained when: $Q^2 = \frac{\omega_0^2}{(p1+p2)^2} = \frac{(1+AoF)p1\, p2}{(p1+p2)^2} = \frac{1}{2}$

And thus: $p2 \approx 2\, Ao\, F\, p1$ if |p1| << |p2|

So, $p2 \approx 2\, Ao\, F\, p1$ is the **condition that must be fulfilled** to obtain an optimal circuit response

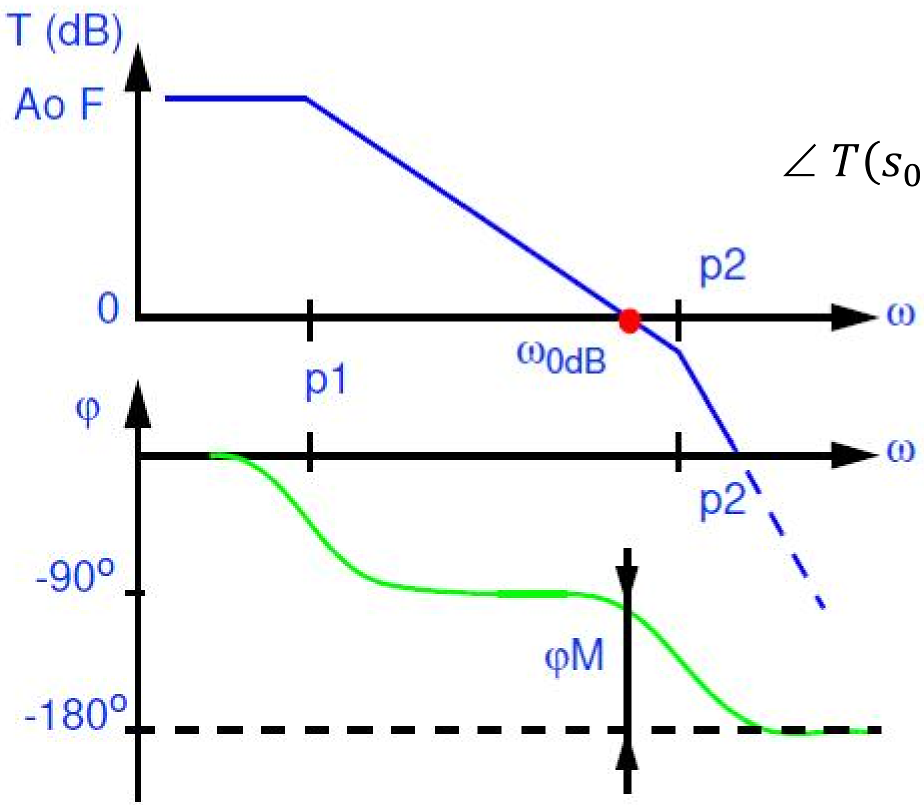
For $p2 \approx 2\, Ao\, F\, p1$, the -3dB bandwidth is $\omega_{-3dB} = \sqrt{2}\, Ao\, F\, p1$ which is 1,4 times larger than the 1st order circuit -3dB bandwidth. It is worth noting that it is achieved at neither expense in circuit complexity nor power consumption.

7. Appendix B: optimal phase margin in 2nd order circuits

Phase margin when $p2 \approx 2\, Ao\, F\, p1$: open-loop study $\quad s_0 = j\, \omega_{0dB}$

$$T(s_0) = \frac{Ao\ F}{\left(1+\frac{s_0}{p1}\right)\left(1+\frac{s_0}{p2}\right)} \approx \frac{Ao\ F}{\left(\frac{s_0}{p1}\right)\left(1+\frac{s_0}{p2}\right)}$$

because |p1| << |p2|



$$\omega_{0dB} \approx Ao\ F\ p1$$

$$\angle\, T(s_0) = -180° + \varphi M = -90° + \tan^{-1}\frac{Ao\ F\ p1}{p2}$$

$$\frac{p2}{p1} = \frac{Ao\ F}{\tan(90° - \varphi M)}$$

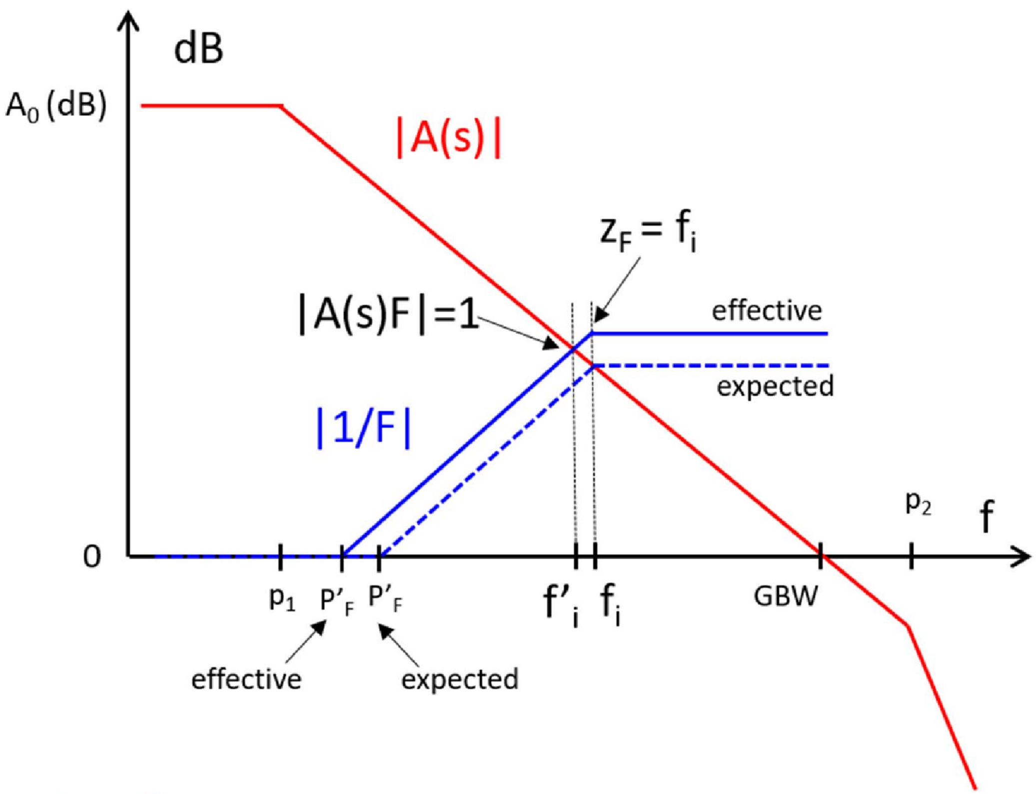
For $p2 \approx 2\ Ao\ F\ p1$:

φM = 63.4° ≈ 60°

8. Appendix C: voltage follower compensation

Assume that z_F is set to the previous value of $f_i \rightarrow$ $$Rz = \frac{1}{2\pi z_F C_t} \approx 20\Omega$$

Therefore, $p'_F = \frac{1}{2\pi(r_0+Rz)C_t} \approx 2.3MHz$ → the situation is not as expected:



$$f'_i = \sqrt{p'_{Feffective}\, GBW} \approx 6.8MHz$$

Amplifier phase shift:

$$\varphi_A = -90 - \tan^{-1}\frac{f'_i}{p_2} \approx -99.6°$$

F block phase shift:

$$\varphi_F = -\tan^{-1}\frac{f'_i}{p'_F} + \tan^{-1}\frac{f'_i}{z_F} \approx -31°$$

$$\varphi_M = (\varphi_A + \varphi_F) - (-180°) \approx 50°$$

→ nearly optimal