# TD n°1 : Correction

## Exercice 1

### 1) Laplace

Calculer les transformées de Laplace suivantes :

a) $\mathscr{L}\left[\left(t^2+t-e^{-3t}\right)\mathscr{U}(t)\right]$

$$f(t)=\left(t^2+t-e^{-3t}\right)\mathscr{U}(t) \;=\; t^2\,\mathscr{U}(t)+t\,\mathscr{U}(t)-e^{-3t}\,\mathscr{U}(t)$$

$$F(p)=\boxed{\frac{2}{p^3}+\frac{1}{p^2}-\frac{1}{p+3}}$$

b) $\mathscr{L}\left[\left(t+2\right)\mathscr{U}(t)+\left(t+3\right)\mathscr{U}(t-2)\right]$

$$f(t)=(t+2)\,\mathscr{U}(t)+(t+3)\,\mathscr{U}(t-2)=\left(t+2\right)\mathscr{U}(t)+\left((t-2)+5\right)\mathscr{U}(t-2)$$

$$\mathscr{L}\left[(t+5)\,\mathscr{U}(t)\right]=\frac{1}{p^2}+\frac{5}{p}$$

$$\mathscr{L}\left[((t-2)+5)\,\mathscr{U}(t-2)\right]=\left(\frac{1}{p^2}+\frac{5}{p}\right)e^{-2p}$$

$$\mathscr{L}\left[(t+2)\,\mathscr{U}(t)\right]=\frac{1}{p^2}+\frac{2}{p}=\frac{2p+1}{p^2}$$

$$F(p)=\boxed{\frac{2p+1}{p^2}+\left(\frac{1}{p^2}+\frac{5}{p}\right)e^{-2p}}$$

## Exercice n°2

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### 3) Équations différentielles

a) $x'(t)+x(t)=t\,\mathscr{U}(t)-t\,\mathscr{U}(t-1)$ condition initiale : $x(0)=0$

$$x'(t)+x(t)=t\,\mathscr{U}(t)-\left((t-1)+1\right)\mathscr{U}(t-1)$$

$$(p\,X(p)-0)+X(p)=\frac{1}{p^2}-\left(\frac{1}{p^2}+\frac{1}{p}\right)e^{-p}$$

$$(p+1)X(p)=\frac{1}{p^2}-\frac{p+1}{p^2}\,e^{-p}$$

$$X(p)=\frac{1}{p^2(p+1)}-\frac{1}{p^2}\,e^{-p}$$

$$X(p)=\frac{1}{p^2}-\frac{1}{p}+\frac{1}{p+1}-\frac{1}{p^2}\,e^{-p}$$

$$\boxed{x(t)=\left(t-1+e^{-t}\right)\mathscr{U}(t)-\left(t-1\right)\mathscr{U}(t-1)}$$

b) $x''(t)+x'(t)=\mathscr{U}(t)$ conditions initiales : $\begin{cases} x(0) = 0 \\ x'(0) = 0 \end{cases}$

$$(p^2\,X(p)-0-0)+(p\,X(p)-0)=\frac{1}{p}$$

$$(p^2+p)X(p)=\frac{1}{p}$$

$$X(p)=\frac{1}{p(p^2+p)}=\frac{1}{p^2}-\frac{1}{p}+\frac{1}{p+1}$$

$$\boxed{x(t)=\left(t-1+e^{-t}\right)\mathscr{U}(t)}$$

**Exercice n°3**

## 2) Laplace inverse

Calculer les originaux suivants :

a) $\mathscr{L}^{-1}\left[\dfrac{p+2}{(p+3)(p+4)}\right]$

$$F(p) = \frac{p+2}{(p+3)(p+4)} = \frac{2}{p+4} + \frac{-1}{p+3}$$

$$f(t) = \boxed{\left(2\,e^{-4t} - e^{-3t}\right)\,\mathscr{U}(t)}$$

b) $\mathscr{L}^{-1}\left[\dfrac{3}{(p+5)^2}\right]$

$$F(p) = \frac{3}{(p+5)^2}$$

$$\mathscr{L}^{-1}\left[\frac{3}{p^2}\right] = 3\,t\,\mathscr{U}(t)$$

$$f(t) = \boxed{3\,t\,e^{-5t}\,\mathscr{U}(t)}$$

**Exercice n°4**

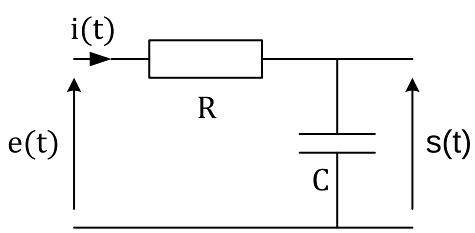



**Figure 1 . Schéma d'un circuit RC**

$$s(t) = \frac{1}{C}\int_0^t i(\alpha)d\alpha$$

$$e(t) = R.i(t) + s(t)$$

**1.**

$e(t) = R.i(t) + s(t) \xrightarrow{\text{Transformée de Laplace}} E(p) = R.I(p) + S(p)$

$s(t) = \frac{1}{C}\int_0^t i(\alpha)d\alpha \xrightarrow{\text{Transformée de Laplace}} S(p) = \frac{1}{C.p} I(p)$

**2.**

$$E(p) = R.I(p) + S(p) = R.C.p.S(p) + S(p) = (1 + R.C.p).S(p)$$

**Donc:**

$$S(p) = \frac{1}{1 + R.C.p} E(p)$$

**3.**

e(t) est un échelon d'amplitude 3 :

$E(p) = 3.\frac{1}{p}$

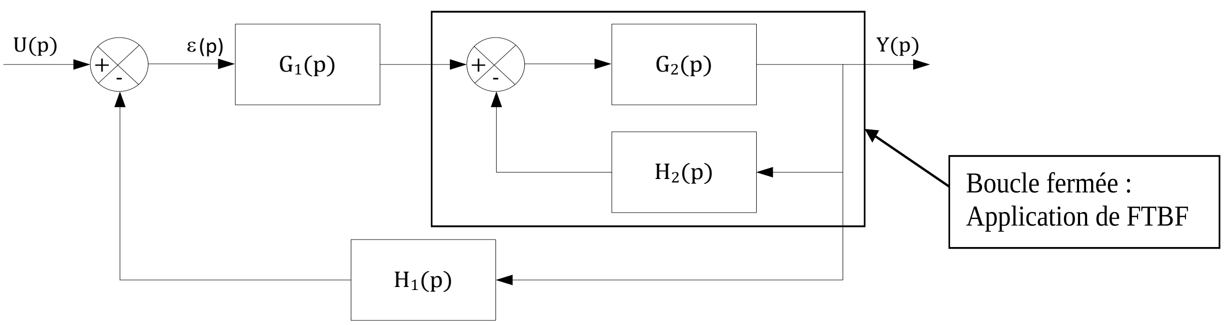
$S(p) = \frac{1}{1+R.C.p} 3.\frac{1}{p}$

Par l'application du théorème de la valeur finale :

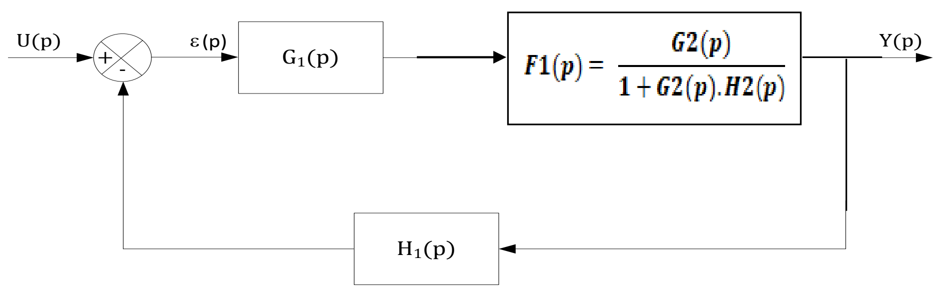
$$\lim_{t \to +\infty} s(t) = \lim_{p \to 0} p.S(p) = \lim_{p \to 0} p.\frac{1}{1 + R.c.p} \times \frac{3}{p} = \mathbf{3}$$

**Exercice n°5**

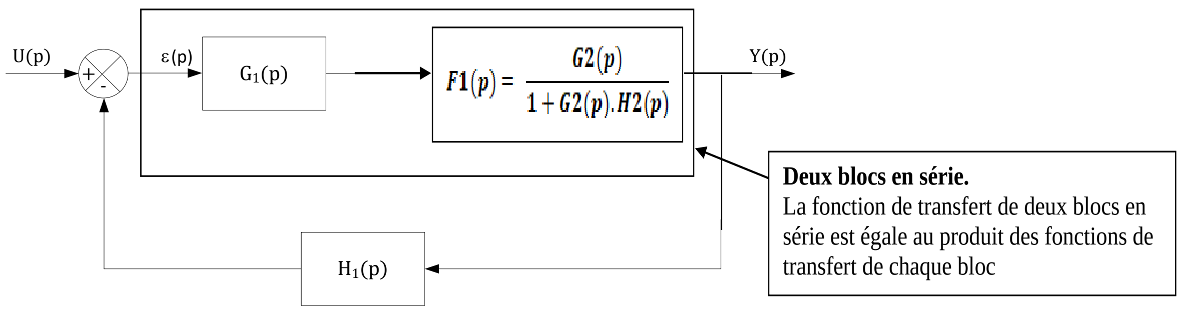
**Etape 1 :**



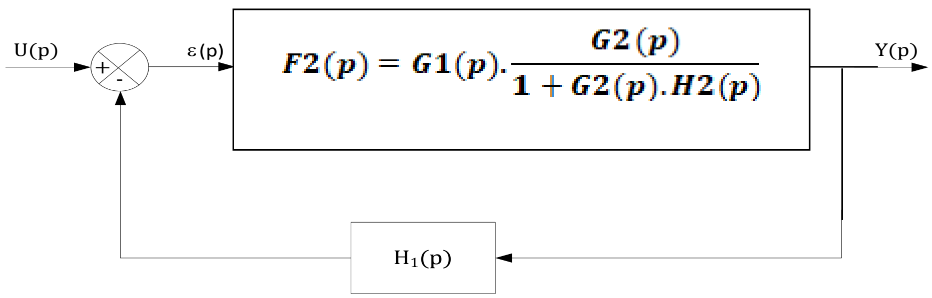

$$\boldsymbol{F1(p) = \frac{G2(p)}{1 + G2(p).H2(p)}}$$




**Etape 2 :**




$$F2(p) = G1(p).\frac{G2(p)}{1 + G2(p).H2(p)}$$




**<u>Etape 3 :</u>**

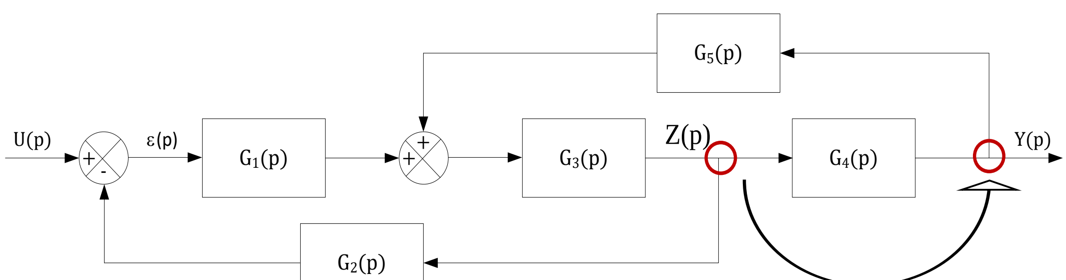
**On obtient finalement une boucle fermée. La fonction de transfert globale est :**

$$F_{TBF}(p) = \frac{Y(p)}{U(p)} = \frac{F2(p)}{1 + F2(p).H1(p)} = \frac{\frac{G1(p).G2(p)}{1 + G2(p).H2(p)}}{1 + H1(p).\frac{G1(p).G2(p)}{1 + G2(p).H2(p)}}$$

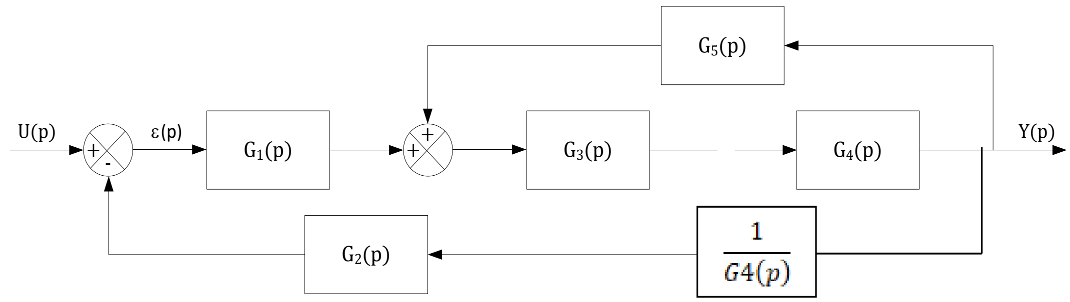
$$= \frac{G1(p).G2(p)}{1 + G2(p).H2(p) + H1(p).G1(p).G2(p)}$$

**Exercice n°6**

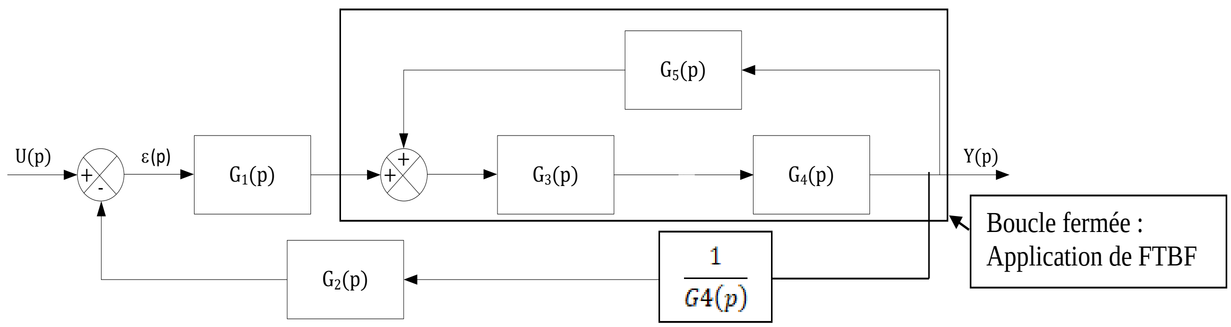
**<u>Etape 1 :</u>**



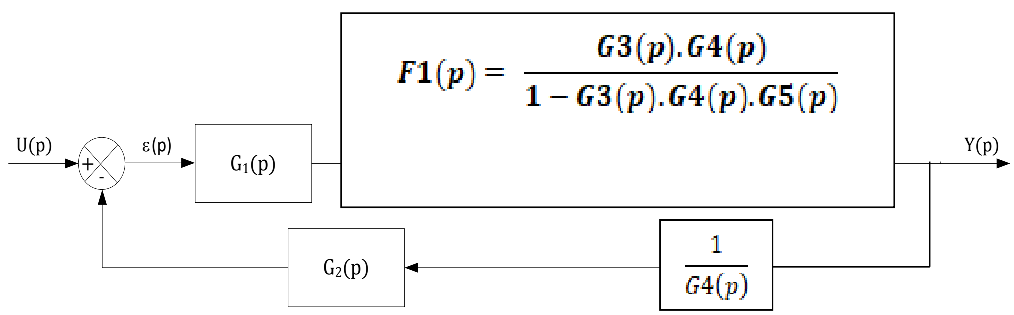

$$\varepsilon(p) = U(p) - G2(p).Z(p) = U(p) - G2(p).\frac{1}{G4(p)}Y(p)$$



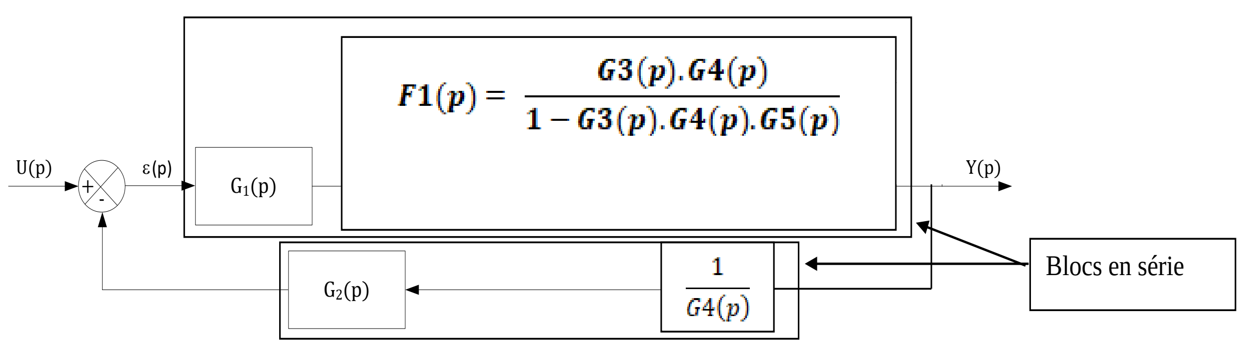

**Etape 2 :**




$$F1(p) = \frac{G3(p).G4(p)}{1 - G3(p).G4(p).G5(p)}$$

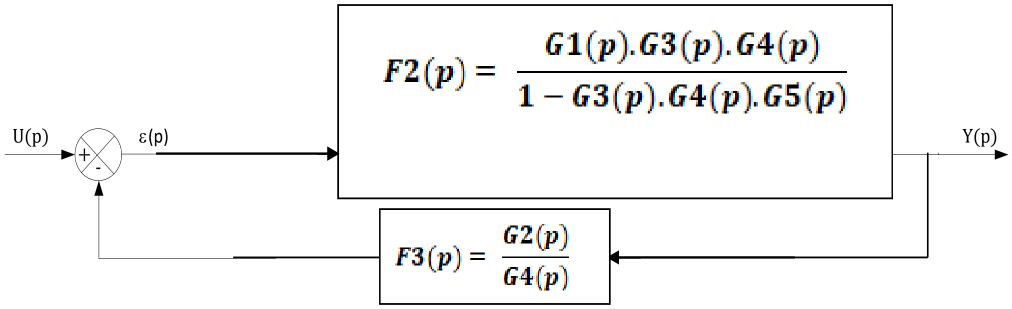



**Etape 3 :**




$$F2(p) = \frac{G1(p).G3(p).G4(p)}{1 - G3(p).G4(p).G5(p)}$$

$$\boldsymbol{F3(p) = \frac{G2(p)}{G4(p)}}$$




**On obtient finalement un système en boucle fermée. La fonction de transfert globale est :**

$$\boldsymbol{F_{TBF}(p) = \frac{Y(p)}{U(p)} = \frac{F2(p)}{1 + F2(p).F3(p)} = \frac{\frac{G1(p).G3(p).G4(p)}{1 - G3(p).G4(p).G5(p)}}{1 + \frac{G2(p)}{G4(p)} \cdot \frac{G1(p).G3(p).G4(p)}{1 - G3(p).G4(p).G5(p)}}}$$

$$\boldsymbol{= \frac{\frac{G1(p).G3(p).G4(p)}{1 - G3(p).G4(p).G5(p)}}{1 + \frac{G2(p).G1(p).G3(p)}{1 - G3(p).G4(p).G5(p)}}}$$

$$\boldsymbol{= \frac{G1(p).G3(p).G4(p)}{1 - G3(p).G4(p).G5(p) + G2(p).G1(p).G3(p)}}$$

**Exercice 7**

Les équations du système sont les suivantes :

1) $e_A(t) = A.(e_r(t) - e_t(t))$

2) $e_A(t) = \frac{R_g}{K_g} \cdot e_g(t) + \frac{L_g}{K_g} \cdot \frac{de_g(t)}{dt}$

3) $e_g(t) = e_m(t) + \frac{r_t}{K_m} \cdot C_m(t) + \frac{l_t}{K_m} \cdot \frac{dC_m(t)}{dt}$

4) $e_m(t) = K_m \cdot \omega(t)$

5) $C_m(t) = C_u(t) + J\frac{d\omega(t)}{dt}$

6) $e_t(t) = K_t \cdot \omega(t)$

Par l'application de la transformée de Laplace aux équations du système, on obtient :

1) $E_a(p) = A.(E_r(p) - E_t(p))$

2) $E_a(p) = \frac{R_g}{K_g} \cdot E_g(p) + \frac{L_g}{K_g} \cdot p . E_g(p) = \left(\frac{R_g}{K_g} + \frac{L_g}{K_g} \cdot p\right) E_g(p)$

$$E_g(p) = \frac{K_g}{R_g + p.L_g} E_a(p)$$

3) $E_g(p) = E_m(p) + \frac{r_t}{K_m} \cdot C_m(p) + \frac{l_t}{K_m} \cdot p . C_m(p) = E_m(p) + \left(\frac{r_t}{K_m} + \frac{l_t}{K_m} \cdot p\right) C_m(p)$

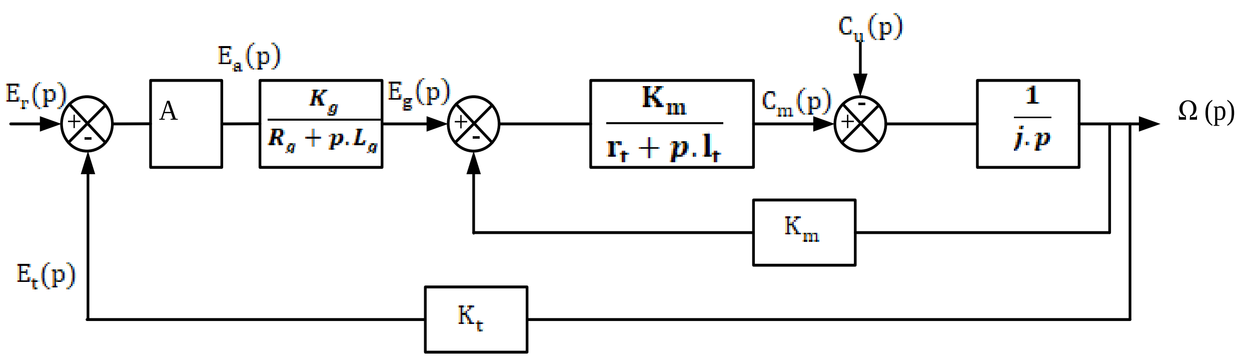
$$C_m(p) = \frac{K_m}{r_t + p.l_t} (E_g(p) - E_m(p))$$

4) $E_m(p) = K_m \cdot \Omega(p)$

5) $C_m(p) = C_u(p) + J . p . \Omega(p)$

$$\Omega(p) = \frac{1}{J.p} (C_m(p) - C_u(p))$$

6) $E_t(p) = K_t \cdot \Omega(p)$




**Exercice 8.**